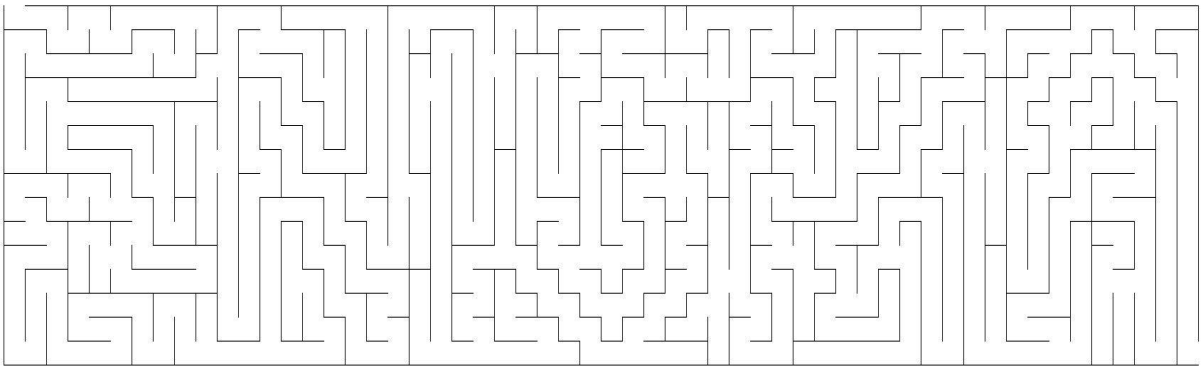
Enigma:
The Newsletter of PuzzleSIG



ENIGMA

245

'Mathematical problems, or puzzles, are important to real mathematics, just as fables, stories, and anecdotes are important to the young in understanding real life'.

Terry Tao

March 2020

Electronic Version of this Newsletter
Email enigma.mensa@yahoo.co.uk and I'll send you a copy

About Enigma
Enigma is the newsletter of Puzzle SIG.
The SIG for anyone interested in puzzles. The scope covers word puzzles, crosswords, logic puzzles, Japanese puzzles, mathematical brain teasers, lateral thinking problems, quizzes and picture quizzes, discussion of physical / mechanical puzzles, computer / internet based puzzles and puzzle games, and puzzle books and publications, and experimentation and innovation of new puzzle forms.

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How to Join
You can join Puzzle SIG by visiting mensa.org.uk/sigs (member login required).

Welcome to Enigma 245

Hello and welcome to another issue of Enigma.

Sorry it's been a while since the last issue, life gets in the way sometimes! But hopefully there are enough puzzles here to keep you occupied until the next issue.
I'm aware that most (but not quite all) of the puzzles I personally construct are mathematical in nature, so I'm always grateful when people such as Rosemary Hodgson or Peter and Jenny Nichols send me some word puzzles. Remember that the scope of this SIG is actually quite wide-ranging, far more so than any one issue would suggest. The scope is always detailed on page two of each issue, but worth mentioning again here. Puzzles, of course, in all their infinite variety. Word puzzles, logic puzzles, maths puzzles. But also items about physical puzzles like Rubik's cube or those intriguing wooden puzzles. Articles also about puzzle games; there are thousands of smart phone games out there that might be worthy of discussion (including my own Android app 'Paddocks'). We can also include quizzes and picture quizzes, particularly the more 'puzzly' type of quiz with teasers like '7 D of the W' or '12 S of the Z'.
If you do feel you can contribute I'd be delighted, but I also understand that some people enjoy the puzzles but don't have the time to contribute themselves, and that is of course fine too.

Please try all of the puzzles, and if you have any feedback, let me know and I'll pass it on to the puzzle's creator.

As ever, if you get stuck and need a hint, drop me a line and I'll be happy to help.
Please keep your puzzles, answers, comments, queries, suggestions, etc coming in.

Happy puzzling
Elliott.

244.01 - COMPETITION: Digital Cube Sums - Elliott Line

SOLUTION: 919 (= 1+64+125+729) & 1459 (= 729+1+729)

WELL DONE TO:
Johann Muller
Michael Kenedy
Abhilash Unnikrishnan
Christa Ramonat
Paul Clark
Roisin Carters
Christa Ramonat
Stuart Nelson

245.01 - COMPETITION: Card Counting - Elliott Line

I have a certain number of cards, with numbers on them counting upwards from 1. I shuffle them and pass them to you. You place them, one by one, on the table. Each time the card you put down is higher than any already on the table, you earn 70 points. Each time the card is lower than any already on the table you earn 30 points. If a card is neither the highest nor the lowest card up to that point it earns no points. When you place the first card on the table, it is both the highest and the lowest so far, and so earns 100 points.

For example, if there were three cards and you dealt them in the following order:

2, 3, 1, you would earn 100 + 70 + 30 = 200 points, or
3, 1, 2, you would earn 100 + 30 + 0 = 130 points.

For three cards, there are six different orders the cards could be dealt, each earning between 130 points for (3,1,2) as shown above, and 240 points (for 1,2,3). The average score is 183 1/3.

However, we are using more than three cards. It turns out that the average score is precisely 245 points (245 being the number of this issue of Enigma).

How many cards are we using?

This is a competition, but not for prizes, only bragging rights. Every correct answer I receive will get an honourable mention in the next issue of Enigma. Send your answer to me at enigma.mensa@yahoo.co.uk .

245.02 – Base 4 Code – Elliott Line

I have written a sentence, and I have replaced each of the letters with the numbers that denote their position in the alphabet. However, I have used the base 4 number system. Be careful, as some sequences of numbers could lead to several words, for instance 31110 could mean CAT (3,1,110), but could equally mean MAD (31,1,10).

