MATH2040C Homework 5

ZHENG Weijia (William, 1155124322)

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1 Section 5.4, Q2(e)

Let $w = \begin{pmatrix} 2 & 4 \\ 4 & 3 \end{pmatrix}$. Note that $w \in W$, because w is symmetric.

Note that $T(w) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 2 & 4 \end{pmatrix}$, which is not symmetric, hence not belongs to W.

Therefore, by definition, W is not a T-invariant subspace of V.

Done.

2 Section 5.4, Q4

$\forall g(t)$ belongs to polynomials, it can be expressed as for some $a_i, i = 0, 1, 2, ..., n$,

$$g(t) = \sum_{i=0}^{n} a_i t^i.$$

Note that $\forall w \in W$, we have

$$g(T)(w) = \sum_{i=0}^{n} a_i T^i(w).$$

Because W is itself a subspace, and note that $T^i(w) \in W, \forall i$. Then

$$\forall w \in W, g(T)(w) \in W.$$

Done.

3 Section 5.4, Q6(d)

Note that $z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $T(z) = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$ and $T^2(z) = 3 \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$.

And hence $T^k(z) = 3^{k-1} \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}, \forall k \geq 1$.

Recall that $span\{z, T(z), T^2(z), \dots\}$ is the T-cyclic subspace of V generated by z.

Claim that $\{z, T(z)\}$ is a ordered basis for $span\{z, T(z), T^2(z), \dots\}$.

Note that $\forall u \in span\{z, T(z), T^2(z), \dots\}$, if $u = z$ or $u = T(z)$, then u are elements inside the basis set.

If $u = T^k(z)$ for some $k \geq 2$, notice that $T^k(z) == 3^{k-1}\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = 3^{k-1}T(z)$.

Therefore, $\{z, T(z)\}$ spans $span\{z, T(z), T^2(z), \dots\}$.

Done.

4 Section 5.4, Q19