MATH 441, HW 3. Due date: 09/16/22.

Problem 1 (20 points)
Consider the IVP

$$xy'(x) = 2y(x), \quad y(0) = 0.$$

Solve it and show that it has an infinite number of solutions. Does it violate the existence and uniqueness theorem for nonlinear first-order ODEs? Justify your answer.

Problem 2 (20 points)
Find two different solutions of the IVP

$$\frac{dy}{dx} = 3y^{\frac{2}{3}}, \quad y(0) = 0.$$

Why does the existence of different solutions not contradict the existence and uniqueness theorem for nonlinear first-order ODEs?

Problem 3 (20 points)
Show that the ODE

$$2xyy' = 4x^2 + 3y^2$$

is a Bernoulli equation and then solve it.

Problem 4 (20 points)
Solve the ODE

$$xy' + 6y = 3xy^{\frac{4}{3}}.$$

Problem 5 (20 points)
A tank initially contains 60 gal of pure water. Brine containing 1 lb of salt per gallon enters the tank at 2 gal/min, and the perfectly mixed solution leaves the tank at 3 gal/min.

(a) Find the volume of fluid in the tank after t minutes.

(b) What is the maximum amount of salt ever in the tank?