Calculus Cheat Sheet

$$\lim_{h \to 0} \frac{\sin h}{h} = 1.$$

$$\frac{dx^n}{dx} = nx^{n-1}.$$

$$\frac{d \ln(x)}{dx} = \frac{1}{x}.$$

$$\frac{da^x}{dx} = \ln(a)a^x.$$

$$\frac{d \sin(x)}{dx} = \cos(x).$$

$$\frac{d \cos(x)}{dx} = -\sin(x).$$

$$\frac{d \tan(x)}{dx} = \sec^2(x).$$

$$\frac{d \cot(x)}{dx} = -\csc^2(x).$$

$$\frac{d \sec(x)}{dx} = \tan(x)\sec(x).$$

$$\frac{d \csc(x)}{dx} = -\cot(x)\csc(x).$$

$$\frac{d \arcsin(x)}{dx} = \frac{1}{\sqrt{1-x^2}}.$$

$$\frac{d \arccos(x)}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

$$\frac{d \arctan(x)}{dx} = \frac{1}{1+x^2}.$$

$$\frac{d \text{arccot}(x)}{dx} = -\frac{1}{1+x^2}.$$

$$\frac{d \text{arcsec}(x)}{dx} = \frac{1}{|x|\sqrt{x^2-1}}.$$

$$\frac{d \text{arccsc}(x)}{dx} = -\frac{1}{|x|\sqrt{x^2-1}}.$$

$$\frac{d \sinh(x)}{dx} = \cosh(x).$$

$$\frac{d \cosh(x)}{dx} = \sinh(x).$$

$$\frac{d \tanh(x)}{dx} = \text{sech}^2(x).$$

$$\frac{d \coth(x)}{dx} = -\text{csch}^2(x), \quad x \neq 0.$$

$$\frac{d \text{sech}(x)}{dx} = -\tanh(x)\text{sech}(x).$$

$$\frac{d \text{csch}(x)}{dx} = -\coth(x)\text{csch}(x), \quad x \neq 0.$$

$$\frac{d\,\operatorname{arcsinh}(x)}{dx} = \frac{1}{\sqrt{x^2+1}}.$$

$$\frac{d\,\operatorname{arccosh}(x)}{dx} = \frac{1}{\sqrt{x^2-1}}, \quad x > 1.$$

$$\frac{d\,\operatorname{arctanh}(x)}{dx} = \frac{1}{1-x^2}, \quad |x| < 1.$$

$$\frac{d\,\operatorname{arccoth}(x)}{dx} = \frac{1}{1-x^2}, \quad |x| > 1.$$

$$\frac{d\,\operatorname{arcsech}(x)}{dx} = -\frac{1}{x\sqrt{1-x^2}}, \quad 0 < x < 1.$$

$$\frac{d\,\operatorname{arccsch}(x)}{dx} = -\frac{1}{|x|\sqrt{1+x^2}}, \quad x \neq 0.$$

$$\int x^n \, dx = \begin{cases} \frac{1}{n+1}x^{n+1} + C, & n \neq -1 \\ \ln|x| + C, & n = -1 \end{cases}.$$

$$\int \ln(x) \, dx = x\ln(x) - x + C.$$

$$\int \left(\ln(x)\right)^n dx = x\left(\ln(x)\right)^n - n\int \left(\ln(x)\right)^{n-1} dx, \quad n \in \mathbb{N}.$$

$$\int e^{nx} \, dx = \frac{1}{n}e^{nx} + C, \quad n \neq 0.$$

$$\int \sin(x) \, dx = -\cos(x) + C.$$

$$\int \sin^2(x) \, dx = \frac{x}{2} - \frac{\sin(2x)}{4} + C.$$

$$\int \sin^n(x) \, dx = -\frac{1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x) \, dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \cos(x) \, dx = \sin(x) + C.$$

$$\int \cos^2(x) \, dx = \frac{x}{2} + \frac{\sin(2x)}{4} + C.$$

$$\int \cos^n(x) \, dx = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x) \, dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \tan(x) \, dx = -\ln|\cos(x)| + C = \ln|\sec(x)| + C.$$

$$\int \tan^2(x) \, dx = \tan(x) - x + C.$$

$$\int \tan^n(x) \, dx = \frac{1}{n-1}\tan^{n-1}(x) - \int \tan^{n-2}(x) \, dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \cot(x) \, dx = \ln|\sin(x)| + C.$$

$$\int \cot^2(x) \, dx = -\cot(x) - x + C.$$

$$\int \cot^n(x) \, dx = -\frac{1}{n-1}\cot^{n-1}(x) - \int \cot^{n-2}(x) \, dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C.$$

