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Comparative Study on Recent Development of Heuristic Optimization Methods

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***Abstract*—In engineering and design problems, various noisy non-linear mathematical optimization problems can't be efficaciously solved by using conventional optimization techniques. But metaheuristic algorithms seem very efficient to approach in these problems and became very popular such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO). Recently, many new metaheuristic algorithms were proposed, but the performance of these algorithms in solving noisy non-linear optimization problems when compared with popular methods still need more of verifications. In this context, two popular algorithms called GA and PSO will be compared with some recent metaheuristic algorithms such as Grey Wolf Optimizer, Firefly Algorithm, and Brain Storm Optimization algorithm in finding optimal solutions of noisy non-linear optimization problems. The results will be compared in terms of accuracy of the best solutions found and the execution time.**

Keywords—Comparative study, nonlinear functions, grey wolf optimizer algorithm, firefly algorithm, brain storm optimization algorithm

I. INTRODUCTION

Metaheuristic optimization techniques now have become very popular and successfully applied to solve problems not only in computer science but also in many different fields related in optimization problems. These techniques already proved their simplicity, flexibility and efficiency in finding out optimal solutions of many complex optimization problems. Therefore recently, many new metaheuristic optimization algorithms were proposed. But the efficiency of these new methods in solving noisy non-linear optimization problems still needs more of verifications.

In this paper, some popular algorithms like Genetic Algorithm (GA) and Particle Swarm Optimization algorithm (PSO) will be compared with recent algorithms like Grey Wolf Optimizer algorithm (GWO), Firefly Algorithm (FA), and Brain Storm Optimization algorithm (BSO). Genetic Algorithm belongs to Evolutionary Algorithms which was invented by John Holland in the early 1970's. GA is a meta-heuristic search algorithm based on the evolutionary ideas of natural selection and genetics. PSO is a population based stochastic optimization technique developed by Dr. Eberhart and Dr. Kennedy in 1995, inspired by simulations of various interpretations of the movement of organisms in a bird flock or fish school [1]. The GWO algorithm mimics the leadership hierarchy and hunting mechanism of grey wolves in nature proposed by Mirjalil in 2014 [2]. FA is a meta-heuristic algorithm, inspired by the natural characteristics of fireflies: flashing behavior proposed by Xin-She in 2008 [3, 4]. BSO is a swarm intelligence algorithm based on the human creative problem solving process proposed by Yuhui Shi in 2011 [5].

In this paper, we use noisy non-linear optimization test functions from [7, 8]. These functions with difference characteristics were used to compare the efficiency of other methods to find out the global optimization values. Some comparisons with GA, PSO, and FA in node localization problem of wireless sensor network was showed in [9].

The rest of this paper is organized as follows: a brief review of other recent heuristic optimization algorithms was used are given in Session II. Our noisy non-linear mathematical functions was used are presented in Session III. Experimental results of the comparison among GA, PSO, GWO, FA, and BSO to solve noisy non-linear mathematical functions are discussed in Session IV. Finally, the conclusion is summarized in Session V.

II. RECENT HEURISTIC OPTIMIZATION ALGORITHMS

A. Grey Wolf Optimizer algorithm

The inspiration of Grey Wolf Optimizer (GWO) algorithm is from the social hierarchy of wolves. Group hunting is another interesting social behavior of grey wolves. The main phases of grey wolf hunting are as follows:

- Tracking, chasing and approaching the prey
- Pursuing, encircling and harassing the prey until it stops moving
- Attack towards the prey

In the social hierarchy of wolves, the fittest solution is the alpha wolf (α). Consequently, the second and third best solutions are named beta wolf (β) and delta wolf (δ) respectively. The rest of the candidate solutions are assumed to be omega wolf (ω). In the GWO algorithm the optimization is guided by: α, β, and δ wolves. The ω wolves

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follow these three wolves [2]. Fig. 1 shows the pseudo code of the GWO algorithm

```
Initialize the grey wolf population (position): X_i (i = 1,2,...,n);
Initialize parameters: a, A, and C;
Calculate the fitness of all current search agents;
Assign X_α, X_β, X_δ is the best, second best and third best search agent respectively;
While (t <Max Iteration)
  For each search agent
    Update the position of the current search agent by equation (3.7) in [2];
  End of For
  Update parameters: a, A, and C;
  Update the new position of all search agents;
  Update the positions X_α, X_β, X_δ ;
  t=t+1;
End of While
Return the best value: X_α;
```

