ID: 51

Question 1

In this question, the universal set is $\mathcal{U} = \{x \in \mathbb{N} \mid x \leq 19\}$.

Let:

$A = \{x \in \mathbb{N} \mid 4 \leq x \leq 17\}$.

$B = \{x \in \mathbb{N} \mid x > 14\}$.

$C = \{x \in \mathbb{N} \mid x \text{ divisible by } 3\}$.

Enumerate the following sets:

a) $A \cup C =$ ________

b) $B^c \cap C =$ ________

c) $A \oplus B^c =$ ________

d) $(A \cap B) \cup C =$ ________

6 marks

Question 2

Prove that

$$15n^2 + n$$

is even for any integer n.

8 marks

Question 3

How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 20$$

(An integer solution to an equation is a solution in which the unknown must have an integer value.)

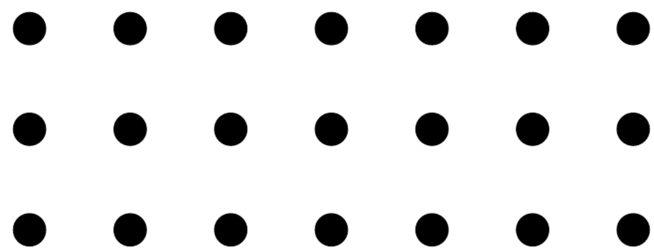
1. where $x_i \geq 0$ for each x_i?
2. where $x_i > 0$ for each x_i?

3. where $x_i \geq 4$ for each x_i?

6 marks

Question 4

Consider the following diagram, consisting of three rows of seven dots.



How many

1. right-angled triangles (with a horizontal base)
2. rectangles

can be drawn using the dots as vertices (corners).

10 marks

Question 5

Consider the relation R defined on the set $A = \{a, b, c, d, e, f\}$ where

$$R = \{(a, a), (a, d), (b, c), (c, b)\}$$

1. Draw the digraph for this relation.
2. Explain why this relation is not symmetric.
3. Determine the symmetric closure of this relation.
4. Determine whether this relation is irreflexive or not. Explain your answer.

10 marks

Question 6

Determine if the following functions are one-to-one and/or onto and use these to determine whether the function is a bijection. Assume both functions are such that $f : \mathbb{R} \to \mathbb{R}$ where

1. $f(x) = 2x + 2$
2. $f(x) = 2x^2 - 4$

10 marks

Question 7

Consider the second-order homogeneous recurrence relation

$$a_n = 9a_{n-1} - 18a_{n-2}$$

with initial conditions $a_0 = -2$ and $a_1 = 4.$

1. Find the next three terms of the sequence.
2. Find the general solution.
3. Find the unique solution with the given initial conditions.

10 marks

Question 8

Use the laws of logic to show that

$$(\neg a \vee c) \wedge (\neg c \vee q) \wedge a \implies q$$

5 marks

Question 9

How many shortest lattice paths start at $(1, 3)$ and
(i) end at $(10, 13)$? ________
(ii) end at $(10, 13)$ and pass through $(6, 4)$? ________
(iii) end at $(10, 13)$ and avoid $(6, 4)$? ________

6 marks

Question 10

Consider the python code

1. $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
2. $R = \{(x, y)$ for x in A for y in A if $x \geq y\}$
3.
4. **print (R.issubset({(a, b) for a in A for b in A}))**

What is the output of line 4? Justify your answer.

4 marks

Question 11

Let

$$A = \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14\} \ .$$

Determine the truth value of each of the following statements, and explain why this is the case.

1. $\forall x \in A, x + 1 \leq 14$
2. $\forall x \in A, \forall y \in A, x + y \leq 41$
3. $\exists x \in A, x^2 + 1 > 198$
4. $\exists x \in A, \exists y \in A, x^2 + y = 198$

10 marks

Question 12

A graph, G, has adjacency matrix,

$$A = \begin{pmatrix} 2 & 1 & 2 & 2 & 0 \\ 0 & 1 & 2 & 0 & 2 \\ 1 & 0 & 2 & 2 & 2 \\ 0 & 1 & 2 & 0 & 1 \\ 1 & 2 & 0 & 0 & 2 \end{pmatrix}$$

1. Is G a simple graph? Explain your answer.
2. State the degree sequence of G.
3. How many edges does G have?

6 marks

Question 13

Expand fully the binomial expansion $(2x + 7y)^9$. Determine the coefficient of the term which has a factor of x^3y^6.

5 marks

Question 14

Construct a truth table for the following compound proposition:

$$((\neg q \to \neg p) \land (q \lor \neg p)) \lor (\neg p \land \neg q).$$

4 marks

ID: 52

Question 1

Let

$$A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\} \ .$$

Determine the truth value of each of the following statements, and explain why this is the case.

1. $\forall x \in A, x + 3 \leq 12$
2. $\forall x \in A, \forall y \in A, x + y \leq 13$
3. $\exists x \in A, x^2 + 1 > 71$
4. $\exists x \in A, \exists y \in A, x^2 + y = 71$

10 marks

Question 2

Use the laws of logic to show that

$$(\neg \mathrm{a} \vee \mathrm{c}) \wedge (\neg \mathrm{c} \vee \mathrm{x}) \wedge \mathrm{a} \implies \mathrm{x}$$

5 marks

Question 3

Expand fully the binomial expansion $(4x + 3y)^9$. Determine the coefficient of the term which has a factor of x^7y^2.

5 marks

Question 4

In this question, the universal set is $\mathcal{U} = \{x \in \mathbb{N} \mid x \leq 14\}$.

Let:

$A = \{x \in \mathbb{N} \mid 3 \leq x \leq 12\}$.

$B = \{x \in \mathbb{N} \mid x > 9\}$.