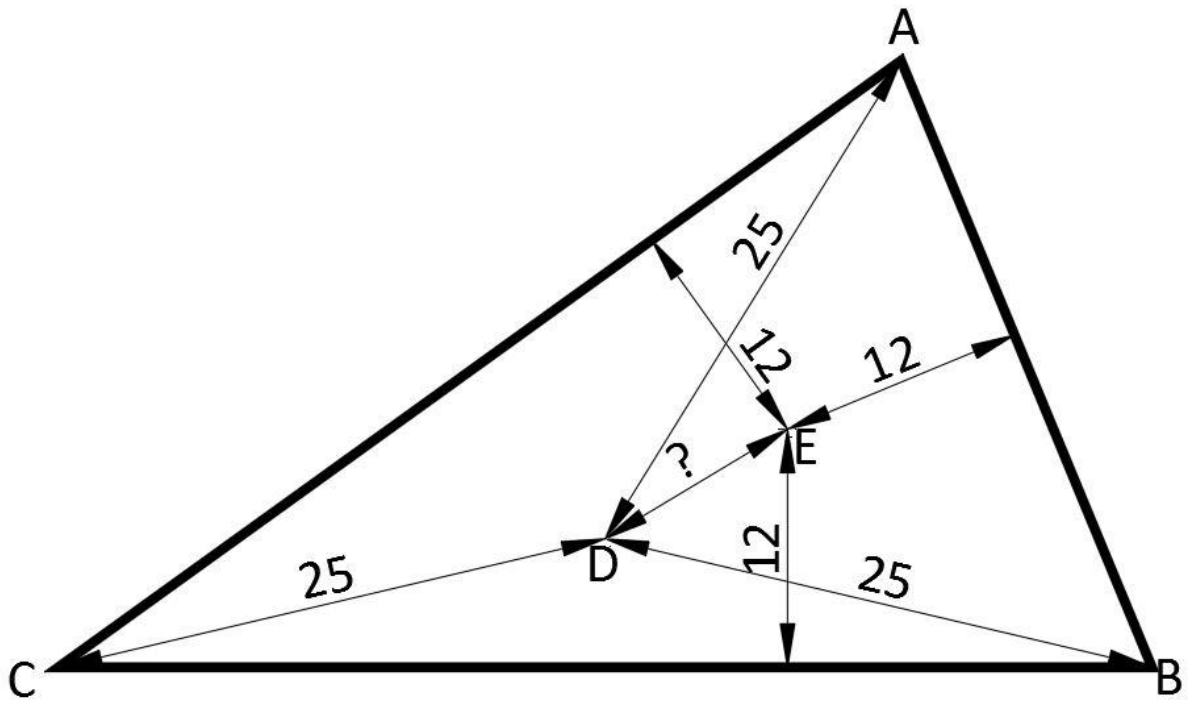
	1 = A	2 = B	3 = C
10 = D	11 = E	12 = F	13 = G
20 = H	21 = I	22 = J	23 = K
30 = L	31 = M	32 = N	33 = O
100 = P	101 = Q	102 = R	103 = S
110 = T	111 = U	112 = V	113 = W
120 = X	121 = Y	122 = Z	

1103 11311 11311 11311 12213210 11311 110211 3233110 3211110230121 1103

1132110311 1103 11311 20331001110 11033 2113333111.

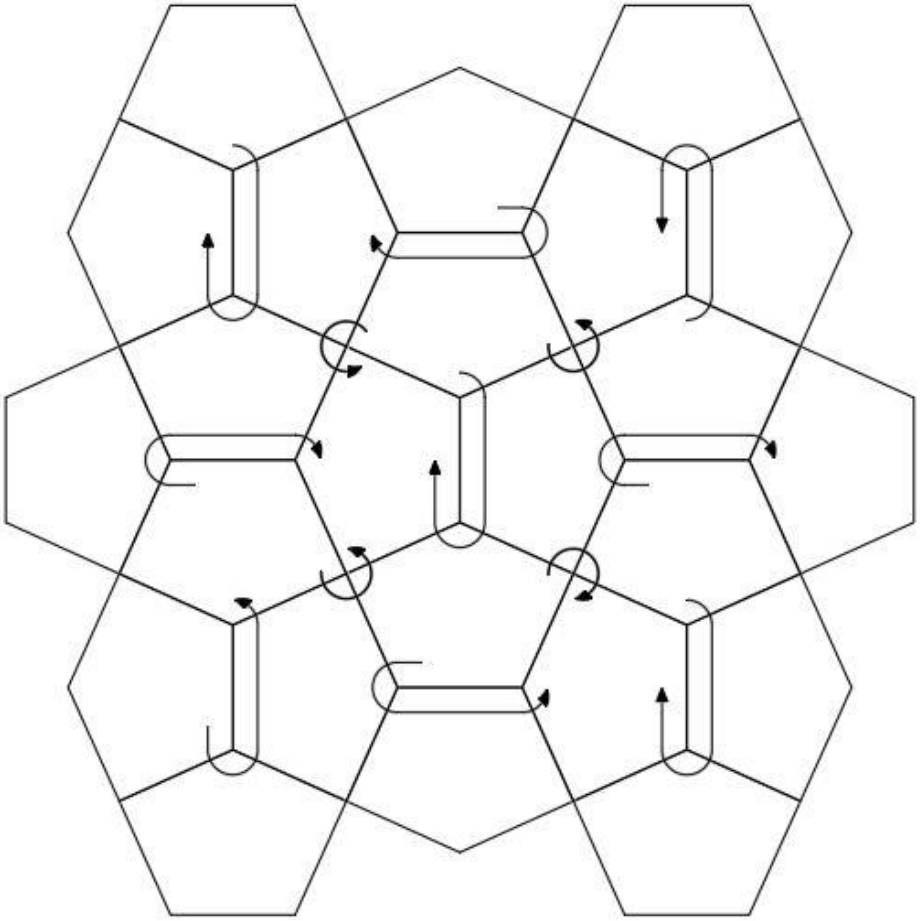
245.03 – Five Towns – Elliott Line

There are five towns: Anderton, Barmby, Calderwood, Dempsey and Edinburgh.
There are straight roads between Anderton, Barmby and Calderwood.
Dempsey is exactly 25 miles from Anderton, Barmby and Calderwood.
Edinburgh is exactly 12 miles to the closest point on each of the roads between Anderton, Barmby and Calderwood.
How far is Dempsey from Edinburgh?



