AER1513H – Notes 1

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The motion model for quadrotor we define in our project is

$$\begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} = \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ z_{k-1} \end{bmatrix} + T \begin{bmatrix} v_{x(k-1)} \\ v_{y(k-1)} \\ v_{z(k-1)} \end{bmatrix} + \frac{1}{2}T^2 \left(R_{3\times3}^T \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} - g \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) \tag{1}$$

$$\begin{bmatrix} v_{x(k)} \\ v_{y(k)} \\ v_{z(k)} \end{bmatrix} = \begin{bmatrix} v_{x(k-1)} \\ v_{y(k-1)} \\ v_{z(k-1)} \end{bmatrix} + T \left(R_{3\times3}^T \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} - g \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) \tag{2}$$

where $(x, y, z)^T$ is the position of quadrotor and (v_X, v_y, v_z) is the velocity. T is the sampling time period, g is the gravity and $R_{3\times3}^T$ is the rotation matrix from body frame to global frame.The motion model can be further expressed into state space equation.

$$\begin{bmatrix} x_k \\ y_k \\ z_k \\ v_{x(k)} \\ v_{y(k)} \\ v_{z(k)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ z_{k-1} \\ v_{x(k-1)} \\ v_{y(k-1)} \\ v_{z(k-1)} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}T^2 & 0 & 0 \\ 0 & \frac{1}{2}T^2 & 0 \\ 0 & 0 & \frac{1}{2}T^2 \\ T & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{bmatrix} \left(R_{3\times3}^T \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} - g \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) \tag{3}$$

Rotation matrix can be expressed by unit quaternion $q = (q_0, \vec{\boldsymbol{q_v}})^T$

$$R_{3\times3}^T = (2q_0^2 - 1)\mathbf{1}_{3\times3} + 2\vec{\boldsymbol{q_v}}\vec{\boldsymbol{q_v}}^T - 2q_0\vec{\boldsymbol{q_v}}^\times$$

where $\vec{\boldsymbol{q_v}}^\times$ is the skew-symmetric cross product matrix of $\vec{\boldsymbol{q_v}}$.

Observation model

$$\begin{bmatrix} z_k \\ v_{x(k)} \\ v_{y(k)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ z_k \\ v_{x(k)} \\ v_{y(k)} \\ v_{z(k)} \end{bmatrix} \tag{4}$$

Data source:

- Input: unit quaternion $q = (q_0, \vec{\boldsymbol{q_v}})^T$ from onboard extended Kalman filter. (collected)
- Input: acceleration $a = (a_x, a_y, a_z)^T$ from onboard IMU. (collected)
- Measurements: $(z, v_x, v_y)^T$ from flowdeck.(collected)