$C = \{x \in \mathbb{N} \mid x \text{ divisible by } 5\}$.

Enumerate the following sets:

a) $A \cup C =$ ________

b) $B^c \cap C =$ ________

c) $A \cap B^c =$ ________

d) $(A \setminus B) \cap C =$ ________

6 marks

Question 5

Construct a truth table for the following compound proposition:

$$((\neg p \vee \neg q) \vee (\neg q \wedge \neg p)) \wedge (\neg p \vee \neg q).$$

4 marks

Question 6

A graph, G, has adjacency matrix,

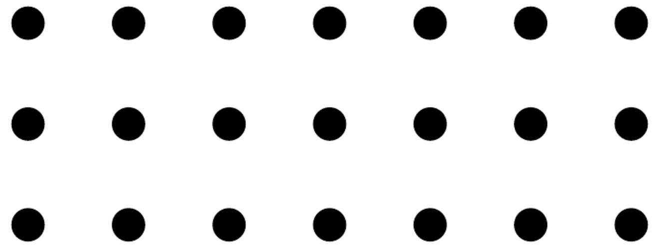
$$A = \begin{pmatrix} 2 & 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 2 & 0 & 2 & 2 & 1 \\ 1 & 0 & 0 & 2 & 2 \\ 0 & 2 & 2 & 2 & 1 \end{pmatrix}$$

1. Is G a simple graph? Explain your answer.
2. State the degree sequence of G.
3. How many edges does G have?

6 marks

Question 7

Consider the following diagram, consisting of three rows of seven dots.



How many

1. triangles (with a horizontal base)
2. squares

can be drawn using the dots as vertices (corners).

10 marks

Question 8

How many shortest lattice paths start at $(1, 0)$ and
(i) end at $(14, 13)$? ________
(ii) end at $(14, 13)$ and pass through $(5, 5)$? ________
(iii) end at $(14, 13)$ and avoid $(5, 5)$? ________

6 marks

Question 9

How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 20$$

(An integer solution to an equation is a solution in which the unknown must have an integer value.)

1. where $x_i \geq 0$ for each x_i?
2. where $x_i > 0$ for each x_i?
3. where $x_i \geq 4$ for each x_i?

6 marks

Question 10

Determine if the following functions are one-to-one and/or onto and use these to determine whether the function is a bijection. Assume both functions are such that $f : \mathbb{R} \to \mathbb{R}$ where

1. $f(x) = 3x + 6$

2. $f(x) = 2x^2 - 7$

10 marks

Question 11

Prove that

$$17n^2 + n$$

is even for any integer n.

8 marks

Question 12

Consider the relation R defined on the set $A = \{a, b, c, d, e, f\}$ where

$$R = \{(a,a), (a,d), (b,c), (c,b)\}$$

1. Draw the digraph for this relation.
2. Explain why this relation is not reflexive.
3. Determine the reflexive closure of this relation.
4. Determine whether this relation is antisymmetric or not. Explain your answer.

10 marks

Question 13

Consider the python code

1. $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
2. $R = \{(x, y)$ for x in A for y in A if $x \geq y\}$
3.
4. **print (R.issubset({(a, b) for a in A for b in A}))**

What is the output of line 4? Justify your answer.

4 marks

Question 14

Consider the second-order homogeneous recurrence relation

$$a_n = 6a_{n-1} - 8a_{n-2}$$

with initial conditions $a_0 = 4$ and $a_1 = 4$.

1. Find the next three terms of the sequence.
2. Find the general solution.
3. Find the unique solution with the given initial conditions.

10 marks

ID: 53

Question 1

Prove that

$$15n^2 + n$$

is even for any integer n.

8 marks

Question 2

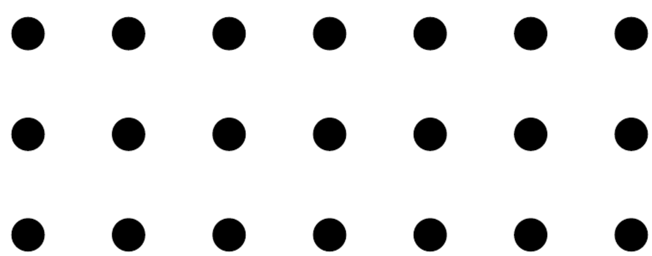
Determine if the following functions are one-to-one and/or onto and use these to determine whether the function is a bijection. Assume both functions are such that $f : \mathbb{R} \to \mathbb{R}$ where

1. $f(x) = 3x + 2$
2. $f(x) = 3x^2 - 5$

10 marks

Question 3

Consider the following diagram, consisting of three rows of seven dots.



How many

1. triangles (with a horizontal base)
2. squares

can be drawn using the dots as vertices (corners).

10 marks

Question 4

In this question, the universal set is $\mathcal{U} = \{x \in \mathbb{N} \mid x \leq 18\}$.

Let:

$A = \{x \in \mathbb{N} \mid 6 \leq x \leq 16\}$.

$B = \{x \in \mathbb{N} \mid x > 13\}$.

$C = \{x \in \mathbb{N} \mid x \text{ divisible by } 6\}$.

Enumerate the following sets:

a) $A \setminus C =$ ________

b) $B^c \cup C =$ ________

c) $A \oplus B^c =$ ________

d) $(A \oplus B) \oplus C =$ ________

6 marks

Question 5

Consider the second-order homogeneous recurrence relation

$$a_n = 11a_{n-1} - 30a_{n-2}$$

with initial conditions $a_0 = 2$ and $a_1 = 1$.

1. Find the next three terms of the sequence.
2. Find the general solution.
3. Find the unique solution with the given initial conditions.