245.04 – Cairo-Word – Elliott Line

I have taken a section of a Cairo tiling pattern and populated it with four-letter-words. To be helpful to you in your task of reconstructing it, I've given you both the list of words and the staring position and the direction of each of the words in the grid.
To give you a hint of where to start, ask yourself where could 'ITCH' go, considering there is only one 'I' in the word list and that any other 'C's appear only at the start of other words.



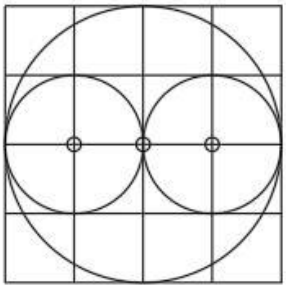
CANT
CAST
CHAT
CHEW
ITCH
NEAT
SAWN
SEWN
SHAM
WANT
WEAN
WEST
WHAT

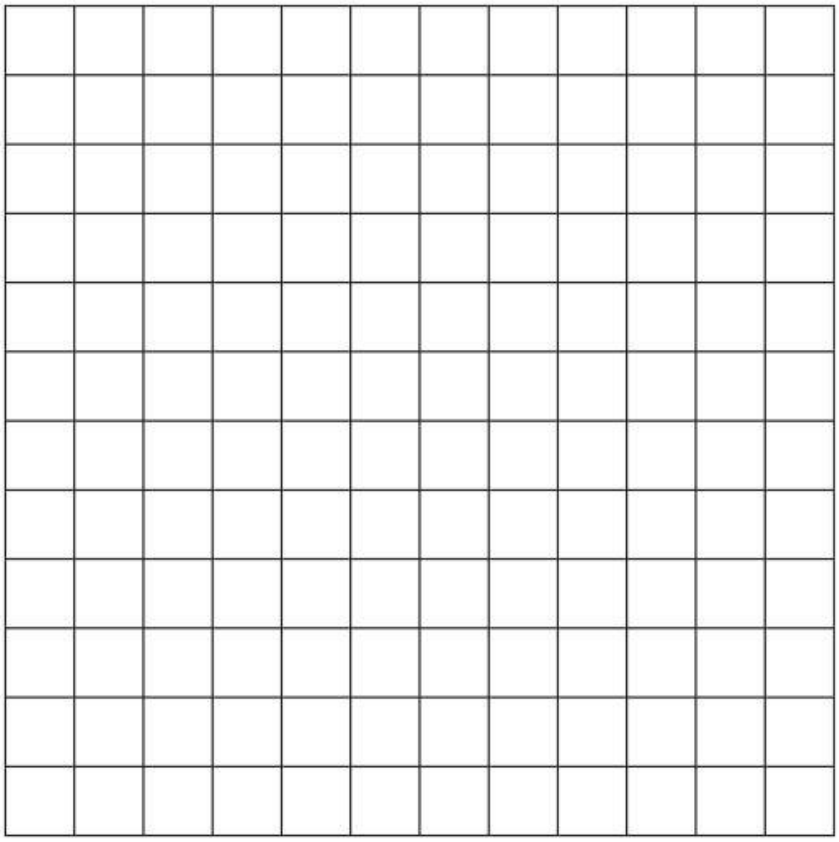
245.05 – Circle Box – Elliott Line

Here's a fun challenge. The task is to fit circles into a grid according to the following rules:

- The centre of every circle must lie on a lattice-point of the grid, and no circles can share the same centre point.
- Every circle must have a whole number as its radius.
- The circles can touch each other and the edge of the grid, but cannot cross. In other words a circle can be entirely inside another, or entirely outside, but not partially inside.
- Your 'score' is the total area of all the circles you can fit in. A radius-1 circle has area of 1(pi), a radius-2 circle has an area of 4(pi), radius-3 = 9(pi), etc.
- The example 4 x 4 unit grid has a score of 6(pi) (4+1+1). I have marked the centre of each circle to show that none coincide.

What score can you achieve in the 12 x 12 unit grid?





245.06 – Irregular Polyhedron – Elliott Line

I have a polyhedron with the properties that every vertex is at the junction of exactly three faces, two of its faces are heptagons, and all its other faces are pentagonal.
How many pentagonal faces are there?

245.07 – Hidden Words – Rosemary Hodgson

Find a colour hidden in each sentence:

The ogre ensured he was not caught.
Andrew hit Edward on the head.
The poor ape aches with loneliness.
Relax, or anger will consume you.
One whole month has passed already.
Please keep ink away from the children.
If awnings are available, use them.
To avoid a scar, let it heal naturally.
Let's scrap ricotta cheese salad.

Find a name hidden in each sentence:

We need wardens here.
Could avid readers spot it?
Don't mar your chances by being daft.
I know I annoy him.
Just the usual ice cream please.
He played his banjo and sang.
Let them make cake.
He is innocent, as am I.

Find a number in each of these:

I like his tone of voice.
Some of our teenagers are here.
No, it won't work.
It's Christmas Eve, nearly.
We will need tinsel eventually.
She sews with reels of red cotton.
Put a coin in each purse.
We'll win if our numbers come up.
He ate no food that day.

245.08 – Nonsense Sentences – Peter and Jenny Nichols

What odd characteristic is shared by all of the following sentences?

I came some way, so I guess I'm messy.
Kiss me quick.
Mucky images make us go away, says Mickey Mouse.
Cows go moo.
My moggies say miaow.
Cake makes mice squeaky.
Some geese squawk magic music.
We swim away.

