More on Tree Based Methodss

ISLR Chapter 8

April 24, 2017

Outline

Ways to improving trees through a multiple trees in some ensemble:

- Bagging

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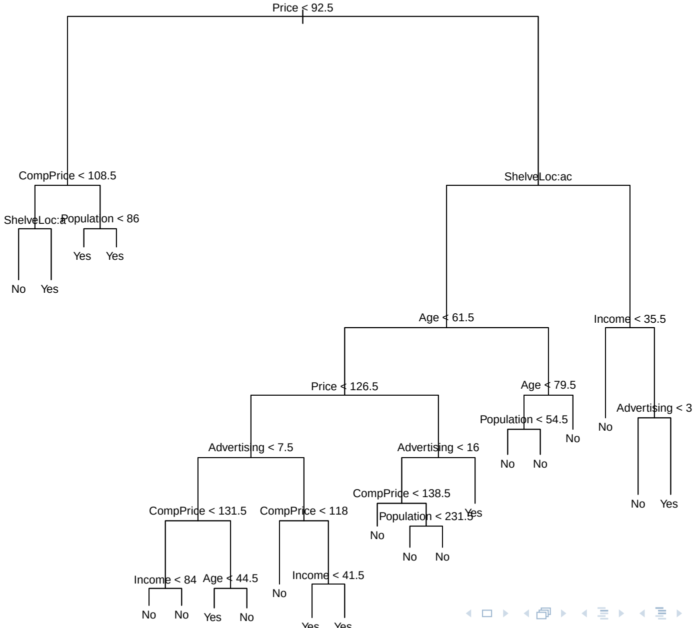
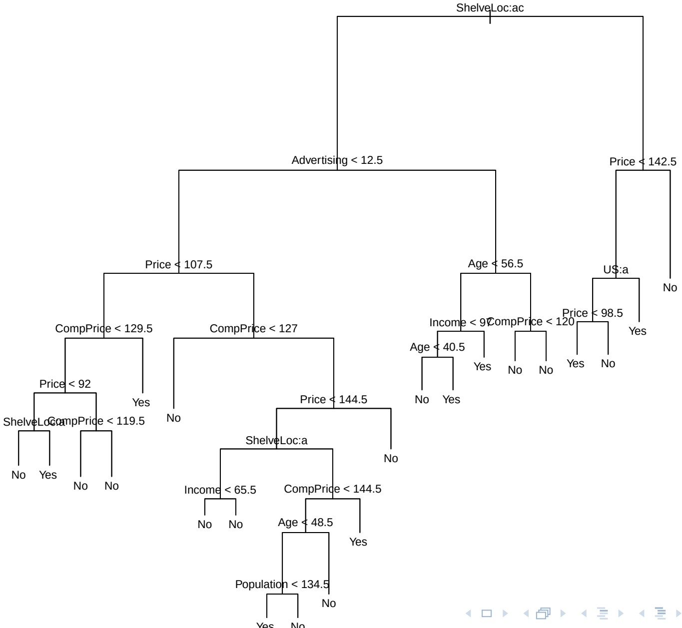
Combining trees will yield improved prediction accuracy, but with loss of interpretability.

Tree with Random Split of Data



Tree with another Random Split of Data



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Bagging Example

```
suppressMessages(library(randomForest))
bag.carseats = randomForest(High ~ . -Sales,
                            data=Carseats, subset = train,
                            mtry=10, importance =TRUE)
#  mtry = 10 (the number of predictors)
yhat.bag = predict(bag.carseats,newdata = Carseats.test)
tab = table(yhat.bag,Carseats.test$High)
tab

##
## yhat.bag No Yes
##      No   96  16
##      Yes  20  68

CE.tab["RandomForests", 1]= (tab[1,1] + tab[2,2])/sum(tab)

CE.tab["RandomForests",]
```

RandomForests

```
 rf.carseats = randomForest(High ~ . -Sales,data=Carseats,
                            subset =train,
                            mtry=3, importance =TRUE)
 yhat.rf= predict(rf.carseats ,newdata =Carseats.test)
tab = table(yhat.rf,Carseats.test$High)

 tab

##
## yhat.rf No Yes
##     No   99  23
##     Yes  17  61

CE.tab["RandomForests", 1]= (tab[1,1] + tab[2,2])/sum(tab)
# (101+63)/200
CE.tab["RandomForests",]

## [1] 0.8
```

Variable Importance Measures in Bagging & RandomForests

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$$G = \sum_{k=1}^{K} \hat{\pi}_{mk}(1 - \hat{\pi}_{mk})$$