10 marks

Question 6

Use the laws of logic to show that

$$(\neg a \lor r) \land (\neg r \lor x) \land a \implies x$$

5 marks

Question 7

A graph, G, has adjacency matrix,

$$A = \begin{pmatrix} 0 & 2 & 0 & 2 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 1 & 0 \\ 2 & 0 & 2 & 2 & 2 \\ 1 & 2 & 2 & 0 & 0 \end{pmatrix}$$

1. Is G a simple graph? Explain your answer.
2. State the degree sequence of G.
3. How many edges does G have?

6 marks

Question 8

How many shortest lattice paths start at $(4, 2)$ and
(i) end at $(10, 14)$? ________
(ii) end at $(10, 14)$ and pass through $(9, 7)$? ________
(iii) end at $(10, 14)$ and avoid $(9, 7)$? ________

6 marks

Question 9

How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 23$$

(An integer solution to an equation is a solution in which the unknown must have an integer value.)

1. where $x_i \geq 0$ for each x_i?
2. where $x_i > 1$ for each x_i?
3. where $x_i \geq 5$ for each x_i?

6 marks

Question 10

Expand fully the binomial expansion $(3x + 6y)^7$. Determine the coefficient of the term which has a factor of x^2y^5.

5 marks

Question 11

Let

$$A = \{4, 5, 6, 7, 8, 9, 10, 11\}\ .$$

Determine the truth value of each of the following statements, and explain why this is the case.

1. $\forall x \in A, x + 3 \leq 12$
2. $\forall x \in A, \forall y \in A, x + y \leq 30$
3. $\exists x \in A, x^2 + 4 > 130$
4. $\exists x \in A, \exists y \in A, x^2 + y = 130$

10 marks

Question 12

Consider the relation R defined on the set $A = \{a, b, c, d, e, f\}$ where

$$R = \{(a, a), (a, d), (b, c), (c, b)\}$$

1. Draw the digraph for this relation.
2. Explain why this relation is not symmetric.
3. Determine the symmetric closure of this relation.
4. Determine whether this relation is antisymmetric or not. Explain your answer.

10 marks

Question 13

Construct a truth table for the following compound proposition:

$$((p \to q) \to (\neg q \vee q)) \vee (\neg p \wedge q).$$

4 marks

Question 14

Consider the python code

1. $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
2. $R = \{(x, y)$ for x in A for y in A if $x == y**2\}$
3.
4. **print (R.issubset({(a, b) for a in A for b in A}))**

What is the output of line 4? Justify your answer.

4 marks

ID: 54

Question 1

Consider the second-order homogeneous recurrence relation

$$a_n = 9a_{n-1} - 20a_{n-2}$$

with initial conditions $a_0 = 5$ and $a_1 = 3$.

1. Find the next three terms of the sequence.
2. Find the general solution.
3. Find the unique solution with the given initial conditions.

10 marks

Question 2

Consider the relation R defined on the set $A = \{a, b, c, d, e, f\}$ where

$$R = \{(a,a),(a,d),(b,c),(c,b)\}$$

1. Draw the digraph for this relation.
2. Explain why this relation is not reflexive.
3. Determine the reflexive closure of this relation.
4. Determine whether this relation is irreflexive or not. Explain your answer.

10 marks

Question 3

In this question, the universal set is $\mathcal{U} = \{x \in \mathbb{N} \mid x \leq 24\}$.

Let:

$A = \{x \in \mathbb{N} \mid 3 \leq x \leq 22\}$.

$B = \{x \in \mathbb{N} \mid x > 19\}$.

$C = \{x \in \mathbb{N} \mid x \text{ divisible by } 3\}$.

Enumerate the following sets:

a) $A \cup C =$ ________

b) $B^c \setminus C =$ _______

c) $A \setminus B^c =$ _______

d) $(A \cup B) \oplus C =$ _______

6 marks

Question 4

A graph, G, has adjacency matrix,

$$A = \begin{pmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 & 2 \\ 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 0 & 2 \end{pmatrix}$$

1. Is G a simple graph? Explain your answer.
2. State the degree sequence of G.
3. How many edges does G have?

6 marks

Question 5

Construct a truth table for the following compound proposition:

$$((q \to \neg p) \to (\neg q \to \neg p)) \to (\neg q \wedge \neg p).$$

4 marks

Question 6

Consider the python code

1. $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
2. $R = \{(x, y)$ for x in A for y in A if $x \leq y\}$
3.
4. **print (R.issubset({(a, b) for a in A for b in A}))**

What is the output of line 4? Justify your answer.

4 marks

Question 7

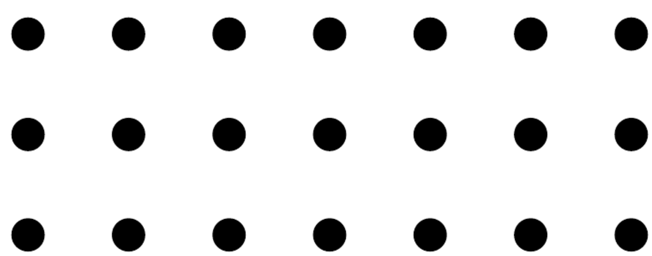
Use the laws of logic to show that

$$(\neg p \vee z) \wedge (\neg z \vee q) \wedge p \implies q$$

5 marks

Question 8

Consider the following diagram, consisting of three rows of seven dots.



How many

1. right-angled triangles (with a horizontal base)
2. rectangles

can be drawn using the dots as vertices (corners).

10 marks

Question 9

Expand fully the binomial expansion $(-3x + 4y)^5$. Determine the coefficient of the term which has a factor of x^2y^3.

5 marks

Question 10

How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 16$$

(An integer solution to an equation is a solution in which the unknown must have an integer value.)

1. where $x_i \geq 0$ for each x_i?
2. where $x_i > 1$ for each x_i?
3. where $x_i \geq 3$ for each x_i?

