Definition of Number Systems

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Contents

1 Definition of Number Systems

$$\mathbb{N} = \{x | x = 1 \vee \exists i, j \in \mathbb{N} : \ x = i + j\}$$

$$\mathbb{Z} = \{a - b | a, b \in \mathbb{N}\}$$

$$\mathbb{Q} = \left\{ \frac{a}{b} \middle| a, b \in \mathbb{Z} \wedge b \neq 0 \right\}$$

$$\begin{aligned}\mathbb{R} = \{b(x) | x \in \mathbb{N} \wedge a, b : \mathbb{N} \to \mathbb{Q} \\ \wedge \forall \epsilon > 0 : \ \exists M \in \mathbb{N} \text{ s.t. } \forall m, n \geq M : \ |a(m) - a(n)| < \epsilon \\ \wedge \forall \epsilon > 0 : \ \exists N \in \mathbb{N} \text{ s.t. } \forall m, n \geq M : \ |a(m) - a(n)| < \epsilon \\ \wedge \forall \epsilon > 0 : \ \exists \delta > 0 \text{ s.t. } a(\delta) - b(\delta) < \epsilon\}\end{aligned}$$

$$\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, \infty\}$$

$$\mathbb{C} = \left\{ a + bi \middle| a, b \in \mathbb{R} \wedge i^2 = -1 \right\}$$