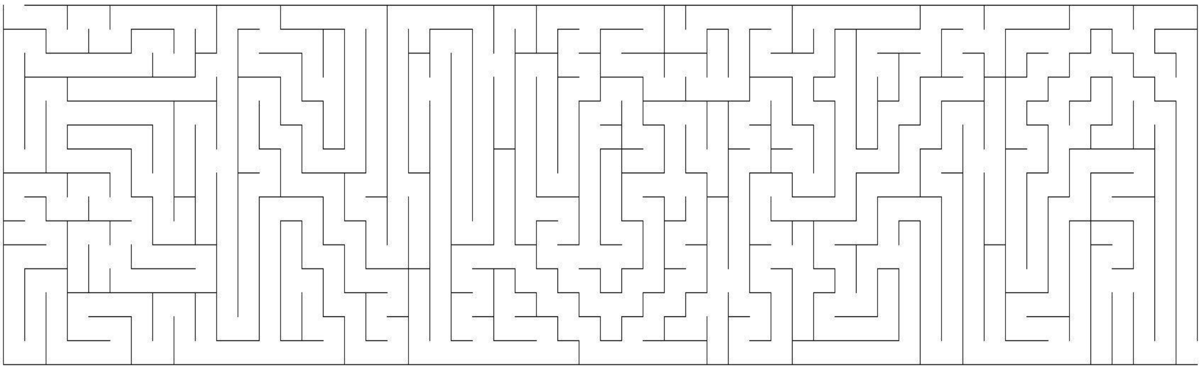
Enigma:
The Newsletter of PuzzleSIG



ENIGMA

247

(ALL DAY EVERY DAY)

'Mathematical problems, or puzzles, are important to real mathematics, just as fables, stories, and anecdotes are important to the young in understanding real life'.

Terry Tao

October 2020

Electronic Version of this Newsletter
Email enigma.mensa@yahoo.co.uk and I'll send you a copy

About Enigma
Enigma is the newsletter of Puzzle SIG.
The SIG for anyone interested in puzzles. The scope covers word puzzles, crosswords, logic puzzles, Japanese puzzles, mathematical brain teasers, lateral thinking problems, quizzes and picture quizzes, discussion of physical / mechanical puzzles, computer / internet based puzzles and puzzle games, and puzzle books and publications, and experimentation and innovation of new puzzle forms.

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How to Join
You can join Puzzle SIG by visiting mensa.org.uk/sigs (member login required).

Welcome to Enigma 247

Hello and welcome to another issue of Enigma.

Thanks to the contributors to this issue: John Causer, Willie Verger, Christa Ramonat and Austin Line.

The day that I'm writing this an exciting puzzle-related thing happened to me personally. Top maths puzzle YouTuber Presh Talwalkar, whose channel 'Mind Your Decisions' featuring short twice-weekly puzzles and having 2 million subscribers, used one of my puzzles, and my worked solution, for one of his videos. The seven minute video, concerning the area of a hexagon given very sparse information, is brilliantly explained with superb simple animations. I feel immensely proud of being a part of it.

You can find it via the following link: https://youtu.be/Yy1EV9XCbtw

Please try all of the puzzles, and if you have any feedback, let me know and I'll pass it on to the puzzle's creator.

As ever, if you get stuck and need a hint, drop me a line and I'll be happy to help.

Please keep your puzzles, answers, comments, queries, suggestions, etc coming in.

Happy puzzling

Elliott.

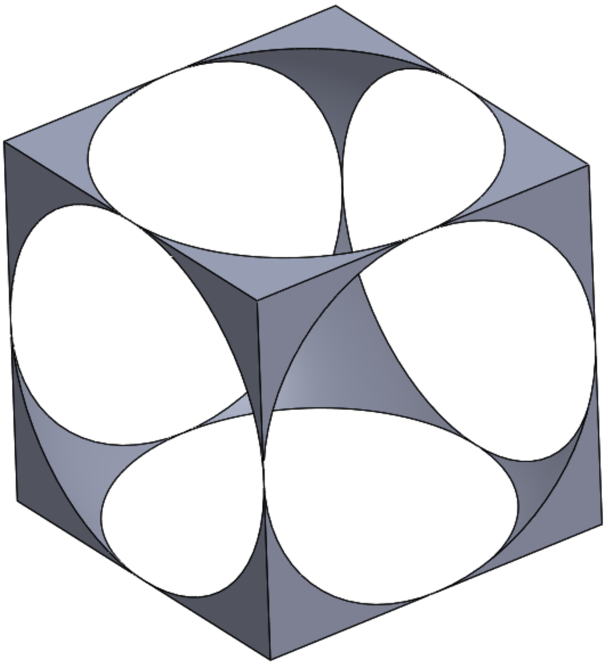
246.01 - COMPETITION: Golomb Ruler - Elliott Line

The missing mark is 187

WELL DONE TO:
Johann Muller
Michael Kenedy
Abhilash Unnikrishnan
Paul Clark
Stuart Nelson
Agnijo Banerjee
Alan Davenport
Karl Billington
Georgia Craven
Matt Francis
Peter Phelps-Jones
Paul Newell
Julie Harkin
Willie Verger

247.01 - COMPETITION: Cube - Elliott Line

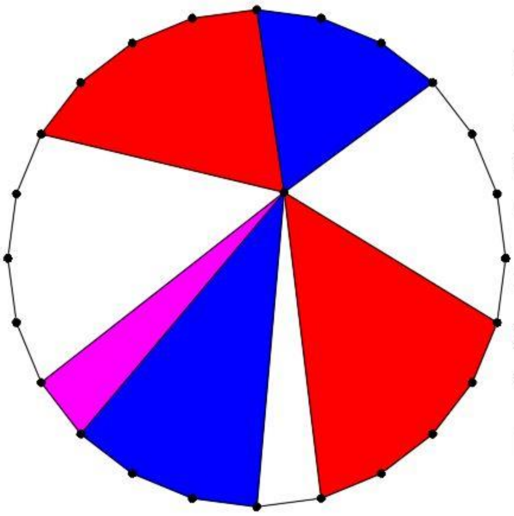
I have taken a cube, and removed from it its intersection with the largest sphere I could before the cube became eight separate corner pieces. The shape (shown below) now has a total volume of $247cm^3$ (247 being the number of this issue of Enigma).



What is the side-length of the cube, to the nearest millimetre? (Note, the volume was given in cubic centimetres).

This is a competition, but not for prizes, only bragging rights. Every correct answer I receive will get an honourable mention in the next issue of Enigma. Send your answer to me at enigma.mensa@yahoo.co.uk .

247.02 – 24-gon – Elliott Line



Given a regular 24-gon with lines from some of the vertices to a point within the polygon: if the combined area of the red regions is 47, and the combined area of the blue regions is 38, what is the area of the magenta region?

247.03 – Base 4 Code – Elliott Line

I have taken a quotation, and I have replaced each of the letters with the numbers that denote their position in the alphabet. However, I have used the base 4 number system. Be careful, as some sequences of numbers could lead to several words, for instance 31110 could mean CAT (3,1,110), but could equally mean MAD (31,1,10).

	1 = A	2 = B	3 = C
10 = D	11 = E	12 = F	13 = G
20 = H	21 = I	22 = J	23 = K
30 = L	31 = M	32 = N	33 = O
100 = P	101 = Q	102 = R	103 = S
110 = T	111 = U	112 = V	113 = W
120 = X	121 = Y	122 = Z	

132121333211 1132033 10311033100103 3011110232213213 21103 333010,

11320111102011102 1101131132110121 33102 11211320110121. 132121333211 1132033

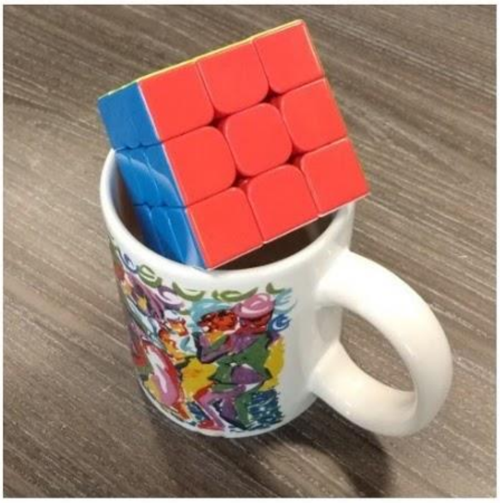
231111100103 3011110232213213 1031101121103 121331113213. 1102011

1310211111011103110 11020213213 12133111 3132 1033 21103 231111100

12133111102 31213210 121331113213.

247.04 – The Cube and the Cup – Elliott Line

This came to me whilst trying to balance a Rubik’s cube on the circular rim of a coffee cup. There are three ways of orienting a cube on top of a cup, which I’ll refer to as face-down, edge-down (resting on the rim in four places) and point-down (resting on the rim in three places. For these latter two cases the cube extends below the level of the rim of the cup. Depending upon the particular cube and the particular cup, the cube might extend deeper into the cup when it is edge-down or when it is point-down.



Assuming that the edge-down and point-down orientations are possible for the given cube and cup, how can you tell, very quickly and without using any measuring devices, only looking at and manipulating the cube and the cup, which of those two orientations extends deeper into the cup?

247.05 – Dice Game – Elliott Line & Austin Line

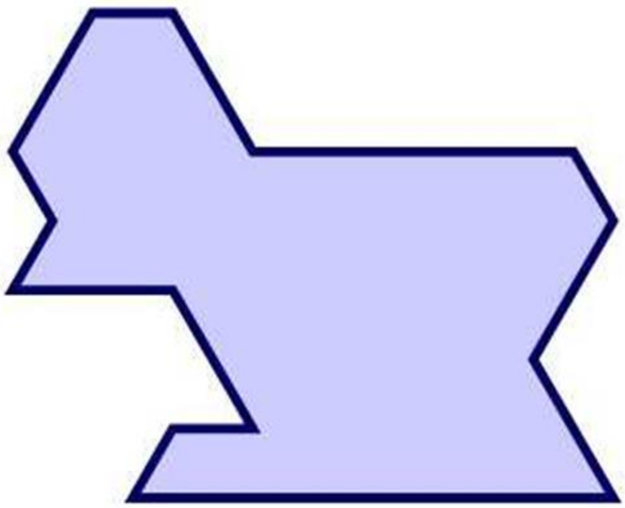
This puzzle is based on a dice game my son and I invented, involving 12-sided, 10-sided, 8-sided and 6-sided dice. (As with modern convention I will be using the word ‘dice’ to refer to both the plural and the singular, rather than the more archaic ‘die’).
In round 1, both players each have a 12-sided dice. They roll them and the player with the lowest number loses that round (if the numbers are equal they both simply roll again). The losing player exchanges their 12-sided dice for a 10-sided dice for the second round.

