An Introduction to Reinforcement Learning

Shivaram Kalyanakrishnan
shivaram@yahoo-inc.com

Yahoo! Labs Bangalore

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What is Reinforcement Learning?

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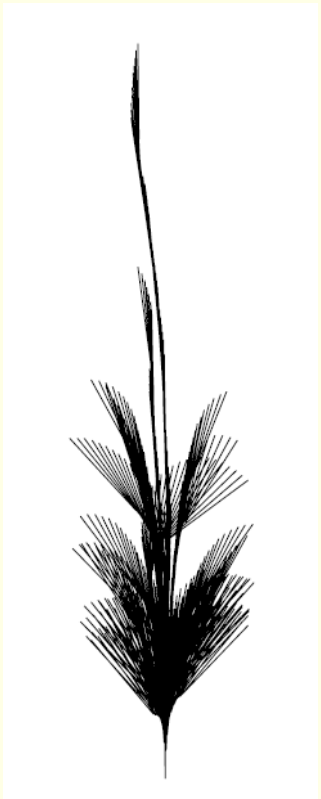

Learning to Drive a Bicycle using Reinforcement Learning and Shaping
Jette Randløv and Preben Alstrøm
In Proc. ICML 1998, pp. 463–471, Morgan Kaufmann, 1998
http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.52.3038&rep=rep1&type=pdf

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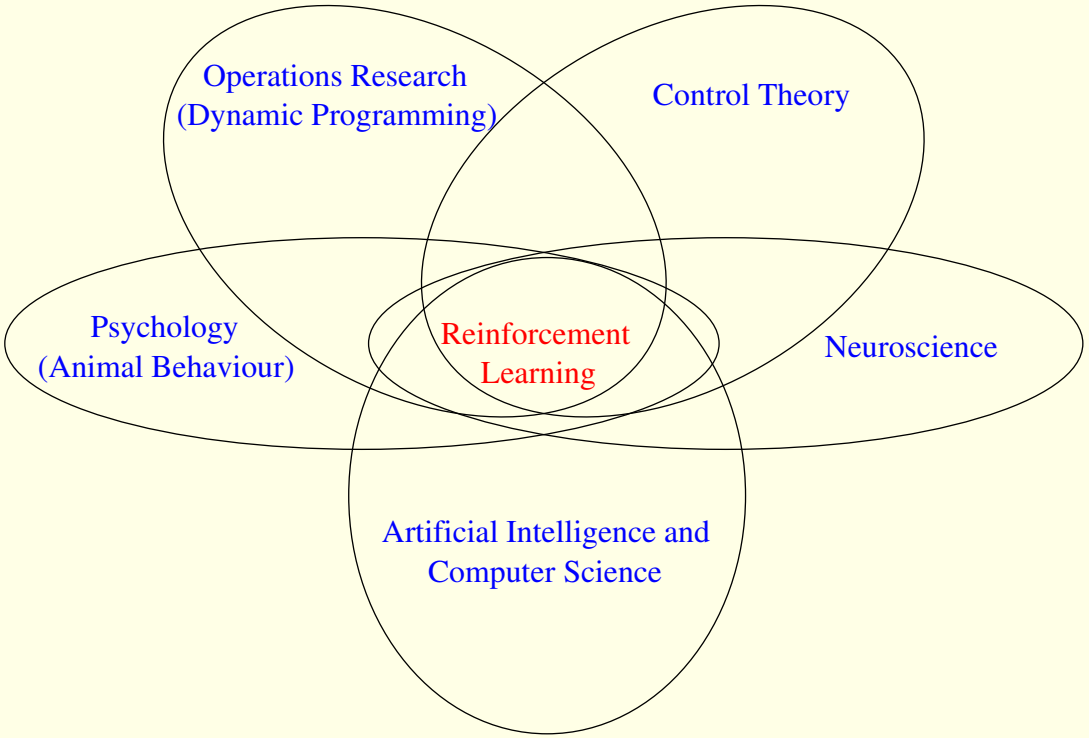
Learning by trial and error to perform *sequential* decision making.

