Bayes Estimators & Ridge Regression

Readings ISLR 6

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Model

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- if smallest eigen value is 0, $\mathbf{X}$ has columns that are linearly dependent

How Good are Various Estimators

Quadratic loss for estimating $\boldsymbol{\beta}$ using estimator $\mathbf{a}$

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- If smallest $\lambda_j \to 0$ then MSE $\to \infty$

Problems

Estimates:

$$\hat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$$

or with g-prior

$$\hat{\beta} = \frac{g}{1+g}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$$

may be unstable
Solutions:

- remove redundant variables (model selection) (AIC, BIC, other approches) 2^p models combinatorial hard problem even with Stochastic Search
- add constant to $\mathbf{X}^T\mathbf{X}$: $\tilde{\beta} = (\mathbf{X}^T\mathbf{X} + k\mathbf{I})^{-1}\mathbf{X}^T\mathbf{Y}$ to stabilise eigenvalues - alternative shrinkage estimator

Independent Prior

- Reference prior $p(\beta_0, \phi) \propto \phi^{-1}$
- Prior Distribution on

$$\boldsymbol{\beta} \mid \phi, \beta_0, k \sim \mathsf{N}(\mathbf{0}_p, \frac{1}{\phi k}\mathbf{I}_p)$$

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- log likelihood (integrated) for $\boldsymbol{\beta}$ plus prior

$$-\frac{\phi}{2}\left(\|\mathbf{Y} - \mathbf{1}\bar{\mathbf{Y}} - \mathbf{X}\boldsymbol{\beta}\|^2 + k\|\boldsymbol{\beta}\|^2\right)$$

- Posterior mean

$$\mathbf{b}_n = (\mathbf{X}^T\mathbf{X} + k\mathbf{I})^{-1}\mathbf{X}^T\mathbf{X}\hat{\boldsymbol{\beta}}$$

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- importance of standardizing
- Choice of k in practice?
- $k = 0$ OLS
- $k = \infty$ estimates are $\mathbf{0}$ (intercept only)

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subject to

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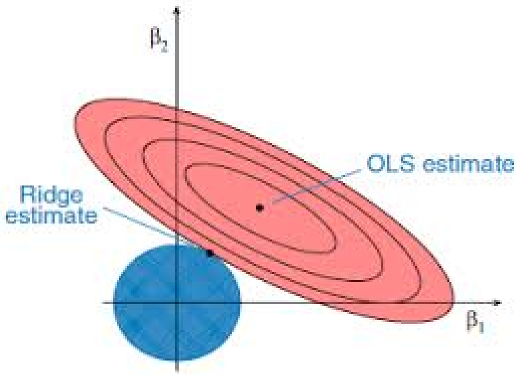
- Ridge Regression

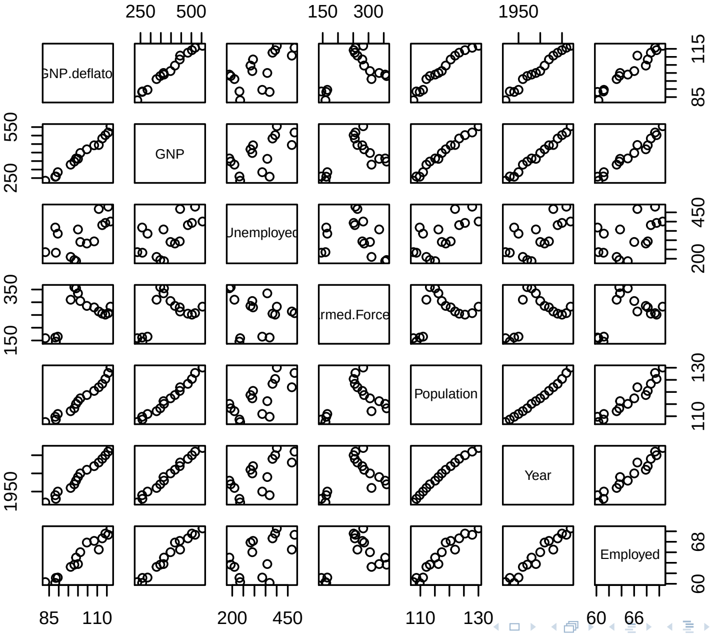
Geometry



[1]

[1] onlinecourses.science.pse.edu

Longley Data: library(MASS); data(longley)



OLS

```
> longley.lm = lm(Employed ~ ., data=longley)
> summary(longley.lm)

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)  -3.482e+03  8.904e+02  -3.911 0.003560 **
GNP.deflator  1.506e-02  8.492e-02   0.177 0.863141
GNP          -3.582e-02  3.349e-02  -1.070 0.312681
Unemployed   -2.020e-02  4.884e-03  -4.136 0.002535 **
Armed.Forces -1.033e-02  2.143e-03  -4.822 0.000944 ***
Population   -5.110e-02  2.261e-01  -0.226 0.826212
Year          1.829e+00  4.555e-01   4.016 0.003037 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3049 on 9 degrees of freedom
Multiple R-squared: 0.9955,^^IAdjusted R-squared: 0.9925
F-statistic: 330.3 on 6 and 9 DF,  p-value: 4.984e-10
```