245.09 – Mastermind – Elliott Line

You may be familiar with the game 'Mastermind', where one player forms a code of coloured pegs and another makes guesses. The coder tells the guesser after each guess how many pegs are fully correct, and how many others are present but in the wrong place.

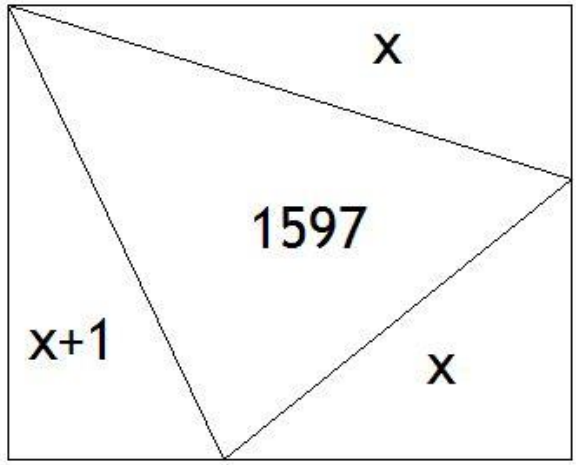
My version is different from the standard version in three respects:

a) Instead of four, the code is now FIVE pegs long.
b) Instead of six different colours to choose from, in this version there are only TWO different colours of pegs: RED and YELLOW. There are therefore 2 to the power of 5, (32), different possible codes the coder might have set.
c) The guesser has decide what their first 'n-1' guesses are going to be straight away, before getting any feedback from the coder, such that when the feedback ultimately comes, the guesser can be certain of guessing correctly on the very next guess, the 'nth' guess (or earlier if one of the original n-1 guesses happened to have been correct). (In other words, if the guesser decides on 9 guesses and knows that whatever the code happens to be, they can use the feedback from those 9 guesses to be certain that the 10^{th} guess will be correct, then n equals 10).

The question is then: what is the minimum value of 'n', such that the nth guess is guaranteed to be correct?

245.10 – Nearly Equal Corners – Elliott Line

A rectangle is split into four triangles as below. The areas of the three corner pieces are almost equal, with one being just 1 square unit larger than the other two. The remaining area is 1597 square units. What is the value of x?



245.11 – Where are they from? – Elliott Line

If Jacob is from Edinburgh, Simon is from Sunderland, and Elliott is from Durham, where is Caroline from?

245.12 – Perfect Power Pairs – Elliott Line

Most numbers cannot be expressed as a^b + c^d, where a and c are prime numbers and b and d are integers greater than 1. However there is one set of 6 consecutive numbers that all can, what are they?

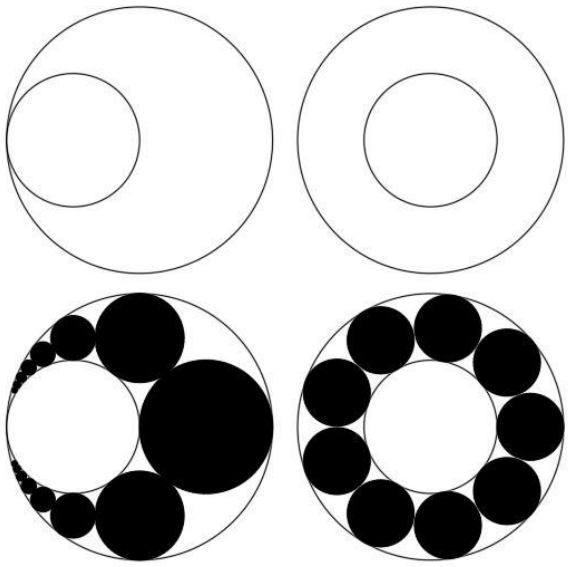
245.13 – Riddle – Rosemary Hodgson

My first's in the Jungle but not at the baker's
My second is hoped for by all exam-takers.
My third is a note on the keyboard and next,
My fourth is a pronoun if written in 'text'.
My fifth is of course the alphabet's leader
And my sixth is the last of these words for the reader.
I command great respect in my normal abode,
And sometimes as well when I'm out on the road.
What am I?

245.14 – Shaded Circles – Elliott Line

A circle of radius 2 has a circular hole of radius 1.
This is done is two alternative ways: in one the hole is tangent to the larger circle forming a crescent shape; in the other the hole is in the exact centre of the larger hole.
In the 'crescent' option a shaded circle of radius 1 is drawn, and then infinitely many smaller and smaller shaded circles are added heading off towards the two tips of the crescent.
In the 'ring' option just nine circles, each of radius ½ are drawn and shaded.

Which version has the largest combined shaded area?



245.15 – Split Squad – Elliott Line

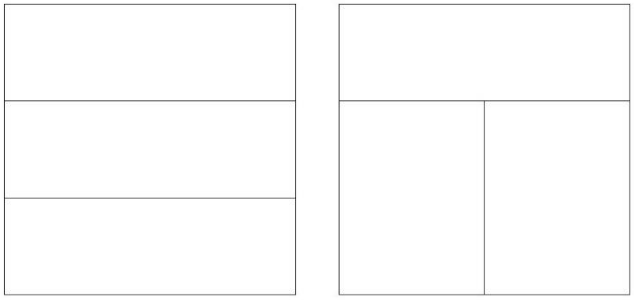
A junior football team has 15 players, with squad numbers from 1 to 15.
During training they split into two teams: a team of 7 and a team of 8, such that:
The sum of the squad numbers on team A is the same as the sum of the squad numbers on team B.
The sum of the *squares* of the squad numbers on team A is the same as the sum of the *squares* of the squad numbers on team B.
The sum of the *cubes* of the squad numbers on team A is the same as the sum of the *cubes* of the squad numbers on team B.
How was the squad split into the two teams?

245.16 – Simple but Complex – Elliott Line

If x + y = 4, and x^2 + y^2 = 6, what are the values of x and y?