Our View of Reinforcement Learning



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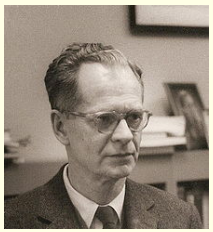

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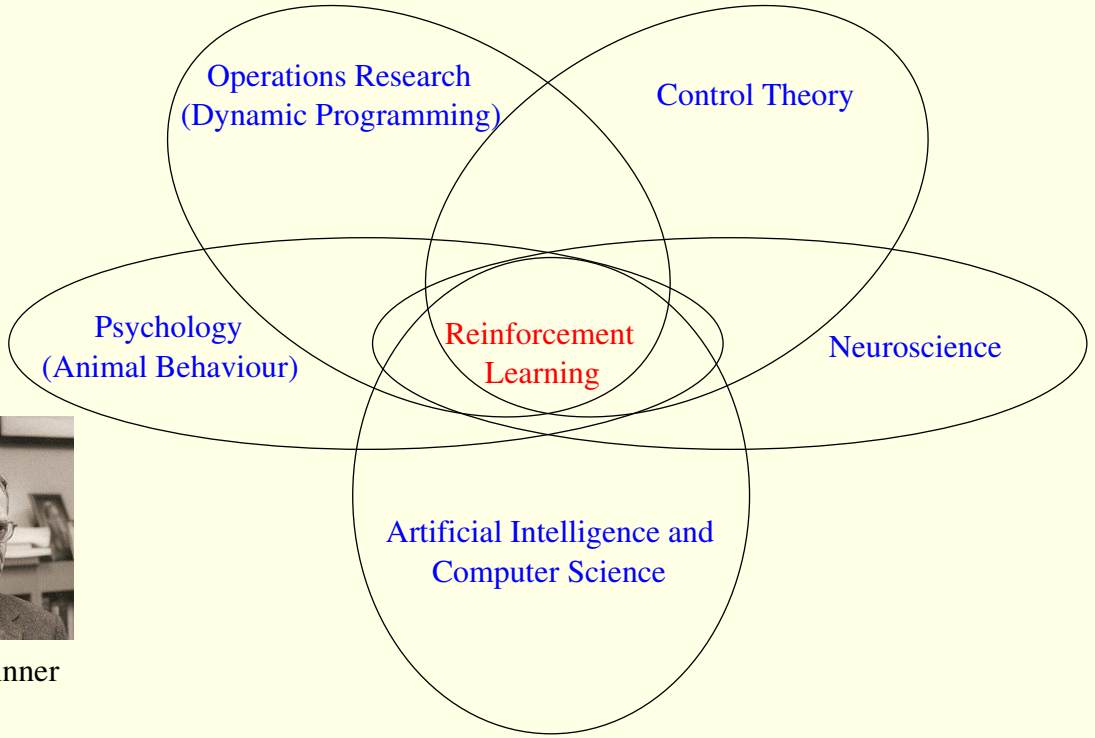

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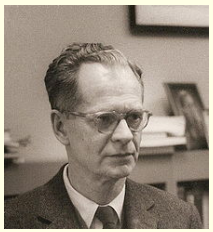

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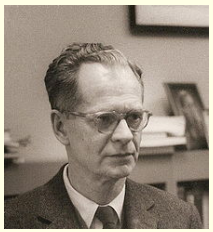

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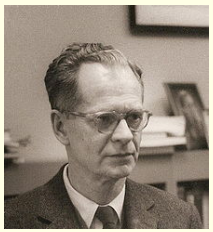

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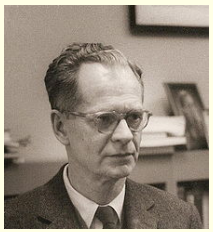

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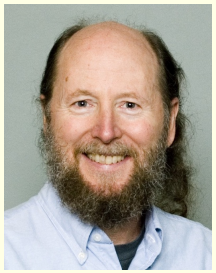


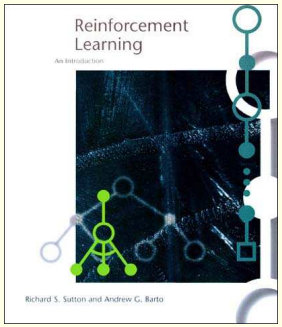
R. S. Sutton

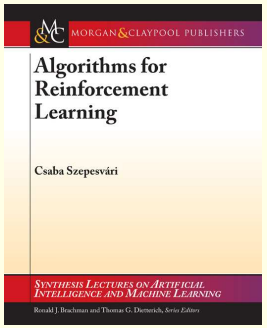
Resources

Tutorial

Reinforcement Learning: A Survey
Leslie Pack Kaelbling, Michael L. Littman, and Andrew W. Moore
Journal of Artificial Intelligence Research 4 (1996) 237–285
http://www.cs.cmu.edu/afs/cs/project/jair/pub/volume4/kaelbling96a.pdf

