

Problem 1 (25 points)

Consider the ODE

$$\frac{dz}{dw} = \frac{w - z - 1}{w + z + 3}. \quad (1)$$

- (a) Find h and k so that the substitutions

$$w = x + h, \quad z = y + k$$

transform it into the homogeneous ODE

$$\frac{dy}{dx} = \frac{x - y}{x + y}. \quad (2)$$

- (b) Find an appropriate change of the dependent variable y and use it to help you solve the homogeneous ODE (2).
(c) Write down the formula for the solution of (1).

Problem 2 (25 points)

Solve the ODE

$$3x^2 + 2y^2 + (4xy + 6y^2)y' = 0.$$

Problem 3 (25 points)

Solve the separable ODE

$$-\frac{1}{2\sqrt{x}} + \frac{1}{\sqrt{1-y^2}}y'(x) = 0.$$

Problem 4 (25 points)

Solve the ODE

$$2xy + (4y^2 + 2x^2 + 1)y' = 0.$$