三、一元函数的积分学

1. 不定积分

① 求导函数的所有原函数过程：$\int f(x)dx = F(x) + C$.

2. 不定积分的性质

① $[\int f(x)\,dx]' = f(x)$; $\int F'(x)dx = F(x) + C$; $d\int f(x)dx = f(x)dx$.

② $\int kf(x)\,dx = k\int f(x)dx$.

③ $\int[f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$.

3. 不定积分的基本公式

① $\int kdx = kx + C$;

② $\int e^x\,dx = e^x + C$;

③ $\int x^\alpha dx = \frac{1}{\alpha+1}x^{\alpha+1} + C$;

④ $\int a^x dx = \frac{a^x}{\ln a} + C$;

⑤ $\int \frac{1}{x}dx = \ln|x| + C$;

⑥ $\int \cos x\,dx = \sin x + C$;

⑦ $\int \sec x \tan x\,dx = \sec x + C$;

⑧ $\int \sin x\,dx = -\cos x + C$;

⑨ $\int \csc x \cot x\,dx = -\csc x + C$;

⑩ $\int sec^2 x dx = \tan x + C$;

⑪ $\int \frac{1}{1+x^2}dx = \arctan x + C = -\operatorname{arccot} x + C$;

⑫ $\int csc^2 x dx = -\cot x + C$;

⑬ $\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + C = -\arccos x + C$.

4. 不定积分的求解

① 凑微分法：设$F(u)$, $u=\varphi(x)$可导，

则$\int f[\varphi(x)]\varphi'(x)dx$=$\int f[\varphi(x)]d\varphi(x)=F[\varphi(x)]+C$.

② 换元法：

$\sqrt[n]{ax+b}\to t$=$\sqrt[n]{ax+b}$;

$\sqrt{a^2-x^2}\to x$=$a\sin t$;

$\sqrt{a^2+x^2}\to x$=$a\tan t$;

$\sqrt{x^2-a^2}\to x$=$a\sec t$.

③ 分部积分法：$\int u\,dv=uv-\int v\,du$

$\int P_n(x)\,e^x dx$=$\int P_n(x)de^x$;

$\int P_n(x)\sin x\,dx$= $-\int P_n(x)d\cos x$;

$\int P_n(x)\cos x\,dx$=$\int P_n(x)d\sin x$;

$\int P_n(x)\ln x\,dx$=$\int \ln x\,dP_{n+1}(x)$;

$\int P_n(x)\arcsin x\,dx$=$\int \arcsin x\,dP_{n+1}(x)$;

$\int P_n(x)\arctan x\,dx$=$\int \arctan x\,dP_{n+1}(x)$;

$\int e^x\sin x\,dx$=$\int \sin x\,de^x$.

$\int e^x\cos x\,dx$=$\int \cos x\,de^x$.