6 marks

Question 11

Determine if the following functions are one-to-one and/or onto and use these to determine whether the function is a bijection. Assume both functions are such that $f : \mathbb{R} \to \mathbb{R}$ where

1. $f(x) = -2x + 5$
2. $f(x) = 2x^2 - 3$

10 marks

Question 12

Let

$$A = \{4, 5, 6, 7, 8, 9, 10, 11, 12\} \ .$$

Determine the truth value of each of the following statements, and explain why this is the case.

1. $\forall x \in A, x + 2 \leq 13$
2. $\forall x \in A, \forall y \in A, x + y \leq 18$
3. $\exists x \in A, x^2 + 3 > 148$
4. $\exists x \in A, \exists y \in A, x^2 + y = 148$

10 marks

Question 13

How many shortest lattice paths start at $(3,4)$ and
(i) end at $(12,12)$? ________
(ii) end at $(12,12)$ and pass through $(11,8)$? ________
(iii) end at $(12,12)$ and avoid $(11,8)$? ________

6 marks

Question 14

Prove that

$$5n^2+n$$

is even for any integer n.

8 marks

ID: 55

Question 1

Use the laws of logic to show that

$$(\neg a \lor z) \land (\neg z \lor x) \land a \implies x$$

5 marks

Question 2

Construct a truth table for the following compound proposition:

$$((q \to p) \to (\neg q \land \neg p)) \to (q \to p).$$

4 marks

Question 3

Consider the second-order homogeneous recurrence relation

$$a_n = 5a_{n-1} - 6a_{n-2}$$

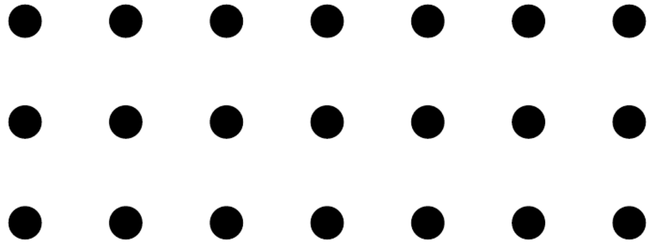
with initial conditions $a_0 = 5$ and $a_1 = 4$.

1. Find the next three terms of the sequence.
2. Find the general solution.
3. Find the unique solution with the given initial conditions.

10 marks

Question 4

Consider the following diagram, consisting of three rows of seven dots.



How many

1. right-angled triangles (with a horizontal base)
2. squares

can be drawn using the dots as vertices (corners).

10 marks

Question 5

A graph, G, has adjacency matrix,

$$A = \begin{pmatrix} 2 & 2 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 2 & 2 \\ 2 & 1 & 2 & 2 & 1 \\ 2 & 2 & 2 & 0 & 2 \end{pmatrix}$$

1. Is G a simple graph? Explain your answer.
2. State the degree sequence of G.
3. How many edges does G have?

6 marks

Question 6

Determine if the following functions are one-to-one and/or onto and use these to determine whether the function is a bijection. Assume both functions are such that $f : \mathbb{R} \to \mathbb{R}$ where

1. $f(x) = -3x + 5$
2. $f(x) = -3x^2 - 3$

10 marks

Question 7

In this question, the universal set is $\mathcal{U} = \{x \in \mathbb{N} \mid x \leq 20\}$.

Let:

$A = \{x \in \mathbb{N} \mid 7 \leq x \leq 18\}$.

$B = \{x \in \mathbb{N} \mid x > 15\}$.

$C = \{x \in \mathbb{N} \mid x \text{ divisible by } 5\}$.

Enumerate the following sets:

a) $A \oplus C =$ ________

b) $B^c \oplus C =$ ________

c) $A \cap B^c =$ ________

d) $(A \setminus B) \cup C =$ ________

6 marks

Question 8

Let

$$A = \{5, 6, 7, 8, 9, 10, 11, 12, 13\} \ .$$

Determine the truth value of each of the following statements, and explain why this is the case.

1. $\exists x \in A, x + 3 \leq 10$
2. $\exists x \in A, \exists y \in A, x + y \leq 33$
3. $\forall x \in A, x^2 + 1 > 171$
4. $\forall x \in A, \forall y \in A, x^2 + y = 171$

10 marks

Question 9

How many shortest lattice paths start at $(2, 5)$ and
(i) end at $(14, 15)$? ________
(ii) end at $(14, 15)$ and pass through $(10, 14)$? ________
(iii) end at $(14, 15)$ and avoid $(10, 14)$? ________

6 marks

Question 10

How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 16$$

(An integer solution to an equation is a solution in which the unknown must have an integer value.)

1. where $x_i \geq 0$ for each x_i?
2. where $x_i > 0$ for each x_i?
3. where $x_i \geq 3$ for each x_i?

6 marks

Question 11

Consider the python code

1. $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
2. $R = \{(x, y)$ for x in A for y in A if $x == y**2\}$
3.
4. **print (R.issubset({(a, b) for a in A for b in A}))**

What is the output of line 4? Justify your answer.

4 marks

Question 12

Expand fully the binomial expansion $(3x + 2y)^9$. Determine the coefficient of the term which has a factor of $x^8 y^1$.

5 marks

Question 13

Consider the relation R defined on the set $A = \{a, b, c, d, e, f\}$ where

$$R = \{(a, a), (a, d), (b, c), (c, b)\}$$

1. Draw the digraph for this relation.

2. Explain why this relation is not symmetric.
3. Determine the symmetric closure of this relation.
4. Determine whether this relation is asymmetric or not. Explain your answer.

10 marks

Question 14

Prove that

$$21n^2 + n$$

is even for any integer n.

