Hyperbolic Function Cheat Sheet

$$\text{sinh}(x) = \frac{e^x - e^{-x}}{2}$$
$$\text{cosh}(x) = \frac{e^x + e^{-x}}{2}$$
$$\text{tanh}(x) = \frac{e^{2x} - 1}{e^{2x} + 1}$$
$$\text{coth}(x) = \frac{e^{2x} + 1}{e^{2x} - 1}$$
$$\text{sech}(x) = \frac{2}{e^x + e^{-x}}$$
$$\text{csch}(x) = \frac{2}{e^x - e^{-x}}$$
$$\text{arcsinh}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$$
$$\text{arccosh}(x) = \ln\left(x + \sqrt{x^2 - 1}\right), \quad x \geq 1$$
$$\text{arctanh}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \quad |x| < 1$$
$$\text{arccoth}(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right), \quad |x| > 1$$
$$\text{arcsech}(x) = \ln\left(\frac{1}{x} + \frac{\sqrt{1 - x^2}}{x}\right), \quad 0 < x \leq 1$$
$$\text{arccsch}(x) = \ln\left(\frac{1}{x} + \frac{\sqrt{1 + x^2}}{|x|}\right), \quad x \neq 0$$

$$\sinh\theta \,\text{csch}\,\theta = 1$$
$$\cosh\theta \,\text{sech}\,\theta = 1$$
$$\tanh\theta \coth\theta = 1$$

$$\tanh\theta \cosh\theta = \sinh\theta$$
$$\sinh\theta \coth\theta = \cosh\theta$$
$$\sinh\theta \,\text{sech}\,\theta = \tanh\theta$$
$$\cosh\theta \,\text{csch}\,\theta = \coth\theta$$
$$\tanh\theta \,\text{csch}\,\theta = \text{sech}\,\theta$$
$$\coth\theta \,\text{sech}\,\theta = \text{csch}\,\theta$$

$$\sinh^2\theta + 1 = \cosh^2\theta$$
$$\tanh^2\theta + \text{sech}^2\theta = 1$$
$$\coth^2\theta + \text{csch}^2\theta = 1$$

$$\cosh x + \sinh x = e^x$$
$$\cosh x - \sinh x = e^{-x}$$

$$\sinh\theta = \frac{2\tanh\frac{\theta}{2}}{1-\tanh^2\frac{\theta}{2}}$$

$$\cosh\theta = \frac{1+\tanh^2\frac{\theta}{2}}{1-\tanh^2\frac{\theta}{2}}$$

$$\tanh\theta = \frac{2\tanh\frac{\theta}{2}}{1+\tanh^2\frac{\theta}{2}}$$

$$\sinh 2\theta = 2\sinh\theta\cosh\theta$$

$$\begin{aligned}\cosh 2\theta &= 1+2\sinh^2\theta \\ &= 2\cosh^2\theta - 1 \\ &= \cosh^2\theta + \sinh^2\theta\end{aligned}$$

$$\sinh\frac{\theta}{2} = \operatorname{sgn}(\theta)\sqrt{\frac{\cosh\theta-1}{2}}$$

$$\cosh\frac{\theta}{2} = \sqrt{\frac{\cosh\theta+1}{2}}$$

$$\begin{aligned}\tanh\frac{\theta}{2} &= \operatorname{sgn}(\theta)\sqrt{\frac{\cosh\theta-1}{\cosh\theta+1}} \\ &= \frac{\sinh\theta}{\cosh\theta+1} \\ &= \frac{\cosh\theta-1}{\sinh\theta} \\ &= \frac{\sinh\theta+\cosh\theta-1}{\sinh\theta+\cosh\theta+1} \\ &= \coth\theta - \operatorname{csch}\theta\end{aligned}$$