The game continues such that at each round the loser changes their dice for one with two fewer sides. The final round occurs when ONE of the players has to roll a 6-sided dice. The winner of this round is the overall winner of the game (even if they lost all of the previous rounds).

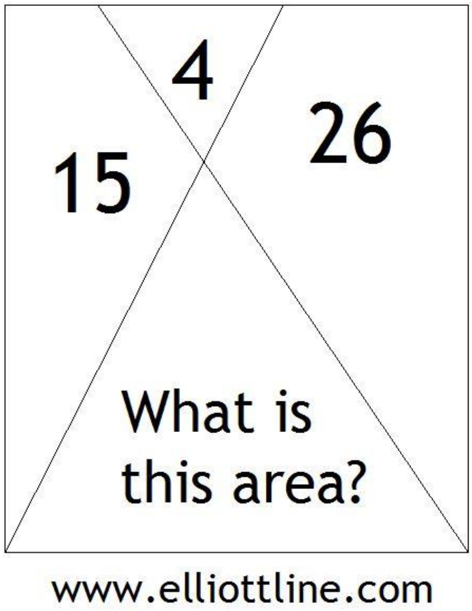
Since the game is symmetrical, the probability of each of the players winning is 50%, but the puzzle question is, how important is the first round? What is the probability that you win the overall game if you win/lose the initial round?

247.06 – Dissected Equilateral Triangle – Guillermo Verger

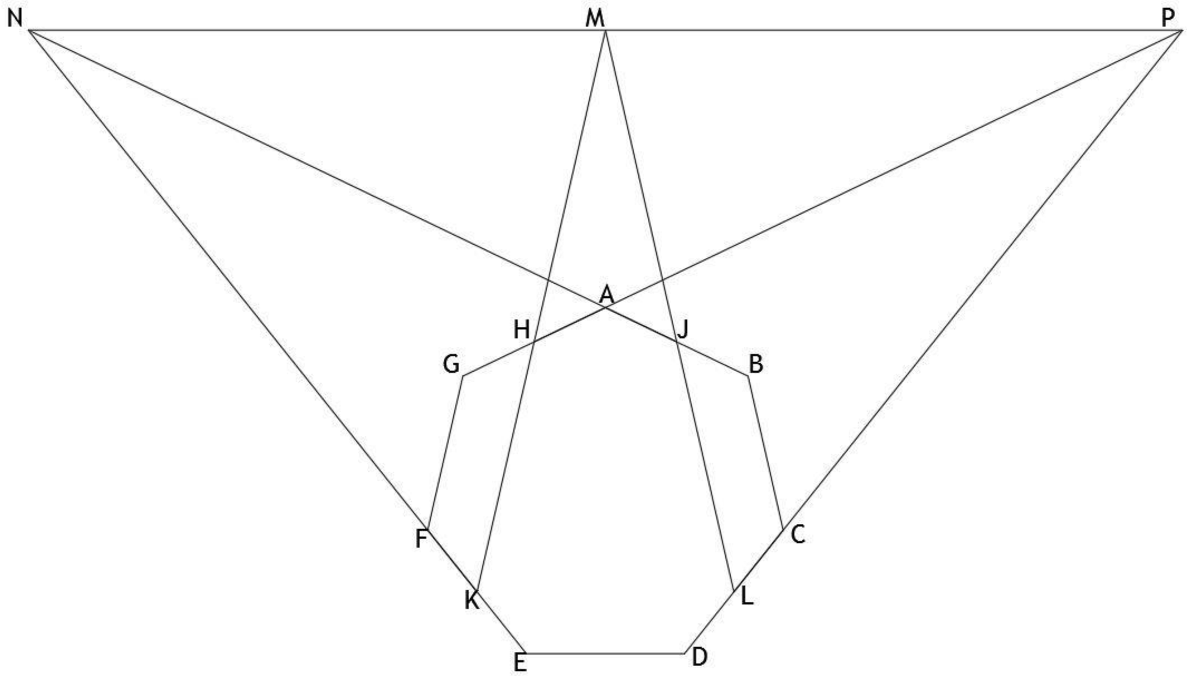
I have taken an equilateral triangle and cut in four pieces. Three of them the same and one different.
Then I joined the pieces to form the given shape without any overlapping.
Your task is to cut the four pieces of this shape to reconstruct the original equilateral triangle.



247.07 – Divided Rectangle – Elliott Line



247.08 – Heptagon Figure – Elliott Line

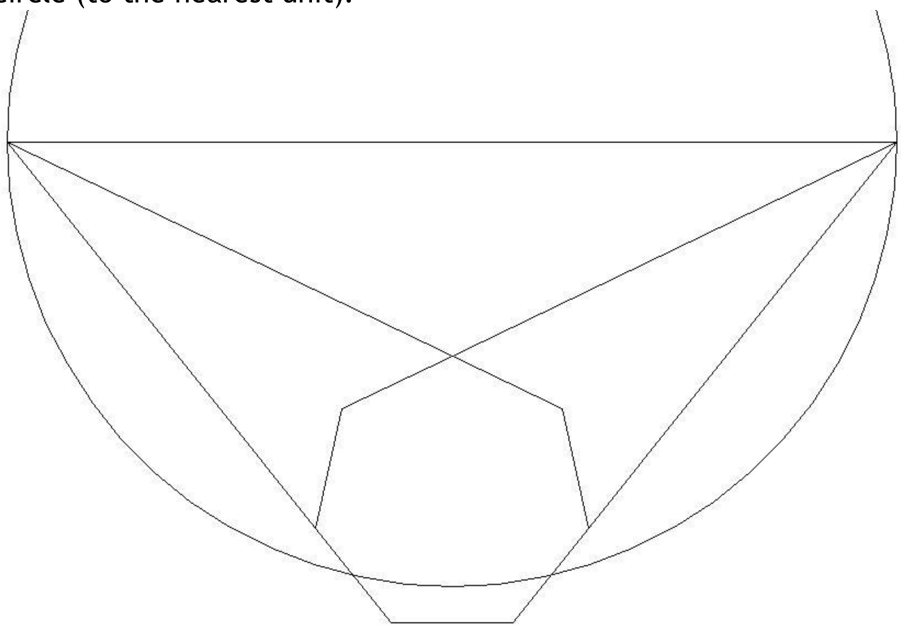


ABCDEFG is a regular heptagon
H, J, K, L are midpoints of sides of the heptagon.
The lines AB and EF coincide at point N.
The lines HK and JL coincide at point M.
The lines AG and CD coincide at point P.
If the length of MK is 7, what is the length of NP?

247.09 – Heptagon Revisited – Elliott Line

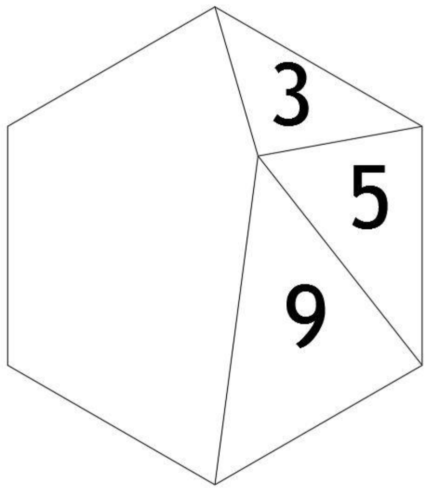
You may remember this heptagon figure from a few weeks ago. One consequence of the solution was that the circle shown here, whose diameter endpoints are also extended intersections of pairs of sides of the heptagon, also passes through two midpoints of sides of the heptagon.
As it happens, for a regular heptagon there is another distinct way in which you can draw a circle whose diameter endpoints are extended intersections of the sides, and which also passes through midpoints of sides.
By distinct I mean not just merely a rotation of the circle shown. The other circle will have a different diameter to the one shown.

If the circle shown has a diameter of 6296, what is the diameter of the other possible circle (to the nearest unit)?



247.10 – Hexagon Area – Elliott Line

Inside a regular hexagon lines are drawn from four of the corners to a point inside as shown. The three triangles that result have areas of 3, 5 and 9 respectively. What is the area of the entire hexagon?



247.11 – Left Right Pairs – Elliott Line

This puzzle was inspired by a real-life problem I had in my job as an engineer. Faced with a number of lengths I needed to come up with a set of left parts and a set of right parts which could combine to form every length on the list. For instance, the length 52 could be made up of 26L + 26R, or 20L + 32R, or any number of different combinations. For reasons of making and stocking the individual left and right parts it was important to minimise the total number of different parts I needed.
For this puzzle I have massively shortened and simplified the list but the idea remains the same: what is the least number of distinct parts you will need, some left-handed, some right-handed (parts that are the same length but different hands count as two different parts), that will fit together in left-right pairs to form any length on this list:

20 23 28 44 46 48 50 52 56 58 70 74

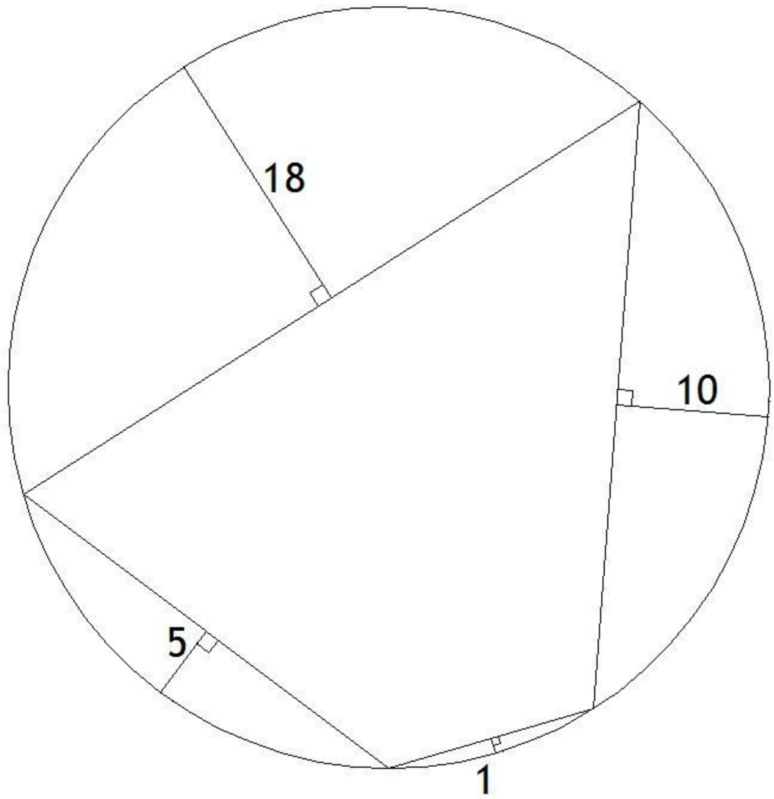
247.12 – Number Hunt – Elliott Line

AB^C = BCBACBDB

What are the digits A, B, C and D?