Figure 1 Pseudo code of the GWO algorithm

```
Randomly generate n potential solutions (individuals);
While (t <Max Iteration)
Cluster n individuals into m clusters using k-means;
Evaluation the fitness values of n individuals;
Rank individuals in each cluster and record the best individual as cluster center in each cluster;
If (random(0,1)<p5a)
  Randomly select a cluster and replace the cluster center with a randomly generated individual ;
End
For each n individuals
   IF (random(0,1)<p6b)
      Randomly select a cluster;
      If (random(0,1)< p6biii)
      Select the cluster center and add random values to it to generate new individual;
      Else
      Randomly select an individual from this cluster and add random value to the individual to generate new individual;
      End
   ELSE
      Randomly select two clusters;
      If (random(0,1)<p6c)
      Two cluster centers are combined and add with random value to generate new individual;
      Else
      Two individuals from each selected cluster are randomly selected and add with random value to generate new individual;
      End
   END of If
End of For
   t=t+1;
End of While
```

Figure 2 Pseudo code of the BSO algorithm

B. *Firefly Algorithm*

The inspiration of Firefly algorithm (FA) is from natural characteristics of fireflies. This algorithm based on three characteristics [3, 4]:

1. Every firefly is unisex so each firefly will be attracted by other fireflies without caring about their gender.
2. The attractiveness of a firefly depends on its brightness proportionally, the brighter will attract the less brighter. The attractiveness will decrease when the distance between two fireflies increase.
3. The brightness of a firefly is affected or determined by the landscape of the objective function.

With β_0 is the attractiveness at $r = 0$, the decrease of attractiveness β depended on the distance r also is defined like in the equation below:

$$\beta = \beta_0 e^{-\gamma r^2} \tag{1}$$

The movement of a firefly when it is attracted by other brighter firefly is determined by:

$$x_i^{t+1} = x_i^t + \beta_0 e^{-\gamma r_{ij}^2}\left(x_j^t - x_i^t\right) + \alpha_t \epsilon_i^t \tag{2}$$

In (2), the second term is due to the attraction. The third term is randomization with α_t being the randomization parameter, and ϵ_i^t is a vector of random numbers drawn from a Gaussian distribution or uniform distribution at the time t.

C. *Brain Storm Optimization algorithm*

Human beings are social animals and are the most intelligent animals in the world. Brain Storm Optimization (BSO) is a new kind of swarm intelligence algorithm inspired by human creative problem solving process. BSO generally uses the grouping, replacing, and creating operators to produce ideas as many as possible to approach the problem global optimum generation by generation [5, 6]. Fig. 2 shows the pseudo code of the BSO algorithm.

III. NOISY NON LINEAR MATHEMATICAL FUNCTIONS

To compare the performance of each method in solving noisy non-linear mathematical optimization problems, these methods will be applied to find out global optimization value (maximum value) of different nature functions. The function with single peak like Parabolic functions, the multi-peak of Four Peak, Rastrigin and Shekel functions, the curved ridge of Styblinski and Rosenbrock functions, the multi-peak with curved ridge of Branin and Goldstein-Price functions.

The typical three-dimensional response surfaces of these functions are show in Fig. 3.

A. *Four Peak function*

$$F_1(x,y) = \exp(-(x-4)^2 - (y-4)^2) + \exp(-(x+4)^2 - (y-4)^2) + 2[\exp(-x^2 - y^2) + \exp(-x^2 - (y+4)^2)] \tag{1}$$

B. *Parabolic Function*

$$F_2(x,y) = 12 - (x^2 + y^2)/100 \tag{2}$$

C. *Goldstein-Price Function*

$$F_3(x,y) = 10 + log_{10}(1/\{[1 + (1 + x + y)^2(19 - 14x + 3x^2 - 14y + 6xy + 3y^2)] * [30 + (2x - 3y)^2(18 - 32x + 12x^2 + 48y - 36xy + 27y^2)]\}) \tag{3}$$

