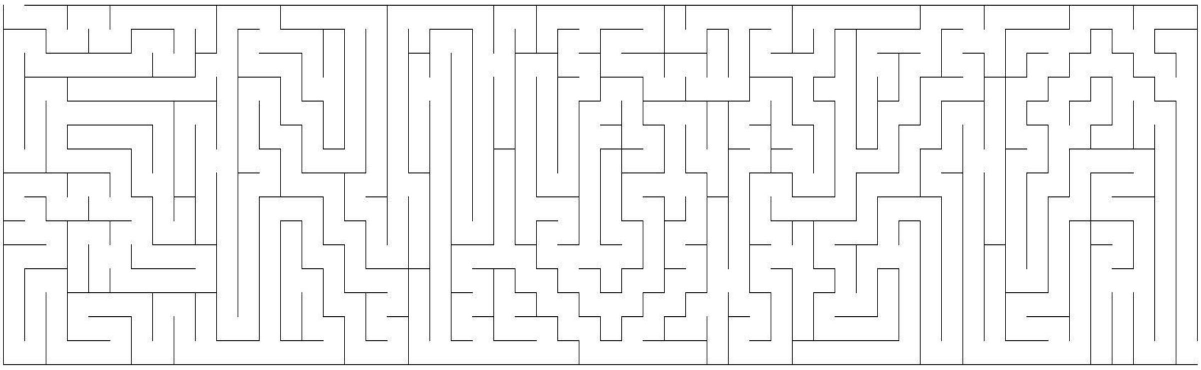
Enigma:
The Newsletter of PuzzleSIG



ENIGMA

246

(...8, never too late. – tom Robinson)

'Mathematical problems, or puzzles, are important to real mathematics, just as fables, stories, and anecdotes are important to the young in understanding real life'.

Terry Tao

July 2020

Electronic Version of this Newsletter
Email enigma.mensa@yahoo.co.uk and I'll send you a copy

About Enigma
Enigma is the newsletter of Puzzle SIG.
The SIG for anyone interested in puzzles. The scope covers word puzzles, crosswords, logic puzzles, Japanese puzzles, mathematical brain teasers, lateral thinking problems, quizzes and picture quizzes, discussion of physical / mechanical puzzles, computer / internet based puzzles and puzzle games, and puzzle books and publications, and experimentation and innovation of new puzzle forms.

SIGSec and Editor: Elliott Line
34 Hillside, Hartshill, CV10 0NN
www.elliottline.com
enigma.mensa@yahoo.co.uk
Deputy SIGSec: Paul Bostock

How to Join
You can join Puzzle SIG by visiting mensa.org.uk/sigs (member login required).

Welcome to Enigma 246

Hello and welcome to another issue of Enigma.

Apologies for the lateness of this issue (I know there hasn't been a strict schedule for some time, but still, this is a long time to wait between issues). In these crazy times I've found myself with not a great deal of time to spend on some things. Luckily, thanks to the fact that I publish a Puzzle of the Week every week on my website, and that I received some great puzzle contributions from readers: John Clarke, Paul Bostock, Agnijo Banerjee, John Causer and Rosemary Hodgson, I was not short of content, just short of the time to put it all into an issue of Enigma. I hope you'll forgive its hasty and chaotic assembly.

Please try all of the puzzles, and if you have any feedback, let me know and I'll pass it on to the puzzle's creator.

As ever, if you get stuck and need a hint, drop me a line and I'll be happy to help.
Please keep your puzzles, answers, comments, queries, suggestions, etc coming in.

Happy puzzling
Elliott.

245.01 - COMPETITION: Card Counting - Elliott Line

We are using 6 cards.

WELL DONE TO:
Johann Muller
Michael Kenedy
Abhilash Unnikrishnan
Paul Clark
Roisin Carters
Stuart Nelson
Agnijo Banerjee
Paul Bostock
Alan Davenport
Seffi Miller
Mick Sulley
Cheylea Hopkinson
Christa Ramonat

246.01 - COMPETITION: Golomb Ruler - Elliott Line

A Golomb Ruler is a fascinating mathematical object. It is a ruler with a series of marks on it, precisely on integers, such that the difference between any two marks is unique for that ruler.

For instance, the shortest possible ruler with 6 marks could have the marks at 0,1,4,10,12,17:
The table to the right shows the 15 possible differences between any two numbers in the list.
As you can see, not every number up to the maximum has to appear (14 and 15 do not appear), but no number can be repeated.

					17
				12	5
			10	2	7
		4	6	8	13
	1	3	9	11	16
0	1	4	10	12	17

It was discovered in 1994 that the shortest possible Golomb Ruler with 19 marks is 246 units long (246 being the number of this issue of Enigma).
I will give you the positions of 18 of those marks, and you have to find one more, between 0 and 246, to complete the ruler.

The 18 points are:

0, 1, 6, 25, 32, 72, 100, 108, 120, 130, 153, 169, 190, 204, 231, 233, 242, 246

What is the missing mark?

This is a competition, but not for prizes, only bragging rights. Every correct answer I receive will get an honourable mention in the next issue of Enigma. Send your answer to me at enigma.mensa@yahoo.co.uk .

246.02 – All Change - John Clarke

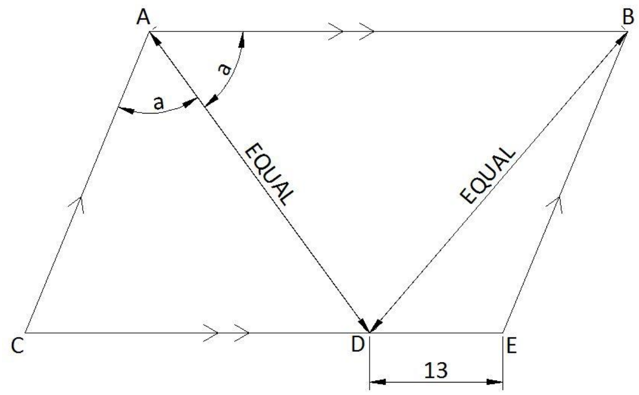
I recently attended a concert by the rock band The Lightning Seeds, where, in the first half, they performed the whole of their album *Jollification* to celebrate the 25th anniversary of its release. But instead of the ten tracks being played in their order as on the album, they were jumbled up, so that only one song, track no.7, appeared in its correct position. From the following information, can you determine, for each song, its track number on the album and the position in which it was performed at the concert?
Four of the songs on the album, *Change*, *Lucky You*, *Marvellous* and *Perfect*, were hit singles. One of these is the lead track on the album, and the concert opened with one of these songs and closed with one of the others.
Marvellous is track 6 on the album, but the sixth song to be played was *Telling Tales*. Track 3 was performed second. *Open Goals* was performed two songs before *Why Why Why*, which is true to their respective positions on the album.
Punch And Judy was performed one place earlier than its position on the album. *Lucky You* is on the first half of the album, two songs before *Change*, but was performed in the second half of the concert, somewhere after *Change*.
Feeling Lazy appears three tracks before *Telling Tales* on the album. *Marvellous* was performed two songs before *My Best Day*.

