STAT3003 Problem Sheet 1

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Note that $L = 4$, $N = N_1+N_2+N_3+N_4 = 225$, with $N_1 = 64, N_2 = 43, N_3 = 92, N_4 = 26$. And $\hat{p_1} = \frac{2}{7}, \hat{p_2} = \frac{1}{3}, \hat{p_3} = \frac{8}{21}, \hat{p_4} = \frac{1}{3}$.

Hence we can have the

$$\hat{p_{st}} = \frac{1}{N}\sum_{i=1}^{L} N_i\hat{p_i} = 0.3393.$$

Also, by

$$\hat{Var}(\hat{p_{st}}) = \frac{1}{N^2}\sum_{i=1}^{L} N_i^2(1 - \frac{n_i}{N_i})\frac{1}{n_i - 1}\hat{p_i}(1 - \hat{p_i}) = \frac{197.979}{225^2} = 3.7724 * 10^{-3}.$$

Therefore, $\sqrt{\hat{Var}(\hat{p_{st}})} = 0.0614$.

Our goal is to find

$$(\ \hat{p_{st}} \pm t_{df,1-\frac{\alpha}{2}} * \sqrt{\hat{Var}(\hat{p_{st}})}\ \ ).$$

By Satterthwaite's Approximation, we have

$$df \approx \frac{(\sum_{i=1}^{L} k_i s_i^2)^2}{\sum_{i=1}^{L}\frac{(k_i s_i^2)^2}{n_i - 1}}, k_i = \frac{N_i(N_i - n_i)}{N^2 n_i},$$

and hence $df = 45$.

Then $t_{df,1-\frac{\alpha}{2}} = t_{45,0.975} = 2.0141$. Therefore, then result should be

$$(\ 0.3393 \pm 2.0141 * 0.0614\ )$$

Done.