D. *Styblinski Function*

$$F_4(x,y) = 275 - [(x^4 - 16x^2 + 5x)/2 + (y^4 - 16y^2 + 5y)/2 + 3] \tag{4}$$

E. *Rastrigin Function*

$$F_5(x,y) = 80 - \{20 + x^2 + y^2 - 10[\cos(2\pi x) + \cos(2\pi y)]\} \tag{5}$$

F. *Rosenbrock Function*

$$F_6(x,y) = 70 * \{[(20 - \{(1 - x/(-7))^2 + [y/6 + (x/(-7))^2]^2\}) + 150]/170\} + 10 \tag{6}$$

G. *Branin Function*

$$F_7(x,y) = 5 - log_{10}[y - 5.1x^2/(4\pi^2) + (5x/\pi - 6)^2 + (10 - 5/4\pi) * cos(x) + 10] \tag{7}$$

H. Shekel Function

$$F_8(x,y) = 100 * \{1/[9+(x-4)^2+(y-6)^2] + 1/(20+x^2+y^2) + 1/[14+(x-8)^2+(y+3)^2] + 1/[11+(x-8)^2+(y-8)^2] + 1/[6+(x+6)^2+(y-7)^2]\} \tag{8}$$

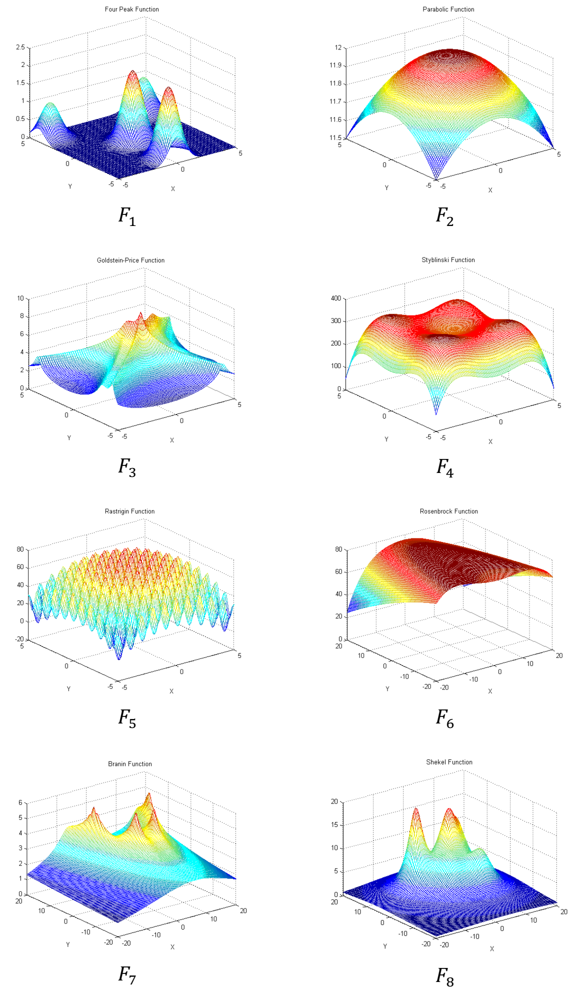

Figure 3 3-D versions of noisy non linear mathematical functions

IV. EXPERIMENTAL RESULTS

The parameters set for GA, PSO, GWO, FA, and BSO were proposed in [10], [11], [2], [4], and [5, 6] respectively.

Genetic Algorithm (GA):

- Crossover Rate (CR): Decrease from 0.9 to 0.5
- Mutation Rate (MR): 0.05

Particle Swarm Optimization (PSO):

- Learning factor: $c_1 = c_2 = 2$
- Inertia weight (w): Decrease from 0.9 to 0.4

Grey Wolf Optimizer (GWO):

- Parameter a: linearly decreased from 2 to 0

Firefly Algorithm (FA):

- Absorption coefficient: γ=1.0
- Randomness reduction: δ=0.97
- Initial randomness scaling factor: α_0=0.2

Brain Storm Optimization (BSO):

- m=5, p5a=0.2, p6b=0.8, p6biii=0.4, p6c=0.5, k=20

In this paper, all the data are obtained by the average of thirty runs of each experiment. And we will examine all the algorithms mentioned above with two cases. In the case 1, we keep the max iteration as a constant which is set to 100 and change the population size from 15 to 45. In contrast with case 1, we keep the population size as a constant which is set to 30 and change the iteration from 10 to 150 in the case 2.

