多元函数微分

16 年试题

8．设二元函数$z=x\ln y$,则$\dfrac{\partial^2 z}{\partial y\partial x}=$ ________

解析: $\dfrac{\partial z}{\partial y}=\dfrac{x}{y},\dfrac{\partial^2 z}{\partial y\partial x}=\dfrac{1}{y}$

16 年试题

15．设$z=u^v$,而$u=2x+y,v=x$,

求$\left.\dfrac{\partial z}{\partial x}\right|_{\substack{x=1\\y=0}}$ 和$\left.\dfrac{\partial z}{\partial y}\right|_{\substack{x=1\\y=0}}$

15.解：

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial x}=2vu^{v-1}+u^v\ln u$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial y}=vu^{v-1}$$

又∵当$x=1,y=0$时，$u=2,v=1$

$$\therefore \left.\frac{\partial z}{\partial x}\right|_{\substack{x=1\\y=0}}=2+\ln 2,\left.\frac{\partial z}{\partial x}\right|_{\substack{x=1\\y=0}}=1$$

15 年试题

19．设二元函数$z=f(x,y)=x^y\ln x(x>0,x\neq 1)$，平面区域

$D=\{(x,y)|2\leq x\leq e,-1\leq y\leq 1\}$。

（1）求全微分dz；

（2）求$\iint\limits_D f(x,y)d\sigma$

19．解：（1）$\because$

$$\frac{\partial z}{\partial x}=x^{y-1}+yx^{y-1}\ln x=x^{y-1}(1+y\ln x),\frac{\partial z}{\partial y}=x^y\ln^2 x$$

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=x^{y-1}(1+y\ln x)dx+x^y\ln^2 xdy$$

（2）$\iint\limits_D f(x,y)d\theta=\int_2^e dx\int_{-1}^1 x^y\ln xdy$

$$=\int_2^e (x^y\big|_{-1}^1)dx$$

（10 分）

$$=\int_2^e (x-\frac{1}{x})dx=(\frac{1}{2}x^2-\ln x)\Big|_2^e=\frac{1}{2}e^2+\ln 2-3。$$

14 年试题

9，设二元函数 $z=\ln(xy)$，

则 $\dfrac{\partial^2 z}{\partial x\partial y}=$ ________________ 。

14 年试题

16．已知三元函数 $f(u,v,w)$ 具有连续偏导数，且 $f_v-f_w\neq 0$。若二元函数 $z=z(x,y)$ 是由三元方程 $f(x-y,y-z,z-x)=0$ 所确定的隐函数，计算 $\dfrac{\partial z}{\partial x}+\dfrac{\partial z}{\partial y}$

16．解：设

$F(x,y,z)=f(x-y,y-z,z-x)=f(u,v,w)$，

其中 $u=x-y,v=y-z,w=z-x$

则 $F_x=f_u-f_w,F_y=-f_u+f_v,F_z=-f_v+f_w$，

（2 分）

$$\therefore\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=\frac{f_u-f_w}{f_v-f_w},\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=\frac{-f_u+f_v}{f_v-f_w}$$

故

$$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=\frac{f_u-f_w-f_u+f_v}{f_v-f_w}=1$$

13 年试题

17．求二元函数 $z=\int_0^{xy}e^{-t^2}dt$ 的全微分 dz 及二阶偏导数 $\frac{\partial^2 z}{\partial x\partial y}$。

17．因为 $\frac{\partial z}{\partial x}=ye^{-x^2y^2},\frac{\partial z}{\partial y}=xe^{-x^2y^2}$，

所以

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=ye^{-x^2y^2}dx+xe^{-x^2y^2}dy,$$

$$\frac{\partial^2 z}{\partial x\partial y}=e^{-x^2y^2}+ye^{-x^2y^2}(-2x^2y)=e^{-x^2y^2}(1-2x^2y^2)$$

12 年试题

10．设函数 $f(u)$ 可微，且 $f'(0)=\frac{1}{2}$，

则 $z=f(4x^2-y^2)$ 在点 $(1,2)$ 处的全微分

$dz\big|_{(1,2)}=$

解： 设$u=4x^2-y^2$， 则$z=f(u)$

$$\frac{\partial z}{\partial x}=f'(u)\cdot\frac{\partial u}{\partial x}=8xf'(u),\left.\frac{\partial z}{\partial x}\right|_{(1,2)}=8f'(0)=8\times\frac{1}{2}=4$$

$$\frac{\partial z}{\partial y}=f'(u)\cdot\frac{\partial u}{\partial y}=-2yf'(u),\left.\frac{\partial z}{\partial y}\right|_{(1,2)}=-4f'(0)=-4\times\frac{1}{2}=-2$$

$$\left.dz\right|_{(1,2)}=\left.\frac{\partial z}{\partial x}\right|_{(1,2)}dx+\left.\frac{\partial z}{\partial y}\right|_{(1,2)}dy=4dx-2dy$$

12 年试题

17．已知二元函数$z=x(2y+1)^x$，求$\left.\frac{\partial^2 z}{\partial y\partial x}\right|_{\substack{x=1\\y=1}}$