246.03 – Biscuit Plates – Roisin Carters

How many ways are there of splitting 10 biscuits across any number of plates?
For each split you can use as many or few plates as you like. Each plate used must contain a whole number of biscuits.
Your plates are identical so for any split, rearranging the order of the plates does not add to the solution.
For example, if you only had 4 biscuits the answer would be 5: 4 - 3,1 - 2,2 - 2,1,1 - 1,1,1,1

246.04 – Diophantine Parallelogram – Elliott Line

All of the lines on this diagram are integer lengths.
DE = 13
AD = BD
AB is parallel to CE
AC is parallel to BE
Angle CAD = angle DAB
What is the length of AD?



246.05 – Build-a-Crossword – Elliott Line

I have taken a completed crossword grid, removed all the consonants to the end of the row and/or column that they belong in, and then changed all the black squares into random vowels.
Your task is to reconstruct the crossword by figuring out which of the vowels are genuine and which need to become black squares, and by putting the consonants all back into place. Good luck!

	A		E	U		E			C F N R T
A	A		O		I		I	I	
		A	I			E			L L R S T W
E	I		E		E		A	E	
E		E		A				U	M N S T Y
	A		U	U	U		O	I	
		O				O			C D R R S S W
A	O		O	E	I		A	O	
	I		E	E		E			B B K L L
B		F		F		C		D	
C		K		R		D		L	
C		M		S		L		L	
S		R		Y		R		T	
S		R				T			
		W							

246.06 – Semi-grams – Elliott Line

Rearrange the letters in each of these five letter words and then pair them up to form ten letter words. I've completed one of the ten letter words to start you off:

First halves:

~~ALERT~~ METRO NOTED TIMES UPSET

Second halves:

GIANT ~~RATIO~~ ROAST SENSE SOUND

Example solution:

ALERT + RATIO = RETAL + IATOR = RETALIATOR

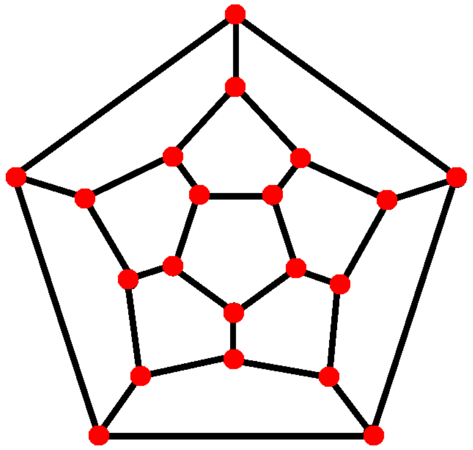
246.07 – Dodecahedron Graph – Paul Bostock

The diagram is a plane representation of a dodecahedron. The red points are the vertices and the black lines are edges.

a) Colouring: we wish to assign a colour to each vertex so that all neighbouring vertices have different colours. What's the minimum number of colours that will suffice? (To be clear 'neighbouring' means sharing an edge, i.e. here, if there's a line directly between two red points in the diagram they must be given a different colour)

b) A path visiting each vertex: Can you find a path that visits every vertex once (and once only) and ends where it started? It does *not* have to use all the edges, but any edge can only be used once.*

*This is called a Hamiltonian path and is similar to the Eulerian type, which tries to visit use all the edges exactly once and return (the well-known Konigsberg Bridge problem).

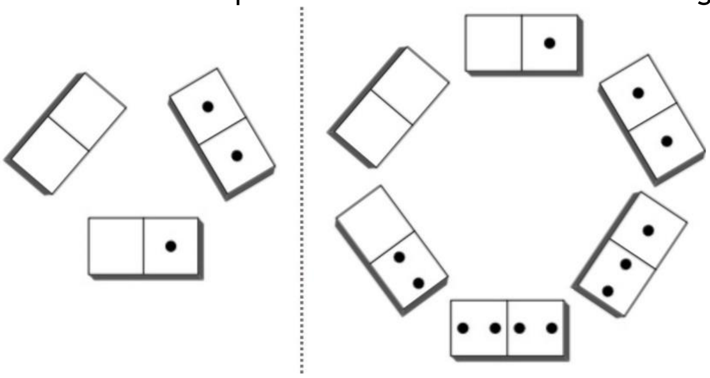


246.08 – Domino Ring – Paul Bostock

The aim is to make a complete loop using a full set of dominos, with the usual domino rule about having to match all the neighbouring ends.

It helps to gain some insight by trying smaller sets. The diagram shows that I cannot make a loop with a three dominos set (0,0) (0,1) and (1,1). Try changing something and make sure you're convinced.

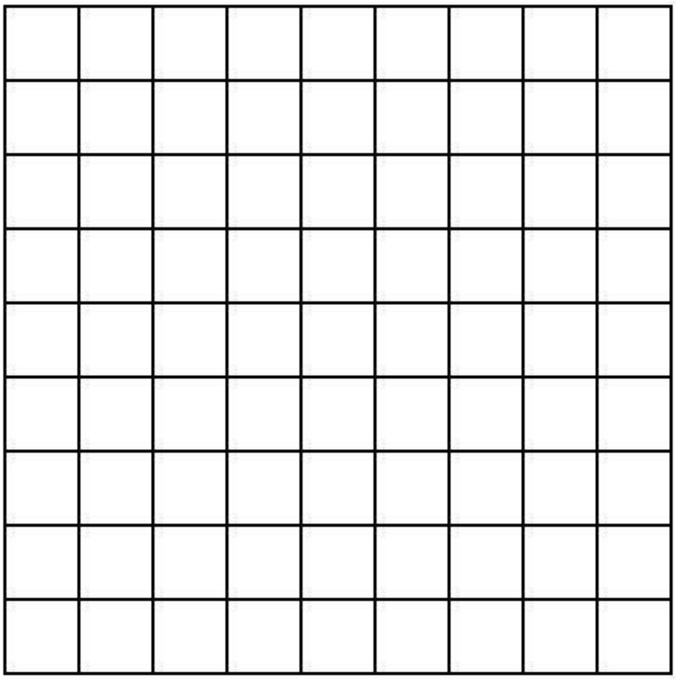
I *can* make a loop with the six domino set shown - going up to (2,2).