Textbooks





Reinforcement Learning: An Introduction
Richard S. Sutton and Andrew G. Barto
MIT Press, Cambridge MA USA, 1998
http://webdocs.cs.ualberta.ca/~sutton/book/the-book.html

Algorithms for Reinforcement Learning
Csaba Szepesvári
Morgan & Claypool, 2010
http://www.sztaki.hu/~szcsaba/papers/RLAlgsInMDPs-lecture.pdf

Resources

Courses

Reinforcement Learning: Theory and Practice (Spring 2013)
Peter Stone, UT Austin
http://www.cs.utexas.edu/~pstone/Courses/394Rspring13/

Reinforcement Learning (Fall 2010)
Balaraman Ravindran, IIT Madras
http://www.cse.iitm.ac.in/academics/cs670.php

Reinforcement Learning for Artificial Intelligence (Winter 2010)
Richard Sutton, Alberta
http://incompleteideas.net/rlai.cs.ualberta.ca/RLAI/RLAIcourse/2010.html

Learning and Sequential Decision Making (Spring 2009)
Michael Littman, Rutgers
http://www.cs.rutgers.edu/~mlittman/courses/seq09/

E-mail List

rl-list@googlegroups.com

RL Competition!

http://2013.rl-competition.org/

Today's Class

1. Markov decision problems
2. Bellman's (Optimality) Equations, planning and learning
3. Challenges
4. Summary

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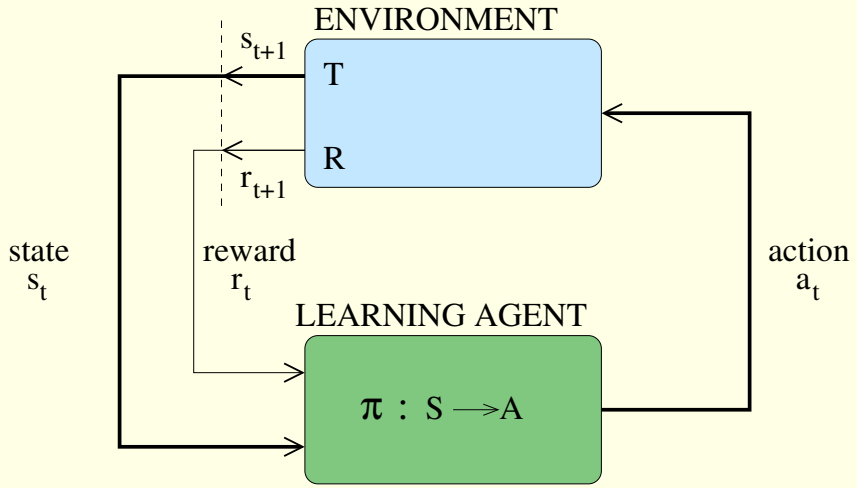

S: set of states.
A: set of actions.
T: transition function. $\forall s \in S, \forall a \in A$, $T(s, a)$ is a distribution over S.
R: reward function. $\forall s, s' \in S, \forall a \in A$, $R(s, a, s')$ is a finite real number.
γ: discount factor. $0 \leq \gamma < 1$.

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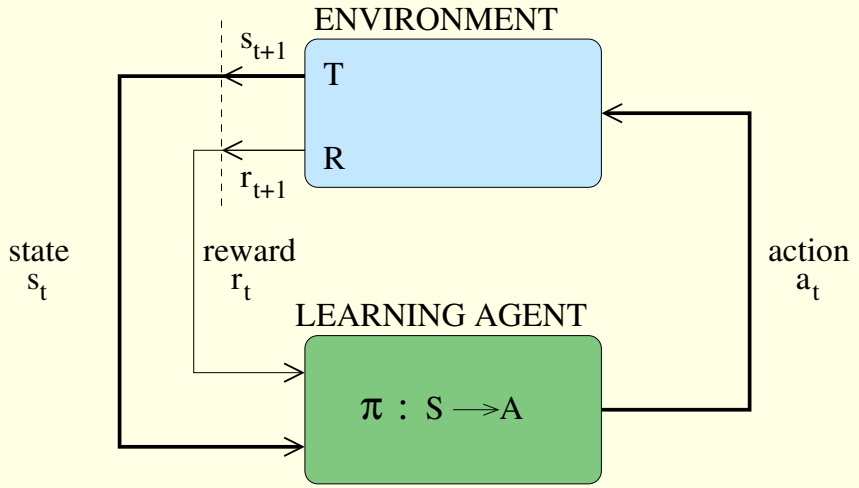