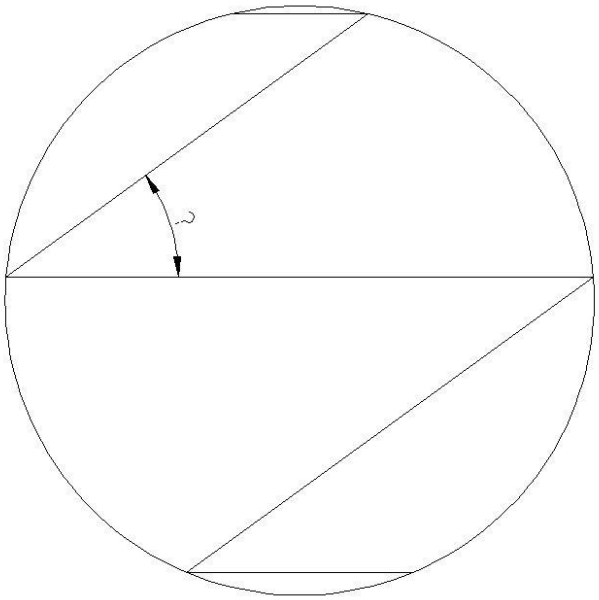
245.17 – Three Way Split – Elliott Line

Given a unit square it is possible to split it into three equal areas in a number of different ways. For instance cutting off the top third and then the bottom third. Or cutting off the top third then making a vertical cut to divide the remaining part in two. The total cut length of that first option is 2, whereas the total cut length of the second option is only 1+2/3.
Can you find a way of dividing the square into three equal areas, which requires even less total cut length?



245.18 – Zigzag – Elliott Line

A zigzag of lines is drawn in a circle. The combined length of the three horizontal lines is exactly equal to the combined length of the two diagonal (parallel) lines. What is the angle formed between the horizontal and diagonal lines?



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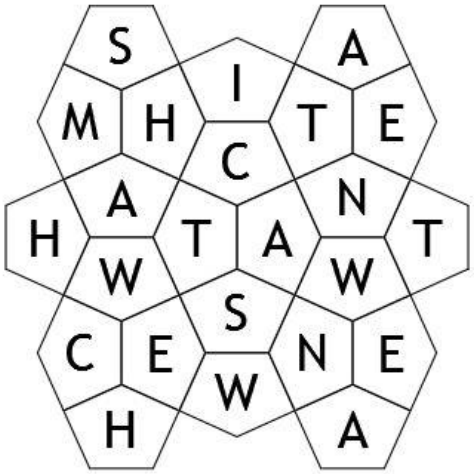
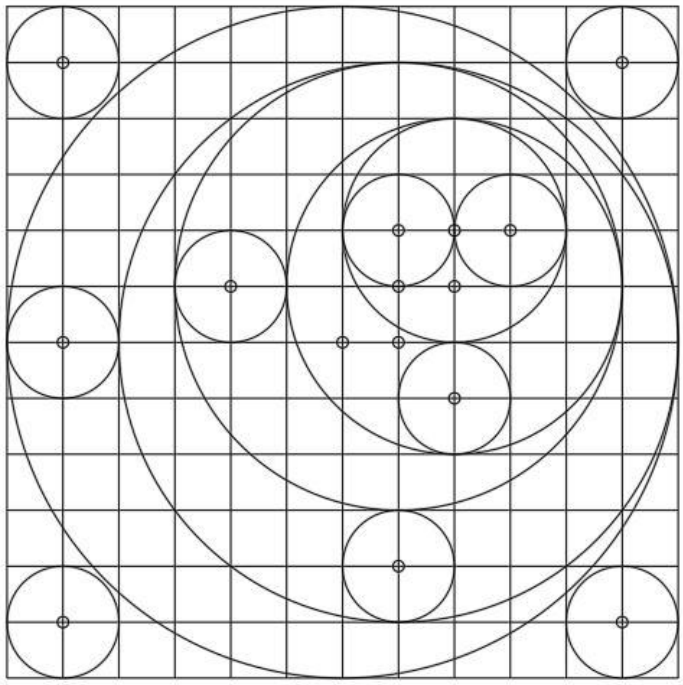
245.02 – Base 4 Code – Elliott Line
1-103 113-11 1-13-11 113-11 12-21-32-10 113-11 1-102-11 32-33-110 32-11-1-102-30-121 1-103 113-21-103-11 1-103 113-11 20-33-100-11-10 110-33 2-11-3-33-31-11.

"As we age we find we are not nearly as wise as we hoped to become." (I deliberately wrote a sentence that would result in ambiguous words, as 'we' and 'age' both encode to 11311).

245.04 – Cairo-Word – Elliott Line



245.05 – Circle Box – Elliott Line
A score of 100(pi) is possible: (36+25+16+9+4+10).



If you're interested, I also did this for a 100x100 grid, which results in a score of 44246(pi). The diagram of this is shown on the back cover.

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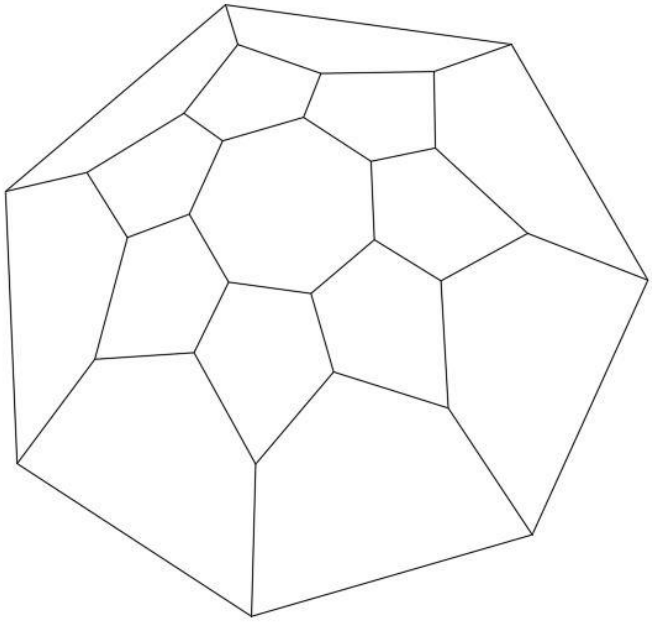
245.03 – Five Towns – Elliott Line

You don't need to know any of the information that hasn't been provided, such as how far it is from Anderton to Barmby, or Edinburgh to Calderwood etc. The distance from Dempsey to Edinburgh is 5 miles.

