

# Chi-Square Test for Categorical Variables

## Introduction

The chi-square test is a statistical method used to determine if there is a significant association between two categorical variables. This test is widely used in various fields, including social sciences, marketing, and healthcare, to analyze survey data, experimental results, and observational studies.

## Concept

The chi-square test is a non-parametric statistical method that evaluates whether the observed frequencies in each category differ significantly from the expected frequencies—assuming no association between the variables.

The test is based on the chi-square distribution, which is a family of distributions defined by degrees of freedom (df). These distributions are right-skewed and vary depending on df. A chi-square distribution table lists critical values for given df and significance levels ( $\alpha$ ), which we use to assess if our computed test statistic is extreme enough to reject the null hypothesis.

## Null Hypothesis and Alternative Hypothesis

The chi-square test involves formulating two hypotheses:

Null Hypothesis ( $H_0$ )(H0) - Assumes that there is no association between the categorical variables, implying that any observed differences are due to random chance.

Alternative Hypothesis ( $H_1$ )(H1) - Assumes that there is a significant association between the variables, indicating that the observed differences are not due to chance alone.

## Formula

The **chi-square statistic** is calculated using the formula:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

where

$O_i$  is the observed frequency for category  $i$ .

$E_i$  is the expected frequency for category  $i$ , calculated as:

$$E_i = \frac{(\text{row total} \times \text{column total})}{\text{grand total}}$$

The sum is taken over all cells in the contingency table.

The calculated chi-square statistic is then compared to a critical value from the chi-square distribution table. This table provides critical values for different degrees of freedom (df) and significance levels ( $\alpha$ ).

The **degrees of freedom** for the test are calculated as:

$$df = (r - 1) \times (c - 1)$$

where  $r$  is the number of rows and  $c$  is the number of columns in the table.

## Chi-Square Distribution Table

A chi-square distribution table provides critical values that vary by degrees of freedom and the significance level ( $\alpha$ ). These values indicate the threshold beyond which the test statistic would be considered statistically significant.

For example:

df = 1,  $\alpha$  = 0.05, the critical value is 3.841

If your calculated  $\chi^2 > 3.841$ , you reject  $H_0$

If  $\chi^2 \leq 3.841$ , you fail to reject  $H_0$

The higher the  $\chi^2$  value, the stronger the evidence against  $H_0$ .

## Python Implementation Example

Below is a Python implementation using `scipy.stats` and `pandas`:

```
import pandas as pd
from scipy.stats import chi2_contingency
# Create the contingency table
data = [[20, 30], # Male: [Like, Dislike]
        [25, 25]] # Female: [Like, Dislike]
# Create a DataFrame for clarity
```

```
df = pd.DataFrame(data, columns=["Like", "Dislike"], index=["Male", "Female"])
# Perform the Chi-Square Test
chi2, p, dof, expected = chi2_contingency(df)
# Display results
print("Chi-square Statistic:", chi2)
print("Degrees of Freedom:", dof)
print("P-value:", p)
print("Expected Frequencies:\n", expected)
```

Output:  
Chi-square Statistic: 1.008  
Degrees of Freedom: 1  
P-value: 0.3156  
Expected Frequencies:[[22.5 27.5] [22.5 27.5]]

**Interpretation:** Since the p-value (0.3156) > 0.05, we fail to reject the null hypothesis—indicating no significant association.

Applications

- 1. **Market Research:** Analyzing the association between customer demographics and product preferences.
- 2. **Healthcare:** Studying the relationship between patient characteristics and disease incidence.
- 3. **Social Sciences:** Investigating the link between social factors (e.g., education level) and behavioral outcomes (e.g., voting patterns).
- 4. **Education:** Examining the connection between teaching methods and student performance.
- 5. **Quality Control:** Assessing the association between manufacturing conditions and product defects.

