Describe deterministic finite-state automata that accept each of the following languages over the alphabet $\Sigma = \{\texttt{0}, \texttt{1}\}$. Describe briefly what each state in your DFAs *means*.

Either drawings or formal descriptions are acceptable, as long as the states Q, the start state s, the accept states A, and the transition function δ are all clear. Try to keep the number of states small.

1. All strings containing the substring **000**.

2. All strings *not* containing the substring **000**.

3. Every string except **000**. *[Hint: Don't try to be clever.]*

4. All strings in which the number of **0**s is even **and** the number of **1**s is *not* divisible by 3.

5. All strings in which the number of **0**s is even **or** the number of **1**s is *not* divisible by 3.

6. Given DFAs M_1 and M_2, all strings in $\overline{L(M_1)} \oplus L(M_2)$.
   Recall that for two sets A and B, their symmetric distance $A \oplus B$ is the set of elements in either A or B, but not both.

Work on these later:

7. All strings w such that *in every prefix of w*, the number of **0**s and **1**s differ by at most 1.

8. All strings containing at least two **0**s and at least one **1**.

9. All strings w such that *in every prefix of w*, the number of **0**s and **1**s differ by at most 2.

*10. All strings in which the substring **000** appears an even number of times.
   (For example, **0001000** and **0000** are in this language, but **00000** is not.)

11. All strings that are **both** the binary representation of an integer divisible by 3 **and** the ternary (base-3) representation of an integer divisible by 4.

    For example, the string **1100** is an element of this language, because it represents $2^3 + 2^2 = 12$ in binary and $3^3 + 3^2 = 36$ in ternary.

*12. All strings w such that $F_{\#(\texttt{10},w)} \bmod 10 = 4$, where $\#(\texttt{10}, w)$ denotes the number of times **10** appears as a substring of w, and F_n is the nth Fibonacci number:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$$