247.13 – Quadrilateral in a Circle – Elliott Line

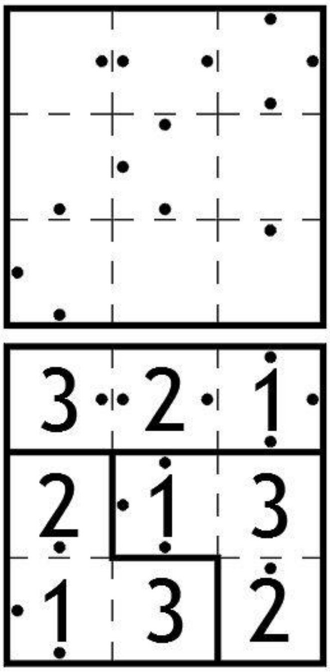
A quadrilateral is drawn inside a circle. Lines are drawn at right angles from the midpoint of each side of the quadrilateral and extended to meet the circle. The lengths of these four lines are 18, 10, 1 and 5 respectively.
What is the area of the quadrilateral?



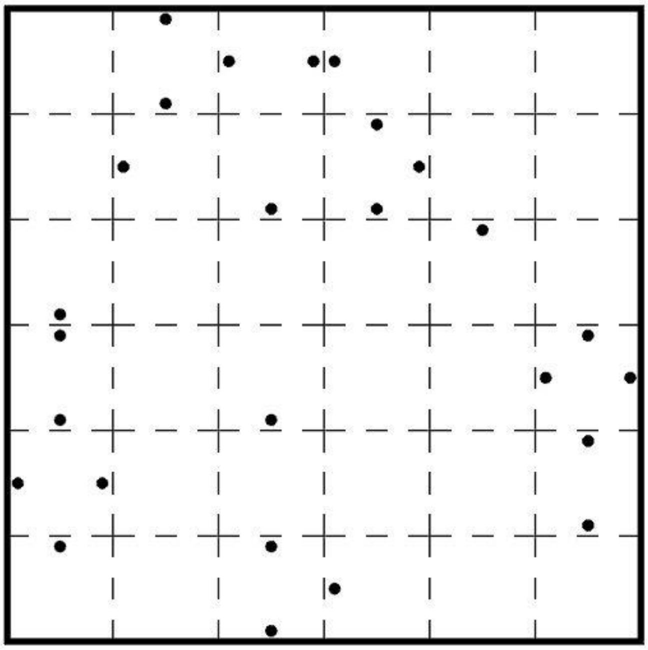
247.14 – Su-Dot-U – Elliott Line

Your task is to solve this irregular sudoku (the digits 1 to 6 appearing once each in every row, column and 'shape').
Except that I haven't told you where the boundaries between the shapes are; instead I've placed dots in any square where the number to be placed in the square denotes how many squares (including the one with the dot in) until you get to the next boundary line in the direction of the dot.
Clear as mud? Hopefully the attached example 3x3 will help. For instance, the 2 in the middle of the top row, combined with the left and right dots, says that in each direction left and right from that square there are two squares (including the one with the 2 and the dots in) before you get to a boundary line (which happens to be the outer boundary of the entire grid).
In both the example and the puzzle, I have placed dots in every position that I could, according to the rules.
A hint to get you started: if a dot appears next to the outside boundary of the grid, then that square must contain a 1.

Example:



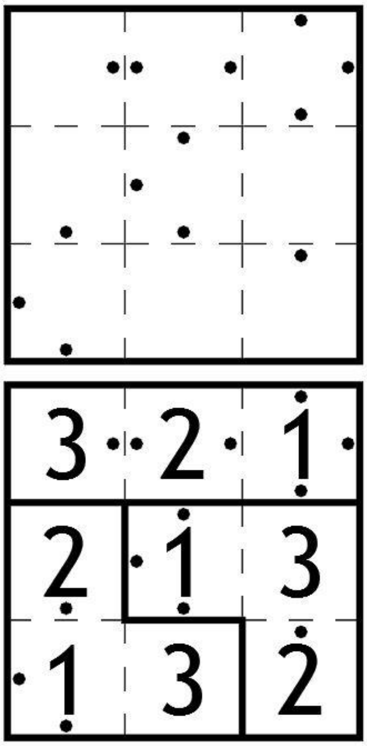
Puzzle:



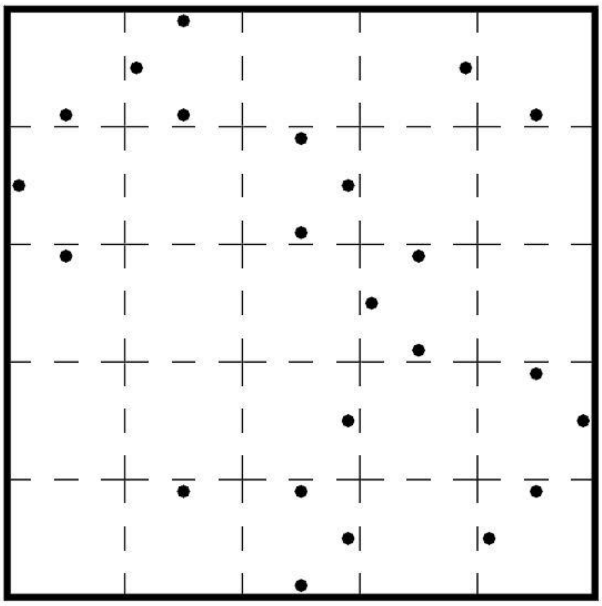
247.15 – Su-Dot-U – Elliott Line

Your task is to solve this irregular sudoku (the digits 1 to 5 appearing once each in every row, column and 'shape').
Except that I haven't told you where the boundaries between the shapes are; instead I've placed dots in any square where the number to be placed in the square denotes how many squares (including the one with the dot in) until you get to the next boundary line in the direction of the dot.
Clear as mud? Hopefully the attached example 3x3 will help. For instance, the 2 in the middle of the top row, combined with the left and right dots, says that in each direction left and right from that square there are two squares (including the one with the 2 and the dots in) before you get to a boundary line (which happens to be the outer boundary of the entire grid).
In both the example and the puzzle, I have placed dots in every position that I could, according to the rules.
A hint to get you started: if a dot appears next to the outside boundary of the grid, then that square must contain a 1.

Example:



Puzzle:



247.16 – Sum of Cubes – Elliott Line

$a + b + c + d = 4$

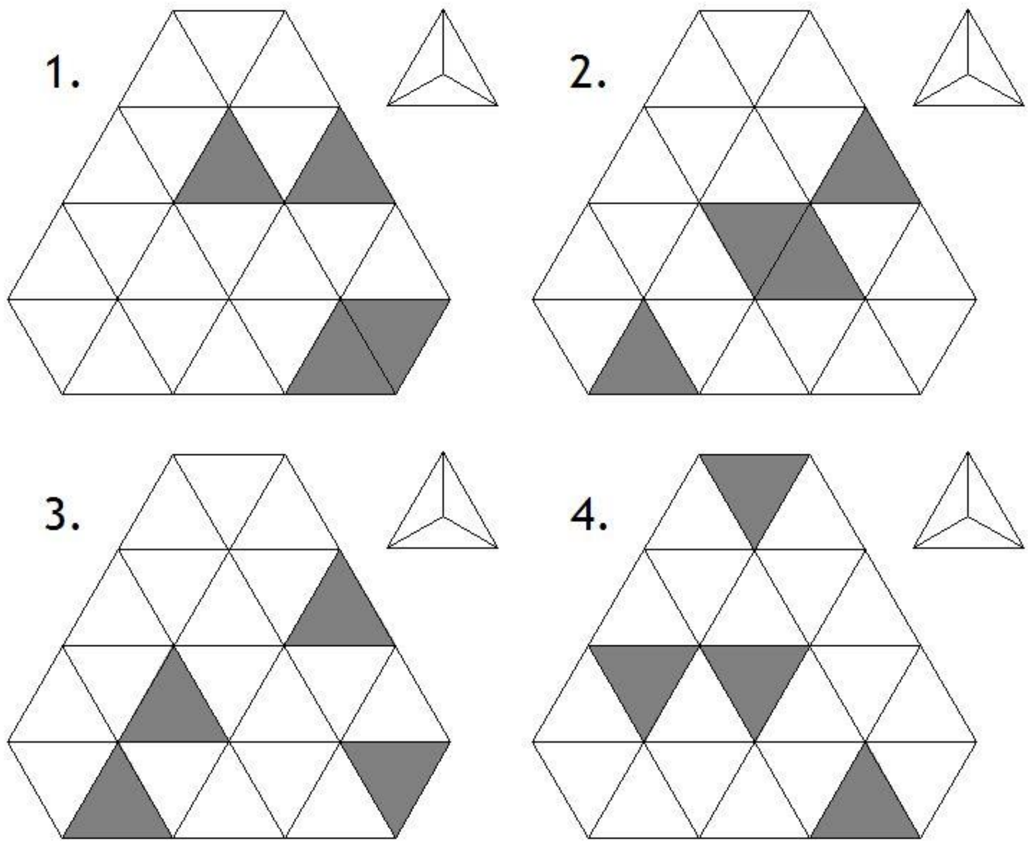
$a^3 + b^3 + c^3 + d^3 = 4$

a b c and d are all integers

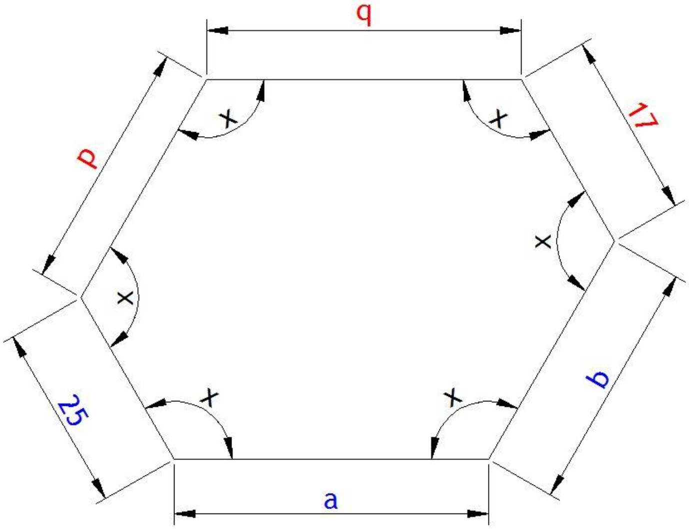
There is a trivial solution where a, b, c and d are all 1, but there is at least one other set of four numbers which satisfy, where none of the numbers are equal to 1. Can you find one?

