

Calculus Cheat Sheet

Limits

$$\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1.$$

Derivatives

$$\frac{dx^n}{dx} = nx^{n-1}.$$

$$\frac{d|x|}{dx} = \text{sgn}(x), \quad x \neq 0.$$

$$\frac{d \ln(x)}{dx} = \frac{1}{x}.$$

$$\frac{da^x}{dx} = \ln(a)a^x.$$

$$\frac{d \sin(x)}{dx} = \cos(x).$$

$$\frac{d \cos(x)}{dx} = -\sin(x).$$

$$\frac{d \tan(x)}{dx} = \sec^2(x).$$

$$\frac{d \cot(x)}{dx} = -\csc^2(x).$$

$$\frac{d \sec(x)}{dx} = \tan(x) \sec(x).$$

$$\frac{d \csc(x)}{dx} = -\cot(x) \csc(x).$$

$$\frac{d \arcsin(x)}{dx} = \frac{1}{\sqrt{1-x^2}}.$$

$$\frac{d \arccos(x)}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

$$\frac{d \arctan(x)}{dx} = \frac{1}{1+x^2}.$$

$$\frac{d \operatorname{arccot}(x)}{dx} = -\frac{1}{1+x^2}.$$

$$\frac{d \operatorname{arcsec}(x)}{dx} = \frac{1}{|x|\sqrt{x^2-1}}.$$

$$\frac{d \operatorname{arccsc}(x)}{dx} = -\frac{1}{|x|\sqrt{x^2-1}}.$$

$$\frac{d \sinh(x)}{dx} = \cosh(x).$$

$$\frac{d \cosh(x)}{dx} = \sinh(x).$$

$$\frac{d \tanh(x)}{dx} = \operatorname{sech}^2(x).$$

$$\frac{d \coth(x)}{dx} = -\operatorname{csch}^2(x).$$

$$\frac{d \operatorname{sech}(x)}{dx} = -\tanh(x) \operatorname{sech}(x).$$

$$\frac{d \operatorname{csch}(x)}{dx} = -\coth(x) \operatorname{csch}(x).$$

$$\frac{d \operatorname{arcsinh}(x)}{dx} = \frac{1}{\sqrt{x^2 + 1}}.$$

$$\frac{d \operatorname{arccosh}(x)}{dx} = \frac{1}{\sqrt{x^2 - 1}}.$$

$$\frac{d \operatorname{arctanh}(x)}{dx} = \frac{1}{1 - x^2}, \quad |x| < 1.$$

$$\frac{d \operatorname{arccoth}(x)}{dx} = \frac{1}{1 - x^2}, \quad |x| > 1.$$

$$\frac{d \operatorname{arcsech}(x)}{dx} = -\frac{1}{x \sqrt{1 - x^2}}.$$

$$\frac{d \operatorname{arccsch}(x)}{dx} = -\frac{1}{|x| \sqrt{1 + x^2}}.$$

Integrals

$$\int x^{-1} dx = \ln|x| + C.$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1$$

$$\int \ln(x) dx = x \ln(x) - x + C.$$

$$\int (\ln(x))^n dx = x (\ln(x))^n - n \int (\ln(x))^{n-1} dx, \quad n \in \mathbb{N}.$$

$$\int e^{nx} dx = \frac{1}{n} e^{nx} + C, \quad n \neq 0.$$

$$\int \sin(x) dx = -\cos(x) + C.$$

$$\int \sin^2(x) dx = \frac{x}{2} - \frac{\sin(2x)}{4} + C.$$

$$\int \sin^n(x) dx = -\frac{1}{n} \sin^{n-1}(x) \cos(x) + \frac{n-1}{n} \int \sin^{n-2}(x) dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \cos(x) dx = \sin(x) + C.$$

$$\int \cos^2(x) dx = \frac{x}{2} + \frac{\sin(2x)}{4} + C.$$

$$\int \cos^n(x) dx = \frac{1}{n} \cos^{n-1}(x) \sin(x) + \frac{n-1}{n} \int \cos^{n-2}(x) dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \tan(x) dx = -\ln |\cos(x)| + C = \ln |\sec(x)| + C.$$

$$\int \tan^2(x) dx = \tan(x) - x + C.$$

$$\int \tan^n(x) dx = \frac{1}{n-1} \tan^{n-1}(x) - \int \tan^{n-2}(x) dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \cot(x) dx = \ln |\sin(x)| + C.$$

$$\int \cot^2(x) dx = -\cot(x) - x + C.$$

$$\int \cot^n(x) dx = -\frac{1}{n-1} \cot^{n-1}(x) - \int \cot^{n-2}(x) dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \sec(x) dx = \ln |\tan(x) + \sec(x)| + C = \ln \left| \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) \right| + C.$$