17．解：$\because\frac{\partial z}{\partial y}=2x^2(2y+1)^{x-1}$，

（2 分）

$\therefore$

$$\frac{\partial^2 z}{\partial y\partial x}=4x(2y+1)^{x-1}+2x^2(2y+1)^{x-1}\ln(2y+1)$$

（4 分）

故$\left.\frac{\partial^2 z}{\partial y\partial x}\right|_{\substack{x=1\\y=1}}=4+2\ln 3$

11 年试题

17．已知二元函数 $z=(3x+y)^{2y}$，

求偏导数 $\frac{\partial z}{\partial x}$ 及 $\frac{\partial z}{\partial y}$。

17．解法一：设 $u=3x+y,\ v=2y,$ 则 $z=u^{v}$

$$\therefore \frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial x}=vu^{v-1}\cdot 3+u^{v}\ln u\cdot 0$$

$$=3vu^{v-1}=6y(3x+y)^{2y-1}\ ;$$

（3 分）

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial y}$$

$$=vu^{v-1}\cdot 1+u^{v}\ln u\cdot 2$$

$$=vu^{v-1}+2u^{v}\ln u$$

$$=2y(3x+y)^{2y-1}+2(3x+y)^{2y}\ln(3x+y)$$

$$=(3x+y)^{2y}\left[\frac{2y}{3x+y}+2\ln(3x+y)\right]$$

（6 分）

解法二：　$\Theta \ln z=2y\ln(3x+y)$，

设 $F(x,y,z)=2y\ln(3x+y)-\ln z$，则

$$F'_x(x,y,z)=\frac{6y}{3x+y},F'_y(x,y,z)=2y\ln(3x+y)-\frac{2y}{3x+y}$$

，$F'_z(x,y,z)=-\frac{1}{z}$。

（4 分）

$$\therefore\frac{\partial z}{\partial x}=-\frac{F'_x(x,y,z)}{F'_z(x,y,z)}$$

$$=\frac{6yz}{3x+y}(3x+y)^{2y}\text{ ；}$$

$$=6y(3x+y)^{2y-1}$$

$$\frac{\partial z}{\partial y}=-\frac{F'_y(x,y,z)}{F'_z(x,y,z)}=z\left[2\ln(3x+y)+\frac{2y}{3x+y}\right]$$

$$=(3x+y)^{2y}\left[2\ln(3x+y)+\frac{2y}{3x+y}\right]$$

11 年试题

9．若二元函数 $z=\frac{4x-3y}{y^2}(y\neq 0)$，

则 $\frac{\partial^2 z}{\partial x\partial y}-\frac{\partial^2 z}{\partial y\partial x}$=________。

11 年试题

5．设 $f(x,y)=\begin{cases}\dfrac{\sin(2x^2-y^2)}{y}, & y\neq 0\\ 0, & y=0\end{cases}$，

则 $f_y'(0,0)$=

A．-1　　B．0　　C．1　　D．2

解：$f_y'(0,0)=\lim\limits_{\vartriangle y\to 0}\dfrac{f(0,\ 0+\vartriangle y)-f(0,\ 0)}{\vartriangle y}$

$$=\lim_{\vartriangle y\to 0}\frac{\dfrac{\sin(2\times 0^2-(0+\vartriangle y)^2)}{\vartriangle y}-0}{\vartriangle y}$$

$$=\lim_{\vartriangle y\to 0}\frac{\sin(-\vartriangle y^2)}{\vartriangle y^2}=-1$$

10 年试题

17．已知隐函数 $z=f(x,y)$ 由方程

$x^z-xy^2+z^3=1$ 所确定，求 $\dfrac{\partial z}{\partial x}$ 和 $\dfrac{\partial z}{\partial y}$。

17．解：设 $F(x,y,z)=x^z-xy^2+z^3-1$，则

$F'_x = zx^{z-1} - y^2, F'_y = -2xy, F'_z = x^z \ln x + 3z^2$

所 以 $\frac{\partial z}{\partial x} = -\frac{F'_x}{F'_z} = \frac{y^2 - zx^{z-1}}{x^z \ln x + 3z^2}$ ，

$\frac{\partial x}{\partial y} = -\frac{F'_y}{F'_z} = \frac{2xy}{x^z \ln x + 3z^2}$

10 年试题

5．设 $f(x+y,xy) = x^2 + y^2 - xy$，则 $\frac{\partial f(x,y)}{\partial y} =$

A． $2y - x$ B. -1

C. $2x - y$ D. -3

解：$f(x+y,xy) = x^2 + y^2 - xy = (x+y)^2 - 3xy$

$f(x,y) = x^2 - 3y$

$\frac{\partial f(x,y)}{\partial y} = -3$

09 年试题

9．已知二元函数 $z = f(x,y)$ 的全微分

$dz = y^2 dx + 2xydy$，则 $\frac{\partial^2 z}{\partial x \partial y} =$ ＿＿。

解：由 $dz = y^2 dx + 2xydy$ 知 $\frac{\partial z}{\partial x} = y^2$,

$$\frac{\partial^2 z}{\partial x \partial y}=2y$$

09 年试题

16．设隐函数$z=f(x,y)$由方程$x^y+z^3+xz=0$所确定，求$\frac{\partial z}{\partial x}$及$\frac{\partial z}{\partial y}$。