In the single peak functions like Parabolic and the curved ridge of Styblinski and Rosenbrock functions, both popular metaheuristic algorithms (GA, PSO) and recent meta-heuristic algorithms (GWO, FA, BSO) almost immediately get the global optimization value even the population size and number of iteration are small. Because the types of these functions will not cause the metaheuristic algorithm is trapped in local peaks. The graph obtained by different algorithms in these functions nearly a unique line and therefore it doesn't necessary to draw in here. The global optimization values of Parabolic, Styblinski and Rosenbrock functions that are obtained by these algorithms are approximately equal to 12, 350 and 80 respectively.

According to the results, the GWO outperforms other algorithms in most of the test functions except $F_8(x,y)$- Shekel function not only in accuracy but also in execution time as shown in Fig. 4 to Fig. 9.

As the results shown in from Fig. 4 to Fig. 8, the performance of GA is good when the population size and the max iteration are large enough. Otherwise, in some test functions, GA will be trapped in a local optimization that causes an inaccurate result. Fig. 4 and Fig. 5 show that GA is not good when the population size is set from 15 to 30, and GA with the max iteration setting from 10 to 60 is not well performed.

PSO algorithm is good in almost of cases excepted in $F_7(x,y)$ - Branin functions shown in Fig. 7.

Generally, the accuracy of FA and BSO is not good as the remained algorithms as shown from Fig. 5 to Fig. 8. Especially when testing in Branin and Shekel functions, FA is easily to trap into local peaks as we can see in Fig. 7 and Fig. 8

Fig. 9 shown the processing time of dealing with $F_1(x,y)$ function for all algorithms in two mentioned cases. We ignore the BSO because the processing time of BSO is longer several times than others. The processing time for testing other functions has form is similar to the $F_1(x,y)$. When considering the processing time, GWO is the best choice for us to solve our proposed optimization problems. Next is PSO, then followed by FA and GA. BSO has the longest execution time than other mentioned algorithms.

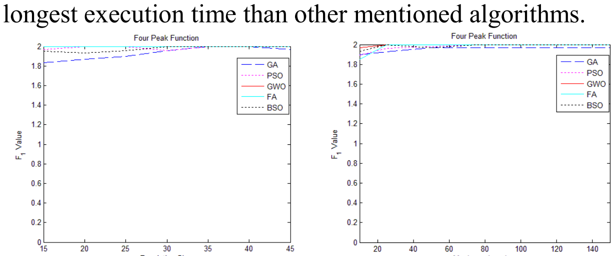

Figure 4 Global optimization value vs Population size (left) and Max Iteration (right) in $F_1(x,y)$

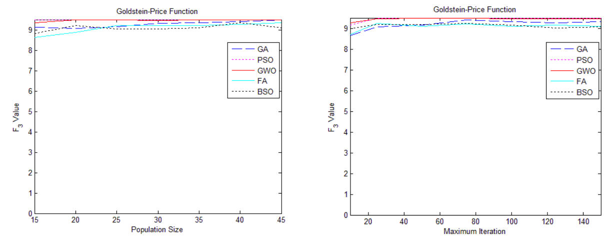

Figure 5 Global optimization value vs Population size (left) and Max Iteration (right) in $F_3(x, y)$

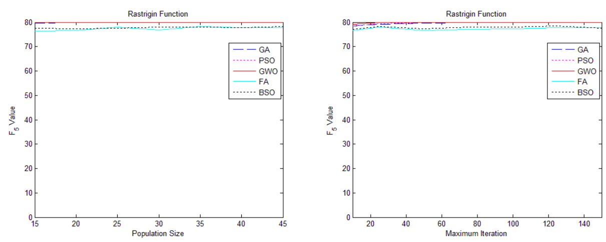

Figure 6 Global optimization value vs Population size (left) and Max Iteration (right) in $F_5(x, y)$

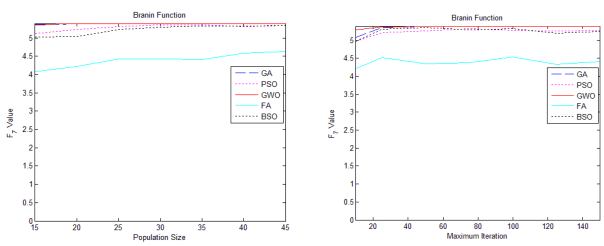

Figure 7 Global optimization value vs Population size (left) and Max Iteration (right) in $F_7(x, y)$

