Continuous quantum computing

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Definition 0.0.1. A **continuous time evolution** of $\mathbb{H}$ is a Hermitian operator H on $\mathbb{H}$, this is the **Hamiltonian**.

Remark 0.0.2. One may be tempted to psychologically project mathematical depth onto: "as *discrete* is to *continuous*, unitary is to Hermitian". This would be pareidolia though. What is suppressed in these notes is that the **evolution** of a continuous time evolution is a vectorial differential equation (Schrödinger's equation)

$$i\hbar\frac{d\,|\psi\rangle}{dt} = H\,|\psi\rangle \tag{1}$$

Single step time evolution can be modelled via continuous time evolution, this involves solving the differnetial equation (1), which is too far abroad from the targetted focus of these notes.

What is important, is the mathematical *definition* 0.0.1. We will not need any continuous analogue to the second half of Definition **??**.

1 introduction

Insert explanation as to why $|0\rangle, |1\rangle$ are labelled the way they are (because they are eigenvalues of particular operators). Then generalise this to continuous quantum variables.

Let $p \in \mathbb{R}$ and consider the function

$$\begin{aligned} \mathbb{C} &\longrightarrow \mathbb{C} \\ x &\longmapsto e^{-ixp} \end{aligned}$$

We notice that

$$-i\frac{d}{dx}e^{-ixp} = pe^{-ixp} \tag{2}$$

That is, p is an eigenvalue of the linear function $-i\frac{d}{dx}$. We therefore let $|p\rangle$ denote the function e^{-ixp}, and this is an example of a continuous quantum variable.

2 Triple modula redundancy for continuous variables

Triple modula redundancy is a simple binary error-correcting routine for classical discrete systems, where each bit $x \in \{0,1\}$ is sent three times xxx so that if a single error occurs $xx\overline{x}$(where $\overline{0} = 1, \overline{1} = 0$) we can achieve correction by looking at the majority bit $xx\overline{x} \longmapsto xxx$. We now adapt this methodology to continuous classical variables.

Let x_1, x_2, x_3 be continuous variables, that is $x_1, x_2, x_3 \in \mathbb{R}$ and assume that they are all initially set to some value $x \in \mathbb{R}$. That is, let $x = x_1 = x_2 = x_3$. Then assume errors occured $x_1 \longmapsto x_1', x_2 \longmapsto x_2', x_3 \longmapsto x_3'$, that is, let $x_1', x_2', x_3' \in \mathbb{R}$ and assume that the classical information $x_1x_2x_3$ was sent through some channel of communication and the received information was $x_1'x_2'x_3'$. Assume without loss of generality that $|x_1'| < |x_2'| < |x_3'|$, that is, x_1' experienced the least dramatic change in error, and x_3' the most, with x_2' in the middle. Then we let $x' = (x_1' + x_2')/2$ and replace $x_1'x_2'x_3'$ by $x'x'x'$. We remark this process does not achieve perfect error correction, nor is it reversible.

We now describe a similar routine which can be used to protect against some forms of quantum error. Consider three continuous quantum variables $|x_1x_2x_3\rangle$ and