245.06 – Irregular Polyhedron – Elliott Line

Using Euler's formula for vertices, edges and faces of a polyhedron:
V+F-E = 2
If we let the number of pentagonal face be n, then the number of faces is (n+2), the number of edges is (14+5n)/2 (since each edge belongs to two faces), and the number of vertices is (14+5n)/3 (since each vertex belongs to three faces).
(14+5n)/3 + (n+2) – (14+5n)/2 = 2
Which boils down to n = 14, so there are 14 pentagonal faces.

Below is such a polyhedron. All of the edges and all of the vertices are visible, just one heptagonal face is on the far side.



245.07 – Hidden Words – Rosemary Hodgson

Green, White, Peach, Orange, Lemon, Pink, Fawn, Scarlet, Apricot.
Edward, David, Mary, Ian, (& Ann) Alice, Joan, Emma, Sam.
One, Fourteen, Two, Seven, Eleven, Three, Nine, Four, Ten.

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245.08 – Nonsense Sentences – Peter and Jenny Nichols
The sentences only use letters in odd positions in the alphabet (A, C, E, G, etc).

245.09 – Mastermind – Elliott Line
It is always possible to solve on the fourth guess if the first three guesses are sufficiently independent.
One such set of initial guesses might be:
YYRRR
YRYRR
RYYYR
With the results of these guesses the true code will be known for certain.

245.14 – Shaded Circles – Elliott Line
The ring option is fairly easy to calculate: each of the nine circles will have an area of pi/4, therefore the shaded are is 9pi/4 = 7.069…
The crescent area is trickier.
The radius of the largest shaded circle is 1, the next is 2/3, the next 1/3, then 2/11… Naming the largest circle the zeroth, in general the radius of the nth circle is:
2/(n^2 + 2). The area of each circle is pi.r^2 or:
pi*(2/(n^2+2))^2. Usefully this also works for negative values of n, so will cover both arms of the crescent.
At this point I consulted the Wolfram Alpha website to see if the infinite sum of this boils down to anything simple:
sum_(n=-∞)^∞ pi (2/(n^2 + 2))^2 =

1/2 pi^2 (sqrt(2) coth(sqrt(2) pi) + 2 pi csch^2(sqrt(2) pi))

Either using Wolfram Alpha or Excel we can estimate the infinite sum as ≈ 6.998…
Therefore the ring option gives a slightly larger shaded area.

Incidentally, they greatest shaded area you could achieve would be by moving the hole 0.3942303… off centre, placing a maximal 0.69711515… radius circle next to it, and then adding nine further circles of varying radii each tangent to the previous circle as well as the inner and outer circles. This gives a shaded are of 7.21792057…

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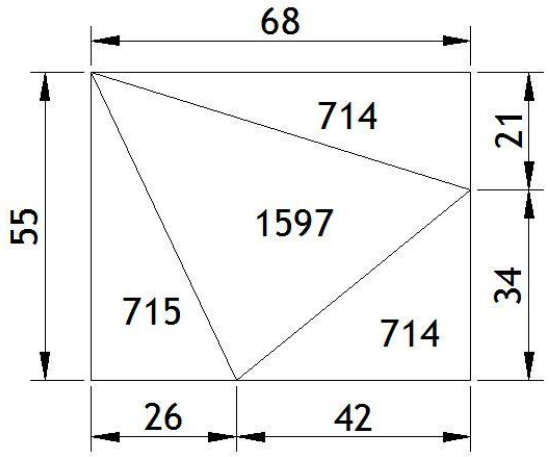
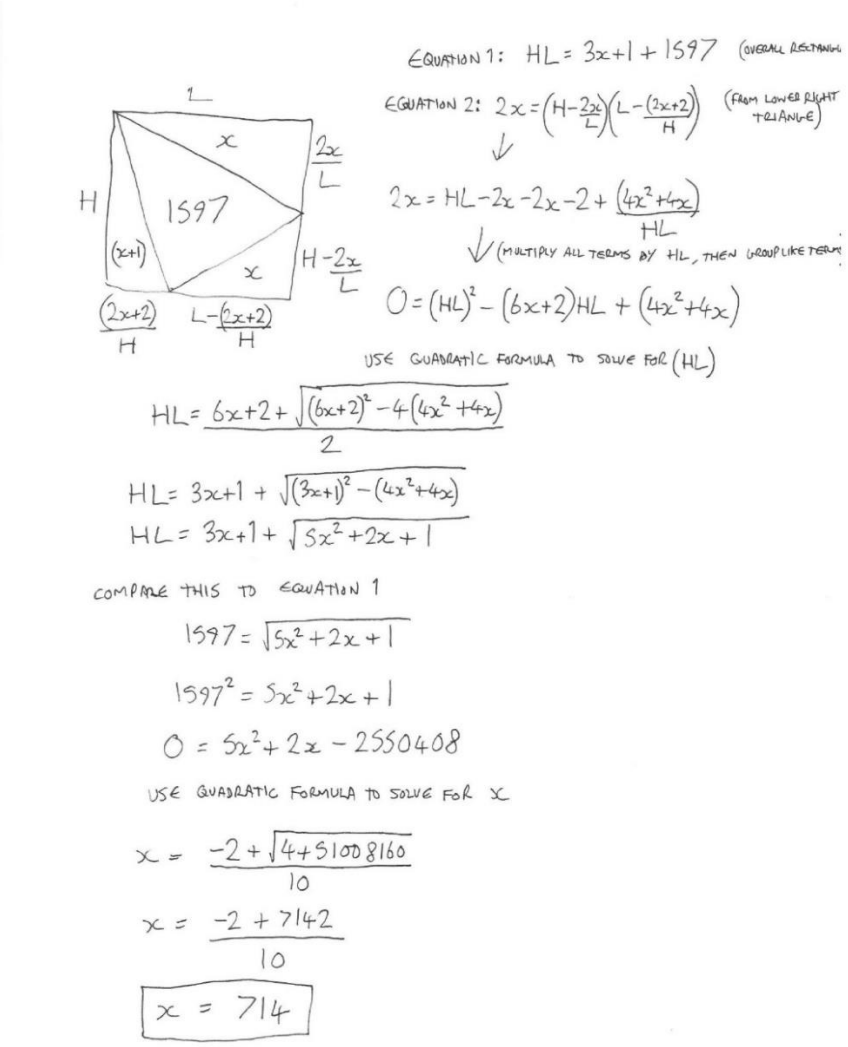
245.10 – Nearly Equal Corners – Elliott Line