Can you find which larger sets will work and which will not?
If you have a standard set of 28 dominos up to (6,6), try making a loop with all of them.

246.09 – Ghost Crossword – Elliott Line

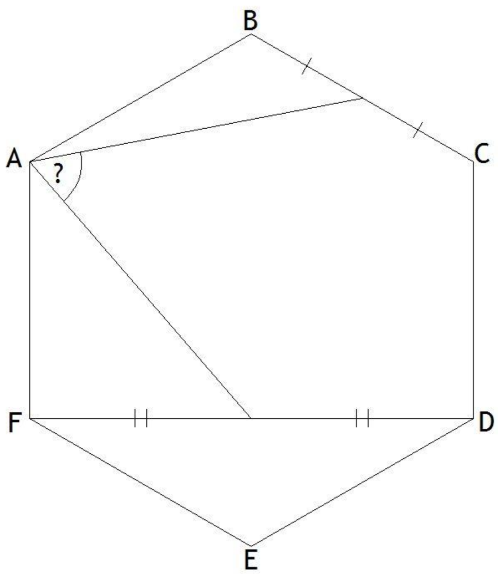
Use logic to fill in the crossword grid given only the clue numbers and the following rules:
1) The crossword is numbered in the usual way.
2) The grid is fully symmetrical.
3) The white area must all be connected together.
4) ‘Words’ are at least three letters long.
5) No 2x2 black squares are allowed.
6) No row or column is entirely white or entirely black.



Across	Down
1	1
4	2
8	3
9	5
10	6
12	7
13	11
15	12
18	14
20	15
22	16
23	17
24	19
25	21

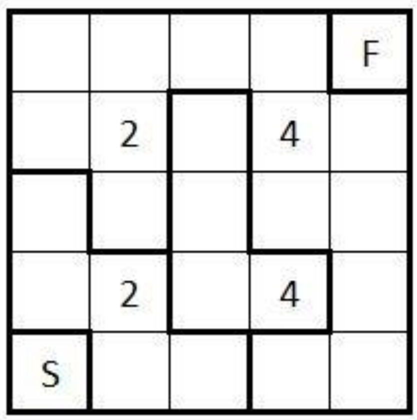
246.10 – Hexagon Angle – Elliott Line

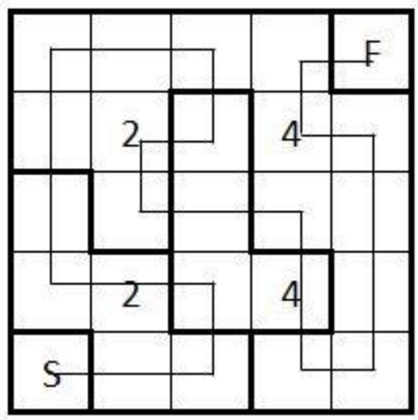
Take a regular hexagon ABCDEF as below. Draw a line from A to a point midway between B and C, and another line from A to a point midway between D and F.
What is the angle between those two lines?

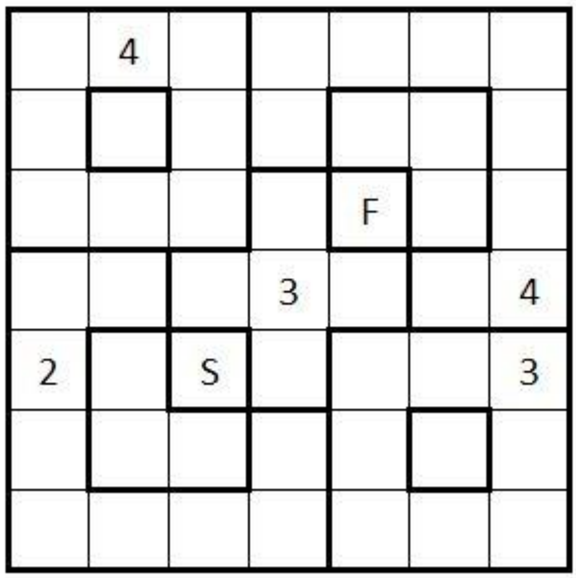


246.11 – Haisu – Agnijo Banerjee

Haisu is a puzzle type developed by my friend William Hu (puzzles.confounded@gmail.com). The rules are simple - draw an orthogonal path from S to F, passing through every cell in the grid exactly once. The grid is divided into several rooms. When your path passes over a cell with a number N, it must be the Nth time you have entered the room. An example Haisu puzzle and its unique solution are shown below.







246.12 – Barber Shop Quintet – John Causer

Apparently hairdressers are going to be among the last businesses to re-open so we are all going to look increasingly Planet of the Apes. Once they do start up again and you wander into an unknown salon to find all five barbers unengaged with scissors poised and full of vim, which one should you choose?

246.13 – Till when? – John Causer

Many supermarkets have an express “10 items or less” till (“or fewer”, in the more grammatical ones). Do these reduce the average queueing time?

246.14 – Find the Link – John Causer

What links:

Stanley Ipkiss goes from Zero to Hero
The King of Sweden gets shot in a ball
Comus
Louis XIV's twin brother
A Song by Billy Bragg

246.15 – Philosophers – John Causer

Can you untangle these anagrams of philosophers:

Wedge until twisting
Slanderers blurt
Oh! A crab unzips
Join small truth
Moaned well or sharp
Jar pleasure ant
Evil manic alcoholic
I am not a squash
Tender scares
A special slab
Boo! I am sure envied

246.16 – Thinking About It – John Causer

These are philosophical works with their vowels and word breaks removed

SYMPSM
KRTKDRRNNVRNNFT
LVTHN
TRCTTSLGCPHLSPHCS

246.17 – Quartet – Elliott Line

The four numbers 2,3,4 and 8 can be combined in pairs in six different ways, and the product of those six pairs will be

2 x 3 = 6
2 x 4 = 8
3 x 4 = 12
2 x 8 = 16
3 x 8 = 24
4 x 8 = 32

The sum of the original four numbers (2, 3, 4 and 8) is 17.

