Heaviside Step Function and Dirac Delta Function

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1 Heaviside Step Function and Dirac Delta Function

I Heaviside step function (黑維塞階躍函數)/unit step function (單位階躍函數)

$$H(x) = \begin{cases} 0, & x < 0 \\ \dfrac{1}{2}, & x = 0 \, , \\ 1, & x > 0 \end{cases}$$

where $H(0)$ is sometimes defined to be 1 or other values instead.

$$H'(x) = \delta(x).$$

II Dirac delta function

$$\delta(x) = \begin{cases} \infty, & x = 0, \\ 0, & x \neq 0. \end{cases}$$

$$\forall \epsilon > 0 : \int_{a-\epsilon}^{a+\epsilon} f(x)\delta(x-a)\,\mathrm{d}x = f(a).$$

$$\int_{-\infty}^{\infty} \delta(x)\,\mathrm{d}x = 1.$$

$$\int_{0}^{x} \delta(t)\,\mathrm{d}t = H(x).$$