$$\int \sec^2(x) dx = \tan(x) + C.$$

$$\int \sec^n(x) dx = \frac{1}{n-1} \tan(x) \sec^{n-2}(x) + \frac{n-2}{n-1} \int \sec^{n-2}(x) dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \csc(x) dx = -\ln |\cot(x) + \csc(x)| + C = -\ln \left| \cot\left(\frac{x}{2}\right) \right| + C.$$

$$\int \csc^2(x) dx = -\cot(x) + C.$$

$$\int \csc^n(x) dx = -\frac{1}{n-1} \cot(x) \csc^{n-2}(x) + \frac{n-2}{n-1} \int \csc^{n-2}(x) dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \arcsin(x) dx = x \arcsin(x) + \sqrt{1-x^2} + C.$$

$$\int \arccos(x) dx = x \arccos(x) - \sqrt{1-x^2} + C.$$

$$\int \arctan(x) dx = x \arctan(x) - \frac{1}{2} \ln(1+x^2) + C.$$

$$\int \operatorname{arccot}(x) dx = x \operatorname{arccot}(x) + \frac{1}{2} \ln(1+x^2) + C.$$

$$\int \operatorname{arcsec}(x) dx = x \operatorname{arcsec}(x) - \operatorname{sgn}(x) \ln \left| x + \sqrt{x^2 - 1} \right| + C, \quad |x| \geq 1.$$

$$\int \operatorname{arccsc}(x) dx = x \operatorname{arccsc}(x) + \operatorname{sgn}(x) \ln \left| x + \sqrt{x^2 - 1} \right| + C, \quad |x| \geq 1.$$

$$\int \sinh(x) dx = \cosh(x) + C.$$

$$\int \sinh^2(x) dx = -\frac{x}{2} + \frac{\sinh(2x)}{4} + C.$$

$$\int \sinh^n(x) dx = \frac{1}{n} \sinh^{n-1}(x) \cosh(x) - \frac{n-1}{n} \int \sinh^{n-2}(x) dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \cosh(x) dx = \sinh(x) + C.$$

$$\int \cosh^2(x) dx = \frac{x}{2} + \frac{\sinh(2x)}{4} + C.$$

$$\int \cosh^n(x) dx = \frac{1}{n} \cosh^{n-1}(x) \sinh(x) + \frac{n-1}{n} \int \cosh^{n-2}(x) dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \tanh(x) dx = \ln(\cosh(x)) + C.$$

$$\int \tanh^2(x) dx = x - \tanh(x) + C.$$

$$\int \tanh^n(x) dx = \frac{1}{n-1} \tanh^{n-1}(x) - \int \tanh^{n-2}(x) dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \coth(x) dx = \ln |\sinh(x)| + C.$$

$$\int \coth^2(x) dx = x - \coth(x) + C.$$

$$\int \coth^n(x) dx = -\frac{1}{n-1} \coth^{n-1}(x) - \int \coth^{n-2}(x) dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \operatorname{sech}(x) dx = 2 \arctan\left(\tanh\left(\frac{x}{2}\right)\right) + C = \arctan(\sinh(x)) + C = \arcsin(\tanh(x)) + C.$$

$$\int \operatorname{sech}^2(x) dx = \tanh(x) + C.$$

$$\int \operatorname{sech}^n(x) dx = \frac{1}{n-1} \tanh(x) \operatorname{sech}^{n-1}(x) + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2}(x) dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \operatorname{csch}(x) dx = \ln \left| \tanh\left(\frac{x}{2}\right) \right| + C = \ln |\coth(x) - \operatorname{csch}(x)| + C.$$

$$\int \operatorname{csch}^2(x) dx = -\coth(x) + C.$$

$$\int \operatorname{csch}^n(x) dx = -\frac{1}{n-1} \coth(x) \operatorname{csch}^{n-1}(x) + \frac{n-2}{n-1} \int \operatorname{csch}^{n-2}(x) dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \operatorname{arcsinh}(x) dx = x \operatorname{arcsinh}(x) - \sqrt{x^2 + 1} + C.$$

$$\int \operatorname{arccosh}(x) dx = x \operatorname{arccosh}(x) - \sqrt{x^2 - 1} + C.$$

$$\int \operatorname{arctanh}(x) dx = x \operatorname{arctanh}(x) + \frac{1}{2} \ln(1 - x^2) + C.$$

$$\int \operatorname{arccoth}(x) dx = x \operatorname{arccoth}(x) + \frac{1}{2} \ln(x^2 - 1) + C.$$

$$\int \operatorname{arcsech}(x) dx = x \operatorname{arcsech}(x) + \arctan\left(\frac{\sqrt{1-x^2}}{x}\right) + C.$$

$$\int \operatorname{arccsch}(x) dx = x \operatorname{arccsch}(x) + \ln \left| x + \sqrt{x^2 + 1} \right| + C.$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C, \quad a > 0.$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln\left(\frac{x-a}{x+a}\right) + C = \frac{1}{a} \operatorname{arccoth}\left(\frac{x}{a}\right) + C, \quad a > 0 \wedge x^2 - a^2 > 0.$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \ln\left(\frac{a-x}{a+x}\right) + C = \frac{1}{a} \operatorname{arctanh}\left(\frac{x}{a}\right) + C, \quad a > 0 \wedge a^2 - x^2 > 0.$$