247.17 – The Tetrahedron Game – Elliott Line

In this game there is a grid of 22 triangles, four of which are painted. You may initially place a tetrahedron on any triangle you want but from there forward you move by ‘rolling’ the tetrahedron over one of its edges so that it will then be on an adjacent triangle on the grid and resting on a different face of the tetrahedron. If you roll onto a painted triangle, the paint transfers to that face of the tetrahedron (unless that face is already painted), and when a painted face of the tetrahedron lands on an unpainted triangle, the paint is transferred from the tetrahedron to the triangular grid.
The object of the game is to continue to roll around the grid until all four faces of the tetrahedron are painted, however only one of the four grids shown can actually be solved. The other three are not solvable regardless of where the tetrahedron is initially placed. Which is one that can be solved?



247.18 – Underdefined Hexagon – Elliott Line



The edges of this hexagon are all different, whereas the angles are all the same.

The red lengths (p,q,17) are each two-digit prime numbers.

The blue lengths (25,a,b) are each two-digit square numbers.

What are the lengths a, b, p and q?

www.elliottline.com

247.19 – Unit Fractions – Elliott Line

1/8 + 1/188 + 1/43992 = 1/a + 1/b, where a and b are both whole numbers.
What are the values of a and b?

247.20 - In Common – John Causer

What have these in common?
PAMELA
BERGERE
BYCOCKET
CAPOTAIN
DOLLY VARDEN
HENNIN
MONTERA
SHTREIMEL
SONGKOK
TOPI
WIDE AWAKE
ZUCHETTO

247.21 - Semi-quotations – John Causer

These quotations have got run together. When it reaches a particular word (marked as 'XXXX' here) it shifts to another quotation using that same word. Can you get them apart again

a) On the east of XXXX demi-paradise
b) Young, gifted and XXXX as her pan you saw e'en now without
c) O cuckoo shall I call thee XXXX thou never wert
d) We live, as we dream – XXXX and palely loitering

247.22 – Names – John Causer
Here is an alphabetical list of names. They have two things in common – what are they?

Ellis Bell
Emma Blair
Isak Dinesen
Deanna Dwyer
Robert Galbraith
Vernon Lee
Dorothea Nile
Jessica Stirling

247.23 - Long Word – John Causer

What English word (11 letters, 4 syllables) changes its meaning according to whether you pronounce its first syllable with a long or a short "e"?

247.24 - Right, you 'orrible little men – John Causer

RSM Alf A. Betts paraded the letters in their usual sequence, then ordered a number of them to fall out of the array and re-form up into a word. When he dismissed these letters and ordered them to return to the line they all became confused and squeezed into the wrong places. This was the result:

A B C G D F H J I K L E N M O P Q R S T U X W V Y Z

What was the word that they had formed?

247.25 - Letter Mixture – John Causer

In a three word phrase one letter has been changed in each word and the words' order scrambled. What is the phrase?

SPARE FACT BANDS

247.26 – Olympians – John Causer
The 1964 Olympic Games were in Tokyo, the 1988 were in Seoul and the 2008 in Beijing. Each of these produced a similar organisational problem over the opening parade of athletes. What was it?

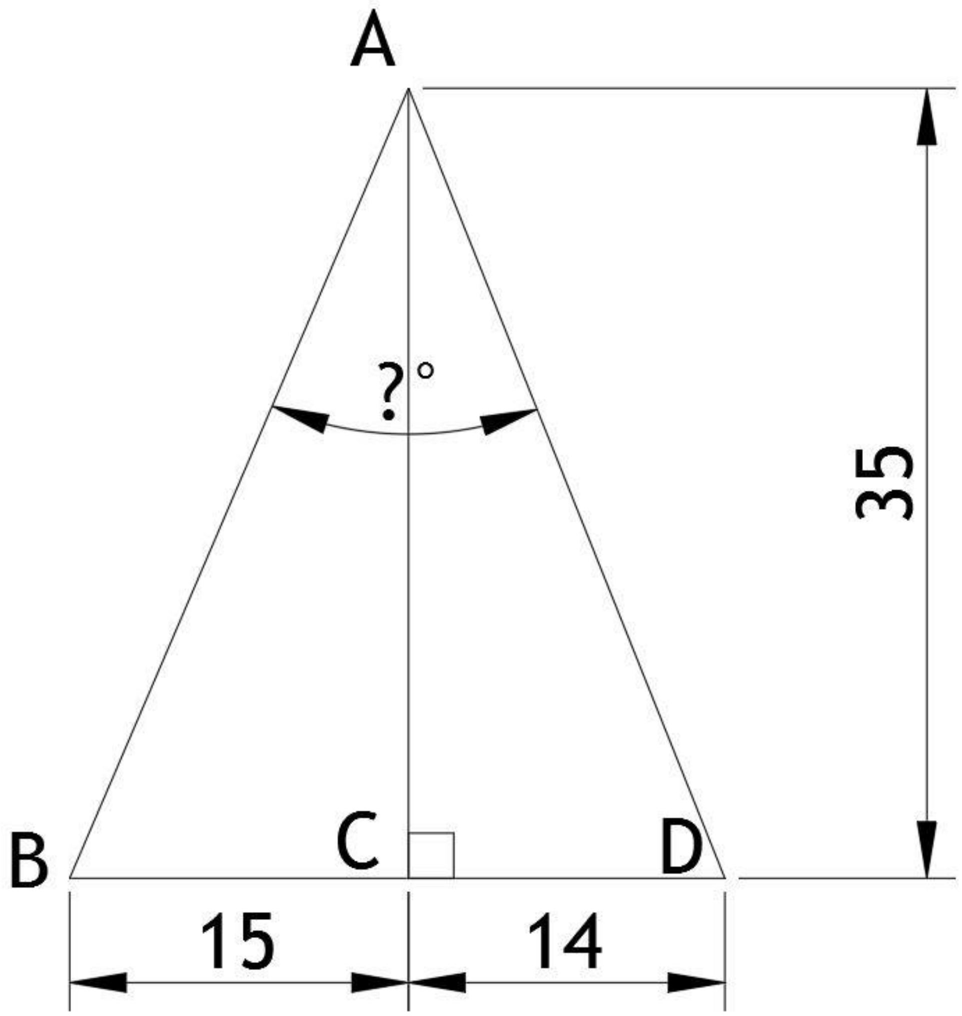
247.27 - Set It Yourself – John Causer

Here are four easy questions.
All you have to do is answer them, and then supply question 5.

1) Whose *Talking Heads* have recently been re-made by the BBC?
2) Who is the Canadian singer whose 1996 album *Falling For You* went 11 X Platinum in USA and who opened the Atlanta Olympic Games the same year?
3) She was a Quaker and an early C19 prison reformer.
4) Which Beatle, who was famously quiet and died in 2001, played lead guitar and wrote a few of the band's songs including '*Here Comes the Sun'*?

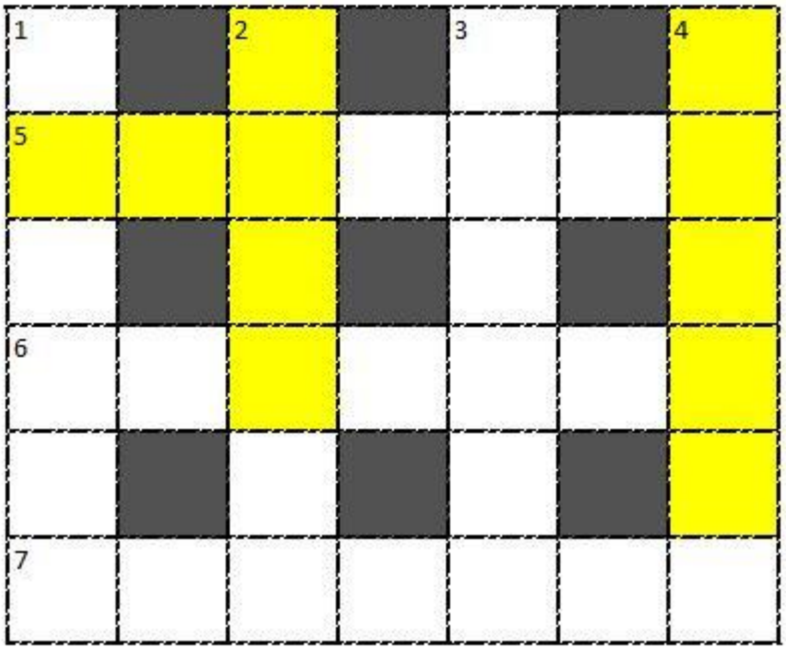
247.28 – What is the Angle? – Elliott Line

Can you work out what the total angle is at the top of this triangle, without using a calculator?



247.29 – Wordwall Crossword – Christa Ramonat

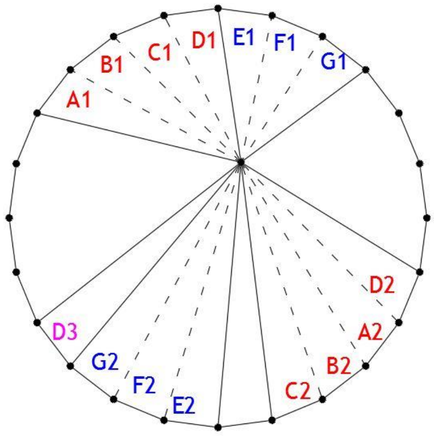
Solve the clues and the part-words in yellow reveal an extra answer.



