Lecture 2 Example

Question 1: Determine the value of r for which the Euler Equation

$$t^2y'' + 4ty' + 2y = 0 \qquad (1)$$

has a solution of the form $y = t^r$ for $t > 0$.

Solution:

Since we know $y = t^r$, then we can know $y' = rt^{r-1}$ $y'' = r(r-1)t^{r-2}$; plug them in the equation (1).

$$t^2y'' + 4ty' + 2y = t^2 \cdot r(r-1)t^{r-2} + 4t \cdot rt^{r-1} + 2t^r$$

$$= r^2 - r t^r + 4rt + 2t^r$$

$$= t^r (r^2 - r + 4r + 2)$$

$$= t^2 (r^2 + 3r + 2) \qquad (2)$$

Since we know $t > 0$, and equation (2) equal to 0, thus $r^2 + 3r + 2 = 0$.

Then we can know the solution of r.

$$r = -1 \quad \text{or} \quad r = -2.$$

Thus, we can know the form of y.

$$y_1 = \frac{1}{t} \qquad y_2 = \frac{1}{t^2}$$

Question 2: Verify that the function $y = 3t + t^2$ is a solution of the first-order linear ODE.

$$ty' - y = t^2 \qquad (1).$$

Solution: Sive we know $y = 3t + t^2$, then $y' = 3 + 2t$, plug it in to equation (1).

$$\begin{aligned} ty' - y &= t(3+2t) - (3t + t^2) \\ &= 3t + 2t^2 - 3t - t^2 \\ &= t^2 \end{aligned}$$

Thus we can know $y = 3t + t^2$ is a solution of equation (1).