ECE 374 B ✧ Fall 2022

Homework 1

- Groups of up to three people can submit joint solutions. Each problem should be submitted by exactly one person, and the beginning of the homework should clearly state the Gradescope names and email addresses of each group member. In addition, whoever submits the homework must tell Gradescope who their other group members are.

- **Submit your solutions electronically on the course Gradescope site as PDF files.** If you plan to typeset your solutions, please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera).

☞ Some important course policies ☜

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.

- **Avoid the Three Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given an ***automatic zero***, unless the solution is otherwise perfect. Yes, we really mean it. We're not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course material.
  - Always give complete solutions, not just examples.
  - Always declare all your variables, in English. In particular, always describe the specific problem your algorithm is supposed to solve.
  - Never use weak induction.

See the course web site for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

1. Find errors in the following proofs:

   (a) Find the error in the following proof that $2 = 1$.

   > Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a+b)(a-b) = b(a-b)$, and divide each side by $(a-b)$, to get $a+b = b$. Finally, let a and b equal 1, which shows that $2 = 1$.

   (b) Find the error in the following proof that all dogs are the same color.

   > **Claim**: In any set of d dogs, all dogs are the same color.
   > **Proof**: By induction on d.
   >
   > ***Basis:*** For $d = 1$. In any set containing just one dog, all dogs clearly are the same color.
   > ***Induction step:*** For $k > 1$ assume that the claim is true for $d = k$ and prove that it is true for $d = k+1$. Take any set D of $k+1$ dogs. We show that all the dogs in this set are the same color. Remove one dog from this set to obtain the set D_1 with just k dogs. By the induction hypothesis, all the dogs in D_1 are the same color. Now replace the removed dog and remove a different one to obtain the set D_2. By the same argument, all the dogs in D_2 are the same color. Therefore all the dogs in D must be the same color, and the proof is complete.

2. Recursively define a set L of strings over the alphabet $\{0, 1\}$ as follows:

   - The empty string ε is in L.
   - For any strings x in L, the strings 0x1 and 1x0 are also in L.
   - For any two strings x and y in L, the string xy is also in L.

   (a) Prove that the string 11000101001110 is in L.

   (b) Prove by induction that every string in L has exactly the same number of 0's and 1's. (Hint: let $\#(a, x)$ be the number of symbol a in string x, you may use the identity $\#(a, xy) = \#(a, x) + \#(a, y)$ for any symbol a and any strings x and y.)

   (c) Prove by induction that L contains every string with the same number of 0's and 1's.

3. For each of the following languages over the alphabet $\Sigma = \{0, 1\}$ give a regular expression that describes that language:

   (a) All strings that begin with a 1 and end with a 0.

   (b) All strings that contain at least three 1s.

   (c) All strings of at least length three where the third symbol is a 0.

   (d) All strings that do not contain the substring 01.

   (e) All strings that do not contain the substring 010.

   (f) All strings not in $(0^* + 1^*)$.

   (g) All strings that contain at least two 0s and at most one 1.

   Hint, you shouldn't need to convert from a DFA for any of these problems. Brevity is the soul of wit and if your solution is too hard to understand, it will be marked incorrect.