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Trajectory over time: $s_0, a_0, r_1, s_1, a_1, r_2, \ldots, s_t, a_t, r_{t+1}, s_{t+1}, \ldots$.

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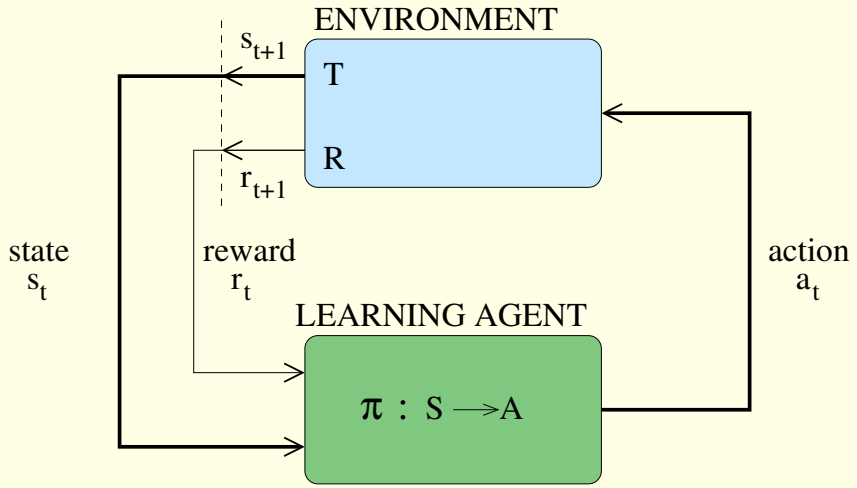

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Value, or expected long-term reward, of state s under policy π:

$$V^{\pi}(s) = \mathbb{E}[r_1 + \gamma r_2 + \gamma^2 r_3 + \ldots \text{ to } \infty | s_0 = s, a_i = \pi(s_i)].$$

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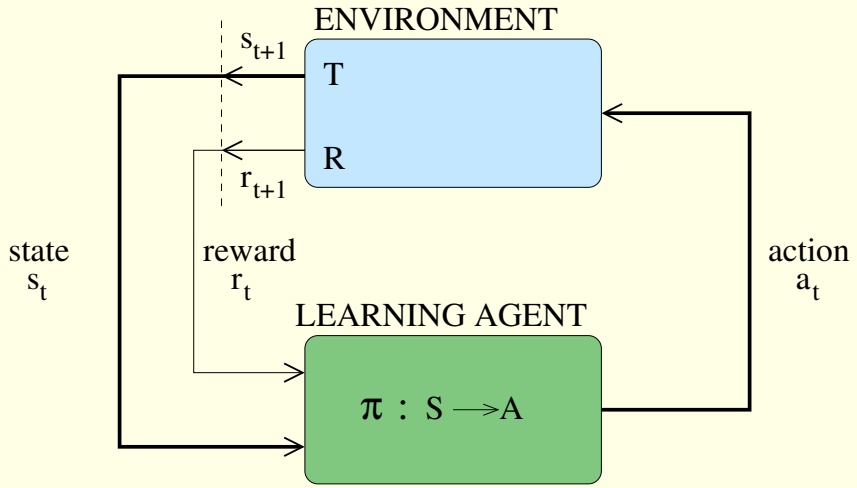

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Objective: "Find π such that $V^{\pi}(s)$ is maximal $\forall s \in S$."

Examples

What are the agent and environment? What are S, A, T, and R?

