Calculus Cheat Sheet

Limits

$$\lim_{h \to 0} \frac{\sin h}{h} = 1.$$

Derivatives

$$\frac{\mathrm{d}x^n}{\mathrm{d}x} = nx^{n-1}.$$

$$\frac{\mathrm{d}|x|}{\mathrm{d}x} = \operatorname{sgn}(x), \quad x \neq 0.$$

$$\frac{\mathrm{d}\ln(x)}{\mathrm{d}x} = \frac{1}{x}.$$

$$\frac{\mathrm{d}a^x}{\mathrm{d}x} = \ln(a)a^x.$$

$$\frac{\mathrm{d}\sin(x)}{\mathrm{d}x} = \cos(x).$$

$$\frac{\mathrm{d}\cos(x)}{\mathrm{d}x} = -\sin(x).$$

$$\frac{\mathrm{d}\tan(x)}{\mathrm{d}x} = \sec^2(x).$$

$$\frac{\mathrm{d}\cot(x)}{\mathrm{d}x} = -\csc^2(x).$$

$$\frac{\mathrm{d}\sec(x)}{\mathrm{d}x} = \tan(x)\sec(x).$$

$$\frac{\mathrm{d}\csc(x)}{\mathrm{d}x} = -\cot(x)\csc(x).$$

$$\frac{\mathrm{d}\arcsin(x)}{\mathrm{d}x} = \frac{1}{\sqrt{1-x^2}}.$$

$$\frac{\mathrm{d}\arccos(x)}{\mathrm{d}x} = -\frac{1}{\sqrt{1-x^2}}.$$

$$\frac{\mathrm{d}\arctan(x)}{\mathrm{d}x} = \frac{1}{1+x^2}.$$

$$\frac{\mathrm{d}\operatorname{arccot}(x)}{\mathrm{d}x} = -\frac{1}{1+x^2}.$$

$$\frac{\mathrm{d}\operatorname{arcsec}(x)}{\mathrm{d}x} = \frac{1}{|x|\sqrt{x^2-1}}.$$

$$\frac{\mathrm{d}\operatorname{arccsc}(x)}{\mathrm{d}x} = -\frac{1}{|x|\sqrt{x^2-1}}.$$

$$\frac{\mathrm{d}\sinh(x)}{\mathrm{d}x} = \cosh(x).$$

$$\frac{\mathrm{d}\cosh(x)}{\mathrm{d}x} = \sinh(x).$$

$$\frac{\mathrm{d}\tanh(x)}{\mathrm{d}x} = \operatorname{sech}^2(x).$$

$$\frac{\mathrm{dcoth}(x)}{\mathrm{d}x} = -\mathrm{csch}^2(x).$$

$$\frac{\mathrm{dsech}(x)}{\mathrm{d}x} = -\tanh(x)\mathrm{sech}(x).$$

$$\frac{\mathrm{dcsch}(x)}{\mathrm{d}x} = -\coth(x)\mathrm{csch}(x).$$

$$\frac{\mathrm{darcsinh}(x)}{\mathrm{d}x} = \frac{1}{\sqrt{x^2+1}}.$$

$$\frac{\mathrm{darccosh}(x)}{\mathrm{d}x} = \frac{1}{\sqrt{x^2-1}}, \quad x > 1.$$

$$\frac{\mathrm{darctanh}(x)}{\mathrm{d}x} = \frac{1}{1-x^2}, \quad |x| < 1.$$

$$\frac{\mathrm{darccoth}(x)}{\mathrm{d}x} = \frac{1}{1-x^2}, \quad |x| > 1.$$

$$\frac{\mathrm{darcsech}(x)}{\mathrm{d}x} = -\frac{1}{x\sqrt{1-x^2}}, \quad 0 < x \leq 1.$$

$$\frac{\mathrm{darccsch}(x)}{\mathrm{d}x} = -\frac{1}{|x|\sqrt{1+x^2}}.$$

Integrals

$$\int x^n \,\mathrm{d}x = \begin{cases} \frac{1}{n+1}x^{n+1} + C, & n \neq -1 \\ \ln|x| + C, & n = -1 \end{cases}.$$

$$\int \ln(x)\,\mathrm{d}x = x\ln(x) - x + C.$$

$$\int \big(\ln(x)\big)^n \,\mathrm{d}x = x\big(\ln(x)\big)^n - n\int \big(\ln(x)\big)^{n-1}\,\mathrm{d}x, \quad n \in \mathbb{N}.$$

