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Due September 9

Use the questions below to identify any topics you should review. See the week 01 notes for background information on these topics.

Trigonometry

Consider the point **p** and angle θ below, where **p** is a distance of 1 unit from the origin and θ is 45 degrees. What is the coordinate of **p**? Hint: what are the values of a and b in terms of θ?

$\sin\theta = \frac{b}{c}$

$\sin 45^\circ = \frac{b}{1}$

$\frac{\sqrt{2}}{2} = b$

$\cos 45^\circ = \frac{a}{c}$

$\frac{\sqrt{2}}{2} = \frac{a}{1}$

$\frac{\sqrt{2}}{2} = a$

$p = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

Consider the point p_1 and angle θ_1 below. Suppose $p_1 = (2, 2, 0)^T$. What is the value of θ_1? Hint: Use tangent.

$\tan\theta_1 = \frac{b}{a}$

$\tan\theta_1 = \frac{2}{2}$

$\tan\theta_1 = 1$

$\theta_1 = \arctan(1)$

$\theta_1 = \frac{\pi}{4}$

Consider the point p_2 and angle θ_2 above. Suppose $p_2 = (3, -2, 0)^T$. What is the value of θ_2? Hint: Use tangent.

$\tan\theta_2 = \frac{-2}{3}$

$\theta_2 = \arctan\left(\frac{-2}{3}\right)$

Vectors

A **vector** is an n-tuple of real numbers. In this class, we will work with 2D, 3D, and 4D vectors. Suppose we have a vector **u**=(-2, 3, 0)^T and **v**=(-1, 4, 0)^T.

- Draw the vectors **u** and **v**, with their tails anchored at the origin below.

- What is the length of **u**?

$u = \sqrt{(-2)^2 + (3)^2}$

$u = \sqrt{4 + 9}$

$u = \sqrt{13}$

- What is the distance between **u** and **v**?

$d(u, v) = \sqrt{(-2 - (-1))^2 + (3 - 4)^2}$

$= \sqrt{(-1)^2 + (-1)^2}$

$= \sqrt{2}$

- Compute and draw **u** + **v**.

$u + v = (-3, 7)$

- Compute and draw **u** - **v**

$(-1, -1)$

$u = (-2, 3, 0)$

- Compute the cross product $u \times v$. $v = (-1, 4, 0)$

$\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} \times \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \cdot 0 - (-4) \cdot 0 \\ 0 \cdot -1 - -2 \cdot 0 \\ -2 \cdot 4 - (-3) \cdot -1 \end{bmatrix}$

$= \begin{bmatrix} 0 - 0 \\ 0 - 0 \\ -8 - 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix}$

- Normalize the vector **u**, e.g. compute $\frac{u}{|u|}$

$|u| = \sqrt{(-2)^2 + (3^2) + (0)^2}$

$= \sqrt{13}$ $\quad \frac{u}{|u|} = \left(\frac{-2}{\sqrt{13}}, \frac{3}{\sqrt{13}}, 0\right)$

- Compute the dot product $u \cdot v$.

$\vec{u} \cdot \vec{v} = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix}$

$= (-2)(-1) + (3)(4) + 0$

$= 16$

Matrices

Consider the following matrices

$$A = \begin{bmatrix} 1 & 3 \\ -0.5 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 0 \\ 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 3 \\ -4 & 5 \\ 3 & -7 \end{bmatrix}$$

- What are the dimensions of A, B, and C?

A = 2×2 matrix

B = 2×2

C = 3×3

- What is the transpose of the matric C?

$C^T = \begin{bmatrix} 1 & -4 & 3 \\ 3 & 5 & -7 \end{bmatrix}$

- Compute the products AB and BA.

$AB = \begin{bmatrix} 1 & 3 \\ -0.5 & 2 \end{bmatrix} \times \begin{bmatrix} -3 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 6 \\ 3.5 & 4 \end{bmatrix}$

$BA = \begin{bmatrix} -3 & 0 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ -0.5 & 2 \end{bmatrix} = \begin{bmatrix} -3 & -9 \\ 0 & 7 \end{bmatrix}$

- Is it possible to multiply C times itself? Why not? What about CC^T?

CC^T is possible since the number of columns in C is equal to the # of rows in C^T. Furthermore it is possible to mult C by C.

- What is the product of AA^{-1}?

$AA^{-1} = \begin{bmatrix} 1 & 3 \\ -0.5 & 2 \end{bmatrix} \begin{bmatrix} \frac{2}{7} & -\frac{3}{7} \\ \frac{1}{7} & \frac{1}{7} \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Consider the following matrix

$$R = \begin{bmatrix} cos(30) & sin(30) & 0 \\ -sin(30) & cos(30) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Suppose we have a vector **u**=(1,0,0)^T. Draw **u** below. Then multiple **u** by **R** and draw **Ru**.

$Ru = \begin{bmatrix} \cos(30) & \sin(30) & 0 \\ -\sin(30) & \cos(30) & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$Ru = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right)$

$= \begin{bmatrix} \frac{\sqrt{3}}{2} + 0 \\ -\frac{1}{2} + 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} \frac{\sqrt{3}}{2} + 0 + 0 \\ -\frac{1}{2} + 0 + 0 \\ 0 + 0 + 0 \end{bmatrix}$

$= \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0\right)$

Polynomials

Consider the polynomial $p(t) = 9t^3 + 6t^2$.

- What is the degree of $p(t)$?

3

- What is the derivative of $p(t)$?

$f'(x) = 9(3t)^2 + 6(2t)$

$= 27t^2 + 12t$

- What is the value of $p(t)$ when t = -1?

$p(-1) = 9(-1)^3 + 6(-1)^2$

$= -9 + 6$

$= -3$

Let $B_0(t) = (t - 1)^2$ and $B_1 = t - 2$.

- Compute an expression for $p(t) = B_0(t) + B_1(t)$ and re-arrange the terms into standard form $p(t) = a_2t^2 + a_1t + a_0$.

Standard form has the following pattern:
$a_nt^n + \ldots + a_2t^2 + at + a_0$.