# HomeWork 1

## Question 1

From the question we can know there is a tangent line to the graph $g$ at the point $(x, y)$ intersects the $x - axis$ at the point $(\frac{x}{3}, 0)$. Thus we can know

$$\begin{aligned} g'(x) &= \frac{-y}{\frac{x}{3} - x} \\ &= \frac{-y}{\frac{x-3x}{3}} \\ &= \frac{3y}{2x} \end{aligned}$$

Thus we find the differential equation $\frac{dy}{dx} = \frac{3y}{2x}$ will have a solution $g$.

## Question 2

First we can simplfy the equation

$$\begin{aligned} y' + 2xy^2 = 0 \Rightarrow \frac{dy}{dx} &= -2xy^2 \\ \frac{dy}{y^2} &= -2xdx \end{aligned}$$

Now we can integrate both sides.

$$
\begin{aligned}
\int \frac{dy}{y^2} &= \int -2x dx \\
-\frac{1}{y} &= -x^2 + c \\
\frac{1}{y} &= x^2 + c
\end{aligned} \tag{1}
$$

Which $c$ is a constant. and it also why when we do equation (1), we didn't add minus sign on $c$.

Finally, we can write the function.

$$
y = \frac{1}{x^2 + c}
$$

# Question 3

(a)

The ODE is a nonlinear equation. We can first simplfy the equation.

$$
\begin{aligned}
e^y y' &= 1 \\
e^y y' - 1 &= 0
\end{aligned}
$$

We cannot find a way to let equation rewrite to linear different equation formula $a(x)y' + b(x)y$. Thus it is a nonlinear equation.

(b)

First we focus on $y = ln(x + C)$. and that its derivative.

$$
\frac{dy}{dx} = \frac{1}{x + C}
$$

Then plug in to equation.

$$e^y y' = e^{ln(x+C)} \cdot \frac{1}{x+C} = |x+C| \cdot \frac{1}{x+C} = 1$$

Thus we can know $y = ln(x+C)$ is the solution of equation when $x > -C$

(c)

$$y(0) = 0 \Leftrightarrow ln(0+C) = 0 \Leftrightarrow ln(C) = 0$$

Clearly, we can know $C = 1$

# Question 4

$$\begin{aligned} 2xy' + y &= 10\sqrt{x} && (1) \\ y' + \frac{y}{2x} &= 10\frac{\sqrt{x}}{2x} && (2) \\ y' + \frac{y}{2x} &= \frac{5}{\sqrt{x}} \end{aligned}$$

We divided $2x$ both side of equation (1), and simplify the equation (2). Since the equation is the first order linear equation so we condifer the form $y' = -p(t)y + g(t)$

$$p(t) = \frac{1}{2x} \quad and \quad g(t) = \frac{5}{\sqrt{x}}$$

Now we can integrate the differential equation.

$$e^{\int p(x)dx} = e^{\int \frac{1}{2x}dx} = e^{2ln(x)} = e^{ln(\sqrt{x})} = \sqrt{x}$$

Now the different equation will be

$$y\sqrt{x} = \int \frac{5}{\sqrt{x}}\sqrt{x}dx + c$$
$$y\sqrt{x} = 5x + c$$
$$y = 5\sqrt{x} + \frac{c}{\sqrt{x}}$$

Since the question give us $y(1) = 2$. we can slove $c = -3$. then the different equation will be.

$$y = 5\sqrt{x} + \frac{-3}{\sqrt{x}}$$

# Question 5

First focus on the equation.

$$2x\frac{dy}{dx} - y = 2x\cos x$$
$$\frac{dy}{dx} - \frac{y}{2x} = \cos x$$

Since, it is a linear different  equation. we can use the formula of the linear different equation.

$$p(k) = -\frac{1}{2x} \; and \; q(x) = cos(x)$$

So we can get

$$e^{\int p(d)dx} = e^{-\frac{1}{2}lnx} = e^{lnx^{-\frac{1}{2}}} = \frac{1}{\sqrt{x}}$$

So we can get

$$y \cdot \frac{1}{\sqrt{x}} = \int \frac{1}{\sqrt{x}} \cdot cosxdx + C$$
$$y = \sqrt{x} \cdot (\int \frac{1}{\sqrt{x}} \cdot cosxdx + C)$$