8 marks

ID: 56

Question 1

Prove that

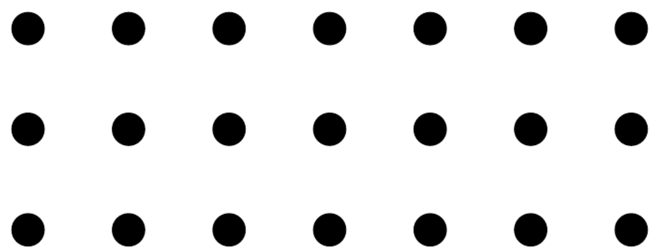
$$15n^2 + n$$

is even for any integer n.

8 marks

Question 2

Consider the following diagram, consisting of three rows of seven dots.



How many

1. right-angled triangles (with a horizontal base)
2. squares

can be drawn using the dots as vertices (corners).

10 marks

Question 3

Construct a truth table for the following compound proposition:

$$((q \wedge p) \vee (q \wedge \neg q)) \rightarrow (p \rightarrow \neg q).$$

4 marks

Question 4

Use the laws of logic to show that

$$(\neg w \vee r) \wedge (\neg r \vee x) \wedge w \implies x$$

5 marks

Question 5

Let

$$A = \{2, 3, 4, 5, 6, 7, 8, 9\}\ .$$

Determine the truth value of each of the following statements, and explain why this is the case.

1. $\forall x \in A, x + 2 \leq 15$
2. $\forall x \in A, \forall y \in A, x + y \leq 21$
3. $\exists x \in A, x^2 + 5 > 81$
4. $\exists x \in A, \exists y \in A, x^2 + y = 81$

10 marks

Question 6

Consider the relation R defined on the set $A = \{a, b, c, d, e, f\}$ where

$$R = \{(a, a), (a, d), (b, c), (c, b)\}$$

1. Draw the digraph for this relation.
2. Explain why this relation is not reflexive.
3. Determine the reflexive closure of this relation.
4. Determine whether this relation is antisymmetric or not. Explain your answer.

10 marks

Question 7

How many shortest lattice paths start at $(3, 2)$ and
(i) end at $(14, 15)$? ________
(ii) end at $(14, 15)$ and pass through $(12, 4)$? ________
(iii) end at $(14, 15)$ and avoid $(12, 4)$? ________

6 marks

Question 8

How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 14$$

(An integer solution to an equation is a solution in which the unknown must have an integer value.)

1. where $x_i \geq 0$ for each x_i?
2. where $x_i > 0$ for each x_i?
3. where $x_i \geq 3$ for each x_i?

6 marks

Question 9

A graph, G, has adjacency matrix,

$$A = \begin{pmatrix} 2 & 1 & 0 & 2 & 2 \\ 2 & 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 1 & 1 \\ 2 & 0 & 2 & 2 & 0 \\ 2 & 0 & 0 & 1 & 0 \end{pmatrix}$$

1. Is G a simple graph? Explain your answer.
2. State the degree sequence of G.
3. How many edges does G have?

6 marks

Question 10

Consider the python code

1. $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$
2. $R = \{(x, y)$ for x in A for y in A if $y == x\}$
3.
4. **print (R.issubset({(a, b) for a in A for b in A}))**

What is the output of line 4? Justify your answer.

4 marks

Question 11

Consider the second-order homogeneous recurrence relation

$$a_n = 5a_{n-1} - 4a_{n-2}$$

with initial conditions $a_0 = 5$ and $a_1 = 1$.

1. Find the next three terms of the sequence.
2. Find the general solution.
3. Find the unique solution with the given initial conditions.

10 marks

Question 12

Expand fully the binomial expansion $(3x + 3y)^4$. Determine the coefficient of the term which has a factor of x^2y^2.

5 marks

Question 13

Determine if the following functions are one-to-one and/or onto and use these to determine whether the function is a bijection. Assume both functions are such that $f : \mathbb{R} \to \mathbb{R}$ where

1. $f(x) = -3x + 6$
2. $f(x) = 3x^2 - 4$

10 marks

Question 14

In this question, the universal set is $\mathcal{U} = \{x \in \mathbb{N} \mid x \leq 24\}$.

Let:

$A = \{x \in \mathbb{N} \mid 5 \leq x \leq 22\}$.

$B = \{x \in \mathbb{N} \mid x > 19\}$.

$C = \{x \in \mathbb{N} \mid x \text{ divisible by } 5\}$.

Enumerate the following sets:

a) $A \cap C =$ ________

b) $B^c \cup C =$ ________

c) $A \cap B^c =$ ________

d) $(A \setminus B) \cup C =$ ________

6 marks

ID: 57

Question 1

Consider the second-order homogeneous recurrence relation

$$a_n = 3a_{n-1} - 2a_{n-2}$$

with initial conditions $a_0 = 5$ and $a_1 = 0$.

1. Find the next three terms of the sequence.
2. Find the general solution.
3. Find the unique solution with the given initial conditions.

10 marks

Question 2

Use the laws of logic to show that

$$(\neg \mathrm{w} \vee \mathrm{c}) \wedge (\neg \mathrm{c} \vee \mathrm{q}) \wedge \mathrm{w} \implies \mathrm{q}$$

5 marks

Question 3

Determine if the following functions are one-to-one and/or onto and use these to determine whether the function is a bijection. Assume both functions are such that $f : \mathbb{R} \to \mathbb{R}$ where

1. $f(x) = -2x + 2$
2. $f(x) = -2x^2 - 4$

10 marks

Question 4

How many shortest lattice paths start at $(1, 1)$ and
(i) end at $(10, 10)$? ________
(ii) end at $(10, 10)$ and pass through $(4, 2)$? ________
(iii) end at $(10, 10)$ and avoid $(4, 2)$? ________

6 marks

Question 5

Let

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\} \ .$$

Determine the truth value of each of the following statements, and explain why this is the case.

