# Tutorial Week 2 - CSCB63 -Complexity

Draw a 2D table with column headers and row headers as follows:

| | $\ln(n)$ | $\lg(n)$ | $\lg(n^2)$ | $(lgn)^2$ | $n$ | $n * lg(n)$ | $2^n$ | $2^{(3n)}$ |
|---|---|---|---|---|---|---|---|---|
| $\ln(n)$ | | | | | | | | |
| $\lg(n)$ | | | | | | | | |
| $\lg(n^2)$ | | | | | | | | |
| $(\lg n)^2$ | | | | | | | | |
| $n$ | | | | | | | | |
| $n * \lg(n)$ | | | | | | | | |
| $2^n$ | | | | | | | | |
| $2^{(3n)}$ | | | | | | | | |

(Remember that lg means log base 2.)

In each cell, fill in "Y" iff (its row function)$\in O$(its column function).

We haven't talked about transitivity (if $f \in O(g)$ and $g \in O(h)$, then simply deduce $f \in O(h)$ and be done), but you may prove on your own and use it.

**Observation**. Even though $3n \in O(n)$, we cannot "exponentiate both sides" to infer $2^{3n} \in O(2^n)$.

Lets look 3 of the more interesting cells to show proofs for.

Fill in proofs for selected big-O cells:

1. $n \in O(n \lg(n))$, using the definition of big-O:
2. $n \lg(n) \notin O(n)$ using the definition of big-O: (Note: This can be explained as a proof by contradiction...other ways possible too).
3. $2^{(3n)} \notin O(2^n)$, using a limit theorem from lecture (you may not have seen the limit theorem if your tutorial is before the Wed. class).

   The limit theorem says:

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty \Rightarrow f(n) \notin O(g(n))$$

Some practice questions:

1. $6n^5 + n^2 - n^3 \in \Theta(n^5)$
2. $3n^2 - 4n \in \Omega(n^2)$

For these the intentions are to use the **definitions** of Theta and Omega not use the limit laws.