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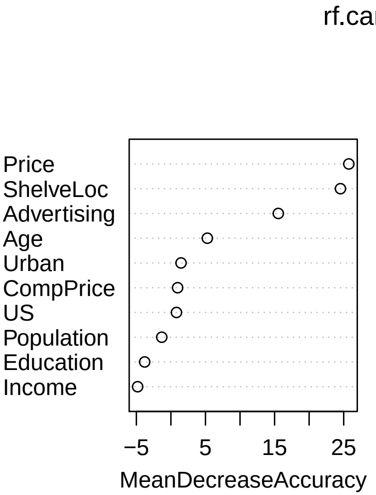
  where K is the number of classes for region m or use the reduction in deviance
- Out of Bag Prediction Error
- May be normalized by dividing by the maximum

Variable Importance Measures in RandomForests

```r
varImpPlot(rf.carseats)
```



Boosting

Algorithm 8.2 *Boosting for Regression Trees*

1. Set $\hat{f}(x) = 0$ and $r_i = y_i$ for all i in the training set.
2. For $b = 1, 2, \ldots, B$, repeat:
   (a) Fit a tree $\hat{f}^b$ with d splits ($d + 1$ terminal nodes) to the training data (X, r).
   (b) Update $\hat{f}$ by adding in a shrunken version of the new tree:
   $$\hat{f}(x) \leftarrow \hat{f}(x) + \lambda \hat{f}^b(x). \tag{8.10}$$
   (c) Update the residuals,
   $$r_i \leftarrow r_i - \lambda \hat{f}^b(x_i). \tag{8.11}$$
3. Output the boosted model,
   $$\hat{f}(x) = \sum_{b=1}^{B} \lambda \hat{f}^b(x). \tag{8.12}$$

Boosting

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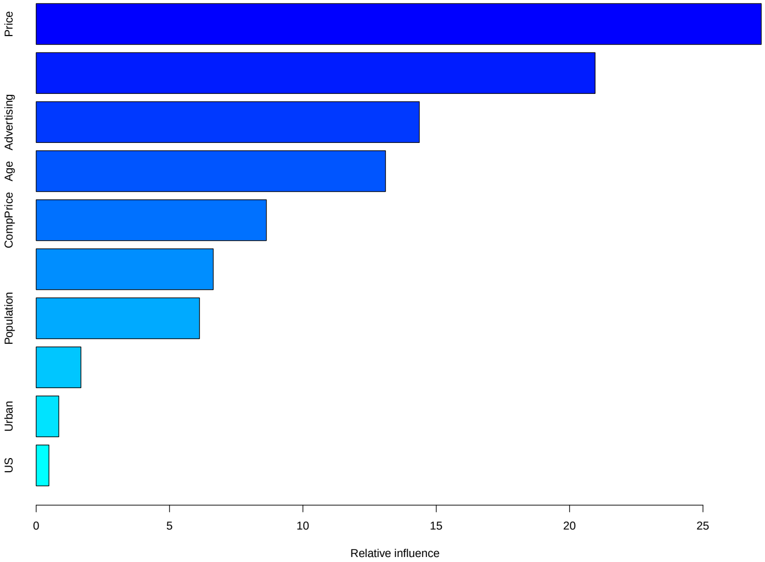
Boosting Example

```
suppressMessages(library(gbm))
boost.car =gbm(I(as.numeric(High)-1) ~ . -Sales,
               data=Carseats[train ,],
               distribution="bernoulli",
               n.trees =5000, interaction.depth =4)
yhat.boost = ifelse(predict(boost.car,
                      newdata=Carseats.test,
                      n.trees=5000,
                      type="response") > .5, 1, 0)
tab =table(yhat.boost, Carseats.test$High)
tab = table(yhat.boost,Carseats.test$High)
tab

##
## yhat.boost  No Yes
##          0 100  16
##          1  16  68
```

Variable Importance: Boosting

```
summary(boost.car, plotit=FALSE)

##                     var    rel.inf
## Price             Price 27.1860994
## ShelveLoc     ShelveLoc 20.9558458
## Advertising Advertising 14.3652543
## Age                 Age 13.0972300
## CompPrice     CompPrice  8.6307712
## Income           Income  6.6391956
## Population   Population  6.1235720
## Education     Education  1.6769694
## Urban             Urban  0.8472994
## US                   US  0.4777628
```

Variable Importance: summary(boost.car)



Bayesian Additive Regression Trees

Gaussian Model: Single Tree model

$$Y = g(x, T, M) + \epsilon$$

Where T is a tree and $M = (\mu_1, \ldots \mu_b)^T$ is the vector of means at the terminal nodes of the tree given by T