$$\int \sec^2(x)\,dx = \tan(x) + C.$$

$$\int \sec^n(x)\,dx = \frac{1}{n-1}\sec^{n-2}(x)\tan(x) + \frac{n-2}{n-1}\int \sec^{n-2}(x)\,dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \csc(x)\,dx = -\ln|\csc(x) + \cot(x)| + C.$$

$$\int \csc^2(x)\,dx = -\cot(x) + C.$$

$$\int \csc^n(x)\,dx = -\frac{1}{n-1}\csc^{n-2}(x)\cot(x) + \frac{n-2}{n-1}\int \csc^{n-2}(x)\,dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \arcsin(x)\,dx = x\arcsin(x) + \sqrt{1-x^2} + C.$$

$$\int \arccos(x)\,dx = x\arccos(x) - \sqrt{1-x^2} + C.$$

$$\int \arctan(x)\,dx = x\arctan(x) - \frac{1}{2}\ln\left(1+x^2\right) + C.$$

$$\int \operatorname{arccot}(x)\,dx = x\operatorname{arccot}(x) + \frac{1}{2}\ln\left(1+x^2\right) + C.$$

$$\int \operatorname{arcsec}(x)\,dx = x\operatorname{arcsec}(x) - \operatorname{sgn}(x)\ln\left|x + \sqrt{x^2-1}\right| + C, \quad |x| \geq 1.$$

$$\int \operatorname{arccsc}(x)\,dx = x\operatorname{arccsc}(x) + \operatorname{sgn}(x)\ln\left|x + \sqrt{x^2-1}\right| + C, \quad |x| \geq 1.$$

$$\int \sinh(x)\,dx = \cosh(x) + C.$$

$$\int \sinh^2(x)\,dx = \frac{\sinh(2x)}{4} - \frac{x}{2} + C.$$

$$\int \sinh^n(x)\,dx = \frac{1}{n}\sinh^{n-1}(x)\cosh(x) - \frac{n-1}{n}\int \sinh^{n-2}(x)\,dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \cosh(x)\,dx = \sinh(x) + C.$$

$$\int \cosh^2(x)\,dx \frac{\sinh(2x)}{4} + \frac{x}{2} + C.$$

$$\int \cosh^n(x)\,dx = \frac{1}{n}\cosh^{n-1}(x)\sinh(x) + \frac{n-1}{n}\int \cosh^{n-2}(x)\,dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \tanh(x)\,dx = \ln\cosh(x) + C.$$

$$\int \tanh^2(x)\,dx = x - \tanh(x) + C.$$

$$\int \tanh^n(x)\,dx = \frac{1}{n-1}\tanh^{n-1}(x) - \int \tanh^{n-2}(x)\,dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \coth(x)\,dx = \ln|\sinh(x)| + C.$$

$$\int \coth^2(x)\,dx = x - \coth(x) + C.$$

$$\int \coth^n(x)\,dx = -\frac{1}{n-1}\coth^{n-1}(x) - \int \coth^{n-2}(x)\,dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \operatorname{sech}(x)\,dx = \operatorname{arctanh}\sinh(x) + C.$$

$$\int \operatorname{sech}^2(x)\,dx = \tanh(x) + C.$$

$$\int \operatorname{sech}^n(x)\,dx = \frac{1}{n-1}\operatorname{sech}^{n-1}(x)\tanh(x) + \frac{n-2}{n-1}\int \operatorname{sech}^{n-2}(x)\,dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \operatorname{csch}(x)\,dx = \ln\left|\tanh\frac{x}{2}\right| + C.$$

$$\int \operatorname{csch}^2(x)\,dx = -\coth(x) + C.$$

$$\int \operatorname{csch}^n(x)\,dx = -\frac{1}{n-1}\operatorname{csch}^{n-1}(x)\coth(x) + \frac{n-2}{n-1}\int \operatorname{csch}^{n-2}(x)\,dx, \quad n \in \mathbb{N}_{>2}.$$

$$\int \operatorname{arcsinh}(x)\,dx = x\operatorname{arcsinh}(x) - \sqrt{x^2+1} + C.$$

$$\int \operatorname{arccosh}(x)\,dx = x\operatorname{arccosh}(x) - \sqrt{x^2-1} + C, \quad (x) \geq 1.$$

$$\int \operatorname{arctanh}(x)\,dx = x\operatorname{arctanh}(x) + \frac{1}{2}\ln\left(1-x^2\right) + C, \quad |x| < 1.$$

$$\int \operatorname{arccoth}(x)\,dx = x\operatorname{arccoth}(x) + \frac{1}{2}\ln\left(x^2-1\right) + C, \quad |x| > 1.$$