1. $\exists x \in A, x + 3 \leq 12$
2. $\exists x \in A, \exists y \in A, x + y \leq 17$
3. $\forall x \in A, x^2 + 5 > 125$
4. $\forall x \in A, \forall y \in A, x^2 + y = 125$

10 marks

Question 6

Consider the python code

1. $A = \{0, 1, 2, 3, 4, 5, 6\}$
2. $R = \{(x, y)$ for x in A for y in A if $x \geq y\}$
3.
4. **print (R.issubset({(a, b) for a in A for b in A}))**

What is the output of line 4? Justify your answer.

4 marks

Question 7

How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 15$$

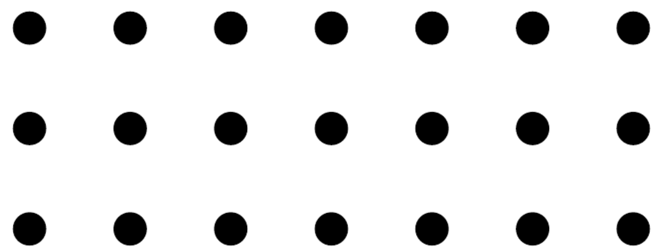
(An integer solution to an equation is a solution in which the unknown must have an integer value.)

1. where $x_i \geq 0$ for each x_i?
2. where $x_i > 0$ for each x_i?
3. where $x_i \geq 3$ for each x_i?

6 marks

Question 8

Consider the following diagram, consisting of three rows of seven dots.



How many

1. right-angled triangles (with a horizontal base)
2. squares

can be drawn using the dots as vertices (corners).

10 marks

Question 9

Prove that

$$3n^2 + n$$

is even for any integer n.

8 marks

Question 10

Consider the relation R defined on the set $A = \{a, b, c, d, e, f\}$ where

$$R = \{(a, a), (a, d), (b, c), (c, b)\}$$

1. Draw the digraph for this relation.
2. Explain why this relation is not reflexive.
3. Determine the reflexive closure of this relation.
4. Determine whether this relation is irreflexive or not. Explain your answer.

10 marks

Question 11

In this question, the universal set is $\mathcal{U} = \{x \in \mathbb{N} \mid x \leq 20\}$.

Let:

$A = \{x \in \mathbb{N} \mid 6 \leq x \leq 18\}$.

$B = \{x \in \mathbb{N} \mid x > 15\}$.

$C = \{x \in \mathbb{N} \mid x \text{ divisible by } 3\}$.

Enumerate the following sets:

a) $A \cap C =$ ________

b) $B^c \cap C =$ ________

c) $A \cup B^c =$ ________

d) $(A \oplus B) \setminus C =$ ________

6 marks

Question 12

Expand fully the binomial expansion $(3x + 2y)^6$. Determine the coefficient of the term which has a factor of x^5y^1.

5 marks

Question 13

Construct a truth table for the following compound proposition:

$$((q \to \neg p) \to (q \wedge \neg p)) \vee (p \wedge q).$$

4 marks

Question 14

A graph, G, has adjacency matrix,

$$A = \begin{pmatrix} 1 & 2 & 0 & 2 & 2 \\ 2 & 2 & 1 & 0 & 2 \\ 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 2 & 1 & 1 & 2 & 1 \end{pmatrix}$$

1. Is G a simple graph? Explain your answer.
2. State the degree sequence of G.
3. How many edges does G have?

6 marks

ID: 58

Question 1

A graph, G, has adjacency matrix,

$$A = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 2 & 2 & 0 & 1 & 0 \\ 2 & 0 & 2 & 0 & 0 \end{pmatrix}$$

1. Is G a simple graph? Explain your answer.
2. State the degree sequence of G.
3. How many edges does G have?

6 marks

Question 2

Let

$$A = \{1,2,3,4,5,6,7,8\}\ .$$

Determine the truth value of each of the following statements, and explain why this is the case.

1. $\forall x \in A, x + 2 \leq 15$
2. $\forall x \in A, \forall y \in A, x + y \leq 20$
3. $\exists x \in A, x^2 + 2 > 68$
4. $\exists x \in A, \exists y \in A, x^2 + y = 68$

10 marks

Question 3

How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 18$$

(An integer solution to an equation is a solution in which the unknown must have an integer value.)

1. where $x_i \geq 0$ for each x_i?
2. where $x_i > 0$ for each x_i?
3. where $x_i \geq 4$ for each x_i?

6 marks

Question 4

How many shortest lattice paths start at $(1,5)$ and
(i) end at $(11,11)$? ________
(ii) end at $(11,11)$ and pass through $(2,7)$? ________
(iii) end at $(11,11)$ and avoid $(2,7)$? ________

6 marks

Question 5

Expand fully the binomial expansion $(-2x+2y)^5$. Determine the coefficient of the term which has a factor of x^4y^1.

5 marks

Question 6

Determine if the following functions are one-to-one and/or onto and use these to determine whether the function is a bijection. Assume both functions are such that $f:\mathbb{R}\to\mathbb{R}$ where

1. $f(x)=3x+3$
2. $f(x)=-3x^2-2$

10 marks

Question 7

Construct a truth table for the following compound proposition:

$$((\neg p\vee q)\to(p\to\neg p))\wedge(\neg p\to\neg q).$$

4 marks

Question 8

Consider the python code

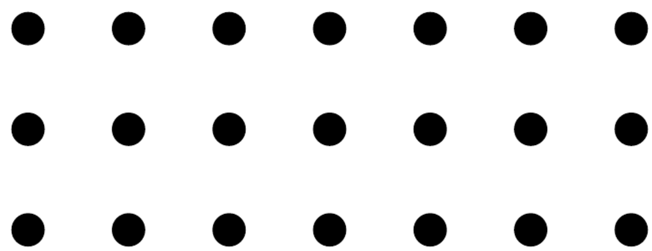
1. $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
2. $R = \{(x, y)$ for x in A for y in A if $y == x ** 2\}$
3.
4. **print (R.issubset({(a, b) for a in A for b in A}))**

What is the output of line 4? Justify your answer.

