MATH 441, HW 1. Due date: 09/02/22.

Problem 1 (10 points)
A function $y = g(x)$ has the following property. The tangent line to the graph of g at the point (x, y) intersects the $x-$axis at the point $(x/3, 0)$. Write a differential equation of the form $dy/dx = f(x, y)$ having the function g as its solution.

Problem 2 (25 points)
Solve the ODE $y' + 2xy^2 = 0$. Use the method of the separation of variables.

Problem 3 (25 points)
Consider the ODE

$$e^y y' = 1.$$

(a) Is it linear or nonlinear? Justify your answer.

(b) Confirm that $y = \ln(x + C)$ satisfies the ODE. For which values of x is it a solution?

(c) Determine a value of the constant C so that $y(x)$ satisfies the IC $y(0) = 0$.

Problem 4 (20 points)
Solve the IVP

$$2xy' + y = 10\sqrt{x}, \quad y(1) = 2.$$

Problem 5 (20 points)
Solve the ODE

$$2xy' - y = 2x \cos x.$$

You may give the solution in terms of an integral.