Identities

沈威宇

2025 年 3 月 17 日

目錄

第一節 Identities (恆等式)

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$$

$$(a^2 + ab + b^2)(a^2 - ab + b^2) = a^4 + a^2b^2 + b^4$$

$$a^4 + b^4 + c^4 - 2(a^2b^2 + b^2c^2 + c^2a^2) = -(a + b + c)(-a + b + c)(a - b + c)(a + b - c)$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{k=0}^{n} r^k = \frac{r^{n+1} - 1}{r - 1}, \quad r \neq 1$$

$$z^n - a^n = (z - a)\sum_{k=0}^{n-1} z^k a^{n-1-k} = \prod_{k=0}^{n-1}\left(z - ae^{i\frac{2\pi k}{n}}\right)$$

$$\frac{1}{|z| - z} + \frac{1}{|z| - \bar{z}} = \frac{1}{|z|}, \quad z \notin \mathbf{R}_{\geq 0}$$

$$\frac{1}{|z| + z} + \frac{1}{|z| + \bar{z}} = \frac{1}{|z|}, \quad z \notin \mathbf{R}_{\leq 0}$$

$$\sum_{k=m}^{m+n-1} e^{i\frac{2\pi k}{n}} = 0$$

$$\sum_{k=m}^{m+n-1} \left| e^{i\frac{2\pi k}{n}} - e^{i\frac{2\pi m}{n}} \right|^2 = 2n$$