Can you find a different quartet of numbers whose products of pairs are also 6, 8, 12, 16, 24 and 32, but whose sum is less than 17?

246.18 – Riddle – Rosemary Hodgson

My first's in the factory and also the farm.
My second's in safety and also in calm.
My third is in careful and also in clean,
And in those words too, my fourth letter's seen.

The one after that is in music, not laughs,
My next's not in numbers, but always in graphs.
My next one's in whisper, but never in yell.
And the next is in knowledge but never in tell.

My aim is your safety; to shield all within.
Just as much as I can so the virus can't win.
What am I?

246.19 – Poem – Rosemary Hodgson

Re-insert the vowels and word spacings to reveal a short poem.
The necessary vowels are given in alphabetical order for each line.

F T D Y Y P L N T D H P	(A A E E I O O O U)
N N Y H P L S S H R T;	(A A E E E I O)
F S M N' S B R D N' S L G H T R	(E E E E I I O O U)
B C S Y P L Y D Y R P R T;	(A A A E E E O O U U U)
F Y C S D L G H	(A A A E I O U U U)
T H T C H S D S M T R S W Y;	(A A A A A E E E O)
F T N G H T Y R N M S N M D	(A A E E I I I O O U)
W H N S M N K N L S T P R Y;	(A E E E E E O O O)
T H N Y R D Y H S B N W L L S P N T.	(A A E E E E E O U)

246.20 – Cryptograms – Rosemary Hodgson

Straightforward cryptograms.

(1)
D ELUG VLWWLQJ LQ D WUHH LV QHYHU DIUDLG RI WKH EUDQFK EUDQFK, EHFDXVH KHU WUXVW LV QRW LQ WKH EUDQFK EXW LQ KHU RZQ ZLQJV. DOZDBV EHOLHYH LQ BRXVHOI.

(2)
EI OMRHIV XLER RIGIWWEVC, FIGEYWI IZIVCSRI CSY QIIX MW JMKLXMRK WSQI OMRH SJ FEXXPI YRWIIR.

(3)
AD Z EHQRS QZSD UDQRHNM NE XNTQRDKE, MNS Z RDBNMC QZSD UDQRHNM NE RNLDNMD DKRD.

246.21 – Aphorisms – Rosemary Hodgson

Straightforward aphorisms, but the letters of each word have been jumbled:

(1)
HET SPHANSIPE NI ROYU ELIF SPEDDEN NOUP ETH QUYLAIT
FO ROUY SHUTTOHG.

(2)
KORW ADHR NI CEEILNS; ELT CCESSSU EB ORUY EINOS.

(3)
ESSUCSC CCORSU EHNW INOOPPRTTUY EEMST AAEINOPPRRT.

(4)
CEEPRST SI ADEENR; YEHNOST SI AACDEEIPPRT;
RSTTU SI ADEGIN; ALLOTYY SI DEENRRTU.

246.22 – Missing Numbers – Rosemary Hodgson

What number should replace "X"?

24,18, 17, 23, ... 2.
9, 12, 7, 3, ...11.
23, 36, 16, 29, ..."X"

Which number should replace "X"?

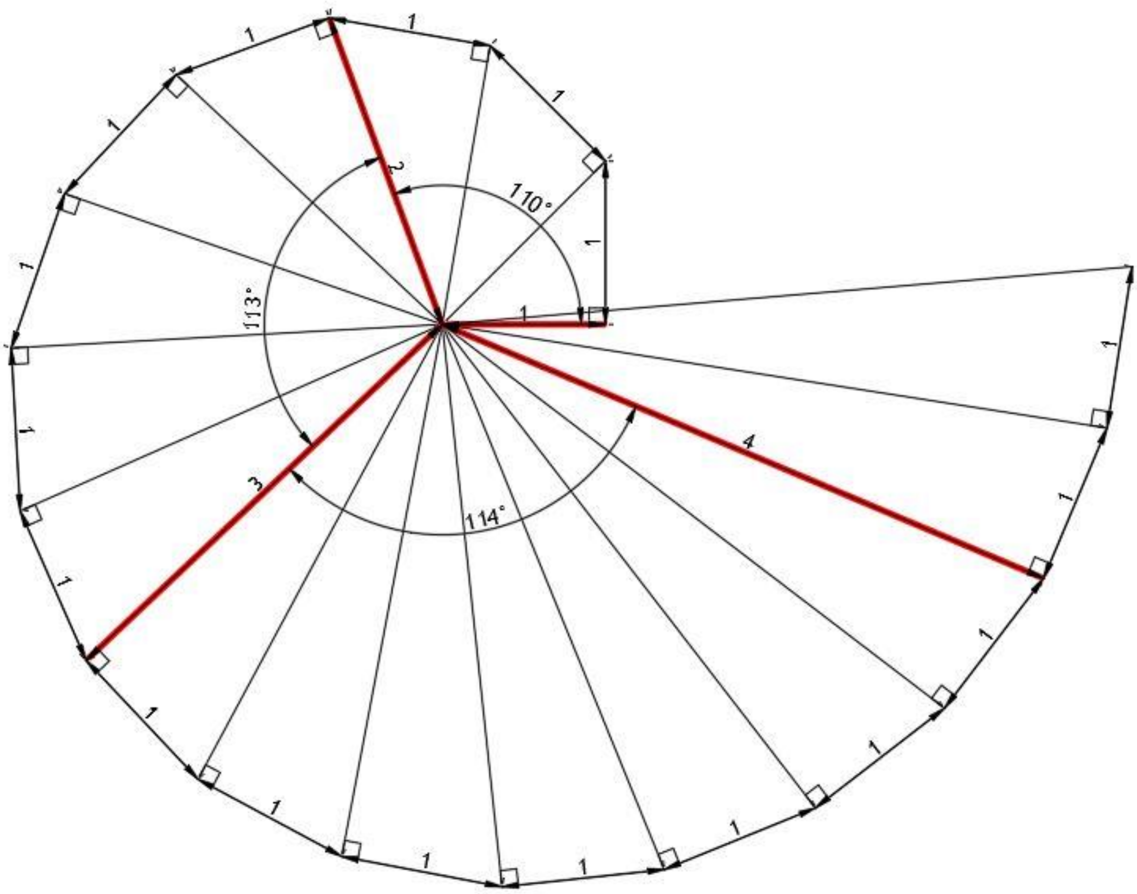
4, 3, 11, 7, 48
9, 2, 12 , 9, 54
7, 1, 13, 2, "X"

246.23 – Shortest Distances on a Cylinder – Paul Bostock

A straight line is famously described as "the shortest distance between two points". While this is true for lines in a plane, it's not so obvious what the shortest paths are on a curved surface. On a sphere the shortest paths between any two points follow great circles (i.e. circles on the surface with the same centre and radius as the sphere).