1: An ideal place
2: Journalism is the ______ estate
3: Mystery
4: Scaled-down version of Rugby
5: Canadian slang for a two-dollar coin
6: The point at which the moon is closest to the earth
7: Large metal barrels for rubbish

~~~SOLUTIONS~~~SOLUTIONS~~~SOLUTIONS~~~

247.02 – 24-gon – Elliott Line



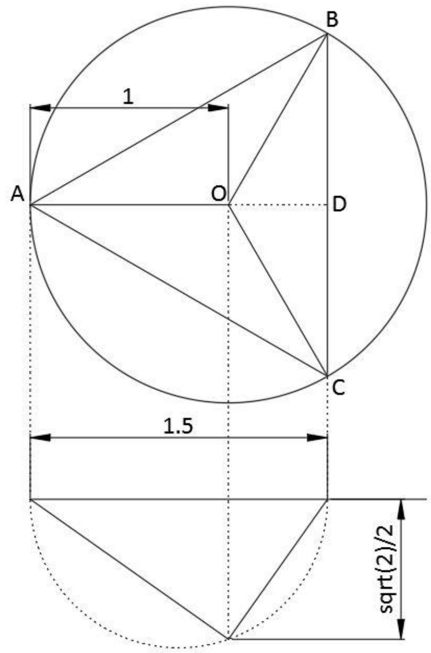
The magenta region (D3) = 10.
Any subsets of triangles whose bases are equally spaced around the polygon have combined areas proportional to the area of the overall polygon. This is obvious for pairs of opposite triangles, which will have a combined area of $\frac{1}{12}$ of the entire figure. But is also true of larger subsets such as the trio of 'D' regions in the figure, which when combined must make up $\frac{1}{8}$ of the area of the entire figure. The rest is just arithmetic.

247.03 – Base 4 Code – Elliott Line

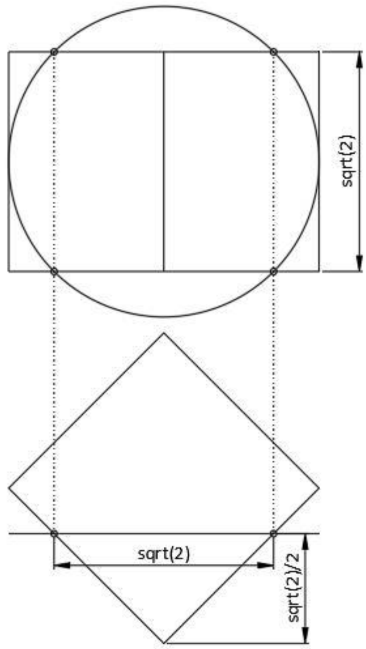
Anyone who stops learning is old, whether twenty or eighty. Anyone who keeps learning stays young. The greatest thing you can do is keep your mind young.
This has been variously attributed to Henry Ford and Mark Twain.

247.04 – The Cube and the Cup – Elliott Line

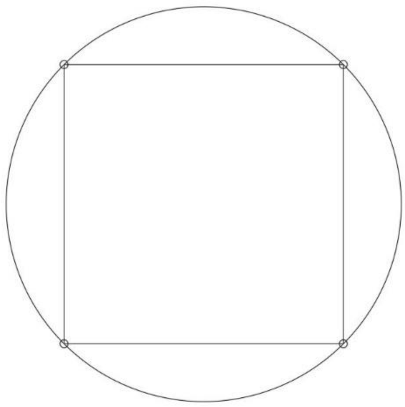
To keep things relatively simple I will assume the circular rim of the cup has a radius of 1.
It's not important to solving the question, but for the two orientations to be possible, the edge length of the cube must be somewhere between sqrt(6)/2 (1.225..) and 2. Any smaller than that and the point-down cube would fall in, any larger and the edge-down cube would not fit within the rim.
So on the assumption that we are within that range and both configurations are possible, it turns out that for the point-down orientation, the cube side length doesn't matter: in theory you could have a mile long cube and the portion below the rim would be exactly the same.
To work out what that depth is for our unit radius cup, consider a cross-section through the cube and cup through point A O and D:



Since the entire face that triangle BCO is part of is perpendicular to edge AO, so is line OD. Projecting down to the side view we can use the fact that an angle in a semicircle is a right angle to ascertain that the depth is sqrt(2)/2 or 0.707…
So now we need to find the cube size for which the edge-down cube will also be at that same depth. By looking side-on at the diamond-shaped end of the cube with the required depth, and then looking at the plan view, we can see that the edge length of the cube must be sqrt(2) = 1.414…. Inevitably, this lies in our range of acceptable cube sizes.



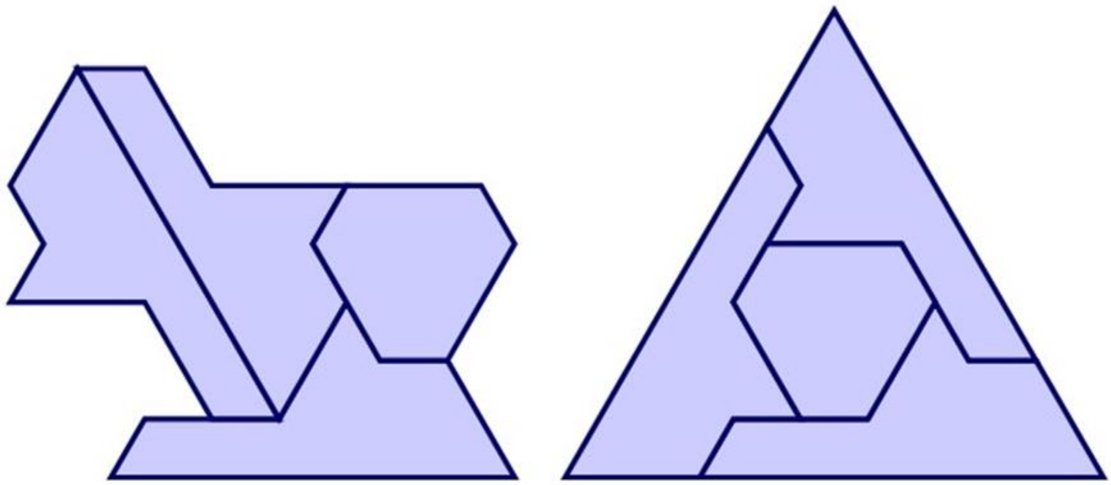
So finally, how can you tell simply and visually if the edge length of the cube (as a ratio of the radius of the cup) exceeds sqrt(2)? Do you remember the forgotten third orientation – face-down? Simply if the face-down cube lies within the rim of the cup, then the ratio of edge length to radius is less than sqrt(2) and the edge-down orientation will be deeper than point-down, whereas if the face-down cube can sit on top of the cup with all four corners outside of the rim, then the point-down orientation will be deeper.



247.05 – Dice Game – Elliott Line & Austin Line
It should come as little surprise that winning the first round gives you an advantage as the game continues. You are already more likely to win the second round, and if you do your advantage is further increased.
It is however by no means a foregone conclusion. Examining all of the possibilities the probability of the winner of the first round going on to win works out at about 63.5%, with the chance of the initial underdog seeing a reversal of fortune as the remaining 36.5%.
The exact values, should you be interested, are as follows:
Initial winner winning: 137009/215622
Initial loser winning: 78613/215622

The most likely specific sequence of results by a long way is for the initial winner to simply win every round. This occurs with a probability of 3315/10648 or just over 31% and so alone accounts for almost half of the initial winner's possible routes to victory.

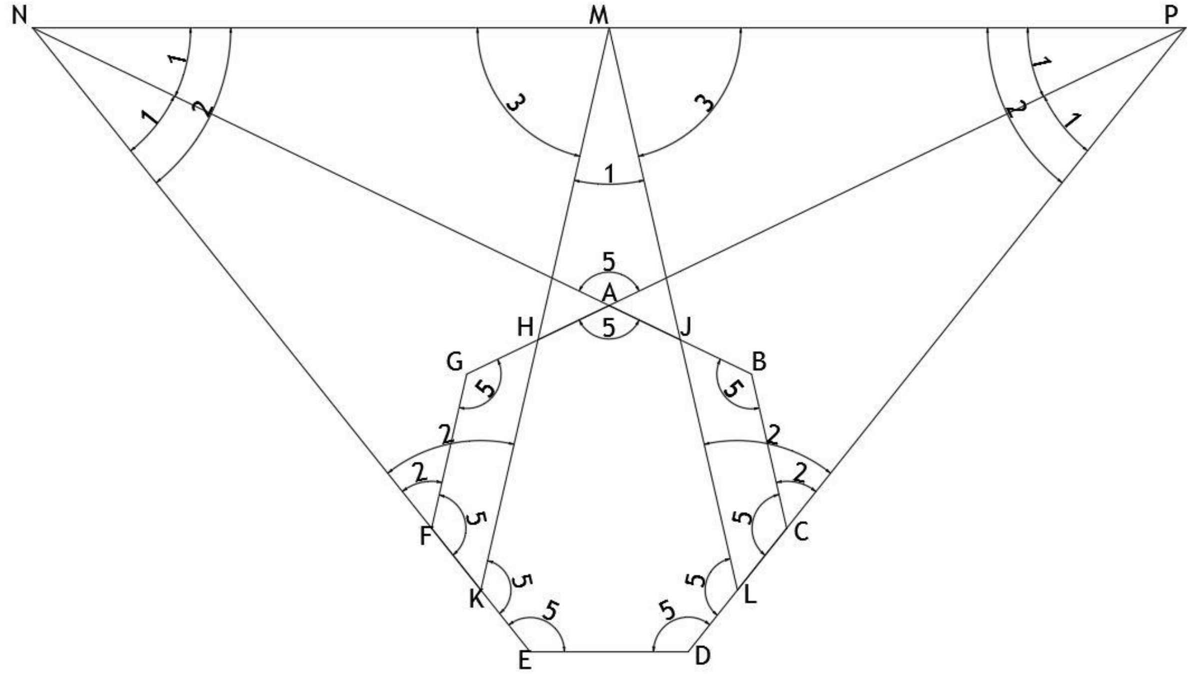
247.06 – Dissected Equilateral Triangle – Guillermo Verger



247.07 – Divided Rectangle – Elliott Line

Call the missing area x^2.
Let the base of the rectangle equal 2x/c (where x is positive, c is just an arbitrary scaling factor).
Since the area of a triangle is half base times height, the height of the missing triangle is xc.
Since the 4-triangle is similar to the missing triangle, the height of that is 2c (as 2^2=4), so the overall height of the rectangle is (xc+2c).
The area of the overall rectangle can now be written is two distinct ways, which must be equal:
Summing the four regions: x^2+15+4+26
