Mid-term presentation

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- Introduction
- Variational inference
- Mean-field approximation
- LOCUS R
- Results
- Future

Introduction

- Estimate the association between SNP s and trait t, $s = 1, \ldots, p$, $t = 1, \ldots, q$
- $y_{n\times q} = x_{n\times p}\beta_{p\times q} + \epsilon_{n\times q}$, $\epsilon \sim \mathcal{N}(0, \tau_t^{-1} I_n)$
- y is response matrix, x are candidate predictors.
- Each response y_t is linearly related with the predictors and has a residual precision $\tau_t \sim \text{Gamma}(\eta_t, \kappa_t)$.
- p and q really large compared n.

Introduction II

- $\beta_{st} \mid \gamma_{st}, \sigma^2, \tau_t \sim \gamma_{st}\mathcal{N}(0, \sigma^2\tau_t^{-1}) + (1 - \gamma_{st})\delta_0$, (spike and slab)
- $\gamma_{st} \mid \omega_s \sim \text{Bernoulli}(\omega_s)$,
- $\omega_s \sim \text{Beta}(a_s, b_s)$,

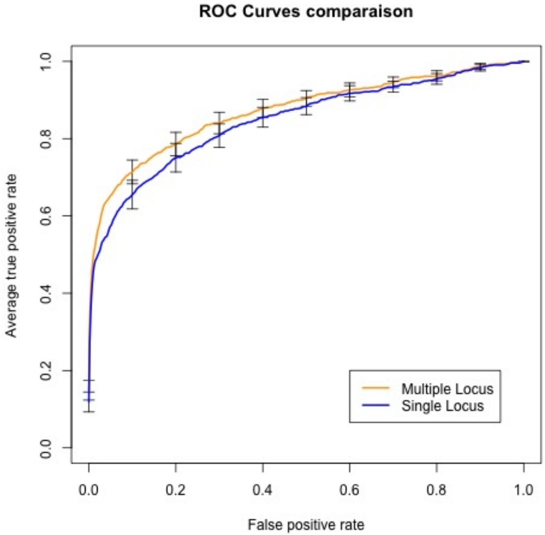
Variational inference

- Observed data $\boldsymbol{y}$, parameters $\boldsymbol{\theta}$, posterior distribution of parameters $p(\boldsymbol{\theta} \mid \boldsymbol{y})$.
- Approximate posterior density with a simpler density q, minimizing a "closeness" measure: the Kullback-Leibler divergence.
- $\mathrm{KL}(q \parallel p) := \int q(\boldsymbol{\theta}) \log \left(\frac{q(\boldsymbol{\theta})}{p(\boldsymbol{\theta} \mid \boldsymbol{y})} \right) \mathrm{d}\boldsymbol{\theta}$.
- Evidence lower bound (ELBO): $\mathcal{L}(q) = \mathbb{E}\left[\log p(\boldsymbol{\theta}, \boldsymbol{y})\right] - \mathbb{E}\left[\log q(\boldsymbol{\theta})\right]$.
- $\mathrm{KL}(q \parallel p) = \log(p) - \mathcal{L}(q)$.
- Minimizing KL is equivalent to maximizing ELBO.

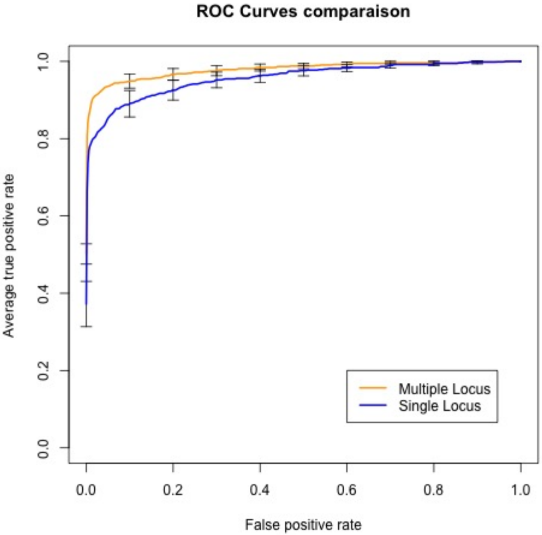
Mean-field approximation

- We assume $q(\boldsymbol{\theta}) = \prod_{j=1}^{J} q_j(\theta_j)$, where $\boldsymbol{\theta}$.

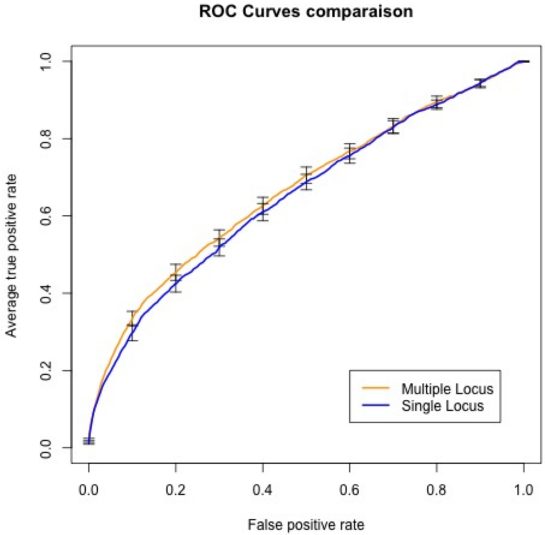
ROC curves comparison, $p_0 = 15$, max var.$= 0.5$



ROC curves comparison, $p_0 = 15$, max var.$= 0.8$



ROC curves comparison, $p_0 = 50$, max var.$= 0.5$



ROC curves comparison, $p_0 = 50$, max var.$= 0.8$