Can you describe the family of shortest distance paths on a circular cylinder?

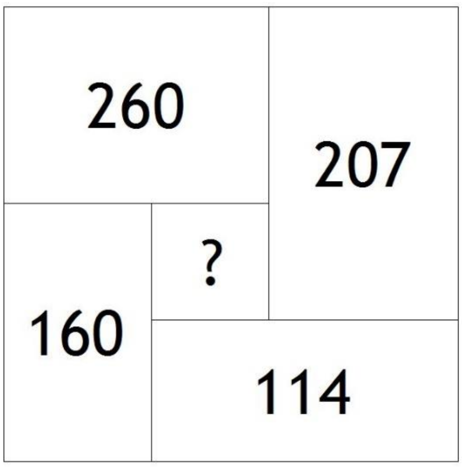
246.24 – Right Angled Triangle Spiral – Elliott Line



I have begun to construct a spiral of right-angled triangles thus:
The first triangle has legs both equal to 1.
Each subsequent triangle uses the hypotenuse of the previous triangle as one leg and a new line of length 1 as the other leg.
Every time the length of a spoke happens to be an exact whole number, I've marked it in red. I happened to notice that the angular distances between the red spokes are of a similar size.
My question is this: is it converging, and if so, to what?

246.25 – Squarespace – Elliott Line

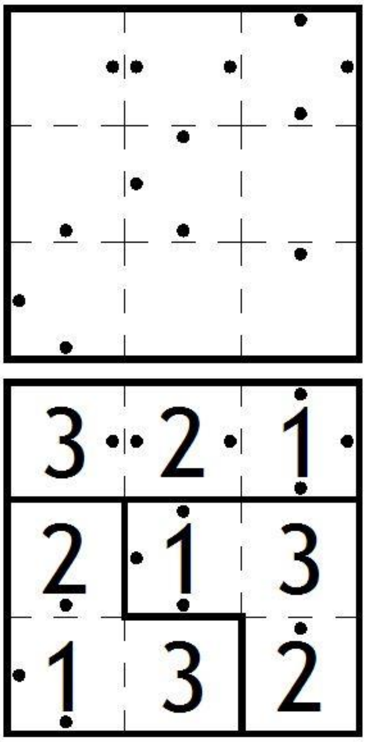
Four rectangles with areas as shown can be arranged to form a SQUARE with a SQUARE space left in the middle.
What is the area of the square space?



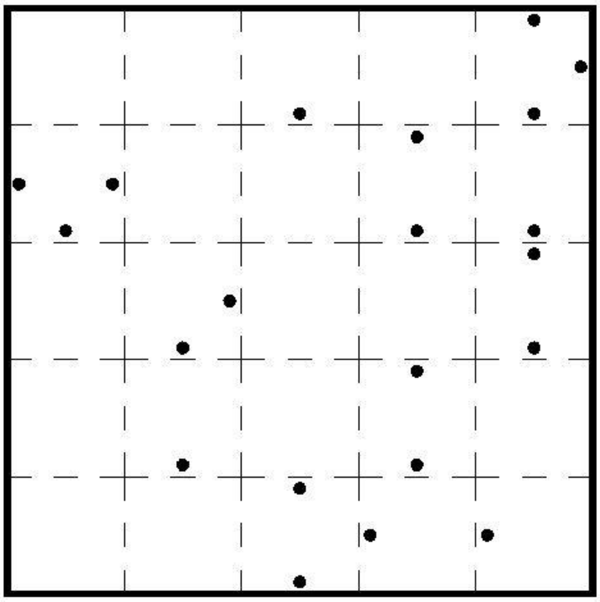
246.26 – Su-Dot-U – Elliott Line

Your task is to solve this irregular sudoku (the digits 1 to 5 appearing once each in every row, column and 'shape').
Except that I haven't told you where the boundaries between the shapes are; instead I've placed dots in any square where the number to be placed in the square denotes how many squares (including the one with the dot in) until you get to the next boundary line in the direction of the dot.
Clear as mud? Hopefully the attached example 3x3 will help. For instance, the 2 in the middle of the top row, combined with the left and right dots, says that in each direction left and right from that square there are two squares (including the one with the 2 and the dots in) before you get to a boundary line (which happens to be the outer boundary of the entire grid).
In both the example and the puzzle, I have placed dots in every position that I could, according to the rules.
A hint to get you started: if a dot appears next to the outside boundary of the grid, then that square must contain a 1.

Example:



Puzzle:



246.27 – Tetrahedron Packing – Elliott Line

You have a tetrahedron-shaped space with a side length of 5 along each edge.
How many solid tetrahedrons of side length 1 can you fit into the space?

246.28 – Three Digits – Elliott Line

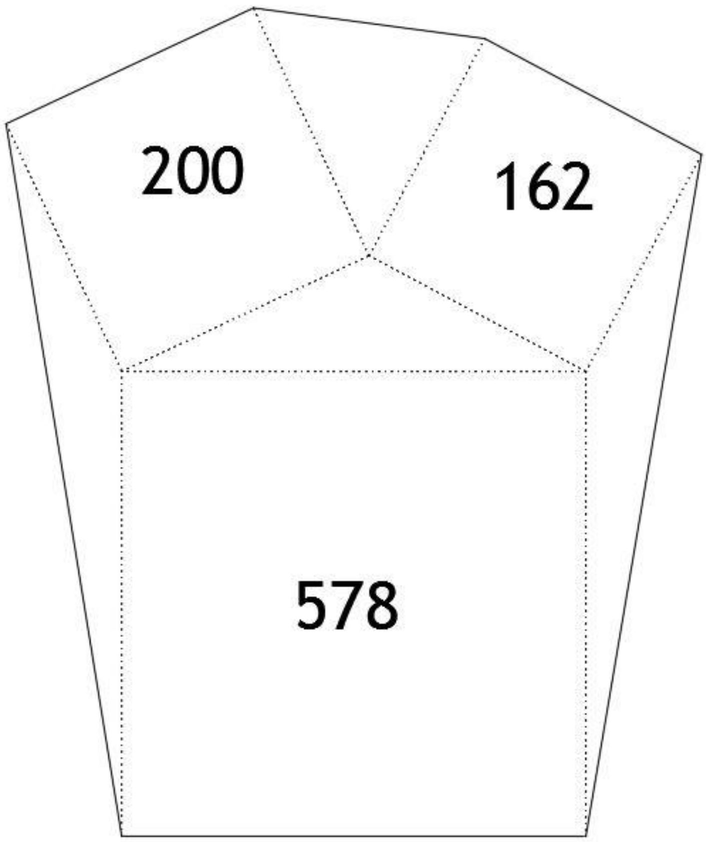
I have three digits A B C, such that A is less than B, which is less than C. I can arrange these to form six different 3-digit numbers.