Rectangle base times height: 2x^2+4x (notice the c vanishes)
Equating those two expressions and bringing everything to one side gives:
x^2+4x-45=0
which we can factorise into:
(x+9)(x-5)=0
The positive root is x=5, so the missing area x^2 is 25.

247.08 – Heptagon Figure – Elliott Line



The angles marked in this figure are in units of 180/7°. So the internal angles in a heptagon will add up to 35, the internal angles in a pentagon will add up to 21, in a quadrilateral will sum to 14 , and in a triangle or on a line will sum to 7.

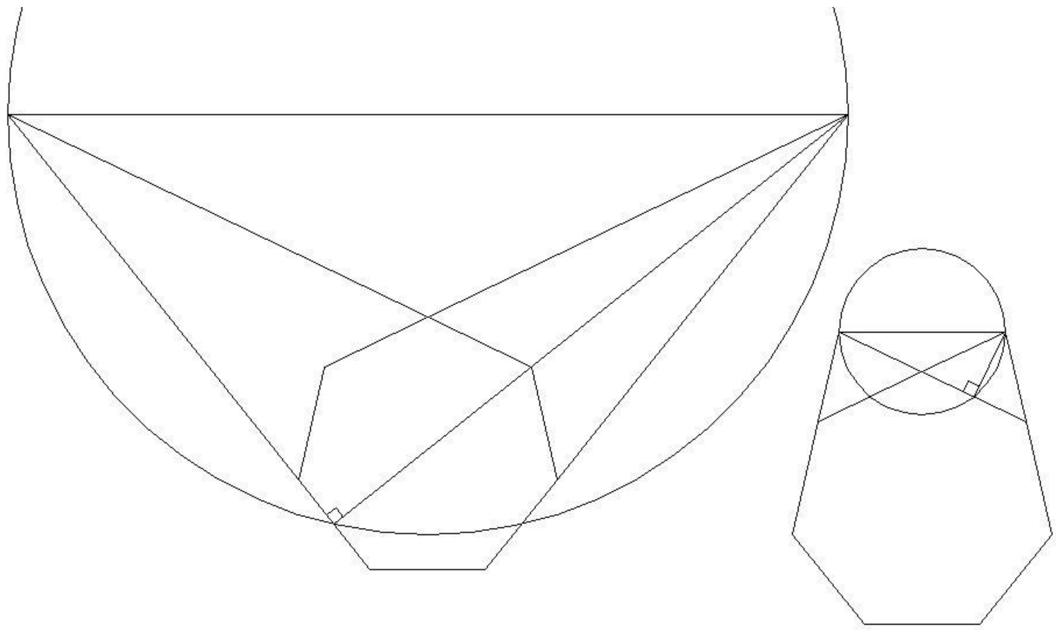
By definition all of the angles in the regular heptagon will be 5. From pentagon NBCDE, angle ENB is 1 (same for angle GPD).

Since H and K are midpoints of the heptagon, FGHK is a trapezoid, therefore HK is parallel to GF, and so angle HKE is 5 (same for JLD). The supplementary angles MKN and MLP are therefore 2.
From the inherent symmetry of the figure, and the angles in the quadrilateral NPDE, angles ENP and DPN are each 2. Therefore from triangle NMK, angle NMK is 3 (same for PML). From the pentagon MKEDL, angle KML is 1. This proves that N M and P are collinear (although the figure already implies this, it's nice to be able to verify it).

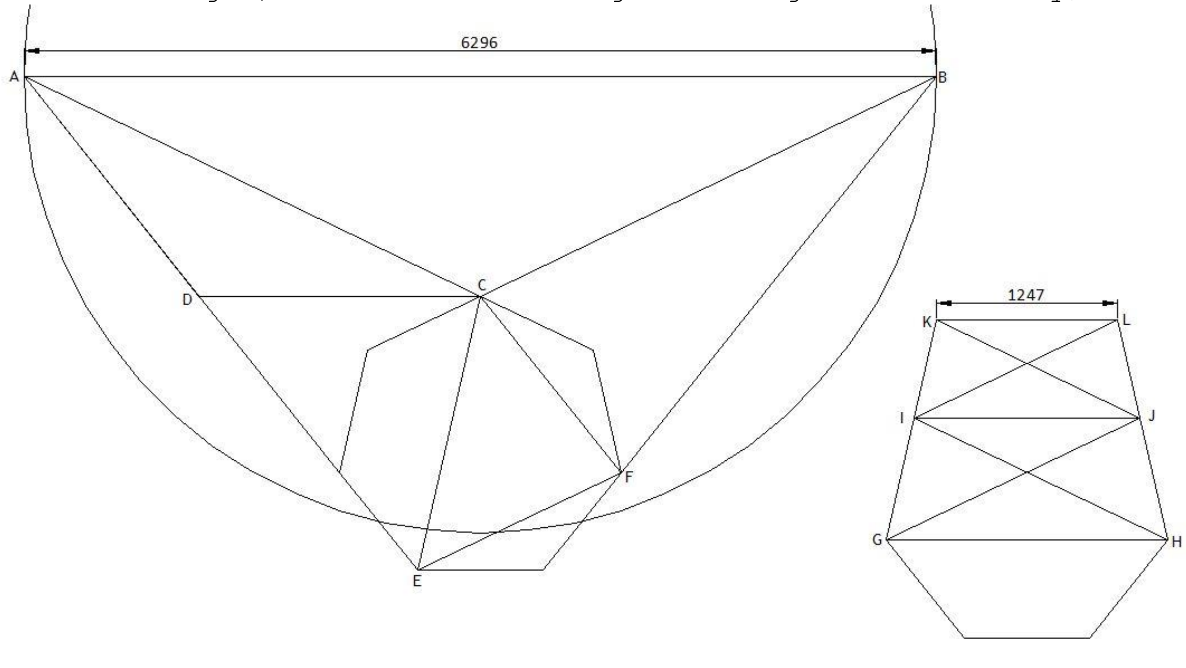
Finally, since NMK is isosceles, NM = MK (same for MP). So if MK is 7, NP = 14.

247.09 – Heptagon Revisited – Elliott Line

Using the fact that an angle in a semi-circle is always a right angle, we can see why the given circle passes through the midpoints shown. Similarly we can see why the smaller circle in the second figure also passes through midpoints of the heptagon. Now we only need to find the diameter of that circle. I only give you an overview of the solution for you to fill in the gaps if you desire.



We can find angle ACB easily enough, and since triangle ABC is isosceles, we can find the length of AC.
Triangle ABC is similar to triangle ACD, therefore AB:AC = AC:CD, so we know length CD.
By examining the angles triangle CDE is isosceles, therefore CE = CD.
Since the heptagon is regular, we know the angles within isosceles triangle CEF, so we can know the length of CF.
Moving to the next figure, GH = CE, and IJ = CF.
GHIJ and IJKL are similar trapezoids (sides and diagonals parallel between the two figures) therefore GH:IJ = IJ:KL.
Therefore if we know AB = 6296, we also know that KL = 1247 (to the nearest integer, 1247.000015… to a greater degree of accuracy).

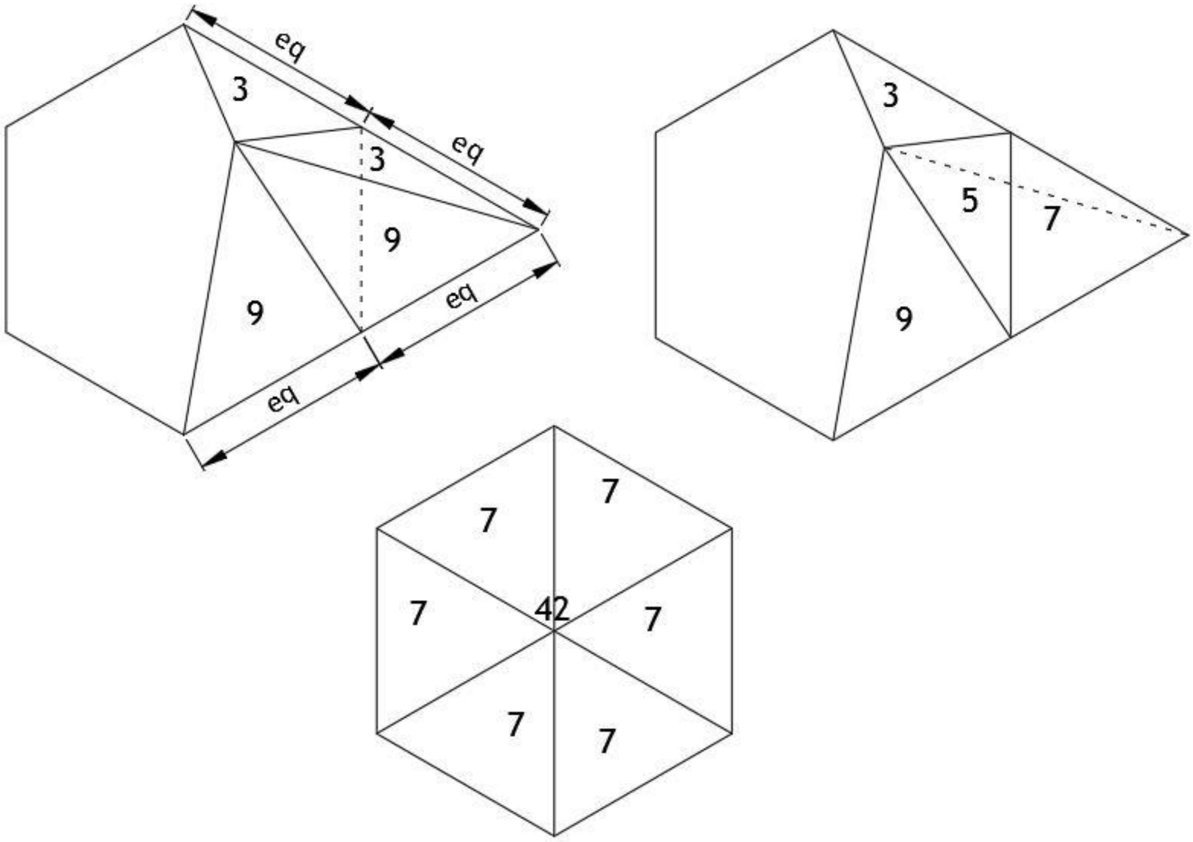


If you’re interested, the exact ratio between AB and KL is:

(1+2sin(270/7))^2

247.10 – Hexagon Area – Elliott Line

If you extend two of the lines until they meet, as shown, and then also draw a line this new point to the point inside the hexagon you will form two new triangles with the same base and height as two of the given triangles. The sum of these, and therefore the area of the quadrilateral, will be 12. But this 12 is also the sum of the other given triangle and the new equilateral triangle formed by extending the lines. This equilateral triangle therefore has an area of 7. This is also one sixth of the area of the original hexagon. The ultimate answer is therefore 42.



A neat little theorem comes in handy in this situation (and applies to any regular polygon, not just hexagons). Any subset of two or more triangles which are equally spaced around the polygon will have a combined area equal to that of any other subset of the same number of equally spaced triangles. For our hexagon, opposite pairs are equal, (and also equal to 1/3 of the overall area), and sets of three equally spaced triangles are also equal (and equal to half the overall area). If we call the triangle directly opposite the '5' triangle x, we have two expressions for the overall area both involving x:

$A = 3(5+x) = 2(3+9+x)$

$15+3x = 24+2x$

$x = 9$, therefore $A = 42$

247.11 – Left Right Pairs – Elliott Line
There are twelve different numbers in the list, so logically it can’t be done in fewer than 7 parts (3 of one hand and four of the other, to combine in 3 x 4 = 12 different ways).
However the fact that 23 is the only odd number in the list throws a spanner in the works as it effectively means one part will help make up the 23 but not be used elsewhere. Assuming that you can find a way of making the other 11 with just 3 + 4 distinct parts, 8 parts in total will be required.

If we assume for the moment that there are only 3 L parts, and that one of them has length 0 (impossible ultimately, but useful right now), then what we are searching for is a subset of four of the 11 lengths (so disregarding 23), and two number a and b such that the remaining 7 numbers can be made by adding either a or b to one of the 4 numbers.

