

# 1 Introduction

Many links between fundamental mathematical concepts and elements of musicology have been found. Sometimes these links offer instructive ways to think about mathematical objects. As a novel example of this, consider the lexicographical ordering of the group  $\mathbb{Z}_4 \oplus \mathbb{Z}_3$ ,

$$(0, 0) < (0, 1) < (0, 2) < (1, 0) < (1, 1) < (1, 2) < (2, 0) < (2, 1) < (2, 2) < (3, 0) < (3, 1) < (3, 2)$$

which induces the following order on  $\mathbb{Z}_{12}$ :

$$0 < 4 < 8 < 9 < 1 < 5 < 6 < 10 < 2 < 3 < 7 < 11 \quad (1)$$

This order is perhaps more easily conceptualised as the order of notes in the arpeggio-like scale depicted in Figure 1. This scale itself can be conceptualised as the C augmented apppegiated triad, followed by the  $C\sharp$  augmented apppegiated triad second inversion, followed by the D augmented apppegiated triad in first inversion, followed by the  $D\sharp$  augmented apppegiated triad. In this way, the elements of  $\mathbb{Z}_{12}$  are interpreted as the 12

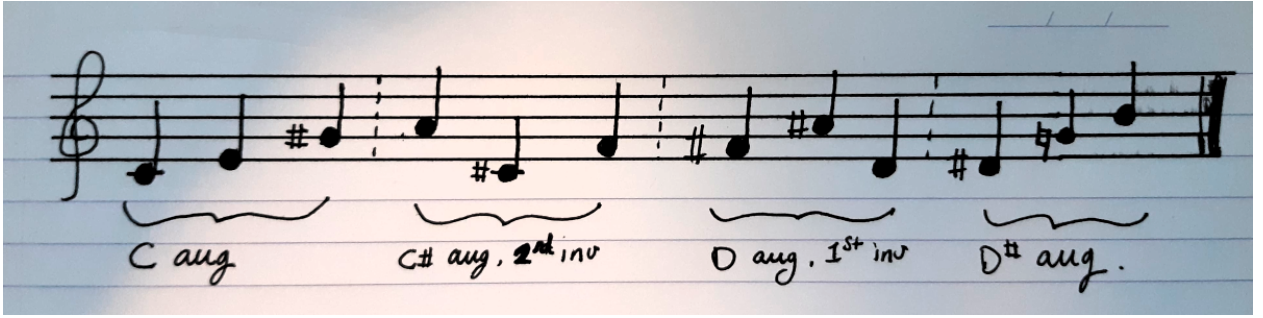


Figure 1: Arpeggio-like scale representing an order on  $\mathbb{Z}_{12}$

tones of an octave (under standard tuning), and  $n < m$  means  $n$  comes before  $m$  in the scale. See [1, §6.8.1] for more details.

The motivating question of this project is the following:

**Question 1.0.1.** *Are there any links between fundamental computational concepts and music?*

The first investigation will be on computation and *musical composition*. The formal objects on the side of musical composition will be *global compositions*, due to Mazzola **need citation to original paper**. In short, a global composition consists of a collection of *local compositions*, ie, small musical snippets, along with *glueing instructions* describing how these snippets fit together. The guiding intuition which will relate this to computation is that just as a musical composer begins with a collection of motifs and organises them into a cohesive whole, a program consists of a collection of smaller programs which are slotted together. In other words, once a musical structure of a particular piece (ie, a global composition) has been written, appropriate local compositions can be *substituted* in to *realise* a complete piece. Since the language of substitution naturally arise here, we adopt the  $\lambda$ -calculus as our formalisation of a *program*. Indeed, the ultimate goal is an appropriate category of *global compositions* lying on the musical side, and an equivalence of categories between this and  $\mathcal{L}_Q$  [2], an appropriate category of  $\lambda$ -terms.

## References

- [1] G. Mazzola, *The Topos of Music I, Theory*, Springer International Publishing AG, part of Springer Nature 2002,2017.
- [2] D. Murfet, W. Troiani, *The Curry-Howard Correspondence*