$$\sinh 3\theta = 3\sinh\theta + 4\sinh^3\theta$$

$$\cosh 3\theta = 4\cosh^3\theta - 3\cosh\theta$$

$$\tanh 3\theta = \frac{3\tanh\theta+\tanh^3\theta}{1+3\tanh^2\theta}$$

$$\sinh(\alpha+\beta) = \sinh\alpha\cosh\beta + \cosh\alpha\sinh\beta$$

$$\sinh(\alpha-\beta) = \sinh\alpha\cosh\beta - \cosh\alpha\sinh\beta$$

$$\cosh(\alpha+\beta) = \cosh\alpha\cosh\beta + \sinh\alpha\sinh\beta$$

$$\cosh(\alpha-\beta) = \cosh\alpha\cosh\beta - \sinh\alpha\sinh\beta$$

$$\tanh(\alpha+\beta) = \frac{\tanh\alpha+\tanh\beta}{1+\tanh\alpha\tanh\beta}$$

$$\tanh(\alpha-\beta) = \frac{\tanh\alpha-\tanh\beta}{1-\tanh\alpha\tanh\beta}$$

$$\coth(\alpha+\beta) = \frac{\coth\alpha\coth\beta+1}{\coth\alpha+\coth\beta}$$

$$\coth(\alpha-\beta) = \frac{\coth\alpha\coth\beta-1}{\coth\beta-\coth\alpha}$$

$$\operatorname{sech}(\alpha+\beta) = \frac{\operatorname{sech}\alpha\operatorname{sech}\beta}{1+\tanh\alpha\tanh\beta} = \frac{\operatorname{csch}\alpha\operatorname{csch}\beta}{\coth\alpha\coth\beta+1}$$

$$\operatorname{sech}(\alpha-\beta) = \frac{\operatorname{sech}\alpha\operatorname{sech}\beta}{1-\tanh\alpha\tanh\beta} = \frac{\operatorname{csch}\alpha\operatorname{csch}\beta}{\coth\alpha\coth\beta-1}$$

$$\operatorname{csch}(\alpha+\beta)=\frac{\operatorname{csch}\alpha\operatorname{csch}\beta}{\coth\alpha+\coth\beta}=\frac{\operatorname{sech}\alpha\operatorname{sech}\beta}{\tanh\alpha+\tanh\beta}$$

$$\operatorname{csch}(\alpha-\beta)=\frac{\operatorname{csch}\alpha\operatorname{csch}\beta}{\coth\beta-\coth\alpha}=\frac{\operatorname{sech}\alpha\operatorname{sech}\beta}{\tanh\alpha-\tanh\beta}$$

$$\sinh^2\theta=\frac{\tanh^2\theta}{1-\tanh^2\theta}$$

$$\cosh^2\theta=\frac{1}{1-\tanh^2\theta}$$

$$\coth^2\theta=\frac{1}{\tanh^2\theta}$$

$$\operatorname{sech}^2\theta=1-\tanh^2\theta$$

$$\operatorname{csch}^2\theta=\frac{1-\tanh^2\theta}{\tanh^2\theta}$$

$$\sinh\alpha+\sinh\beta=2\sinh\frac{\alpha+\beta}{2}\cosh\frac{\alpha-\beta}{2}$$

$$\sinh\alpha-\sinh\beta=2\cosh\frac{\alpha+\beta}{2}\sinh\frac{\alpha-\beta}{2}$$

$$\cosh\alpha+\cosh\beta=2\cosh\frac{\alpha+\beta}{2}\cosh\frac{\alpha-\beta}{2}$$

$$\cosh\alpha-\cosh\beta=2\sinh\frac{\alpha+\beta}{2}\sinh\frac{\alpha-\beta}{2}$$

$$2\sinh\alpha\cosh\beta=\sinh(\alpha+\beta)+\sinh(\alpha-\beta)$$

$$2\cosh\alpha\sinh\beta=\sinh(\alpha+\beta)-\sinh(\alpha-\beta)$$

$$2\cosh\alpha\cosh\beta=\cosh(\alpha+\beta)+\cosh(\alpha-\beta)$$

$$2\sinh\alpha\sinh\beta=\cosh(\alpha+\beta)-\cosh(\alpha-\beta)$$