Missing Data and Imputation

Hoff Chapter 7, GH Chapter 25

April 21, 2017

Bednets and Malaria

- `Y`:presence or absence of parasites in a blood smear
- `AGE`: age of child
- `BEDNET`: bed net use (exposure)
- `GREEN`:greenness of the surrounding vegetation based on satellite photography
- `PHC`: whether a village is part of a primary health-care system

Bednets and Malaria

```
malaria = read.csv("gambia.dat", header=TRUE)
summary(malaria)

       Y               AGE              BEDNET           GREEN
 Min.    :0.0000   Min.    :1.000   Min.    :0.0000   Min.    :28.85   Min.
 1st Qu.:0.0000   1st Qu.:1.000   1st Qu.:0.0000   1st Qu.:40.85   1st
 Median :0.0000   Median :2.000   Median :1.0000   Median :40.85   Medi
 Mean    :0.3093   Mean    :2.399   Mean    :0.7049   Mean    :39.84   Mean
 3rd Qu.:1.0000   3rd Qu.:3.000   3rd Qu.:1.0000   3rd Qu.:40.85   3rd
 Max.    :1.0000   Max.    :4.000   Max.    :1.0000   Max.    :47.65   Max.
                                    NA's    :317
```

39% missing

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- Analysis depends on assumptions about missingness

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Cannot tell from data

Modeling

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- Model Based Methods

Observed Data

- $(Y_{i,1}, Y_{i,2}, Y_{i,3}, Y_{i,4}, Y_{i,5})$
- $(O_{i,1}, O_{i,2}, O_{i,3}, O_{i,4}, O_{i,5})$

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Marginal Model for observed data

$$\begin{aligned} p(o_i, y[o_i = 1] \mid \boldsymbol{\theta}) &= p(o_i) p(y[o_i = 1] \mid \boldsymbol{\theta}) \\ &= p(o_i) \int \left\{ p(y_{i,1}, y_{i,2}, y_{i,3}, y_{i,4}, y_{i,5} \mid \boldsymbol{\theta}) \prod_{y_{i,j} \ni o_{i,j}=0} dy_{i,j} \right. \end{aligned}$$

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Integrate over the missing variables to obtain the likelihood

Use the Gibbs Sampler to Integrate

If we had "complete data" then we would draw $\boldsymbol{\theta}$ from the condition distribution of $\boldsymbol{\theta} \mid \mathbf{Y}$ class for sampling $\boldsymbol{\mu}$ and Σ. Add a step at each iteration to generate the missing data:

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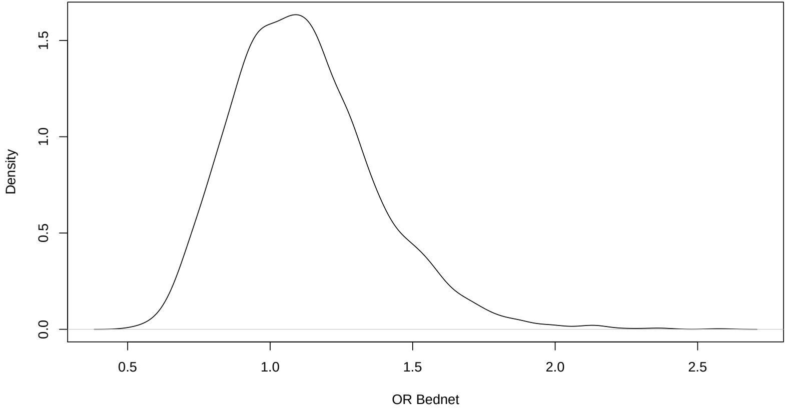
Averaging over the draws of Y_{miss} "integrates" marginalizes over the missing dimensions

JAGS Model

```
model = function() {
  for (i in 1:N) {
  Y[i] ~ dbern(p[i])
  logit(p[i]) <- alpha + beta.age*AGE[i] + beta.bednet*BEDNET[i]
                 +beta.green*GREEN[i] + beta.phc*PHC[i]
}
   # model for missing exposure variable
for (i in 1:N) {
  BEDNET[i] ~ dbern(q) #prior model for whether or not child
                       # sleeps under treated bednet
}
  #uniform prior (uniform) on prob of sleeping under bednet
  q ~ dbeta (1,1)
 #vague priors on regression coefficients
   alpha ~ dnorm(0,0.00000001)
   beta.age ~ dnorm(0,0.00000001)
   beta.bednet ~ dnorm(0,0.00000001)
   beta.green ~ dnorm(0,0.00000001)
   beta.phc ~ dnorm(0,0.00000001)
  # calculate odds ratios of interest
  OR.bednet <- exp(beta.bednet) #OR of malaria for children using bedne
}
```