An Application of Reinforcement Learning to Aerobatic Helicopter Flight
Pieter Abbeel, Adam Coates, Morgan Quigley, and Andrew Y. Ng
In Proc. NIPS 2006, pp. 1–8, MIT Press, 2006
http://www.cs.stanford.edu/~pabbeel/pubs/AbbeelCoatesQuigleyNg_aaorltahf_nips2006.pdf

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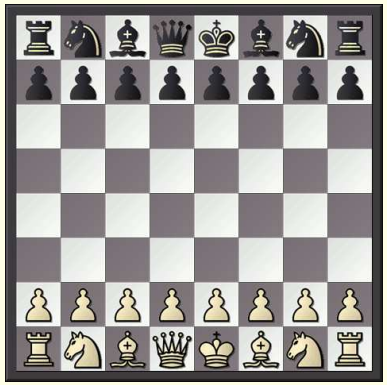

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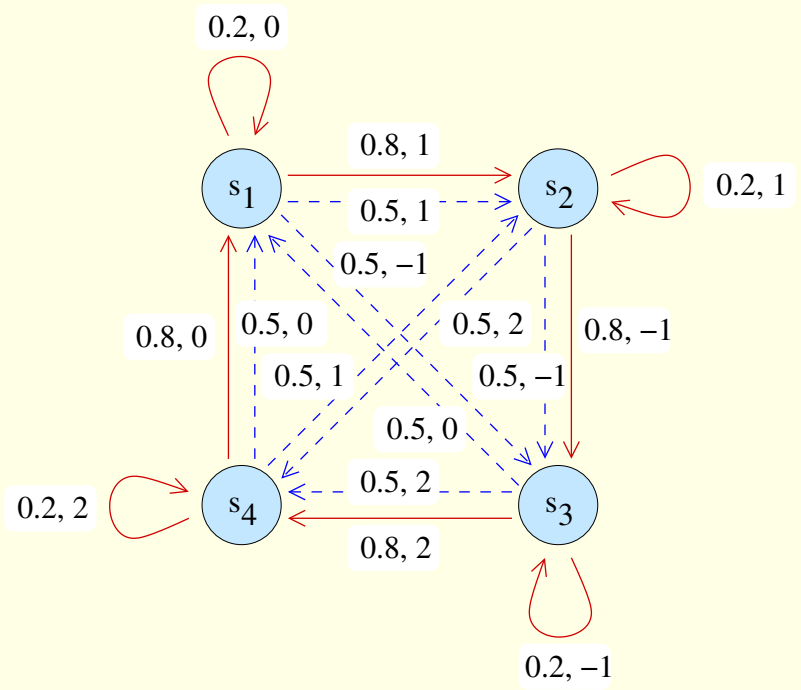
Illustration: MDPs as State Transition Diagrams



Notation: "transition probability, reward" marked on each arrow

States: s_1, s_2, s_3, and s_4.

Actions: Red (solid lines) and blue (dotted lines).

Transitions: Red action leads to same state with 20% chance, to next-clockwise state with 80% chance. Blue action leads to next-clockwise state or 2-removed-clockwise state with equal (50%) probability.

Rewards: $R(*, *, s_1) = 0$, $R(*, *, s_2) = 1$, $R(*, *, s_3) = -1$, $R(*, *, s_4) = 2$.

Discount factor: $\gamma = 0.9$.

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1. Markov decision problems
2. Bellman's (Optimality) Equations, planning and learning
3. Challenges
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Bellman's Equations

Recall that

$$V^{\pi}(s) = \mathbb{E}[r_1 + \gamma r_2 + \gamma^2 r_3 + \ldots | s_0 = s, a_i = \pi(s_i)].$$

Bellman's Equations ($\forall s \in S$):

$$\boxed{V^{\pi}(s) = \sum_{s' \in S} T(s, \pi(s), s') \left[R(s, \pi(s), s') + \gamma V^{\pi}(s')\right].}$$

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Define ($\forall s \in S, \forall a \in A$):

$$Q^{\pi}(s, a) = \sum_{s' \in S} T(s, a, s') \left[R(s, a, s') + \gamma V^{\pi}(s')\right].$$

Q^{π} is called the action value function of π.

$V^{\pi}(s) = Q^{\pi}(s, \pi(s))$.

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The variables in Bellman's equation are $V^{\pi}(s)$. $|S|$ linear equations in $|S|$ unknowns.