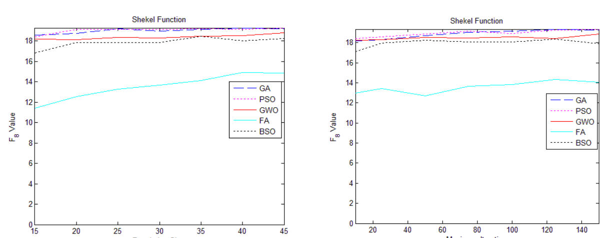

Figure 8 Global optimization value vs Population size (left) and Max Iteration (right) in $F_8(x, y)$

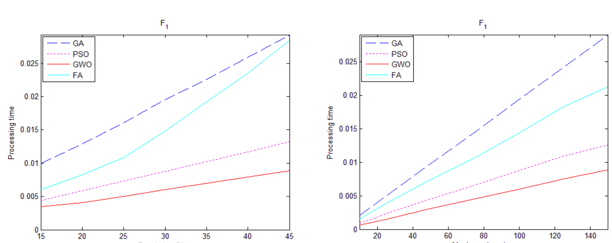

Figure 9 Processing time vs Population size (left) and Max Iteration in (right) $F_1(x, y)$

V. CONCLUSION

This paper compared the efficiency of some recent heuristic optimization algorithms with GA and PSO in finding out global optimization value of noisy non-linear optimization problems. The test functions used in this paper have different characteristics. And we already tried to change the input parameters when applying different algorithms in order to get overall views and objectively judges. GA and PSO showed a quite good performance on almost of the testing cases. GWO also showed that it is a very competitive and potential algorithm in solving noisy non-linear optimization problems in both accuracy and execution time. FA also is a quite competitive algorithm in execution time. While in accuracy part, FA showed the results not really good in some of test functions. Finally, BSO shown is an admissible algorithm in accuracy but not effective in execution time part. This comparison results in this paper will be a helpful reference for choosing the accordant heuristic optimization algorithms to solve engineering and design problems.

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REFERENCES

[1] J. Kennedy, "Particle swarm optimization," in *Encyclopedia of Machine Learning*, ed: Springer, 2010, pp. 760-766.

[2] S. Mirjalili, S. M. Mirjalili, and A. Lewis, "Grey Wolf Optimizer," *Advances in Engineering Software,* vol. 69, pp. 46-61, 2014.

[3] X.-S. Yang, "Firefly algorithms for multimodal optimization," in *Stochastic algorithms: foundations and applications*, ed: Springer, 2009, pp. 169-178.

[4] X. S. Yang and X. He, "Firefly algorithm: recent advances and applications," *International Journal of Swarm Intelligence,* vol. 1, p. 36, 2013.

[5] Y. Shi, "Brain storm optimization algorithm," in *Advances in Swarm Intelligence*, ed: Springer, 2011, pp. 303-309.

[6] Z.-h. Zhan, J. Zhang, Y.-h. Shi, and H.-l. Liu, "A modified brain storm optimization," in *Evolutionary Computation (CEC), 2012 IEEE Congress on*, 2012, pp. 1-8.

[7] N. Chai-Ead, P. Aungkulanon, and P. Luangpaiboon, "Bees and firefly algorithms for noisy non-linear optimization problems," in *Proceedings of the international multi conference of engineering and computer scientists*, 2011.

[8] S. K. Pal, C. S. Rai, and A. P. Singh, "Comparative Study of Firefly Algorithm and Particle Swarm Optimization for Noisy Non-Linear Optimization Problems," *International Journal of Intelligent Systems and Applications,* vol. 4, pp. 50-57, 2012.

[9] Van-Oanh Sai, Chin-Shiuh Shieh, Trong-The Nguyen, Yuh-Chung Lin, Mong-Fong Horng, and Quang-Duy Le, "Parallel Firefly Algorithm for Localization Algorithm in Wireless Sensor Network," *2015 Third International Conference on Robot, Vision and Signal Processing,* pp. 300-305, November 18-20, 2015.

[10] W.-Y. Lin, W.-Y. Lee, and T.-P. Hong, "Adapting crossover and mutation rates in genetic algorithms," *J. Inf. Sci. Eng.,* vol. 19, pp. 889-903, 2003.

[11] J. Wang, "Particle swarm optimization with adaptive parameter control and opposition," *Journal of Computational Information Systems,* vol. 7, pp. 4463-4470, 2011.