MATH2040C Homework 6

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1 Section 6.1, Q8

8. Provide reasons why each of the following is not an inner product on the given vector spaces.

(a) $\langle (a, b), (c, d) \rangle = ac - bd$ on R^2.

(b) $\langle A, B \rangle = \operatorname{tr}(A + B)$ on $\mathsf{M}_{2 \times 2}(R)$.

(c) $\langle f(x), g(x) \rangle = \int_0^1 f'(t)g(t)\, dt$ on $\mathsf{P}(R)$, where $'$ denotes differentiation.

Figure 1: The caption of this figure.

Let $w = \begin{pmatrix} 2 & 4 \\ 4 & 3 \end{pmatrix}$. Note that $w \in W$, because w is symmetric.

Note that $T(w) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 2 & 4 \end{pmatrix}$, which is not symmetric, hence not belongs to W.

Therefore, by definition, W is not a T-invariant subspace of V.

Done.