Exercise 1. *Mystery Function*

```
Function F(a, n):
  If n = 0 : Return (1, 0)
  Else:
    b <- 0
    For i from 1 to n:
      b <- b + a
    (u, v) <- F(a, n - 1)
    Return (u * b, v + n * b)
```

- *What are the results of*
  - *F(a, 2) :* $\langle 2a^2, 5a \rangle$
  - *F(a, 3)*
  - *F(a, 4)*
- *What does the code do?*

  Proof. Hypothesis: F(a, n) $= (n!n^n, \frac{n(n+1)(2n+1)a}{6})$ Base Case: Substituting n=1, F(a, 1) $= (1!1^1, \frac{1(1+1)(2(1)+1)a}{6}) \rightarrow (1, 0)$ Induction Step:
  If statement proven by base case.
  $b = n \cdot a$
  (u,v) = F(a, n - 1)
  $u = (n-1)!a^{n-1}$
  $v = \frac{(n-1)(n)(2(n-1)+1)a}{6}$
  Return statement (x, y):

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  □

Exercise 2. *Tiling with tetrominoes.*

Proof.

Corollary 1. *A tromino has four legal rotational orientations, I will denote these as* $T_{0,1,2,3}$ *to*

Base Case n=1: For a 2×2 grid, the □