$$\int e^{nx}\,\mathrm{d}x = \frac{1}{n}e^{nx} + C, \quad n \neq 0.$$

$$\int \sin(x)\,\mathrm{d}x = -\cos(x) + C.$$

$$\int \sin^2(x)\,\mathrm{d}x = \frac{x}{2} - \frac{\sin(2x)}{4} + C.$$

$$\int \sin^n(x)\,\mathrm{d}x = -\frac{1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)\,\mathrm{d}x, \quad n \in \mathbb{N}_{>2}.$$

$$\int \cos(x)\,\mathrm{d}x = \sin(x) + C.$$

$$\int \cos^2(x)\,\mathrm{d}x = \frac{x}{2} + \frac{\sin(2x)}{4} + C.$$

$$\int \cos^n(x)\,\mathrm{d}x = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,\mathrm{d}x, \quad n \in \mathbb{N}_{>2}.$$

$$\int \tan(x)\,\mathrm{d}x = -\ln|\cos(x)| + C = \ln|\sec(x)| + C.$$

$$\int \tan^2(x)\,\mathrm{d}x = \tan(x) - x + C.$$

$$\int \tan^n(x)\,\mathrm{d}x = \frac{1}{n-1}\tan^{n-1}(x) - \int \tan^{n-2}(x)\,\mathrm{d}x\,, \quad n \in \mathbb{N}_{>2}.$$

$$\int \cot(x)\,\mathrm{d}x = \ln|\sin(x)| + C.$$

$$\int \cot^2(x)\,\mathrm{d}x = -\cot(x) - x + C.$$

$$\int \cot^n(x)\,\mathrm{d}x = -\frac{1}{n-1}\cot^{n-1}(x) - \int \cot^{n-2}(x)\,\mathrm{d}x\,, \quad n \in \mathbb{N}_{>2}.$$

$$\int \sec(x)\,\mathrm{d}x = \ln|\tan(x) + \sec(x)| + C = \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C.$$

$$\int \sec^2(x)\,\mathrm{d}x = \tan(x) + C.$$

$$\int \sec^n(x)\,\mathrm{d}x = \frac{1}{n-1}\tan(x)\sec^{n-2}(x) + \frac{n-2}{n-1}\int \sec^{n-2}(x)\,\mathrm{d}x\,, \quad n \in \mathbb{N}_{>2}.$$

$$\int \csc(x)\,\mathrm{d}x = -\ln|\cot(x) + \csc(x)| + C = -\ln\left|\cot\left(\frac{x}{2}\right)\right| + C.$$

$$\int \csc^2(x)\,\mathrm{d}x = -\cot(x) + C.$$

$$\int \csc^n(x)\,\mathrm{d}x = -\frac{1}{n-1}\cot(x)\csc^{n-2}(x) + \frac{n-2}{n-1}\int \csc^{n-2}(x)\,\mathrm{d}x\,, \quad n \in \mathbb{N}_{>2}.$$

$$\int \arcsin(x)\,\mathrm{d}x = x\arcsin(x) + \sqrt{1-x^2} + C.$$

$$\int \arccos(x)\,\mathrm{d}x = x\arccos(x) - \sqrt{1-x^2} + C.$$

$$\int \arctan(x)\,\mathrm{d}x = x\arctan(x) - \frac{1}{2}\ln\left(1+x^2\right) + C.$$

$$\int \operatorname{arccot}(x)\,\mathrm{d}x = x\operatorname{arccot}(x) + \frac{1}{2}\ln\left(1+x^2\right) + C.$$

$$\int \operatorname{arcsec}(x)\,\mathrm{d}x = x\operatorname{arcsec}(x) - \operatorname{sgn}(x)\ln\left|x + \sqrt{x^2-1}\right| + C, \quad |x| \geq 1.$$

$$\int \operatorname{arccsc}(x)\,\mathrm{d}x = x\operatorname{arccsc}(x) + \operatorname{sgn}(x)\ln\left|x + \sqrt{x^2-1}\right| + C, \quad |x| \geq 1.$$

$$\int \sinh(x)\,\mathrm{d}x = \cosh(x) + C.$$

$$\int \sinh^2(x)\,\mathrm{d}x = -\frac{x}{2} + \frac{\sinh(2x)}{4} + C.$$

$$\int \sinh^n(x)\,\mathrm{d}x = \frac{1}{n}\sinh^{n-1}(x)\cosh(x) - \frac{n-1}{n}\int \sinh^{n-2}(x)\,\mathrm{d}x\,, \quad n \in \mathbb{N}_{>2}.$$