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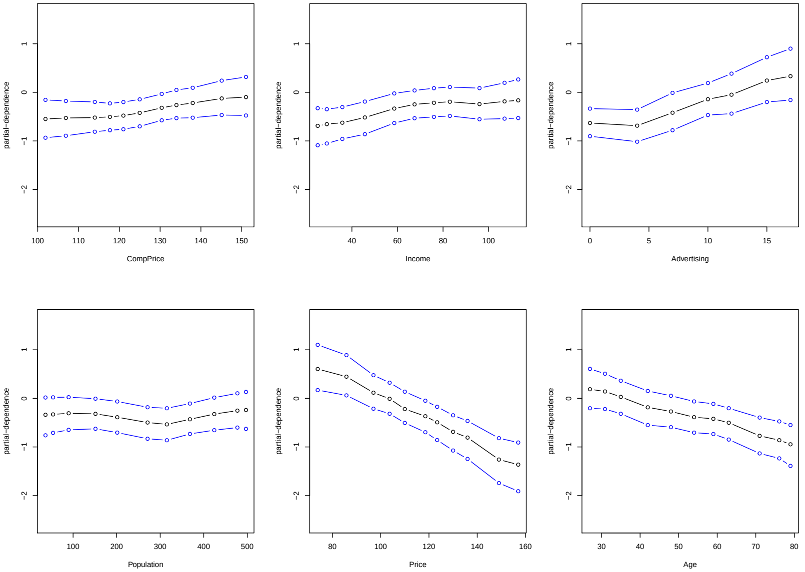
BART: Bayesian Additive Regression Trees

```
library(BayesTree)

## Error in library(BayesTree):  there is no package
called 'BayesTree'

set.seed(42)
bart.carseats = bart(x.train=Carseats[train,-c(1,12)],
                     y.train=Carseats$High[train],
                     x.test=Carseats[-train,-c(1,12)],
                     verbose=FALSE    )

## Error in eval(expr, envir, enclos):  could not
find function "bart"

pihat.bart = apply(pnorm(bart.carseats$yhat.test), 2, mean)

## Error in pnorm(bart.carseats$yhat.test):  object
'bart.carseats' not found
```

Uncertainty in Mean Function

```
plot(bart.carseats)

## Error in plot(bart.carseats):  object
'bart.carseats' not found
```

Partial Dependendence (computationally intensive!)



BartMachine

```r
suppressMessages(library(bartMachine))
set.seed(42)
bart.carseats = bartMachine(X=Carseats[train,-c(1,12)],
                            y=Carseats$High[train],
                            verb=FALSE, serialize=TRUE)

## serializing in order to be saved for future R sessions.

yhat.bart = predict(bart.carseats,
                    Carseats.test[, -c(1,12)],
                    type="class")
table(yhat.bart, Carseats.test$High)

##
## yhat.bart  No Yes
##       No  103  20
##       Yes  13  64
```

Variable Importance

```
## Error in library(bartMachine): there is no
package called 'bartMachine'
```

```
investigate_var_importance(bart.carseats)

## Error in eval(expr, envir, enclos): could not
find function "investigate_var_importance"
```

GLMs and Bayesian Variable Selection

```
set.seed(42); library(BAS)
bas.carseats = bas.glm(High ~ . - Sales, data=Carseats,
                       subset=train, family=binomial(),
                       method="MCMC",  n.models=10000,
                       betaprior=bic.prior(n=200))
yhat = predict(bas.carseats, newdata=Carseats[-train,])
tab = table(ifelse(yhat$fit > .5, 1, 0), Carseats.test$Hig
CE.tab["BMA", 1]= (tab[1,1] + tab[2,2])/sum(tab)

CE.tab["BMA",]

## [1] 0.905

#(107 + 74)/200
```

LASSO Variable Selection

```
set.seed(42); library(glmnet)

## Loading required package:  Matrix
## Loading required package:  foreach
## Loaded glmnet 2.0-5

glmnet.carseats = cv.glmnet(
  x=model.matrix(High ~ . - Sales, data=Carseats[train,]),
  y=(as.numeric(Carseats$High[train]) - 1), family="binomia
yhat = predict(glmnet.carseats,
               newx=model.matrix(High ~ . - Sales, data=Ca
tab = table(ifelse(yhat > .0, 1, 0), Carseats.test$High)
CE.tab["LASSO", 1]= (tab[1,1] + tab[2,2])/sum(tab)

CE.tab["LASSO",]

## [1] 0.89
```

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Interpretability? For which methods can you explain how changes in a predictor change the response?

```r
save(CE.tab, file="OoS.accuracy")
```