Dirac  $\delta$  function:  $\delta(x)$ 

$$\delta(x) = \begin{cases} \infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$
 
$$\forall \epsilon > 0, f(a) \text{ is defined as : } \int_{a - \epsilon}^{a + \epsilon} f(x) \delta(x - a) \, \mathrm{d}x = f(a)$$
 
$$\int_{0^{-}}^{0^{+}} \delta(x) \, \mathrm{d}x = \frac{1}{2}$$
 
$$\int_{-\infty}^{\infty} \delta(x) \, \mathrm{d}x = 1$$