Thus, given S, A, T, R, γ, and a fixed policy π, we can solve Bellman's equations to obtain, $\forall s \in S, \forall a \in A$, $V^{\pi}(s)$ and $Q^{\pi}(s, a)$.

Bellman's Optimality Equations

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It can be shown that there exists a policy $\pi^* \in \Pi$ such that

$$\forall \pi \in \Pi \, \forall s \in S: \, V^{\pi^*}(s) \geq V^{\pi}(s).$$

V^{π^*} is denoted V^*, and Q^{π^*} is denoted Q^*.

There could be multiple optimal policies π^*, but V^* and Q^* are unique.

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Bellman's Optimality Equations ($\forall s \in S$):

$$V^*(s) = \max_{a \in A} \sum_{s' \in S} T(s, a, s') \left[R(s, a, s') + \gamma V^*(s') \right].$$

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Given S, A, T, R, γ, how can we find an optimal policy π^*? We need to be computationally efficient.

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Learning problem:

Given S, A, γ, and the facility to follow a trajectory by sampling from T and R, how can we find an optimal policy π^*? We need to be sample-efficient.

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One option. We can pose Bellman's optimality equations as a linear program, solve for V^*, derive Q^*, and induce $\pi^*(s) = \text{argmax}_a \, Q^*(s, a)$.

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Another option. We can apply the policy iteration algorithm, which is typically more efficient in practice.

- Pick an initial policy π arbitrarily.
- Compute Q^π using Bellman's equations.
- converged $\leftarrow$ `false`.
- Repeat
  - Set π' as: $\forall s \in S, \pi'(s) = \text{argmax}_a\, Q^\pi(s, a)$ (break ties arbitrarily).[Improvement]
  - Compute $Q^{\pi'}$ using Bellman's equations. [Evaluation]
  - If ($Q^{\pi'} = Q^\pi$), converged $\leftarrow$ `true`.
  - $\pi \leftarrow \pi'$, $Q^\pi \leftarrow Q^{\pi'}$.
- Until converged.
- Return π (which is provably optimal).

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Other options. Value iteration and its various "mixtures" with policy iteration.

Learning

Given S, A, γ, and the facility to follow a trajectory by sampling from T and R, how can we find an optimal policy π^*?

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Given S, A, γ, and the facility to follow a trajectory by sampling from T and R, how can we find an optimal policy π^*?

Various classes of learning methods exist. We will consider a simple one called Q-learning, which is a temporal difference learning algorithm.

- Let Q be our "guess" of Q^*: for every state s and action a, initialise $Q(s, a)$ arbitrarily. We will start in some state s_0.
- For $t = 0, 1, 2, \ldots$
  - Take an action a_t, chosen uniformly at random with probability ϵ, and to be $\text{argmax}_a\, Q(s_t, a)$ with probability $1 - \epsilon$.
  - The environment will generate next state s_{t+1} and reward r_{t+1}.
  - Update: $Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha_t(r_{t+1} + \gamma \max_{a \in A} Q(s_{t+1}, a) - Q(s_t, a_t))$.

  [ϵ: parameter for "ϵ-greedy" exploration] [α_t: learning rate]
  [$r_{t+1} + \gamma \max_{a \in A} Q(s_{t+1}, a) - Q(s_t, a_t)$: temporal difference prediction error]

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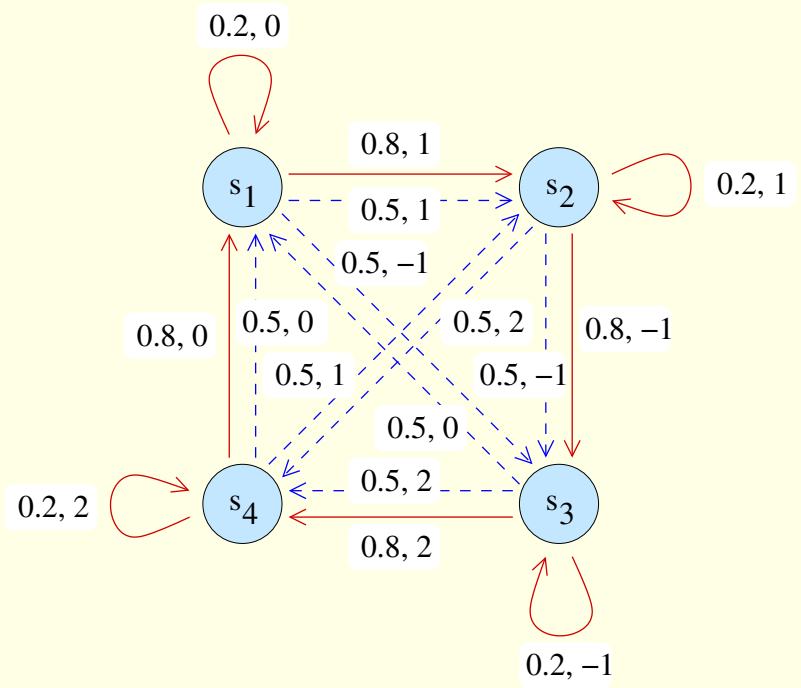
For $\epsilon \in (0, 1]$ and $\alpha_t = \frac{1}{t}$, it can be proven that as $t \to \infty$, $Q \to Q^*$.

