Identities

沈威宇

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1 Identities (恒等式)

$$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$a^3+b^3=(a+b)(a^2-ab+b^2)=(a+b)^3-3ab(a+b)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)=(a-b)^3+3ab(a-b)$$

$$(a^2+ab+b^2)(a^2-ab+b^2)=a^4+a^2b^2+b^4$$

$$a^4+b^4+c^4-2(a^2b^2+b^2c^2+c^2a^2)=-(a+b+c)(-a+b+c)(a-b+c)(a+b-c)$$

$$\sum_{k=1}^{n} k=\frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2=\frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3=\left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{k=0}^{n} r^k=\frac{r^{n+1}-1}{r-1},\quad r\neq 1$$

$$z^n-a^n=(z-a)\sum_{k=0}^{n-1} z^k a^{n-1-k}=\prod_{k=0}^{n-1}\left(z-ae^{i\frac{2\pi k}{n}}\right),\quad z,a\in\mathbb{C}$$

$$\frac{1}{|z|-z}+\frac{1}{|z|-\bar{z}}=\frac{1}{|z|},\quad z\in\mathbb{C}\setminus\mathbf{R}_{\geq 0}$$

$$\frac{1}{|z|+z}+\frac{1}{|z|+\bar{z}}=\frac{1}{|z|},\quad z\in\mathbb{C}\setminus\mathbf{R}_{\leq 0}$$

$$\sum_{k=m}^{m+n-1} e^{i\frac{2\pi k}{n}}=0$$

$$\sum_{k=m}^{m+n-1}\left|e^{i\frac{2\pi k}{n}}-e^{i\frac{2\pi p}{n}}\right|^2=2n,\quad p\in\mathbb{N}\wedge m\leq p\leq m+n-1$$