

# Chi-Square Test for Categorical Variables

## Introduction

The chi-square test is a statistical method used to determine if there is a significant association between two categorical variables. This test is widely used in various fields, including social sciences, marketing, and healthcare, to analyze survey data, experimental results, and observational studies.

## Concept

The chi-square test is a non-parametric statistical method that evaluates whether the observed frequencies in each category differ significantly from the expected frequencies—assuming no association between the variables.

The test is based on the chi-square distribution, which is a family of distributions defined by degrees of freedom (df). These distributions are right-skewed and vary depending on df. A chi-square distribution table lists critical values for given df and significance levels ( $\alpha$ ), which we use to assess if our computed test statistic is extreme enough to reject the null hypothesis.

## Null Hypothesis and Alternative Hypothesis

The chi-square test involves formulating two hypotheses:

Null Hypothesis ( $H_0$ ) - Assumes that there is no association between the categorical variables, implying that any observed differences are due to random chance.

Alternative Hypothesis ( $H_1$ ) - Assumes that there is a significant association between the variables, indicating that the observed differences are not due to chance alone.

## Formula

The **chi-square statistic** is calculated using the formula:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \quad \chi^2 = \sum E_i(O_i - E_i)^2$$

where

$O_i$  is the observed frequency for category  $i$ .

$E_i$  is the expected frequency for category  $i$ , calculated as:

$$E_i = \frac{(row\ total \times column\ total)}{grand\ total} \quad E_i = \text{grand total}(\text{row total} \times \text{column total})$$

The sum is taken over all cells in the contingency table.

The calculated chi-square statistic is then compared to a critical value from the chi-square distribution table. This table provides critical values for different degrees of freedom (df) and significance levels ( $\alpha$ ).

The **degrees of freedom** for the test are calculated as:

$$df = (r - 1) \times (c - 1) \quad df = (r - 1) \times (c - 1)$$

where  $r$  is the number of rows and  $c$  is the number of columns in the table.

## Chi-Square Distribution Table

A chi-square distribution table provides critical values that vary by degrees of freedom and the significance level ( $\alpha$ ). These values indicate the threshold beyond which the test statistic would be considered statistically significant.

For example:

$df = 1, \alpha = 0.05$ , the critical value is 3.841

If your calculated  $\chi^2 > 3.841$ , you reject  $H_0$

If  $\chi^2 \leq 3.841$ , you fail to reject  $H_0$

The higher the  $\chi^2$  value, the stronger the evidence against  $H_0$ .

## Python Implementation Example

Below is a Python implementation using `scipy.stats` and `pandas`:

```
import pandas as pd
from scipy.stats import chi2_contingency
# Create the contingency table
data = [[20, 30], # Male: [Like, Dislike]
        [25, 25]] # Female: [Like, Dislike]
# Create a DataFrame for clarity
```

```

df = pd.DataFrame(data, columns=["Like", "Dislike"], index=["Male", "Female"])
# Perform the Chi-Square Test
chi2, p, dof, expected = chi2_contingency(df)
# Display results
print("Chi-square Statistic:", chi2)
print("Degrees of Freedom:", dof)
print("P-value:", p)
print("Expected Frequencies:\n", expected)

```

**Output:**

```

Chi-square Statistic: 1.008
Degrees of Freedom: 1
P-value: 0.3156
Expected Frequencies:[[22.5 27.5] [22.5 27.5]]

```

**Interpretation:** Since the p-value (0.3156) > 0.05, we fail to reject the null hypothesis—indicating no significant association.

## Applications

1. **Market Research:** Analyzing the association between customer demographics and product preferences.
2. **Healthcare:** Studying the relationship between patient characteristics and disease incidence.
3. **Social Sciences:** Investigating the link between social factors (e.g., education level) and behavioral outcomes (e.g., voting patterns).
4. **Education:** Examining the connection between teaching methods and student performance.
5. **Quality Control:** Assessing the association between manufacturing conditions and product defects.

### Practical Example - Weak Correlation

Suppose a researcher wants to determine if there is an association between gender (male, female) and preference for a new product (like, dislike). The researcher surveys 100 people and records the following data:

Category	Like	Dislike	Total
Male	20	30	50
Female	25	25	50
Total	45	55	100

#### Step 1: Calculate Expected Frequencies

Using the formula for expected frequencies:

$$E_{Male, Like} = \frac{(50 \times 45)}{100} = 22.5 \quad E_{Male, Like} = 100(50 \times 45) / 100 = 22.5$$

$$E_{Male, Dislike} = \frac{(50 \times 55)}{100} = 27.5 \quad E_{Male, Dislike} = 100(50 \times 55) / 100 = 27.5$$

$$E_{Female, Like} = \frac{(50 \times 45)}{100} = 22.5 \quad E_{Female, Like} = 100(50 \times 45) / 100 = 22.5$$

$$E_{Female, Dislike} = \frac{(50 \times 55)}{100} = 27.5 \quad E_{Female, Dislike} = 100(50 \times 55) / 100 = 27.5$$