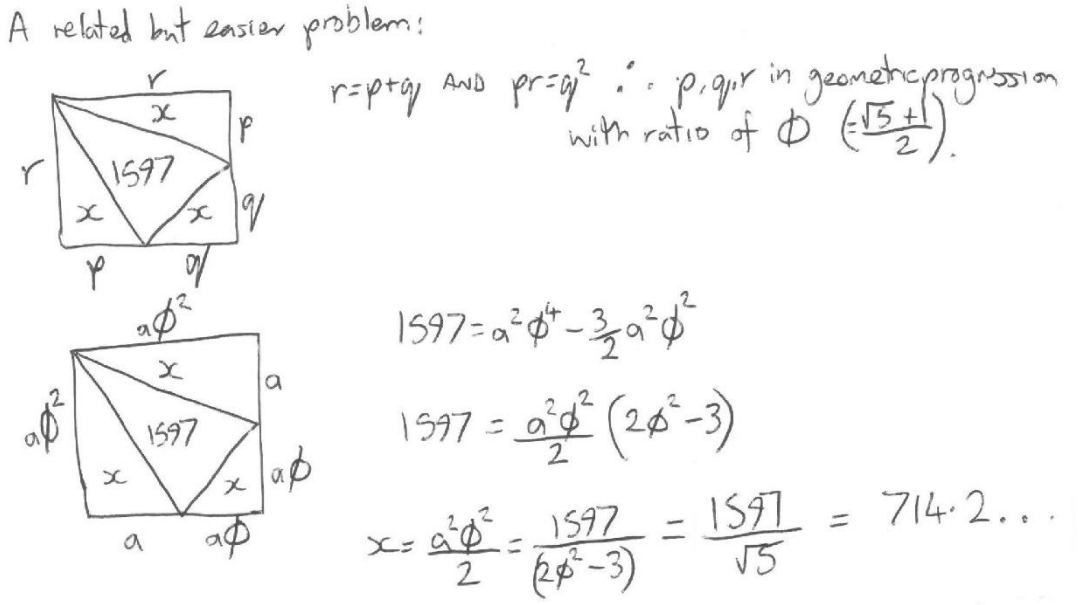
x is 714. One possible set of dimensions are as below:



My expectation being that the solver would try some numbers and arrive at this or another integer solution. However it is solvable using algebra as below (excuse my handwriting):



Alternatively, there is a far quicker way to arrive at an approximate solution by looking at a similar but easier problem:



245.11 – Where are they from? – Elliott Line

Each person+place contains the five vowels exactly once each. Caroline is only missing the letter U so she is from Hull (or anywhere else with a 'u' and no other vowels).

245.12 – Perfect Power Pairs – Elliott Line

31 = 3^3 + 2^2
32 = 2^4 + 2^4
33 = 5^2 + 2^3
34 = 5^2 + 3^2
35 = 3^3 + 2^3
36 = 3^3 + 3^2 = 2^2 + 2^5

245.13 – Riddle – Rosemary Hodgson

A Jaguar.

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245.15 – Split Squad – Elliott Line
Team A: 3,5,6,9,10,12,15
Team B: 1,2,4,7,8,11,13,14
(or vice versa)

I have a machine which can turn any A into AB, and B into BA. I don't have any letters to begin with, so I borrow an A off my Grandad:
A
I run it through the machine and obtain:
AB
Then I run it through again:
ABBA
And again:
ABBABAAB
Then one final time:
ABBABAABBAABABBA

I remove the initial A and return it to my Grandad, thanking him profusely.

I now have 15 letters, all either A or B. I assign the players in the squad either team A or B accordingly:

B B A B A A B B A A B A B B A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

And hey presto, that's the answer!