$$\int \operatorname{arcsech}(x)\,dx = x\operatorname{arcsech}(x) + \arctan\left(\frac{\sqrt{1-x^2}}{x}\right) + C, \quad 0 < x \leq 1.$$

$$\int \operatorname{arccsch}(x)\,dx = x\operatorname{arccsch}(x) + \ln\left|x + \sqrt{x^2+1}\right| + C, \quad x \neq 0.$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C, \quad a \neq 0.$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) + C, \quad a \neq 0.$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left(\frac{a-x}{a+x}\right) + C, \quad a \neq 0.$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C, \quad a \neq 0.$$

$$\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n-1} + C, \quad a \neq 0 \wedge n \neq -1.$$

$$\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + C, \quad a \neq 0.$$

$$\int \frac{x}{(ax+b)^2}\,dx = \frac{1}{a^2}\ln|ax+b| + \frac{b}{a^2(ax+b)} + C, \quad a \neq 0.$$

$$\int x(ax+b)^n\,dx = \frac{a(n+1)x-b}{a^2(n+1)(n+2)}(ax+b)^{n+1} + C, \quad a \neq 0 \wedge n \notin \{-1,-2\}.$$

$$\int \sqrt{a^2 + x^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\ln\left(x + \sqrt{a^2 + x^2}\right) + C.$$

$$\int x\sqrt{a^2 + x^2}\,\mathrm{d}x = \frac{1}{3}(a^2 + x^2)^{3/2} + C.$$

$$\int \frac{1}{\sqrt{a^2 + x^2}}\,\mathrm{d}x = \operatorname{arcsinh}\frac{x}{|a|} + C = \ln\left(x + \sqrt{a^2 + x^2}\right) + C.$$

$$\int \frac{x}{\sqrt{a^2 + x^2}}\,\mathrm{d}x = \sqrt{a^2 + x^2} + C.$$

$$\int \sqrt{x^2 - a^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left(x + \sqrt{x^2 - a^2}\right) + C, \quad x^2 > a^2.$$

$$\int x\sqrt{x^2 - a^2}\,\mathrm{d}x = \frac{1}{3}(x^2 - a^2)^{3/2} + C, \quad x^2 > a^2.$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,\mathrm{d}x = \arcsin\left(\frac{x}{|a|}\right) + C, \quad x^2 > a^2.$$

$$\int \frac{x}{\sqrt{a^2 - x^2}}\,\mathrm{d}x = -\sqrt{a^2 - x^2} + C, \quad x^2 > a^2.$$

$$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\arcsin\frac{x}{|a|} + C, \quad a^2 \geq x^2.$$

$$\int x\sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{3}(a^2 - x^2)^{3/2} + C, \quad a^2 \geq x^2.$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,\mathrm{d}x = \arcsin\left(\frac{x}{|a|}\right) + C, \quad a^2 \geq x^2.$$

$$\int \frac{x}{\sqrt{a^2 - x^2}}\,\mathrm{d}x = \sqrt{a^2 - x^2} + C, \quad a^2 \geq x^2.$$

$$\int \sqrt{ax + b}\,\mathrm{d}x = \frac{2}{3a}(ax + b)^{3/2} + C, \quad a \neq 0.$$

$$\int x^n \sqrt{ax + b}\,\mathrm{d}x = \frac{2}{a(2n + 3)}\left(x^n(ax + b)^{3/2} - bn\int x^{n-1}\sqrt{ax + b}\,\mathrm{d}x\right), \quad a \neq 0 \wedge n \in \mathbb{N}.$$

$$\int \frac{1}{\sqrt{ax + b}}\,\mathrm{d}x = \frac{2\sqrt{ax + b}}{a} + C, \quad a \neq 0.$$

$$\int \frac{x^n}{\sqrt{ax + b}}\,\mathrm{d}x = \frac{2}{a}\left(x^n\sqrt{ax + b} - n\int x^{n-1}\sqrt{ax + b}\,\mathrm{d}x\right), \quad a \neq 0 \wedge n \in \mathbb{N},$$

$$= \frac{2}{a(2n + 1)}\left(x^n\sqrt{ax + b} - bn\int \frac{x^{n-1}}{\sqrt{ax + b}}\,\mathrm{d}x\right).$$

$$\int_{-\infty}^{\infty} e^{-t^2}\,\mathrm{d}t = \sqrt{\pi}.$$

$$\int_{0}^{\infty} e^{-t^2}\,\mathrm{d}t = \frac{\sqrt{\pi}}{2}.$$