When A is at the start, both possible numbers (ABC and ACB) are prime.
When A is in the middle, both numbers are semi-prime (the product of two prime numbers).
When A is at the end, the number is either abundant by 71 (the sum of its proper divisors is 71 more than the number itself), or deficient by 226 (the sum of its proper divisors is 226 less than the number itself).

What are the three digits?

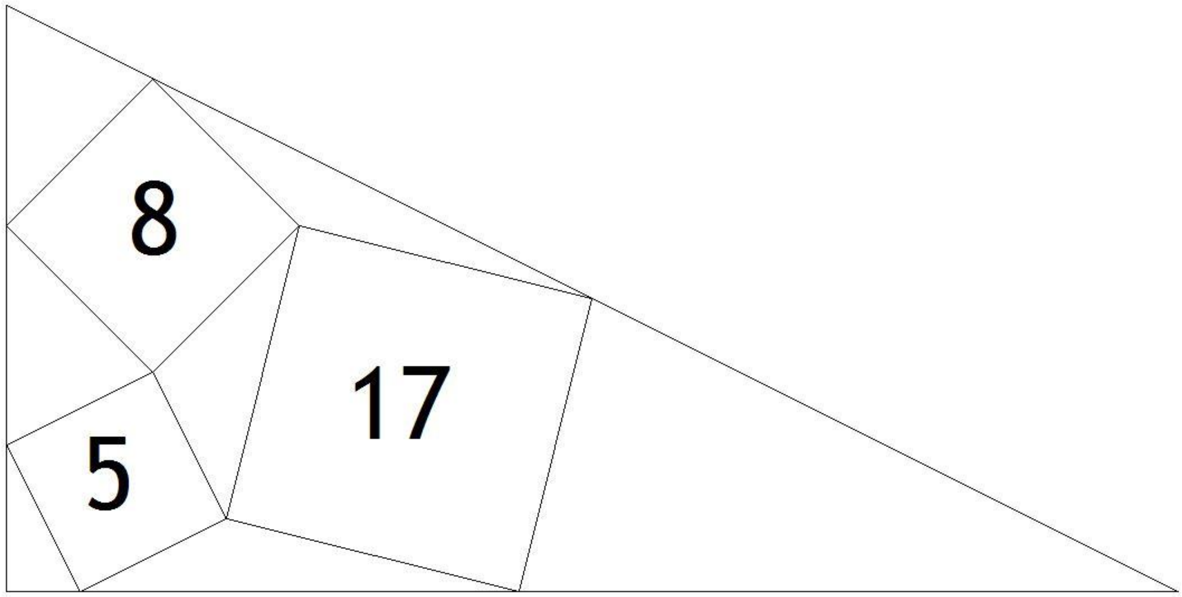
246.29 – Three Squares in a Hexagon – Elliott Line

Three squares fit in a hexagon as shown. The areas of the three squares are 200, 162 and 578 respectively.
What is the area of the entire hexagon?

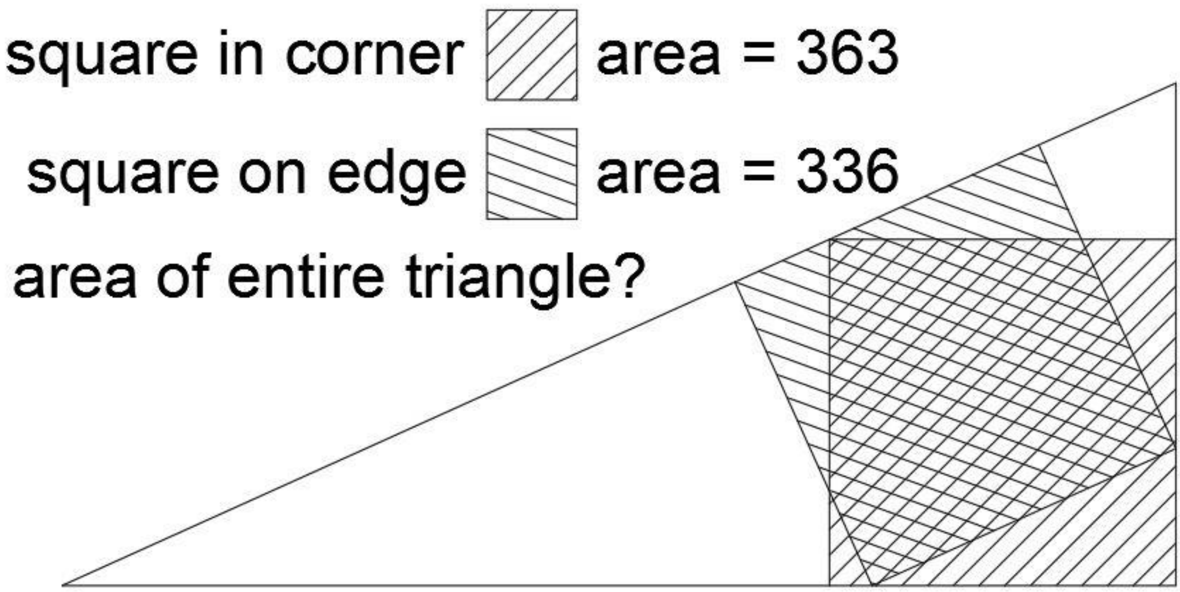


246.30 – Three Squares in a Triangle – Elliott Line

Three squares of area 5, 8 and 17 respectively, fit within a triangle as shown.
What is the area of the overall triangle?