Q-Learning
Christopher J. C. H. Watkins and Peter Dayan
Machine Learning 8(3–4) (1992) 279–292
http://www.springerlink.com/content/p120815501618373/fulltext.pdf

Exercise

Illustration from Section 2.



Notation: "transition probability, reward" marked on each arrow

1. **Planning**: Compute V^*, Q^*, and π^* using policy iteration.
2. **Learning**: Implement Q-learning with ϵ-greedy exploration, and a constant learning rate α. Experiment with values of ϵ and α in the range $[10^{-5}, \frac{1}{2}]$. Do the learned Q-values converge towards Q^*; how quickly? Try annealing α and ϵ as a function of t (the number of samples seen).

Implementing and Evaluating a Learning Algorithm

Play: http://www.youtube.com/watch?v=mRpX9DFCdwI.

Generalized Model Learning for Reinforcement Learning on a Humanoid Robot
Todd Hester, Michael Quinlan, and Peter Stone
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We desire both good learning speed (high rewards soon) and good asymptotic performance.

Today's Class

1. Markov decision problems
2. Bellman's (Optimality) Equations, planning and learning
3. Challenges
4. Summary

Important Questions for the Theory and Practice of RL

- Exploration
- Generalisation (over states and actions)
- State aliasing (partial observability)
- Multiple agents, nonstationary rewards and transitions
- Abstraction (over states and over time)
- Proofs of convergence, bounding sub-optimality

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My thesis question:

"How well do different learning methods for sequential decision making perform in the presence of state aliasing and generalization; can we develop methods that are both sample-efficient and capable of achieving high asymptotic performance in their presence?"

Learning Methods for Sequential Decision Making with Imperfect Representations
Shivaram Kalyanakrishnan
Ph.D. dissertation, published as UT Austin Computer Science Technical Report TR-11-41, 2011
http://www.cs.utexas.edu/~shivaram/papers/k_diss_2011.pdf

Practice $\Longrightarrow$ Imperfect Representations

Task	State Aliasing	State Space	Policy Representation (Number of features)
Backgammon (T1992)	Absent	Discrete	Neural network (198)
Job-shop scheduling (ZD1995)	Absent	Discrete	Neural network (20)
Tetris (BT1906)	Absent	Discrete	Linear (22)
Elevator dispatching (CB1996)	Present	Continuous	Neural network (46)
Acrobot control (S1996)	Absent	Continuous	Tile coding (4)
Dynamic channel allocation (SB1997)	Absent	Discrete	Linear (100's)
Active guidance of finless rocket (GM2003)	Present	Continuous	Neural network (14)
Fast quadrupedal locomotion (KS2004)	Present	Continuous	Parameterized policy (12)
Robot sensing strategy (KF2004)	Present	Continuous	Linear (36)
Helicopter control (NKJS2004)	Present	Continuous	Neural network (10)
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Adaptive job routing/scheduling (WS2004)	Present	Discrete	Tabular (4)
Robot soccer keepaway (SSK2005)	Present	Continuous	Tile coding (13)
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Optimized trade execution (NFK2007)	Present	Discrete	Tabular (2-5)
Blimp control (RPHB2007)	Present	Continuous	Gaussian Process (2)
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Perfect representations (fully observable, enumerable states) are impractical.

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Thank you! Questions?