Practical Example - Weak Correlation

Suppose a researcher wants to determine if there is an association between gender (male, female) and preference for a new product (like, dislike). The researcher surveys 100 people and records the following data:

Category	Like	Dislike	Total
Male	20	30	50
Female	25	25	50
Total	45	55	100

Step 1: Calculate Expected Frequencies

Using the formula for expected frequencies:

$E_{Male, Like} = \frac{(50 \times 45)}{100} = 22.5$   
 $E_{Male, Dislike} = \frac{(50 \times 55)}{100} = 27.5$   
 $E_{Female, Like} = \frac{(50 \times 45)}{100} = 22.5$   
 $E_{Female, Dislike} = \frac{(50 \times 55)}{100} = 27.5$

Step 2: Compute Chi-Square Statistic

$\chi^2 = \frac{(20 - 22.5)^2}{22.5} + \frac{(30 - 27.5)^2}{27.5} + \frac{(25 - 22.5)^2}{22.5} + \frac{(25 - 27.5)^2}{27.5}$   
 $\chi^2 = 22.5(20 - 22.5)^2 + 27.5(30 - 27.5)^2 + 22.5(25 - 22.5)^2 + 27.5(25 - 27.5)^2$   
 $\chi^2 = \frac{(2.5)^2}{22.5} + \frac{(2.5)^2}{27.5} + \frac{(2.5)^2}{22.5} + \frac{(2.5)^2}{27.5}$   
 $\chi^2 = 22.5(2.5)^2 + 27.5(2.5)^2 + 22.5(2.5)^2 + 27.5(2.5)^2$   
 $\chi^2 = \frac{6.25}{22.5} + \frac{6.25}{27.5} + \frac{6.25}{22.5} + \frac{6.25}{27.5}$   
 $\chi^2 = 0.277 + 0.227 + 0.277 + 0.227$   
 $\chi^2 = 1.008$

Step 3: Determine Degrees of Freedom

$df = (2 - 1) \times (2 - 1) = 1$   
 $df = (2 - 1) \times (2 - 1) = 1$

Step 4: Interpret the Result

Using a chi-square distribution table, we compare the calculated chi-square value (1.008) with the critical value at one degree of freedom and a significance level (e.g., 0.05). The critical value, as determined from chi-square distribution tables, is approximately 3.841.

Since  $1.008 < 3.841$ , we fail to reject the null hypothesis. Thus, there is no significant association between gender and product preference in this sample.

Practical Example - Strong Association

Consider a study investigating the relationship between smoking status (smoker, non-smoker) and the incidence of lung disease (disease, no disease). The researcher collects data from 200 individuals and records the following information:

Category	Disease	No Disease	Total
Smoker	50	30	80
Non-Smoker	20	100	120
Total	70	130	200

Step 1: Calculate Expected Frequencies

Using the formula for expected frequencies:

$E_{Smoker, Disease} = \frac{(80 \times 70)}{200} = 28$   
 $E_{Smoker, No Disease} = \frac{(80 \times 130)}{200} = 52$   
 $E_{Non-Smoker, Disease} = \frac{(120 \times 70)}{200} = 42$   
 $E_{Non-Smoker, No Disease} = \frac{(120 \times 130)}{200} = 78$

Step 2: Compute Chi-Square Statistic

$\chi^2 = \frac{(50 - 28)^2}{28} + \frac{(30 - 52)^2}{52} + \frac{(20 - 42)^2}{42} + \frac{(100 - 78)^2}{78}$   
 $\chi^2 = 28(50 - 28)^2 + 52(30 - 52)^2 + 42(20 - 42)^2 + 78(100 - 78)^2$   
 $\chi^2 = \frac{(22)^2}{28} + \frac{(22)^2}{52} + \frac{(22)^2}{42} + \frac{(22)^2}{78}$   
 $\chi^2 = 28(22)^2 + 52(22)^2 + 42(22)^2 + 78(22)^2$   
 $\chi^2 = \frac{484}{28} + \frac{484}{52} + \frac{484}{42} + \frac{484}{78}$   
 $\chi^2 = 28(17.29) + 52(9.31) + 42(11.52) + 78(6.21)$   
 $\chi^2 = 44.33$

Step 3: Determine Degrees of Freedom

$df = (2 - 1) \times (2 - 1) = 1$   
 $df = (2 - 1) \times (2 - 1) = 1$

Step 4: Interpret the Result

Using a chi-square distribution table, we compare the calculated chi-square value (44.33) with the critical value at one degree of freedom and a significance level (e.g., 0.05), approximately 3.841. Since  $44.33 > 3.841$ , we reject the null hypothesis. This indicates a significant association between smoking status and the incidence of lung disease in this sample.

Conclusion

The chi-square test is a powerful tool for analyzing the relationship between categorical variables. By comparing observed and expected frequencies, researchers can determine if there is a statistically significant association, providing valuable insights in various fields of study.



# Skills Network