$$\begin{aligned}\int \frac{1}{ax+b} dx &= \frac{1}{a} \ln |ax+b| + C, \quad a > 0. \\ \int (ax+b)^n dx &= \frac{1}{a(n+1)} (ax+b)^{n+1} + C, \quad a \neq 0 \wedge n \neq -1. \\ \int \frac{x}{ax+b} dx &= \frac{x}{a} - \frac{b}{a^2} \ln |ax+b| + C, \quad a > 0. \\ \int \frac{x}{(ax+b)^2} dx &= \frac{1}{a^2} \ln |ax+b| + \frac{b}{a^2(ax+b)} + C, \quad a > 0. \\ \int x(ax+b)^n dx &= \frac{a(n+1)x-b}{a^2(n+1)(n+2)} (ax+b)^{n+1} + C, \quad a \neq 0 \wedge n \notin \{-1, -2\}.\end{aligned}$$

Let $r = \sqrt{a^2 + x^2}$.

$$\begin{aligned}\int r dx &= \frac{1}{2} (xr + a^2 \ln(x+r)) + C. \\ \int xr dx &= \frac{1}{3} r^3 + C. \\ \int \frac{1}{r} dx &= \operatorname{arcsinh}\left(\frac{x}{|a|}\right) + C = \ln(x+r) + C. \\ \int \frac{x}{r} dx &= r + C.\end{aligned}$$

Let $s = \sqrt{x^2 - a^2}$, $x^2 > a^2$.

$$\begin{aligned}\int s dx &= \frac{1}{2} (xs - a^2 \ln(x+s)) + C. \\ \int xs dx &= \frac{1}{3} s^3 + C. \\ \int \frac{1}{s} dx &= \operatorname{arcsin}\left(\frac{x}{|a|}\right) + C. \\ \int \frac{x}{s} dx &= -s + C.\end{aligned}$$

Let $q = \sqrt{a^2 - x^2}$, $a^2 \geq x^2$.

$$\begin{aligned}\int q dx &= \frac{1}{2} \left(q + a^2 \operatorname{arcsin}\left(\frac{x}{|a|}\right) \right) + C. \\ \int xq dx &= \frac{1}{3} q^3 + C. \\ \int \frac{1}{q} dx &= \operatorname{arcsin}\left(\frac{x}{|a|}\right) + C. \\ \int \frac{x}{q} dx &= q + C.\end{aligned}$$

Let $R = \sqrt{ax+b}$, $a \neq 0$.

$$\begin{aligned}\int R dx &= \frac{2}{3a} R^3 + C. \\ \int x^n R dx &= \frac{2}{a(2n+3)} \left(x^n R^3 - bn \int x^{n-1} R dx \right), \quad n \in \mathbb{N}. \\ \int \frac{1}{R} dx &= \frac{2R}{a} + C. \\ \int \frac{x^n}{R} dx &= \frac{2}{a} \left(x^n R - n \int x^{n-1} R dx \right) = \frac{2}{a(2n+1)} \left(x^n R - bn \int \frac{x^{n-1}}{R} dx \right), \quad n \in \mathbb{N}.\end{aligned}$$

Maclaurin Series and Taylor Series

$$x^n = \sum_{k=0}^{\infty} \binom{n}{k} a^{n-k} (x-a)^k, \quad |x-a| < |a|.$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1.$$

$$\frac{1}{1-x} = \frac{1}{1-a} \sum_{n=0}^{\infty} \left(\frac{x-a}{1-a} \right)^n, \quad |x-a| < |1-a|.$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n, \quad |x| < 1.$$

$$\frac{1}{1+x} = \frac{1}{1+a} \sum_{n=0}^{\infty} \left(\frac{-(x-a)}{1+a} \right)^n, \quad |x-a| < |1+a|.$$

$$\ln(x) = \ln(a) + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{na^n} (x-a)^n, \quad x \leq a.$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n, \quad x \leq 1.$$

$$\ln(1+x) = \ln(1+a) + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(1+a)^n} (x-a)^n, \quad x \leq 1+a.$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

$$e^x = e^a \sum_{n=0}^{\infty} \frac{(x-a)^n}{n!}.$$

$$b^x = \sum_{n=0}^{\infty} \frac{(\ln b)^n}{n!} x^n, \quad b > 0.$$

$$b^x = b^a \sum_{n=0}^{\infty} \frac{(\ln b)^n}{n!} (x-a)^n, \quad b > 0.$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}.$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{\sin\left(a + \frac{n\pi}{2}\right)}{n!} (x-a)^n.$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}.$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{\cos\left(a + \frac{n\pi}{2}\right)}{n!} (x-a)^n.$$