4 marks

Question 9

Consider the following diagram, consisting of three rows of seven dots.



How many

1. triangles (with a horizontal base)
2. squares

can be drawn using the dots as vertices (corners).

10 marks

Question 10

Prove that

$$17n^2 + n$$

is even for any integer n.

8 marks

Question 11

Consider the second-order homogeneous recurrence relation

$$a_n = 7a_{n-1} - 6a_{n-2}$$

with initial conditions $a_0 = 5$ and $a_1 = 1$.

1. Find the next three terms of the sequence.
2. Find the general solution.
3. Find the unique solution with the given initial conditions.

10 marks

Question 12

In this question, the universal set is $\mathcal{U} = \{x \in \mathbb{N} \mid x \leq 21\}$.

Let:

$A = \{x \in \mathbb{N} \mid 5 \leq x \leq 19\}$.

$B = \{x \in \mathbb{N} \mid x > 16\}$.

$C = \{x \in \mathbb{N} \mid x \text{ divisible by } 5\}$.

Enumerate the following sets:

a) $A \cap C =$ ________

b) $B^c \cap C =$ ________

c) $A \setminus B^c =$ ________

d) $(A \cup B) \cup C =$ ________

6 marks

Question 13

Use the laws of logic to show that

$$(\neg \mathrm{a} \vee \mathrm{z}) \wedge (\neg \mathrm{z} \vee \mathrm{x}) \wedge \mathrm{a} \implies \mathrm{x}$$

5 marks

Question 14

Consider the relation R defined on the set $A = \{a, b, c, d, e, f\}$ where

$$R = \{(a,a),(a,d),(b,c),(c,b)\}$$

1. Draw the digraph for this relation.
2. Explain why this relation is not reflexive.
3. Determine the reflexive closure of this relation.
4. Determine whether this relation is irreflexive or not. Explain your answer.

10 marks

ID: 59

Question 1

Use the laws of logic to show that

$$(\neg p \lor c) \land (\neg c \lor b) \land p \implies b$$

5 marks

Question 2

Construct a truth table for the following compound proposition:

$$((\neg q \land \neg p) \land (\neg q \lor q)) \land (\neg q \to q).$$

4 marks

Question 3

Let

$$A = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13\} \ .$$

Determine the truth value of each of the following statements, and explain why this is the case.

1. $\exists x \in A, x + 4 \leq 12$
2. $\exists x \in A, \exists y \in A, x + y \leq 26$
3. $\forall x \in A, x^2 + 2 > 179$
4. $\forall x \in A, \forall y \in A, x^2 + y = 179$

10 marks

Question 4

A graph, G, has adjacency matrix,

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 2 & 2 & 0 & 2 \\ 2 & 2 & 1 & 1 & 1 \end{pmatrix}$$

1. Is G a simple graph? Explain your answer.
2. State the degree sequence of G.
3. How many edges does G have?

6 marks

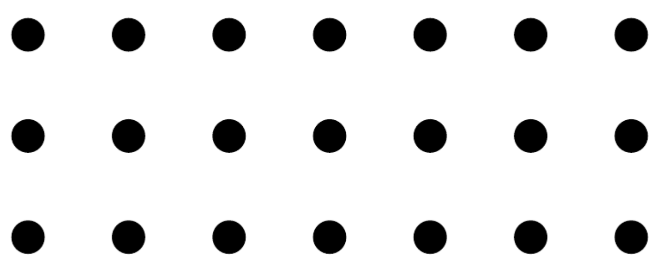
Question 5

Expand fully the binomial expansion $(-2x+5y)^9$. Determine the coefficient of the term which has a factor of x^5y^4.

5 marks

Question 6

Consider the following diagram, consisting of three rows of seven dots.



How many

1. triangles (with a horizontal base)
2. rectangles

can be drawn using the dots as vertices (corners).

10 marks

Question 7

How many shortest lattice paths start at $(0,2)$ and
(i) end at $(10,12)$? ________
(ii) end at $(10,12)$ and pass through $(6,3)$? ________
(iii) end at $(10,12)$ and avoid $(6,3)$? ________

6 marks

Question 8

Determine if the following functions are one-to-one and/or onto and use these to determine whether the function is a bijection. Assume both functions are such that $f : \mathbb{R} \to \mathbb{R}$ where

1. $f(x) = 3x + 6$
2. $f(x) = 2x^2 - 3$

10 marks

Question 9

Consider the second-order homogeneous recurrence relation

$$a_n = 6a_{n-1} - 8a_{n-2}$$

with initial conditions $a_0 = 2$ and $a_1 = 5$.

1. Find the next three terms of the sequence.
2. Find the general solution.
3. Find the unique solution with the given initial conditions.

10 marks

Question 10

Consider the python code

1. $A = \{0,1,2,3,4,5,6,7,8\}$
2. $R = \{(x,y)$ for x in A for y in A if $y == x**2\}$
3.
4. **print (R.issubset({(a, b) for a in A for b in A}))**

What is the output of line 4? Justify your answer.

4 marks

Question 11

Consider the relation R defined on the set $A = \{a, b, c, d, e, f\}$ where

$$R = \{(a, a), (a, d), (b, c), (c, b)\}$$

1. Draw the digraph for this relation.
2. Explain why this relation is not transitive.
3. Determine the transitive closure of this relation.
4. Determine whether this relation is irreflexive or not. Explain your answer.