Posterior Density

```
theta = as.data.frame(sim$BUGSoutput$sims.matrix)
plot(density(theta[,1]), xlab="OR Bednet", main="")
```



JAGS Model

```
model2 = function() {
  for (i in 1:N) {
  Y[i] ~ dbern(p[i])
  logit(p[i]) <- alpha + beta.age*AGE[i] + beta.bednet*BEDNET[i]
                 +beta.green*GREEN[i] + beta.phc*PHC[i]
}
   # model for missing exposure variable
for (i in 1:N) {
  BEDNET[i] ~ dbern(q[i]) #prior model for  bednet use
  logit(q[i]) <- gamma[1] + gamma[2]*PHC[i] #allow prob depend on PHC
}

 #vague priors on regression coefficients
   gamma[1] ~ dnorm(0,0.00000001)
   gamma[2] ~ dnorm(0,0.00000001)
   alpha ~ dnorm(0,0.00000001)
   beta.age ~ dnorm(0,0.00000001)
   beta.bednet ~ dnorm(0,0.00000001)
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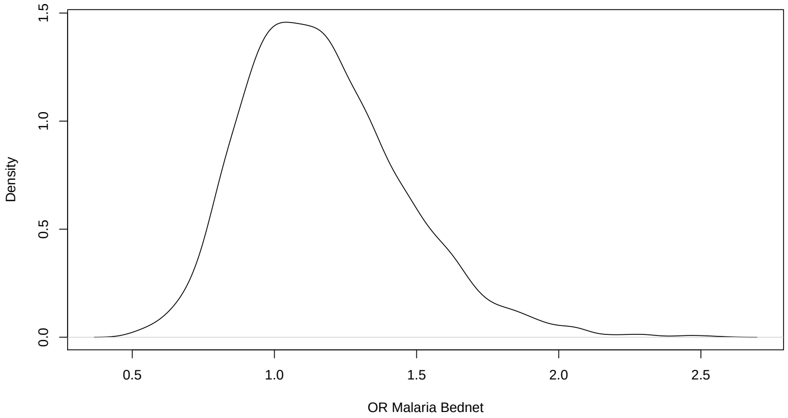
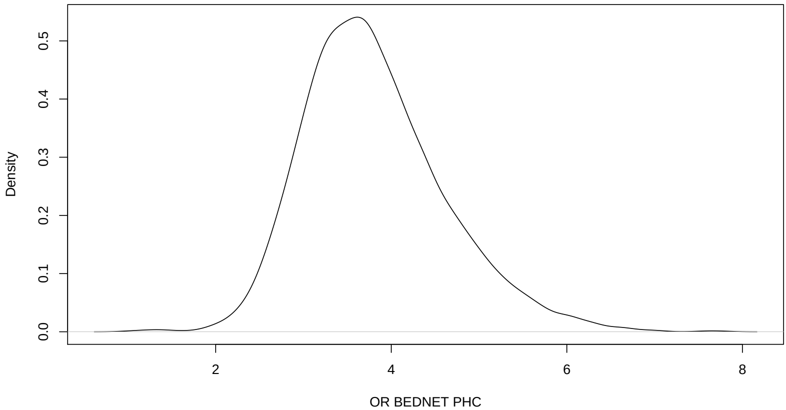
Posterior Density

```
theta.phc = as.data.frame(sim.phc$BUGSoutput$sims.matrix)
plot(density(theta.phc[,1]), xlab="OR Malaria Bednet", main="")
```



Posterior Density

```
plot(density(theta.phc[,"OR.bednet.PHC"]), xlab="OR BEDNET PHC", main="
```



intervals

```
exp(confint(glm(Y ~ . , data=malaria, family=binomial), parm="BEDNET"))

    2.5 %    97.5 %
0.7104643 1.7646674

HPDinterval(as.mcmc(theta))

                   lower        upper
OR.bednet       0.6730938    1.6168561
beta.bednet    -0.3414251    0.5189801
deviance     1564.7933630 1579.4668447
attr(,"Probability")
[1] 0.95

HPDinterval(as.mcmc(theta.phc))

                     lower       upper
OR.bednet         0.6752977    1.742158
OR.bednet.PHC     2.4186453    5.499151
deviance       1524.1955374 1539.457613
attr(,"Probability")
[1] 0.95
```

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- model missingness given variables

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- need more information !

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