ECE 374 B ✧ Fall 2022

Homework 2

- Groups of up to three people can submit joint solutions. Each problem should be submitted by exactly one person, and the beginning of the homework should clearly state the Gradescope names and email addresses of each group member. In addition, whoever submits the homework must tell Gradescope who their other group members are.

- **Submit your solutions electronically on the course Gradescope site as PDF files.** Submit a separate PDF file for each numbered problem. If you plan to typeset your solutions, please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera).

☞ Some important course policies ☜

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.

- **Avoid the Three Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given an ***automatic zero***, unless the solution is otherwise perfect. Yes, we really mean it. We're not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course material.
  - Always give complete solutions, not just examples.
  - Always declare all your variables, in English. In particular, always describe the specific problem your algorithm is supposed to solve.
  - Never use weak induction.

See the course web site for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

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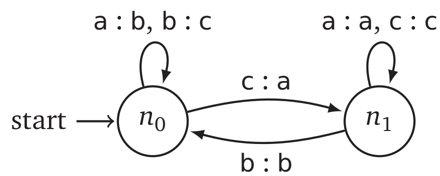
1. Let
$$\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$
Consider each row to be a binary number and let
$$D = \{w \in \Sigma^* \mid \text{the top row of } w \text{ is larger than the bottom row.}\}$$
For example
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} \in D, \text{ but } \begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} \notin D.$$
Show that D is regular.

2. A *finite-state transducer* (FST) is a type of deterministic finite automaton whose output is a string instead of just *accept* or *reject*. The following is the state diagram of finite state transducer FST_0.



Each transition of an FST is labeled at least an input symbol and an output symbol, separated by a colon (:). There can also be multiple input-output pairs for each transitions, separated by a comma (,). For instance, the transition from n_0 to itself can either take a or b as an input, and outputs b or c respectively.

When an FST computes on an input string $s := \overline{s_0 s_1 \ldots s_{n-1}}$ of length n, it takes the input symbols $s_0, s_1, \ldots, s_{n-1}$ one by one, starting from the starting state, and produces corresponding output symbols. For instance, the input string abccba produces the output string bcacbb, while cbaabc produces abbbca.

(a) Each of the following strings is the input of FST_0. Give the sequence of states entered and the output produced.
   - aaca
   - cbbc
   - bcba
   - acbbca

(b) Assume that FST_1 has an input alphabet Σ_1 and an output alphabet Γ_1, give a formal definition of this model and its computation. (Hint: An FST is a 5-tuple with no accepting states. Its transition function is of the form $\delta : Q \times \Sigma \to Q \times \Gamma$.)

(c) Give a formal description of FST_0.

(d) Give a state diagram of an FST with the following behavior. Its input and output alphabets are $\{\mathsf{T}, \mathsf{F}\}$. Its output string is inverted on the positions with indices divisible by 3 and is identical on all the other positions. For instance, on an input TFTTFTFT it should output FFTFFTTT.

3. Given an arbitrary regular language L on some alphabet Σ, prove that it is closed under the following operations. In other words, prove the following languages are regular.

   (a) $\mathsf{palin}(L) := \{w \in \Sigma^* | ww^R \in L\}$,

   (b) $\mathsf{cycle}(L) := \{xy | x, y \in \Sigma^*, yx \in L\}$,

   (c) $\mathsf{subseq}(L) := \{x \in \Sigma^* | x \text{ is a subsequence of some } y \in L\}$.

   [*Hint*: given an NFA (or DFA) for L, construct an NFA for $func(L)$. Give a formal description of your construction. Provide an explanation of how your NFA works, including the meaning of each state. A formal proof of correctness of your NFA is not required.]