By experimenting, the subset 20, 44, 48 and 50, with a and b values 8 and 26 fits the bill. To avoid the zero length part, we can simply subtract 10 from each of (20,44,48,50) and call those the L parts, and add 10 to (0,8,26) to find the R parts, adding in an extra R part to make 23 possible. Thus:

20 = 10L + 10R
23 = 10L + 13R
28 = 10L + 18R
44 = 34L + 10R
46 = 10L + 36R
48 = 38L + 10R
50 = 40L + 10R
52 = 34L + 18R
56 = 38L + 18R
58 = 40L + 18R
70 = 34L + 36R
74 = 38L + 36R

which uses the following eight parts: 10L, 34L, 38L, 40L, 10R, 13R, 18R, 36R.

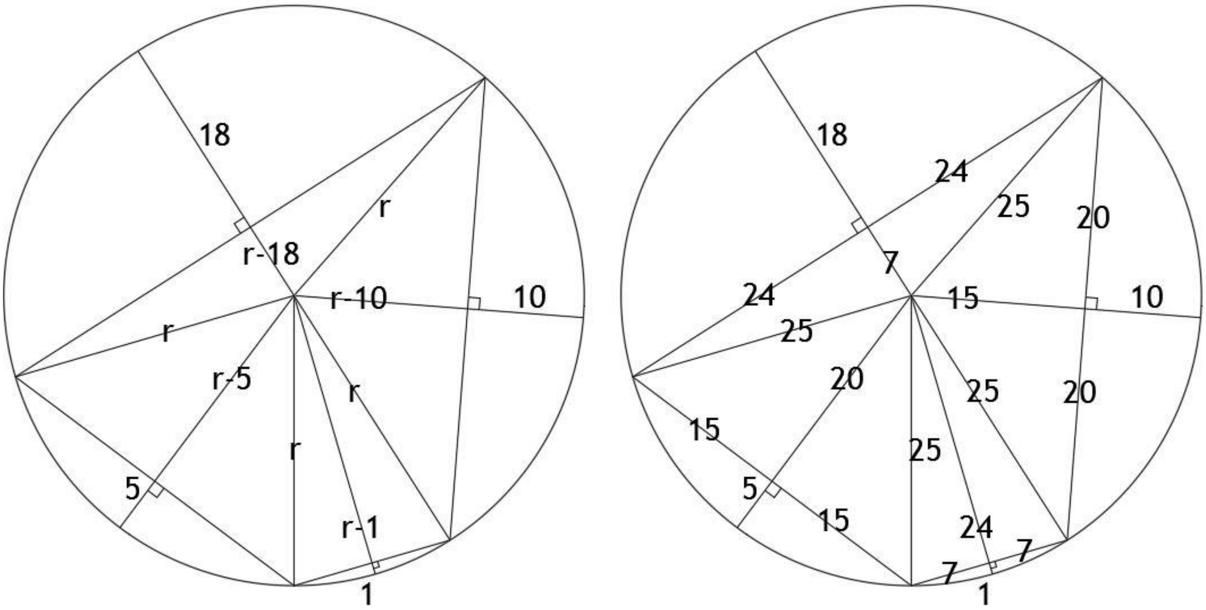
247.12 – Number Hunt – Elliott Line
A, B, C and D are 3, 4, 5 and 2 respectively, since 34^5 = 45435424

247.13 – Quadrilateral in a Circle – Elliott Line

Area = 936.

To work through the problem in trigonometry is a bit of a nightmare so I'd envisage people solving the problem by drawing lines from all vertices and midpoints to the centre of the circle, calculating the cosine of each of the angles at the centre in terms of 'r', and then varying the value of r until the angles add up to 180 degrees. This occurs when r is precisely 25. From there we can easily work out the side lengths of all 8 of the internal right-angled triangles and tot up their areas to get 936.

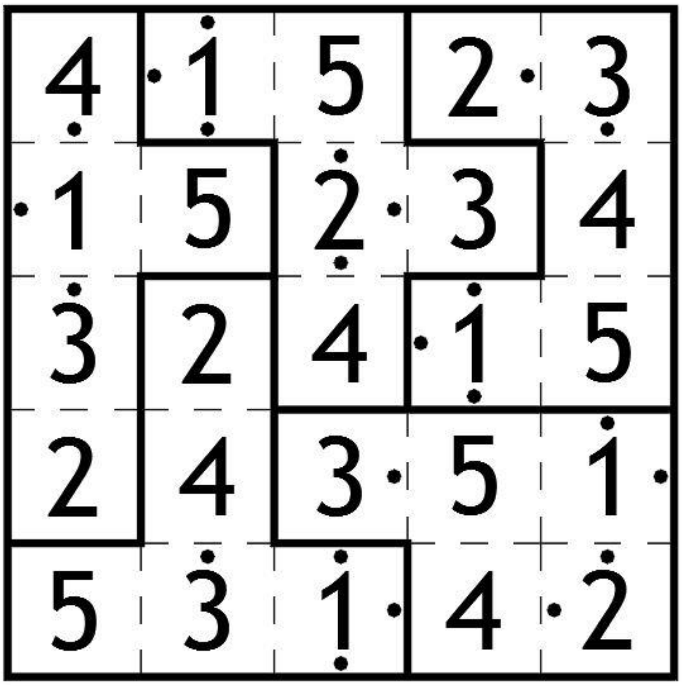
If you do that you might be surprised to discover that the eight triangles are two sets of four congruent triangles: (7,24,25) and (15,20,25).



247.14 – Su-Dot-U – Elliott Line

2	1	3	4	6	5
6	2	4	1	5	3
4	3	5	2	1	6
3	5	2	6	4	1
1	4	6	5	3	2
5	6	1	3	2	4

247.15 – Su-Dot-U – Elliott Line



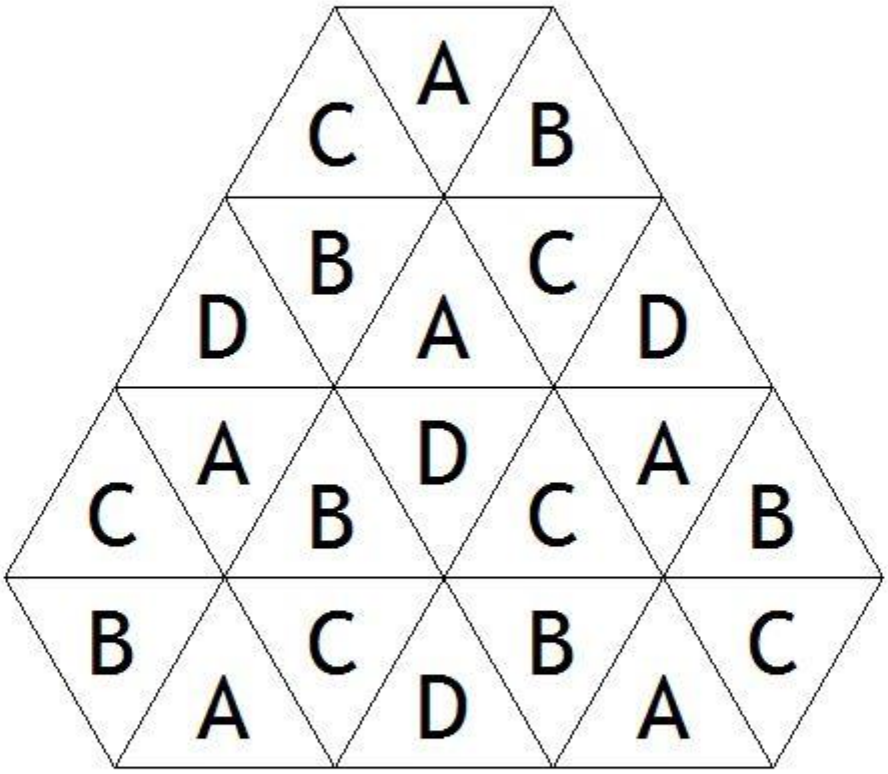
247.16 – Sum of Cubes – Elliott Line
a, b, c and d are (in some order) -11,-2,7 and 10.
Their sum is 4, and the sum of their cubes (-1331, -8, 343, 1000) is also 4.

I arrived at this answer partly by narrowing the scope of where I was looking, and thereafter trial and inspection. I remembered that ALL cube numbers are either 0,1 or 8 modulo 9 (in other words they all cube numbers are either a multiple of 9, or one away either side). You can verify this astonishing fact by just trying the first nine cube numbers and observing that this is in fact true for those, and because we are working in module 9, it is therefore true for all cube numbers.
For our four cube numbers to add to 4, they must all be of the +1 variety. This happens whenever the number to be cubed is 1 more than a multiple of 3, ie: 1, 4, 7, 10, etc and -2, -5, -8, -11, etc. So we only need to look at the cubes of these numbers. Obviously the trivial 1 appears in this list, as do each of the numbers in my answer: -11, -2, 7 and 10.

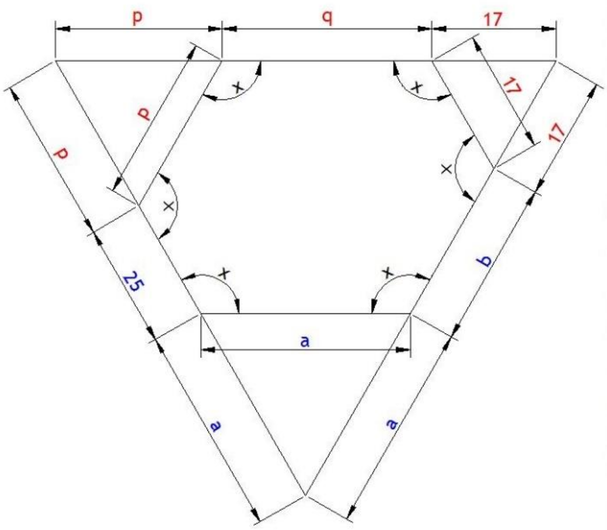
247.17 – The Tetrahedron Game – Elliott Line

As it rolls around the grid, the tetrahedron will always present the same face to particular triangle of the grid. In fact the grid can be subdivided into four classes of triangles, each of which can only be landed on by one particular face of the tetrahedron. That being the case, a selection of painted triangles is solvable if and only if it contains one each of the four classes of triangles. Grid 3 is the only such grid.

Interestingly, so long as the grid is solvable, it doesn’t matter where you initially place the tetrahedron, and you can also decide to end up on any particular triangle of the grid (even if it isn’t initially painted).



247.18 – Underdefined Hexagon – Elliott Line



If we extend three of the edges as shown, because the angles of the hexagon are all equal, we will have an equilateral triangle in each corner and the overall figure becomes an equilateral triangle, so we can write the equation:

$p+q+17 = 17+a+b = 25+a+p$

Taking 17 from each part:

$p+q = a+b = a+p+8$

therefore $b-8=p$ and $a+8=q$

So we are looking for two square numbers less than 100, where if we add 8 to one and subtract 8 from the other we get two prime numbers.

The two square numbers will need to be odd, since adding/subtracting 8 will not change the parity and the prime numbers need to be odd, and since we have already used 25 we are only left with 49 and 81. 49+8 is not prime, so the prime numbers we are looking for must be 49-8 and 81+8.

So a=81, b=49, p=41 and q=89.

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247.19 – Unit Fractions – Elliott Line

Since both 8 and 188 divide evenly into 43992, the original sum of three fractions can be written as (5499+234+1)/43992. This further cancels to 61/468. So we have:

61/468 = 1/a + 1/b   multiply each term by 468ab to eliminate denominators:

61ab = 468b + 468a   multiply each term by 61, and move everything over to the left:

61x61ab – 61x468a – 61x468b = 0   add 468x468 to both sides:

61x61ab – 61x468a – 61x468b + 468x468 = 468x468