#### Step 2: Compute Chi-Square Statistic

$$\begin{aligned} \chi^2 &= \frac{(20 - 22.5)^2}{22.5} + \frac{(30 - 27.5)^2}{27.5} + \frac{(25 - 22.5)^2}{22.5} + \frac{(25 - 27.5)^2}{27.5} \chi^2 = 22.5(20 - 22.5)^2 / 22.5 + 27.5(30 - 27.5)^2 / 27.5 \\ &+ 22.5(25 - 22.5)^2 / 22.5 + 27.5(25 - 27.5)^2 / 27.5 \end{aligned}$$

$$\begin{aligned} \chi^2 &= \frac{(2.5)^2}{22.5} + \frac{(2.5)^2}{27.5} + \frac{(2.5)^2}{22.5} + \frac{(2.5)^2}{27.5} \chi^2 = 22.5(2.5)^2 / 22.5 + 27.5(2.5)^2 / 27.5 + 22.5(2.5)^2 / 22.5 + 27.5(2.5)^2 / 27.5 \end{aligned}$$

$$\begin{aligned} \chi^2 &= \frac{6.25}{22.5} + \frac{6.25}{27.5} + \frac{6.25}{22.5} + \frac{6.25}{27.5} \chi^2 = 22.5 \cdot 6.25 / 22.5 + 27.5 \cdot 6.25 / 27.5 + 22.5 \cdot 6.25 / 22.5 + 27.5 \cdot 6.25 / 27.5 \end{aligned}$$

$$\chi^2 = 0.277 + 0.227 + 0.277 + 0.227 \chi^2 = 0.277 + 0.227 + 0.277 + 0.227$$

$$\chi^2 = 1.008 \chi^2 = 1.008$$

#### Step 3: Determine Degrees of Freedom

$$df = (2 - 1) \times (2 - 1) = 1$$

#### Step 4: Interpret the Result

Using a chi-square distribution table, we compare the calculated chi-square value (1.008) with the critical value at one degree of freedom and a significance level (e.g., 0.05). The critical value, as determined from chi-square distribution tables, is approximately 3.841.

Since  $1.008 < 3.841$ , we fail to reject the null hypothesis. Thus, there is no significant association between gender and product preference in this sample.

#### Practical Example - Strong Association

Consider a study investigating the relationship between smoking status (smoker, non-smoker) and the incidence of lung disease (disease, no disease). The researcher collects data from 200 individuals and records the following information:

Category	Disease	No Disease	Total
Smoker	50	30	80
Non-Smoker	20	100	120
Total	70	130	200

#### Step 1: Calculate Expected Frequencies

Using the formula for expected frequencies:

$$E_{Smoker, Disease} = \frac{(80 \times 70)}{200} = 28 E_{Smoker, No Disease} = 200(80 \times 70) = 28$$

$$E_{Smoker, No Disease} = \frac{(80 \times 130)}{200} = 52 E_{Smoker, Disease} = 200(80 \times 130) = 52$$

$$E_{Non-Smoker, Disease} = \frac{(120 \times 70)}{200} = 42 E_{Non-Smoker, No Disease} = 200(120 \times 70) = 42$$

$$E_{Non-Smoker, No Disease} = \frac{(120 \times 130)}{200} = 78 E_{Non-Smoker, Disease} = 200(120 \times 130) = 78$$

#### Step 2: Compute Chi-Square Statistic

$$\chi^2 = \frac{(50 - 28)^2}{28} + \frac{(30 - 52)^2}{52} + \frac{(20 - 42)^2}{42} + \frac{(100 - 78)^2}{78} \chi^2 = 28(50-28)^2 + 52(30-52)^2 + 42(20-42)^2 + 78(100-78)^2$$

$$\chi^2 = \frac{(22)^2}{28} + \frac{(22)^2}{52} + \frac{(22)^2}{42} + \frac{(22)^2}{78} \chi^2 = 28(22)^2 + 52(22)^2 + 42(22)^2 + 78(22)^2$$

$$\chi^2 = \frac{484}{28} + \frac{484}{52} + \frac{484}{42} + \frac{484}{78} \chi^2 = 28484 + 52484 + 42484 + 78484$$

$$\chi^2 = 17.29 + 9.31 + 11.52 + 6.21 \chi^2 = 17.29 + 9.31 + 11.52 + 6.21$$

$$\chi^2 = 44.33 \chi^2 = 44.33$$

#### Step 3: Determine Degrees of Freedom

$$df = (2 - 1) \times (2 - 1) = 1$$

#### Step 4: Interpret the Result

Using a chi-square distribution table, we compare the calculated chi-square value (44.33) with the critical value at one degree of freedom and a significance level (e.g., 0.05), approximately 3.841. Since  $44.33 > 3.841$ , we reject the null hypothesis. This indicates a significant association between smoking status and the incidence of lung disease in this sample.

#### Conclusion

The chi-square test is a powerful tool for analyzing the relationship between categorical variables. By comparing observed and expected frequencies, researchers can determine if there is a statistically significant association, providing valuable insights in various fields of study.



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