245.16 – Simple but Complex – Elliott Line
2+i and 2-i

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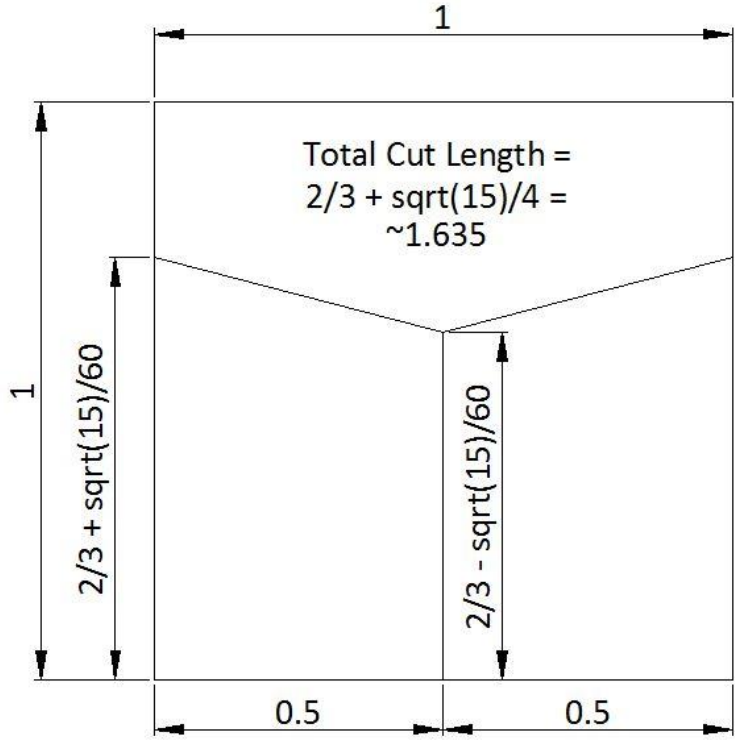
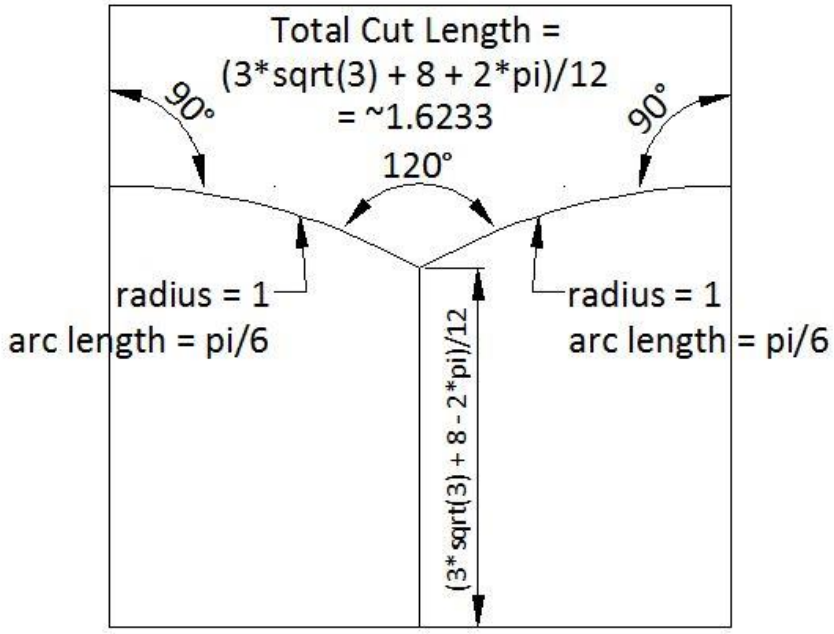
245.17 – Three Way Split – Elliott Line

The following is simple proof that the 1 2/3 solution can be improved upon:



However, we can do even better using circular arcs, such that any junctions with the edge of the square are at right angles and any junctions within the square are at 120 degrees:



As far as I can ascertain, this is now the shortest overall cut length but if you can do better I would be interested in knowing about it!

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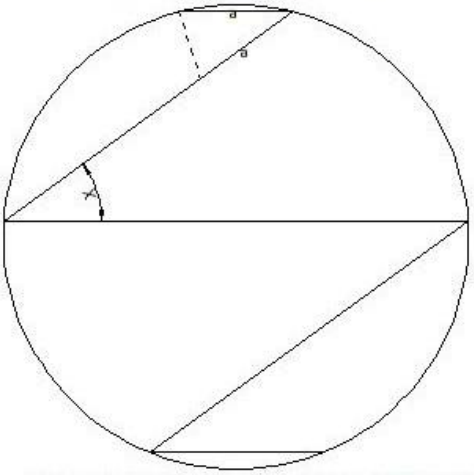
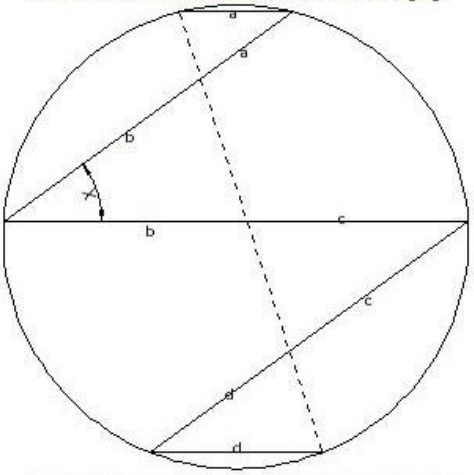
245.18 – Zigzag – Elliott Line

36 degrees

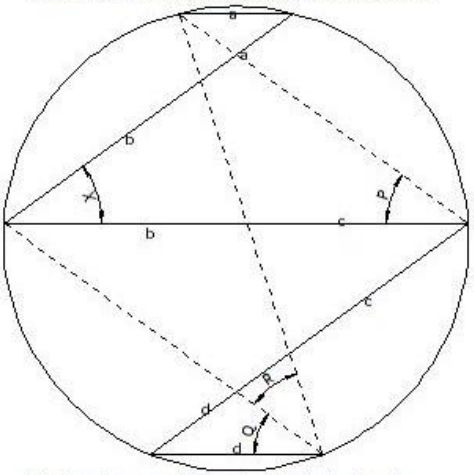
Draw a line from the top end of the zigzag, that meets the next line segment to form an isosceles triangle:



Because of similar triangles, combined with the fact that the sum of the horizontal lines is equal to the sum of the diagonal lines, continuing this line will result in three more isosceles triangles and the line will coincide with the other end of the zigzag:



Because of the inherent symmetry of the figure, the angles P and Q will also be equal to X. And because of the angle-chord theorem, R will be equal to P and therefor also equal to X:



This therefore means that each of the similar isosceles triangles will have angles X, 2X and 2X. X is therefore 180/5 = 36 degrees.