Step by step guide to Simplex Algorithm

Simplex Method for Standard Maximization Problems

Maximize: $P = 2x + 5y$

$$2x + y \leq 5$$
$$x + 2y \leq 4$$
$$x \geq 0, y \geq 0$$

1. Convert the system of inequalities (constraints) to equations using slack variables.
2. Set the objective function equal to zero.
3. Create a simplex table or tableau and label the active or basic variables.

Step 1 and 2)

$$2x + y + s = 5$$
$$x + 2y + t = 4$$
$$-2x - 5y + P = 0$$

Step 3) expand the variables for each row and then construct the Tableau or "Simplex Table"

Include all the variables in every equation:

$$2x + 1y + 1s + 0t + 0P = 5$$
$$1x + 2y + 0s + 1t + 0P = 4$$
$$-2x - 5y + 0s + 0t + 1P = 0$$

Construct Tableau

Our active variables are the ones that have only 1s and 0s, here, we have three, s, t, and P"

	x	y	s	t	P	
s	2	1	1	0	0	5
t	1	2	0	1	0	4
P	−2	−5	0	0	1	0

Each Tableau shows a possible solution but not necessarily the maximum P. For instance, here, we have the following, but p is 0 so not the maximum we needed!

Note that inactive variables, x and y, are 0.

$s = 5 \qquad x = 0$

$t = 4 \qquad y = 0$

$P = 0$

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Prepared by: W.P
Last update: 30/July/2025

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4. Select the pivot column. This is the column with the most negative number on the left side of the bottom row. (If all numbers on the bottom row are nonnegative, we are done.)
5. Select the pivot row. Divide each entry in the constant column by the corresponding positive entry in the pivot column. The smallest positive ratio indicates the pivot row.
6. Select the pivot, which is the entry in the pivot column and pivot row. The pivot must be positive. It can't be zero.

Step 4) Select Pivot column:

	x	y	s	t	P	
s	2	1	1	0	0	5
t	1	2	0	1	0	4
P	-2	-5	0	0	1	0

Step 5) Select Pivot row: The pivot row will never be this last row. We divide the last column (constant column) by corresponding row in the pivot column. Here 2, the smallest outcome indicate the pivot row.

	x	y	s	t	P		
s	2	1	1	0	0	5	$\frac{5}{1}=5$
t	1	2	0	1	0	4	$\frac{4}{2}=2$
P	-2	-5	0	0	1	0	

Step 6) Select Pivot Value (intersection of pivot row and column):

	x	y	s	t	P	
s	2	1	1	0	0	5
t	1	$\boxed{2}$	0	1	0	4
P	-2	-5	0	0	1	0

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7. Perform row operations to make the pivot equal to 1 and the remaining elements in the pivot column equal to zero. Making the pilot equal to 1 is optional.

Step 7)

Let's make 2 → 1 and 1 and -5 should become also 0.

	x	y	s	t	P	
s	2	1	1	0	0	5
t	1	$\boxed{2}$	0	1	0	4
P	-2	-5	0	0	1	0

But how? Only with row operation

New Row 2 is defined as ½ of Row 2.

	x	y	s	t	P	
s	2	1	1	0	0	5
t	$\frac{1}{2}$	$\boxed{1}$	0	$\frac{1}{2}$	0	2
P	-2	-5	0	0	1	0

Now , we want 1 and -5 to become 0, let's do row operation:

		x	y	s	t	P	
$R_1 \to R_1 - R_2$	s	$\frac{3}{2}$	0	1	$-\frac{1}{2}$	0	3
	y	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	2
$R_3 \to R_3 + 5R_2$	P	$\frac{1}{2}$	0	0	$\frac{5}{2}$	1	10

Notice that **t** is no longer active variable (exiting variable) and y is now active (entering variable). Hence we replace t, with y.

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8. Repeat the process by identifying the most negative entry in the last row.

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	x	y	s	t	P	
s	$\frac{3}{2}$	0	1	$-\frac{1}{2}$	0	3
y	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	2
P	$\frac{1}{2}$	0	0	$\frac{5}{2}$	1	10

But there is nothing left as negative on the last row, so we are done with step 8, moving to step 9.

9. Once the left side of last row is all nonnegative, the solution can be found. The value of each row variables or active variables is equal to the right most entry in the row. All inactive variables equal zero.

Step 9)

Max of P is 10 when y = 2 and x = 0

$s = 3$ $\quad$ $y = 2$

$t = 0$ $\quad$ $x = 0$

$P = 10$

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