$$\int \cosh(x)\,\mathrm{d}x = \sinh(x) + C.$$

$$\int \cosh^2(x)\,\mathrm{d}x = \frac{x}{2} + \frac{\sinh(2x)}{4} + C.$$

$$\int \cosh^n(x)\,\mathrm{d}x = \frac{1}{n}\cosh^{n-1}(x)\sinh(x) + \frac{n-1}{n}\int \cosh^{n-2}(x)\,\mathrm{d}x\,, \quad n \in \mathbb{N}_{>2}.$$

$$\int \tanh(x)\,\mathrm{d}x = \ln\big(\cosh(x)\big) + C.$$

$$\int \tanh^2(x)\,\mathrm{d}x = x - \tanh(x) + C.$$

$$\int \tanh^n(x)\,\mathrm{d}x = \frac{1}{n-1}\tanh^{n-1}(x) - \int \tanh^{n-2}(x)\,\mathrm{d}x\,, \quad n \in \mathbb{N}_{>2}.$$

$$\int \coth(x)\,\mathrm{d}x = \ln|\sinh(x)| + C.$$

$$\int \coth^2(x)\,\mathrm{d}x = x - \coth(x) + C.$$

$$\int \coth^n(x)\,\mathrm{d}x = -\frac{1}{n-1}\coth^{n-1}(x) - \int \coth^{n-2}(x)\,\mathrm{d}x\,, \quad n \in \mathbb{N}_{>2}.$$

$$\int \operatorname{sech}(x)\,\mathrm{d}x = 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + C = \arctan\big(\sinh(x)\big) + C = \arcsin\big(\tanh(x)\big) + C.$$

$$\int \operatorname{sech}^2(x)\,\mathrm{d}x = \tanh(x) + C.$$

$$\int \operatorname{sech}^n(x)\,\mathrm{d}x = \frac{1}{n-1}\tanh(x)\operatorname{sech}^{n-1}(x) + \frac{n-2}{n-1}\int \operatorname{sech}^{n-2}(x)\,\mathrm{d}x\,, \quad n \in \mathbb{N}_{>2}.$$

$$\int \operatorname{csch}(x)\,\mathrm{d}x = \ln\left|\tanh\left(\frac{x}{2}\right)\right| + C = \ln|\coth(x) - \operatorname{csch}(x)| + C.$$

$$\int \operatorname{csch}^2(x)\,\mathrm{d}x = -\coth(x) + C.$$

$$\int \operatorname{csch}^n(x)\,\mathrm{d}x = -\frac{1}{n-1}\coth(x)\operatorname{csch}^{n-1}(x) + \frac{n-2}{n-1}\int \operatorname{csch}^{n-2}(x)\,\mathrm{d}x\,, \quad n \in \mathbb{N}_{>2}.$$

$$\int \operatorname{arcsinh}(x)\,\mathrm{d}x = x\operatorname{arcsinh}(x) - \sqrt{x^2+1} + C.$$

$$\int \operatorname{arccosh}(x)\,\mathrm{d}x = x\operatorname{arccosh}(x) - \sqrt{x^2-1} + C.$$

$$\int \operatorname{arctanh}(x)\,\mathrm{d}x = x\operatorname{arctanh}(x) + \frac{1}{2}\ln\big(1-x^2\big) + C.$$

$$\int \operatorname{arccoth}(x)\,\mathrm{d}x = x\operatorname{arccoth}(x) + \frac{1}{2}\ln\big(x^2-1\big) + C.$$

$$\int \operatorname{arcsech}(x)\,\mathrm{d}x = x\operatorname{arcsech}(x) + \arctan\left(\frac{\sqrt{1-x^2}}{x}\right) + C.$$

$$\int \operatorname{arccsch}(x)\,\mathrm{d}x = x\operatorname{arccsch}(x) + \ln\left|x + \sqrt{x^2+1}\right| + C.$$

$$\int \frac{1}{x^2+a^2}\,\mathrm{d}x = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C, \quad a > 0.$$

$$\int \frac{1}{x^2-a^2}\,\mathrm{d}x = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) + C, \quad a > 0 \wedge x^2 - a^2 > 0.$$

$$\int \frac{1}{a^2-x^2}\,\mathrm{d}x = \frac{1}{2a}\ln\left(\frac{a-x}{a+x}\right) + C, \quad a > 0 \wedge a^2 - x^2 > 0.$$

$$\int \frac{1}{ax+b}\,\mathrm{d}x = \frac{1}{a}\ln|ax+b| + C, \quad a > 0.$$

$$\int (ax+b)^n \,\mathrm{d}x = \frac{1}{a(n+1)}(ax+b)^{n+1} + C, \quad a \neq 0 \wedge n \neq -1.$$

$$\int \frac{x}{ax+b} \,\mathrm{d}x = \frac{x}{a} - \frac{b}{a^2} \ln|ax+b| + C, \quad a > 0.$$

$$\int \frac{x}{(ax+b)^2} \,\mathrm{d}x = \frac{1}{a^2} \ln|ax+b| + \frac{b}{a^2(ax+b)} + C, \quad a > 0.$$

$$\int x(ax+b)^n \,\mathrm{d}x = \frac{a(n+1)x - b}{a^2(n+1)(n+2)}(ax+b)^{n+1} + C, \quad a \neq 0 \wedge n \notin \{-1,-2\}.$$