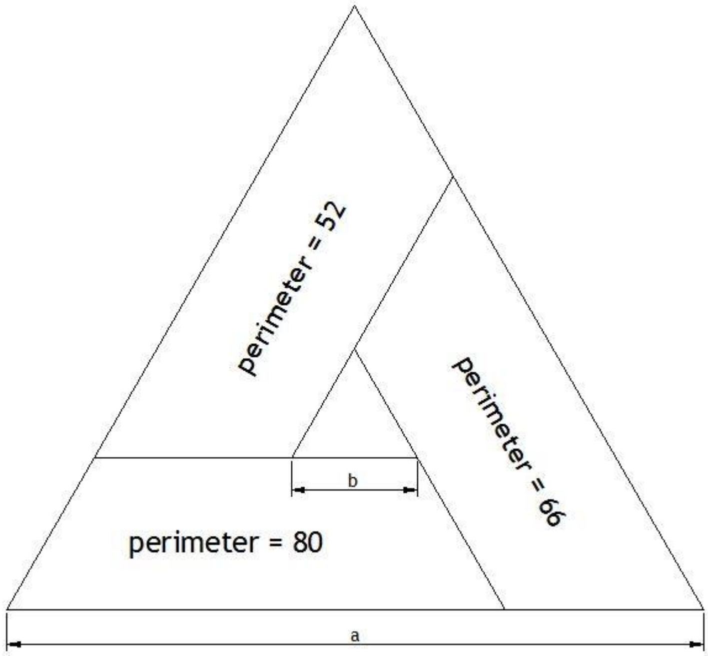


246.31 – Triangle Area (Tough) – Elliott Line



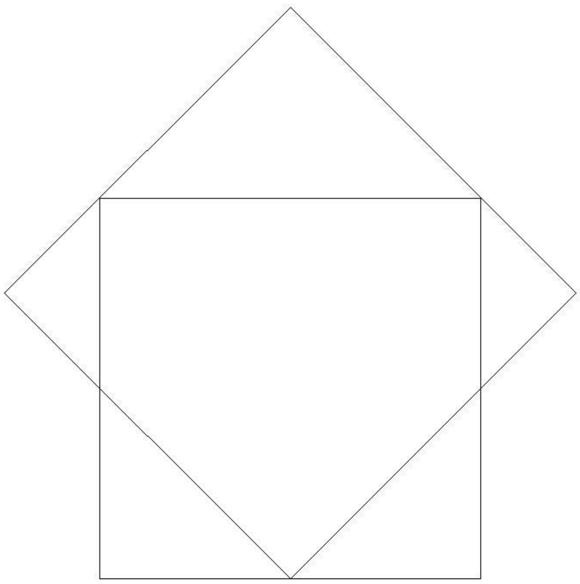
246.32 – Trispace – Elliott Line

I have an equilateral triangle, dissected into three trapezoids and a smaller equilateral triangle. The PERIMETERS of the three trapezoids are 52, 66 and 80 respectively. The ratio of the side of the overall equilateral triangle (a) is precisely 13 times the side length of the smaller equilateral triangle (b).
What is the value of a?



