Definition of Number Systems

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January 7, 2025

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1 Definition of Number Systems

$$\begin{split} \mathbb{N} &= \{x \mid x = 1 \text{ or } (\exists \, i, \, j \in \mathbb{N} \, \colon \, x = i + j) \} \\ \mathbb{N}_0 &= \mathbb{N} \cup \{0\} \\ \mathbb{Z} &= \{a - b \mid a, \, b \in \mathbb{N} \} \\ \mathbb{Q} &= \left\{ \frac{a}{b} \mid a \in \mathbb{Z} \text{ and } b \in \mathbb{Z} \setminus \{0\} \right\} \\ \mathbb{R} &= \left\{ \lim_{n \to \infty} a(n) \mid a \colon \mathbb{N} \to \mathbb{Q} \text{ s.t. } \forall \, \epsilon > 0 \colon \exists \, \{N, \, n\} \subseteq \mathbb{N} \text{ s.t. } \forall \, n > N, |a(n) - a(N)| < \epsilon \right\} \\ \mathbb{R} &= \mathbb{R} \cup \{-\infty, \, +\infty \} \\ \mathbb{C} &= \left\{ a + bi \mid a, b \in \mathbb{R} \text{ and } i = \sqrt{-1} \right\} \end{split}$$