Let $r = \sqrt{a^2+x^2}$.

$$\int r \,\mathrm{d}x = \frac{1}{2}\big(xr + a^2 \ln(x+r)\big) + C.$$

$$\int xr \,\mathrm{d}x = \frac{1}{3}r^3 + C.$$

$$\int \frac{1}{r} \,\mathrm{d}x = \operatorname{arcsinh}\left(\frac{x}{|a|}\right) + C = \ln(x+r) + C.$$

$$\int \frac{x}{r} \,\mathrm{d}x = r + C.$$

Let $s = \sqrt{x^2-a^2}, \quad x^2 > a^2.$

$$\int s \,\mathrm{d}x = \frac{1}{2}\big(xs - a^2 \ln(x+s)\big) + C.$$

$$\int xs \,\mathrm{d}x = \frac{1}{3}s^3 + C.$$

$$\int \frac{1}{s} \,\mathrm{d}x = \arcsin\left(\frac{x}{|a|}\right) + C.$$

$$\int \frac{x}{s} \,\mathrm{d}x = -s + C.$$

Let $q = \sqrt{a^2-x^2}, \quad a^2 \geq x^2.$

$$\int q \,\mathrm{d}x = \frac{1}{2}\left(q + a^2 \arcsin\left(\frac{x}{|a|}\right)\right) + C.$$

$$\int xq \,\mathrm{d}x = \frac{1}{3}q^3 + C.$$

$$\int \frac{1}{q} \,\mathrm{d}x = \arcsin\left(\frac{x}{|a|}\right) + C.$$

$$\int \frac{x}{q} \,\mathrm{d}x = q + C.$$

Let $R = \sqrt{ax+b}, \quad a \neq 0.$

$$\int R \,\mathrm{d}x = \frac{2}{3a}R^3 + C.$$

$$\int x^n R \,\mathrm{d}x = \frac{2}{a(2n+3)}\left(x^n R^3 - bn \int x^{n-1} R \,\mathrm{d}x\right), \quad n \in \mathbb{N}.$$

$$\int \frac{1}{R} \,\mathrm{d}x = \frac{2R}{a} + C.$$

$$\int \frac{x^n}{R} \,\mathrm{d}x = \frac{2}{a}\left(x^n R - n \int x^{n-1} R \,\mathrm{d}x\right) = \frac{2}{a(2n+1)}\left(x^n R - bn \int \frac{x^{n-1}}{R} \,\mathrm{d}x\right), \quad n \in \mathbb{N}.$$

Maclaurin Series and Taylor Series

$$x^n = \sum_{k=0}^{\infty} \binom{n}{k} a^{n-k}(x-a)^k, \quad |x-a| < |a|.$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1.$$

$$\frac{1}{1-x} = \frac{1}{1-a}\sum_{n=0}^{\infty} \left(\frac{x-a}{1-a}\right)^n, \quad |x-a| < |1-a|.$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n, \quad |x| < 1.$$

$$\frac{1}{1+x} = \frac{1}{1+a}\sum_{n=0}^{\infty} \left(\frac{-(x-a)}{1+a}\right)^n, \quad |x-a| < |1+a|.$$

$$\ln(x) = \ln(a) + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{na^n}(x-a)^n, \quad x \leq a.$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n, \quad x \leq 1.$$

$$\ln(1+x) = \ln(1+a) + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(1+a)^n}(x-a)^n, \quad x \leq 1+a.$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

$$e^x = e^a \sum_{n=0}^{\infty} \frac{(x-a)^n}{n!}.$$

$$b^x = \sum_{n=0}^{\infty} \frac{(\ln b)^n}{n!} x^n, \quad b > 0.$$

$$b^x = b^a \sum_{n=0}^{\infty} \frac{(\ln b)^n}{n!} (x-a)^n, \quad b > 0.$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}.$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{\sin\left(a + \frac{n\pi}{2}\right)}{n!}(x-a)^n.$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}.$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{\cos\left(a + \frac{n\pi}{2}\right)}{n!}(x-a)^n.$$