紙張的彎曲

A.1 $$x_0 = \left(\frac{6Ehd^2}{\rho g}\right)^{1/4}$$ 1.2 pt

A.2 $$z\left(x\right) = -\frac{3h}{x_0^4}\left(x^4 - \frac{8}{3}x_0x^3 + 2x_0^2x^2 - \frac{1}{3}x_0^4\right)$$ 2.0 pt

液滴的形變

A 部分　球座標中的曲率半徑 (1.0 pt)

A.2 $$f\left(\tilde{R},\theta,\phi\right)=\frac{2}{R_0}-\frac{2\tilde{R}}{R_0^2}-\frac{1}{R_0^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\tilde{R}}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2\tilde{R}}{\partial\phi^2}\right]$$ 0.4 pt

B 部分　液滴的表面波 (3.0 pt)

B.1 $$P=P_0+\frac{2\sigma}{R_0}$$ 0.2 pt

B.2 $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\varphi}{\partial\phi^2}=0$$ 0.8 pt

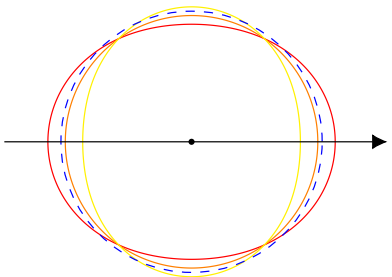
在 $r=R_0$ 處的邊界條件為

$$\rho\frac{\partial^2\varphi}{\partial t^2}=\frac{\sigma}{R_0^2}\left\{2\frac{\partial\varphi}{\partial r}+\frac{\partial}{\partial r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2\varphi}{\partial\phi^2}\right]\right\}$$

B.3 $$\omega_l=\sqrt{\frac{\sigma}{\rho R_0^3}l\left(l-1\right)\left(l+2\right)}$$ 0.8 pt

B.4 $$r\left(\theta,t\right)=R_0+a\left[-\frac{1}{3}+\frac{2}{3}\left(3\cos^2\theta-1\right)\cos\omega_2 t\right]$$ 0.6 pt

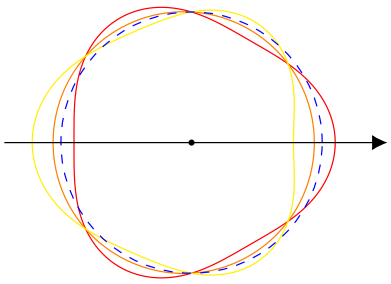
其中 $\omega_2=\sqrt{8\sigma/\rho R_0^3}$。



B.5 0.6 pt

$$r(\theta, t) = R_0 + a\left[-\frac{3}{5}\cos\theta + \frac{4}{5}\left(5\cos^3\theta - 3\cos\theta\right)\cos\omega_3 t\right]$$

其中 $\omega_3 = \sqrt{30\sigma/\rho R_0^3}$。



C 部分　電場作用下的拉伸 (2.8 pt)

C.1 2.0 pt

$$\tilde{R}_0 = -\frac{3}{8}\frac{\epsilon_0 E^2 R_0^2}{\sigma}$$

$$\tilde{R}_2 = \frac{9}{8}\frac{\epsilon_0 E^2 R_0^2}{\sigma}$$

C.2 0.8 pt

$$\Delta P = -\frac{3}{4}\epsilon_0 E^2$$