MATH2040C Homework 6

ZHENG Weijia (William, 1155124322)

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1 Section 6.1, Q8

8. Provide reasons why each of the following is not an inner product on the given vector spaces.

(a) $\langle (a,b),(c,d)\rangle = ac - bd$ on R^2.
(b) $\langle A, B\rangle = \mathrm{tr}(A+B)$ on $\mathsf{M}_{2\times 2}(R)$.
(c) $\langle f(x), g(x)\rangle = \int_0^1 f'(t)g(t)\,dt$ on $\mathsf{P}(R)$, where $'$ denotes differentiation.

Figure 1: Section 6.1 Q8

1.1 (a)

Suppose this is an inner product. Then $\langle x, x\rangle \geq 0$ should hold $\forall x \in \mathbb{R}^2$.
Let $x = (1, 10)$. Then $\langle x, x\rangle = \langle (1, 10), (1, 10)\rangle = 1^2 - 10^2 = -99 < 0$.
Therefore, this is not an inner product.

1.2 (b)

Suppose this is an inner product. Then $\langle x, x\rangle \geq 0$ should hold $\forall x \in M_{2\times 2}(R)$.
Let $x = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$. Then

$$\langle x, x\rangle = tr(x + x) = -2 - 2 = -4 < 0.$$

Therefore, this is not an inner product.

1.3 (c)

Suppose this is an inner product. Then $\forall f, g \in P(\mathbb{R}), \overline{\langle g, f\rangle} = \langle f, g\rangle$ should hold.
Let $f(x) = x, g(x) = x^2 + x$.
Then

$$\langle f, g\rangle = \int_0^1 1(x^2 + x)\,dx = \frac{5}{6}.$$

$$\overline{\langle g, f \rangle} = \overline{\int_0^1 (2x+1)x\,dx} = \frac{7}{6}.$$

Therefore $\overline{\langle g, f \rangle} \neq \langle f, g \rangle$ for some $f, g \in P(\mathbb{R})$.
Hence, this is not an inner product.
Done.

2 Section 6.1, Q17