10 marks

Question 12

In this question, the universal set is $\mathcal{U} = \{x \in \mathbb{N} \mid x \leq 18\}$.

Let:

$A = \{x \in \mathbb{N} \mid 5 \leq x \leq 16\}$.

$B = \{x \in \mathbb{N} \mid x > 13\}$.

$C = \{x \in \mathbb{N} \mid x \text{ divisible by } 5\}$.

Enumerate the following sets:

a) $A \cap C =$ ________

b) $B^c \cap C =$ ________

c) $A \cap B^c =$ ________

d) $(A \cup B) \oplus C =$ ________

6 marks

Question 13

How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 16$$

(An integer solution to an equation is a solution in which the unknown must have an integer value.)

1. where $x_i \geq 0$ for each x_i?
2. where $x_i > 1$ for each x_i?
3. where $x_i \geq 3$ for each x_i?

6 marks

Question 14

Prove that

$$3n^2 + n$$

is even for any integer n.

8 marks

ID: 60

Question 1

Consider the relation R defined on the set $A = \{a, b, c, d, e, f\}$ where

$$R = \{(a, a), (a, d), (b, c), (c, b)\}$$

1. Draw the digraph for this relation.
2. Explain why this relation is not reflexive.
3. Determine the reflexive closure of this relation.
4. Determine whether this relation is irreflexive or not. Explain your answer.

10 marks

Question 2

Consider the second-order homogeneous recurrence relation

$$a_n = 8a_{n-1} - 15a_{n-2}$$

with initial conditions $a_0 = 3$ and $a_1 = 3$.

1. Find the next three terms of the sequence.
2. Find the general solution.
3. Find the unique solution with the given initial conditions.

10 marks

Question 3

A graph, G, has adjacency matrix,

$$A = \begin{pmatrix} 2 & 0 & 2 & 0 & 1 \\ 1 & 1 & 2 & 2 & 2 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 \end{pmatrix}$$

1. Is G a simple graph? Explain your answer.
2. State the degree sequence of G.
3. How many edges does G have?

6 marks

Question 4

Determine if the following functions are one-to-one and/or onto and use these to determine whether the function is a bijection. Assume both functions are such that $f : \mathbb{R} \to \mathbb{R}$ where

1. $f(x) = -3x + 7$
2. $f(x) = -2x^2 - 5$

10 marks

Question 5

Prove that

$$5n^2 + n$$

is even for any integer n.

8 marks

Question 6

Construct a truth table for the following compound proposition:

$$((p \wedge q) \wedge (\neg p \vee \neg q)) \to (\neg q \to p).$$

4 marks

Question 7

In this question, the universal set is $\mathcal{U} = \{x \in \mathbb{N} \mid x \leq 19\}$.

Let:

$A = \{x \in \mathbb{N} \mid 6 \leq x \leq 17\}$.

$B = \{x \in \mathbb{N} \mid x > 14\}$.

$C = \{x \in \mathbb{N} \mid x \text{ divisible by } 5\}$.

Enumerate the following sets:

a) $A \oplus C =$ ________

b) $B^c \cap C =$ ________

c) $A \cap B^c =$ ________

d) $(A \cup B) \cup C =$ ________

6 marks

Question 8

Consider the python code

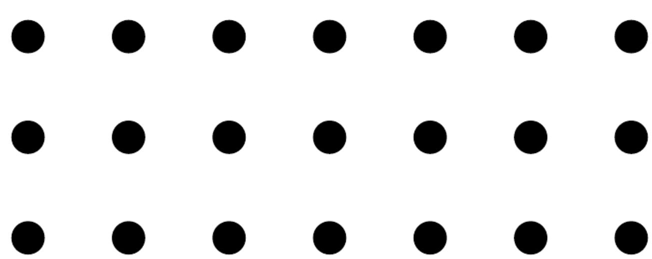
1. $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$
2. $R = \{(x, y)$ for x in A for y in A if $y == x\}$
3.
4. **print (R.issubset({(a, b) for a in A for b in A}))**

What is the output of line 4? Justify your answer.

4 marks

Question 9

Consider the following diagram, consisting of three rows of seven dots.



How many

1. right-angled triangles (with a horizontal base)
2. squares

can be drawn using the dots as vertices (corners).

10 marks

Question 10

Use the laws of logic to show that

$$(\neg \mathrm{p} \vee \mathrm{z}) \wedge (\neg \mathrm{z} \vee \mathrm{b}) \wedge \mathrm{p} \implies \mathrm{b}$$

5 marks

Question 11

Let

$$A = \{1, 2, 3, 4, 5, 6, 7\} \ .$$

Determine the truth value of each of the following statements, and explain why this is the case.

1. $\exists x \in A, x + 1 \leq 10$
2. $\exists x \in A, \exists y \in A, x + y \leq 7$
3. $\forall x \in A, x^2 + 4 > 49$
4. $\forall x \in A, \forall y \in A, x^2 + y = 49$

10 marks

Question 12

Expand fully the binomial expansion $(4x + 8y)^9$. Determine the coefficient of the term which has a factor of x^2y^7.

5 marks

Question 13

How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 11$$

(An integer solution to an equation is a solution in which the unknown must have an integer value.)

1. where $x_i \geq 0$ for each x_i?
2. where $x_i > 0$ for each x_i?
3. where $x_i \geq 2$ for each x_i?

6 marks

Question 14

How many shortest lattice paths start at $(2, 4)$ and
(i) end at $(13, 15)$? ________
(ii) end at $(13, 15)$ and pass through $(4, 14)$? ________
(iii) end at $(13, 15)$ and avoid $(4, 14)$? ________

6 marks