MATH2040C Homework 3

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1 Section 2.2, Q3

According to the question, $\beta = \{(1,0),(0,1)\}$.
Therefore, $T((1,0)) = (1,1,2), T((0,1)) = (-1,0,1)$.
Then we need to find $[T((1,0))]_\gamma = [(1,1,2)]_\gamma$ and $[T((0,1))]_\gamma = [(-1,0,1)]_\gamma$.
By the question, $\gamma = \{(1,1,0),(0,1,1),(2,2,3)\}$.

Note that $\begin{pmatrix}1\\1\\2\end{pmatrix} = \begin{pmatrix}1&0&2\\1&1&2\\0&1&3\end{pmatrix}\begin{pmatrix}-\frac{1}{3}\\0\\\frac{2}{3}\end{pmatrix}$.

Hence $[T((1,0))]_\gamma = [(1,1,2)]_\gamma = (-\frac{1}{3},0,\frac{2}{3})$.

Also note that $\begin{pmatrix}-1\\0\\1\end{pmatrix} = \begin{pmatrix}1&0&2\\1&1&2\\0&1&3\end{pmatrix}\begin{pmatrix}-1\\1\\0\end{pmatrix}$.

Therefore, $[T]_\beta^\gamma = \begin{pmatrix}-\frac{1}{3}&-1\\0&1\\\frac{2}{3}&0\end{pmatrix}$.

Note that $\alpha = \{(1,2),(2,3)\}$. And $T(1,2) = (-1,1,4)$ and $T(2,3) = (-1,2,7)$. Then we will find $[(-1,1,4)]_\gamma$ and $[-1,2,7]_\gamma$.

Note that $\begin{pmatrix}-1\\1\\4\end{pmatrix} = \begin{pmatrix}1&0&2\\1&1&2\\0&1&3\end{pmatrix}\begin{pmatrix}-\frac{7}{3}\\2\\\frac{2}{3}\end{pmatrix}$.

Hence $[T(1,2)]_\gamma = [(-1,1,4)]_\gamma = (-\frac{7}{3},2,\frac{2}{3})$.

Also note that $\begin{pmatrix}-1\\2\\7\end{pmatrix} = \begin{pmatrix}1&0&2\\1&1&2\\0&1&3\end{pmatrix}\begin{pmatrix}-\frac{11}{3}\\3\\\frac{4}{3}\end{pmatrix}$.

Hence $[T(2,3)]_\gamma = [(-1,2,7)]_\gamma = (-\frac{11}{3},3,\frac{4}{3})$.

Therefore, $[T]_\alpha^\gamma = \begin{pmatrix}-\frac{7}{3}&-\frac{11}{3}\\2&3\\\frac{2}{3}&\frac{4}{3}\end{pmatrix}$.

Done.

2 Section 2.2, Q5