So far we just seem to be needlessly complicating it, but in fact we've been leading up to the next step, which is to factorise the left hand side:

(61a-468)x(61b-468) = 468x468

Now here's the really clever bit. Since a and b have to be whole numbers, the remainder when divisible by 61 of (61a-468) or (61b-468) will remain the same regardless of the value of a and b. Their 'modulo 61' value will each be 20. The point of doing this is that however we factorise the right hand side, the two parts that correspond to (61a-468) and (61b-468) must also have a value of 20, modulo 61.

468x468 = 219024 = 2x2x2x2x3x3x3x3x13x13, so there are plenty of options for pairs of numbers that multiply to make 219024:

1x219024, 2x109512, 3x73008, 4x54756, 6x36504, 8x27378, 9x24336, 12x18252, 13x16848, 16x13689, 18x12168, 24x9126, 26x8424, 27x8112, 36x6084, 39x5616, 48x4563, 52x4212, 54x4056, 72x3042, 78x2808, 81x2704, 104x2106, 108x2028, 117x1872, 144x1521, 156x1404, 162x1352, 169x1296, 208x1053, 216x1014, 234x936, 312x702, 324x676, 338x648, 351x624, 432x507, 468x468,

However we are only interested in numbers that are equal to 20 mod 61. 81 is such a number, and its complement 2704 is the only other.

So we can let 61a-468=81 and 61b-468=2704 (or vice versa). These solve simply to a=9 and b=52 (or vice versa), and so these are the only solutions to the puzzle.

To conclude:

1/8 + 1/188 + 1/43992 = 1/9 + 1/52

247.20 - In Common – John Causer
They are all hats (a Dolly Varden is also a dress)

247.21 - Semi-quotations – John Causer
a) EDEN – Genesis 4: 16, and Richard 2nd, II.i.42
b) BLACK – Song title by Weldon J Irvine (1969), and Bartholomew Fair, II.i
c) BIRD – To the Cuckoo by William Wordsworth (1807); and To a Skylark, by PB Shelley (1819)
d) ALONE – Heart of Darkness, by Joseph Conrad (1902); and La Belle Dame Sans Merci, by John Keats (1820)

247.22 – Names – John Causer
They are all literary pseudonyms, and all designed to reverse (as opposed to conceal) the author's sex. the authors are Emily Bronte, Iain Blair, Karen Blixen, Dean Koontz, JK Rowling, Violet Paget, Michael Avallone, and Hugh Rae

247.23 - Long Word – John Causer
Rĕformation = Amendment, improvement (esp. religious or moral), amelioration
Rēformation = Rearrangement, re-ordering, reintegration (of sth. previously severed)

247.24 - Right, you 'orrible little men – John Causer
VEXING

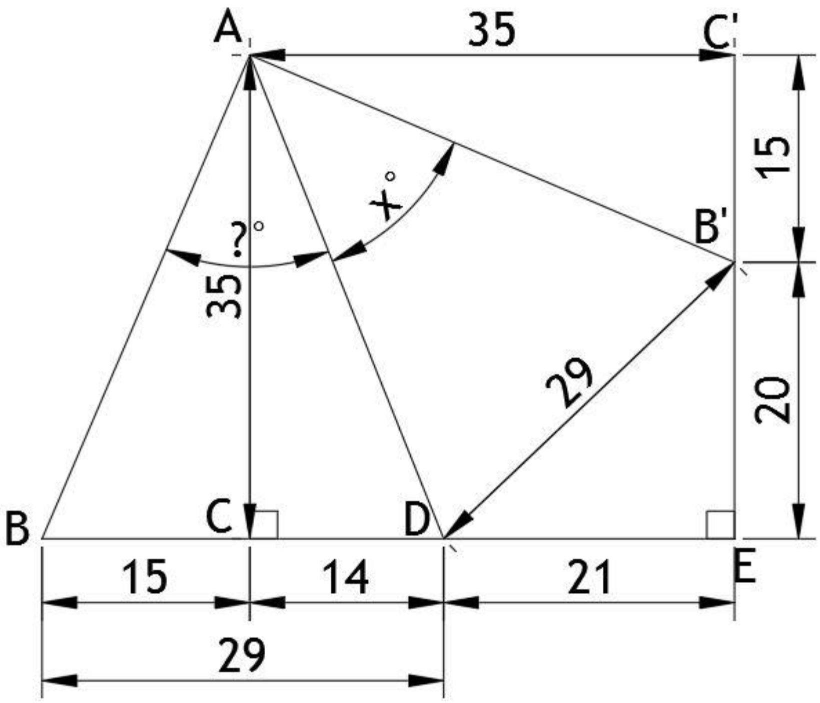
247.25 - Letter Mixture – John Causer
Hands, face, space

247.26 – Olympians – John Causer
The 1921 Olympic Charter provides that teams shall parade in alphabetical order, clarified in 1949 to be 'in the language of the host country'. But what is alphabetical order in a language which does not have letters? In Japan the problem was ignored and the English order used; in South Korea, Ghana led followed by Gabon as *ga* is first in the *han'gul* syllabary; in China a fourth century sorting method was used to order ideograms by their number of radicals followed by their type of brush-strokes (resulting in the last two teams being Australia followed by Zambia).

247.27 - Set It Yourself – John Causer
The answers are Alan Bennett, Celine Dion, Elizabeth Fry, George Harrison, and so have initials AB, CD, EF, GH so a possible question 5 is "Who in 1616 designed the Queen's House in Greenwich?", to which the answer is of course INIGO JONES

247.28 – What is the Angle?

Here are a couple of different approaches:
Firstly I'm going to make a copy of triangle ABC, rotated 90 degrees anti-clockwise around point A to form triangle AB'C'.

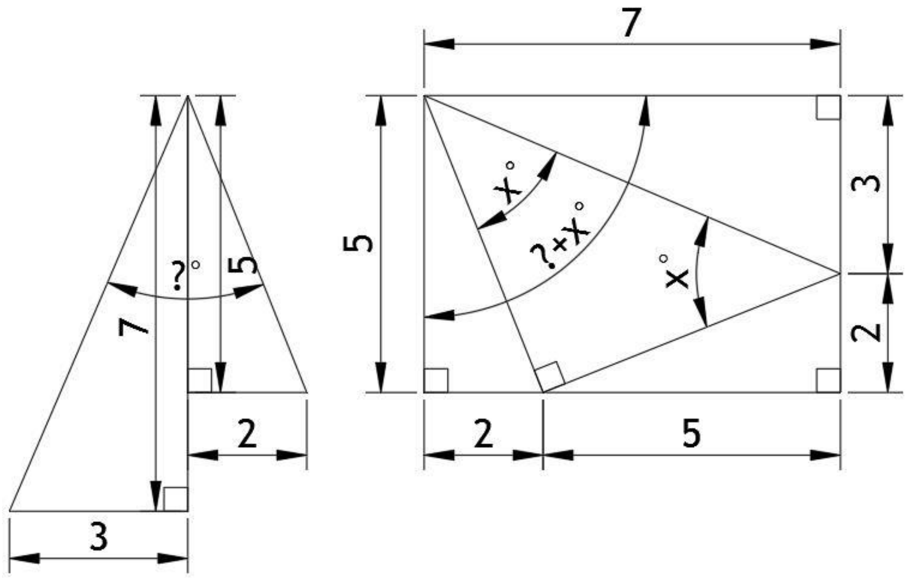


Since ACEC' is a 35 by 35 square, we can easily find out what DE and B'E are: 21 and 20 respectively. This forms a Pythagorean triple with the hypotenuse, B'D, equal to 29.

However since BD is also (14+15=) 29, AB equals AB' and AD is shared by both, triangles ABD and AB'D are similar and angle x is equal to the angle we seek.

But since angle BAB' = 90 degrees (as B' was formed by rotating B by 90 degrees), the angle we seek must be half of this, namely 45 degrees.

An alternative approach is to scale down the two parts of the puzzle triangle as below. This can be done because it doesn't change the angle we are seeking to find.



Similar to before we can rotate the left hand triangle 90 degrees anti-clockwise. Now to complete a rectangle we just need a copy of the 5x2 triangle, which means the triangular space in the middle is a right angled isosceles triangle, so x is 45 degrees. Since x + ? = 90 degrees, ? is also 45 degrees.

247.29 – Wordwall Crossword – Christa Ramonat

TWO-FOUR-SEVEN, the number of this issue of Enigma.

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T	W	O	O	N	I	E
O		U		I		V
P	E	R	I	G	E	E
I		T		M		N
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