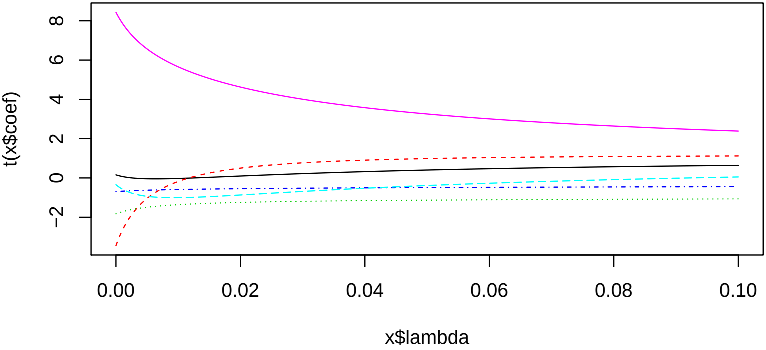
Ridge Regression

```
# from library MASS
longley.ridge = lm.ridge(Employed ~ ., data=longley,
                         lambda=seq(0, 0.1, 0.0001))
# lambda = k in notes

summary(longley.ridge)

##        Length Class  Mode
## coef   6006   -none- numeric
## scales    6   -none- numeric
## Inter     1   -none- numeric
## lambda 1001   -none- numeric
## ym        1   -none- numeric
## xm        6   -none- numeric
## GCV    1001   -none- numeric
## kHKB      1   -none- numeric
## kLW       1   -none- numeric
```

Ridge Trace Plot



Choice of k

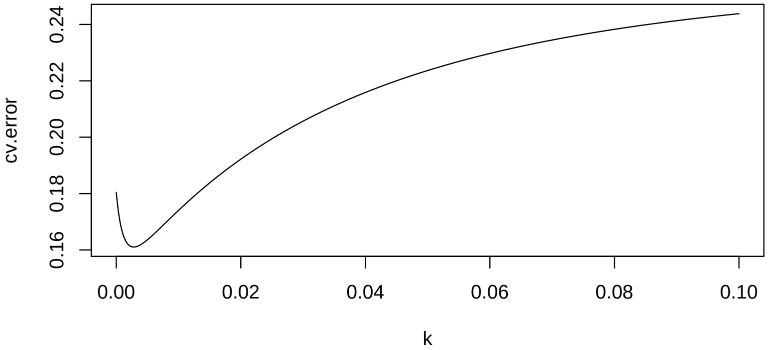
```
k = seq(0, 0.1, 0.0001)
n.k = length(k); n = nrow(longley)
cv.lambda = matrix(NA, n, n.k)

rmse.ridge = function(data, i, j, k) {
  m.ridge = lm.ridge(Employed ~ ., data = data, lambda=k[j]
                                 subset = -i)
  yhat = scale(data[i,1:6, drop=F],center = m.ridge$xm,
                              scale = m.ridge$scales) %*%
                    m.ridge$coef + m.ridge$ym
(yhat - data$Employed[i])^2
}

for (i  in 1:n)  {
  for (j in 1:n.k) {
    cv.lambda[i,j] = rmse.ridge(longley, i, j, k)
  }
}
```

Cross Validation Error

```
cv.error = apply(cv.lambda, 2, mean)
plot(k, cv.error, type="l")
```



Best $k = 0.0028$

Generalized Cross-validation

```
select(lm.ridge(Employed ~ ., data=longley,
        lambda=seq(0, 0.1, 0.0001)))

## modified HKB estimator is 0.004275357
## modified L-W estimator is 0.03229531
## smallest value of GCV  at 0.0028

best.k = longley.ridge$lambda[which.min(longley.ridge$GCV)]
longley.RReg = lm.ridge(Employed ~ ., data=longley,
                              lambda=best.k)
coef(longley.RReg)

##                GNP.deflator           GNP    Unemployed  Arme
## -2.950348e+03 -5.381450e-04 -1.822639e-02 -1.761107e-02 -9.60
##    Population          Year
## -1.185103e-01  1.557856e+00
```

Priors on k

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$$\kappa \mid \phi \sim \mathsf{Gamma}(1/2, 1/2)$$

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- What is induced prior on $\boldsymbol{\beta} \mid \phi$?

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1. Sample $\kappa^{(t)} \sim p(\kappa \mid \beta_0^{(t-1)}, \boldsymbol{\beta}^{(t-1)}, \phi^{(t-1)}, \mathbf{Y})$

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2. Sample $\beta_0^{(t)}, \boldsymbol{\beta}^{(t)}, \phi^{(t)} \mid \kappa(t), \mathbf{Y}$

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Use Samples $\beta_0^{(t)}, \boldsymbol{\beta}^{(t)}, \phi^{(t)}, \kappa^{(t)}$ for $t = B, \ldots, T$ for inference

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How would you compare Bayes predictions with Ridge with Cross-validation?