三、多元函数的微积分学

1. 一阶偏导数

①定义式：对x求偏导$f_x, z_x, \frac{\partial z}{\partial x}$。 $f_x = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$

对y求偏导$f_y, z_y, \frac{\partial z}{\partial y}$。 $f_y = \lim\limits_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$

2. 全微分方程

① 已知$z = f(x,y)$，则 $dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$.

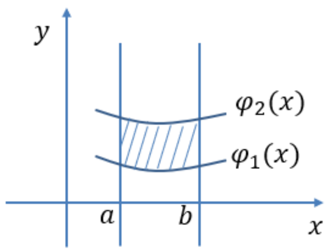
3. 复合函数求偏导（链式法则）

已知z是关于u和v函数，其中，u和v都是关系x和y的函数。

① $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$. ② $\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$.

4. 二重积分

① 直角坐标系下的二重积分

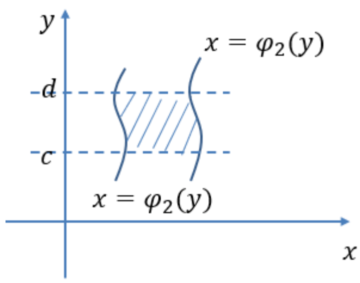


（1）X型：$D: a \le x \le b$，$\varphi_1(x) \le y \le \varphi_2(x)$

$$\therefore \iint_D f(x,y)dxdy = \int_a^b dx \int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)dy$$

（2）Y型：$D: c \le y \le d$，$\varphi_1(y) \le x \le \varphi_2(y)$

$$\therefore \iint_D f(x,y)d\sigma = \int_c^d dy \int_{\varphi_1(y)}^{\varphi_2(y)} f(x,y)dx$$

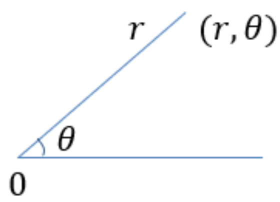


② 极坐标系下的二重积分

$x = r\cos\theta$，$y = r\sin\theta$，$x^2 + y^2 = r^2$

$D: \alpha \le \theta \le \beta$，$r_1 \le r \le r_2$

$$\therefore \iint_D f(x,y)d\sigma = \int_\alpha^\beta d\theta \int_{r_1}^{r_2} f(r\cos\theta, r\sin\theta) r dr$$



5. 三重积分

①直角坐标系下的三重积分：若$a < x < b, c < y < d, z_1 < z < z_2$，则：

$\iiint f(x,y,z)dv = \int_a^b dx \int_c^d dy \int_{z_1}^{z_2} f(x,y,z)dz.$

②柱面坐标系下的三重积分：若$\alpha \leq \theta \leq \beta, r_1 \leq r \leq r_2, z_1 < z < z_2$，则：

$\iiint f(x,y,z)dv = \int_\alpha^\beta d\theta \int_{r_1}^{r_2} rdr \int_{z_1}^{z_2} f(rcos\theta, rsin\theta, z)dz$

6. 重积分的应用

① 二重积分求面积 $\iint_D d\sigma = S_D$

② 三重积分求体积 $\iiint_\Omega dv = V_\Omega$

③ 求空间曲面的面积 $A = \iint_D \sqrt{1 + (\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2}\, dxdy$