16．【解析】设$F(x,y,z)=x^y+z^3+xz$, 则

$$F_x'=yx^{y-1}+z，\ F_y'=x^y\ln x，\ F_z'=3z^2+x.$$

所以

$$\frac{\partial z}{\partial x}=-\frac{F_x'}{F_z'}=-\frac{yx^{y-1}+z}{3z^2+x},\frac{\partial z}{\partial y}=-\frac{F_y'}{F_z'}=-\frac{x^y\ln x}{3z^2+x}.$$

08 试题

16. 设方程$e^{-xy}-2z+e^z=0$确定隐函数$z=z(x,y)$，求$\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$。

16.【解析】设$F(x,y,z)=e^{-xy}-2z+e^z$,

则

$$\frac{\partial F}{\partial x}=-ye^{-xy},\frac{\partial F}{\partial y}=-xe^{-xy},\frac{\partial F}{\partial z}=-2+e^z,$$

$$\therefore \frac{\partial z}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}=\frac{ye^{-xy}}{e^{z}-2},\frac{\partial z}{\partial y}=-\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}=\frac{xe^{-xy}}{e^{z}-2}.$$

08 试题

9. 设$u=e^{x}\cos y,v=e^{x}\sin y$，

则$\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=$____________。

07 年试题

5. 设 $f(x,y)=\begin{cases}\frac{x^{3}-y^{3}}{x^{2}+y^{2}},(x,y)\neq(0,0),\\ 0, \qquad (x,y)=(0,0).\end{cases}$

则 $f_x'(0,0)$

A. 等于 1　　　　B. 等于-1

C. 等于 0　　　　D. 不存在

07 年试题

17. 设 $f(x+y,x-y)=\arctan\frac{x+y}{x-y}$,

计算$y\dfrac{\partial f(x,y)}{\partial x}-x\dfrac{\partial f(x,y)}{\partial y}$的值。

17.【解析】由题意知$f(x,y)=\arctan\dfrac{x}{y}$，

$$\therefore \frac{\partial f(x,y)}{\partial x}=\frac{1}{1+\dfrac{x^2}{y^2}}\cdot\frac{1}{y}=\frac{y}{x^2+y^2},$$

$$\frac{\partial f(x,y)}{\partial x}=\frac{1}{1+\dfrac{x^2}{y^2}}\cdot\left(-\frac{x}{y^2}\right)=\frac{-x}{x^2+y^2}.$$

故$y\dfrac{\partial f(x,y)}{\partial x}-x\dfrac{\partial f(x,y)}{\partial y}=\dfrac{y^2}{x^2+y^2}+\dfrac{x^2}{x^2+y^2}=1.$

07 年试题

10. 设$u=\ln(x^2+y^2+z^2)$，

则全微分$du=$________________。

06 年试题

4．设$z=\ln(xy)$，则$dz=$（　　）

A. $\dfrac{1}{x}dx+\dfrac{1}{y}dy$　　　B. $\dfrac{1}{y}dx+\dfrac{1}{x}dy$

C. $\dfrac{dx+dy}{xy}$　　　D. $ydx+xdy$

06 年试题

17．设函数$z=x\arctan\dfrac{x}{y}$，求$\left.\dfrac{\partial^2 x}{\partial y\partial x}\right|_{\substack{x=1\\y=1}}$。

17.【解析】$\because \dfrac{\partial z}{\partial y}=x\dfrac{1}{1+(\dfrac{x}{y})^2}(-\dfrac{x}{y^2})$

$$=-\frac{x^2}{x^2+y^2},$$

$$\therefore \frac{\partial^2 z}{\partial y\partial x}=\frac{-2x(x^2+y^2)+x^2\cdot 2x}{(x^2+y^2)}$$

$$=\frac{-2xy^2}{(x^2+y^2)^2}$$

$$\therefore \left.\frac{\partial^2 z}{\partial y\partial x}\right|_{\substack{x=1\\y=1}}=\frac{-2}{4}=-\frac{1}{2}.$$

05 年试题

20. 已知$z=\sin(xy)+xe^{-xy}$，求全微分 dz。

20.【解析】$\dfrac{\partial z}{\partial x}=y\cos(xy)+e^{-xy}-xye^{-xy}$

$$\frac{\partial z}{\partial y}=x\cos(xy)-x^2e^{-xy}$$

故

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy$$

$$=\left[y\cos(xy)+e^{-xy}\left(1-xy\right)\right]dx+\left[x\cos(xy)-x^2e^{-xy}\right]dy$$

05 年试题

5. 已知$u=(xy)^x$，则$\frac{\partial u}{\partial y}$=（　　）

A. $x^2(xy)^{x-1}$　　B. $x^2\ln(xy)$

C. $x(xy)^{x-1}$　　D. $y^2\ln(xy)$