246.33 – Two Squares – Elliott Line

Two squares are arranged as shown, one of them rotated 45 degrees with respect to the other, and the perimeters of the two squares coinciding at three points.
What is the ratio of the areas of the two squares?



~~~~*****~~~~   **SOLUTIONS**   ~~~~*****~~~~

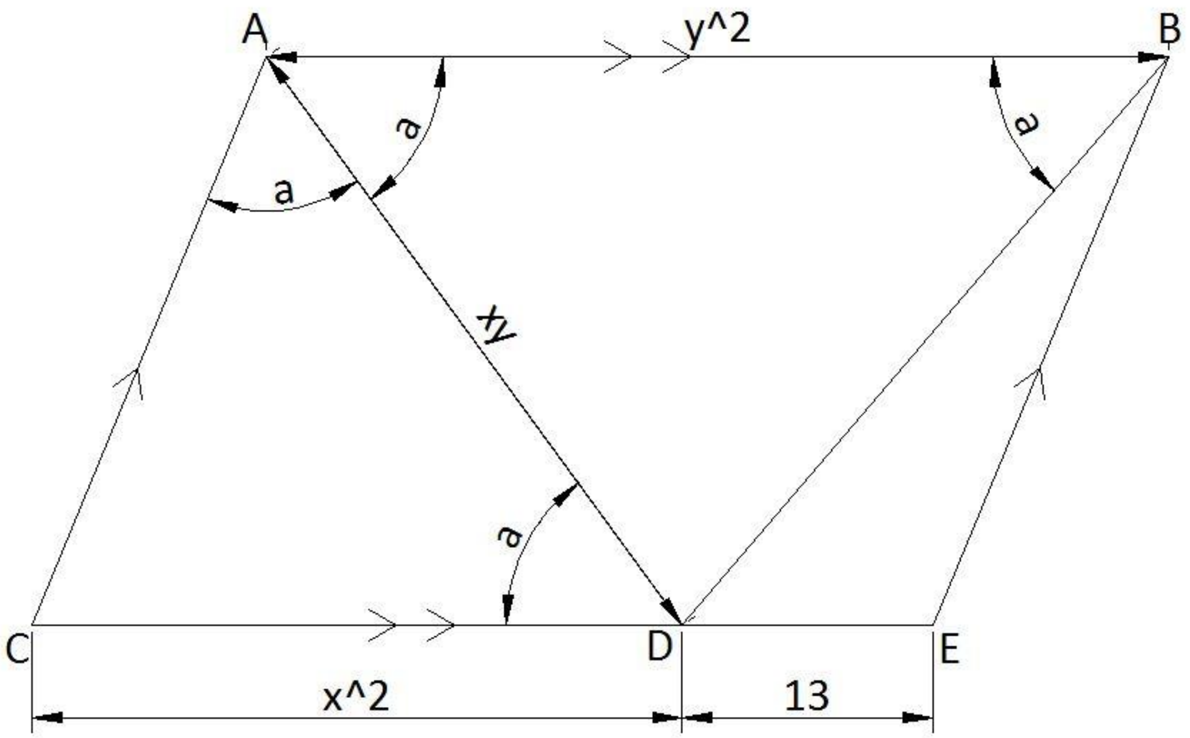
246.03 – Biscuit Plates – Roisin Carters

There are 42 ways:
1 with 1 plate
5 with 2 plates
8 with 3 plates
9 with 4 plates
7 with 5 plates
5 with 6 plates
3 with 7 plates
2 with 8 plates
1 with 9 plates
1 with 10 plates

246.04 – Diophantine Parallelogram – Elliott Line

By alternate angles ADC is also equal to the two equal corners at point A. By isosceles triangle, ABD is also the same angle. Therefore triangles ADB and ACD are similar. AD:AB = CD:AD.
Let CD = x^2 and AB = y^2. It follows that AD =xy.
We are told y^2 – x^2 = 13, therefore (y+x)(y-x) = 13.
The only integer solutions to this is y=7, x=6, therefore AD = 42.

(Pedantic note: x and y don't necessarily need to be integers for x^2, xy, y^2 and (y^2)-(x^2) all to be integers, eg sqrt(2) and sqrt(8) would satisfy. Basically either x and y are integers, or y/x is an integer, (or both). However since they represent the sides of a triangle, y/x must be between 1 and 2, so y=7, x=6 is the only solution).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246.05 – Build-a-Crossword – Elliott Line

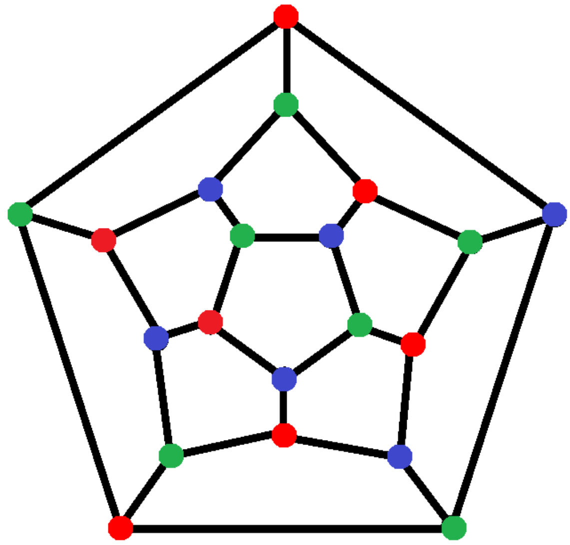
C	A	F	E		R	E	N	T
A		R		F		L		I
S	T	A	I	R	W	E	L	L
E		M		Y		C		E
	M	E	N		S	T	Y	
S		W		U		R		I
C	R	O	S	S	W	O	R	D
A		R		E		D		O
B	I	K	E		B	E	L	L

246.06 – Semi-grams – Elliott Line

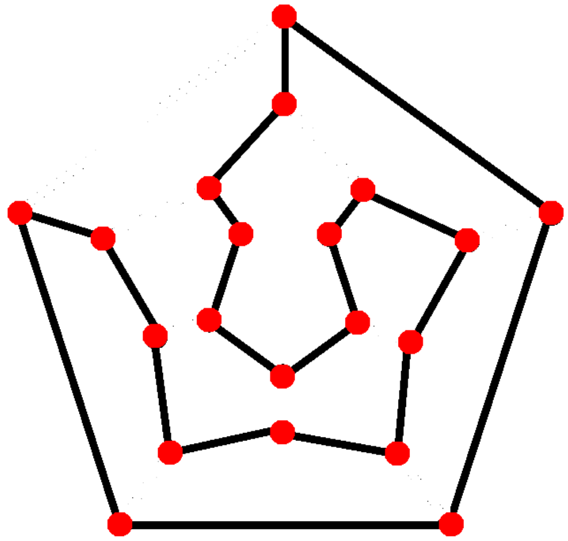
ALERT + RATIO = RETAL + IATOR = RETALIATOR
METRO + SENSE = REMOT + ENESS = REMOTENESS
NOTED + ROAST = DETON + ATORS = DETONATORS
TIMES + GIANT = ESTIM + ATING = ESTIMATING
UPSET + SOUND = STUPE + NDOUS = STUPENDOUS
DETONATING and ESTIMATORS also works

246.07 – Dodecahedron Graph – Paul Bostock

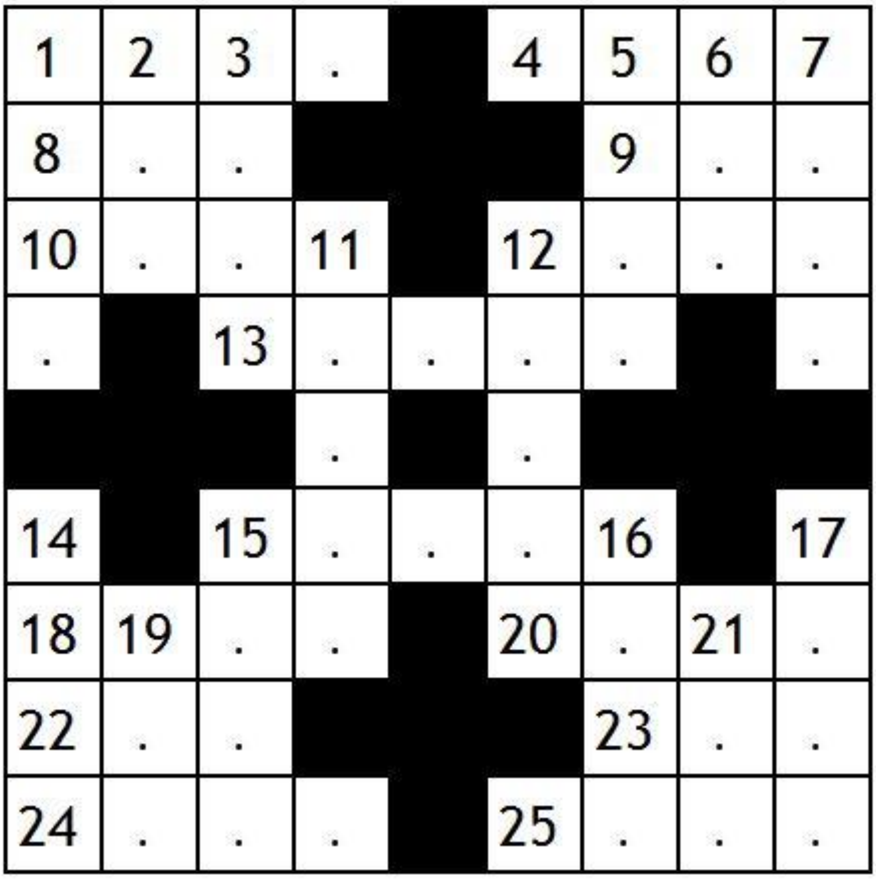
a) It can be coloured with three colours, for example:



b) Yes it is possible, for example:



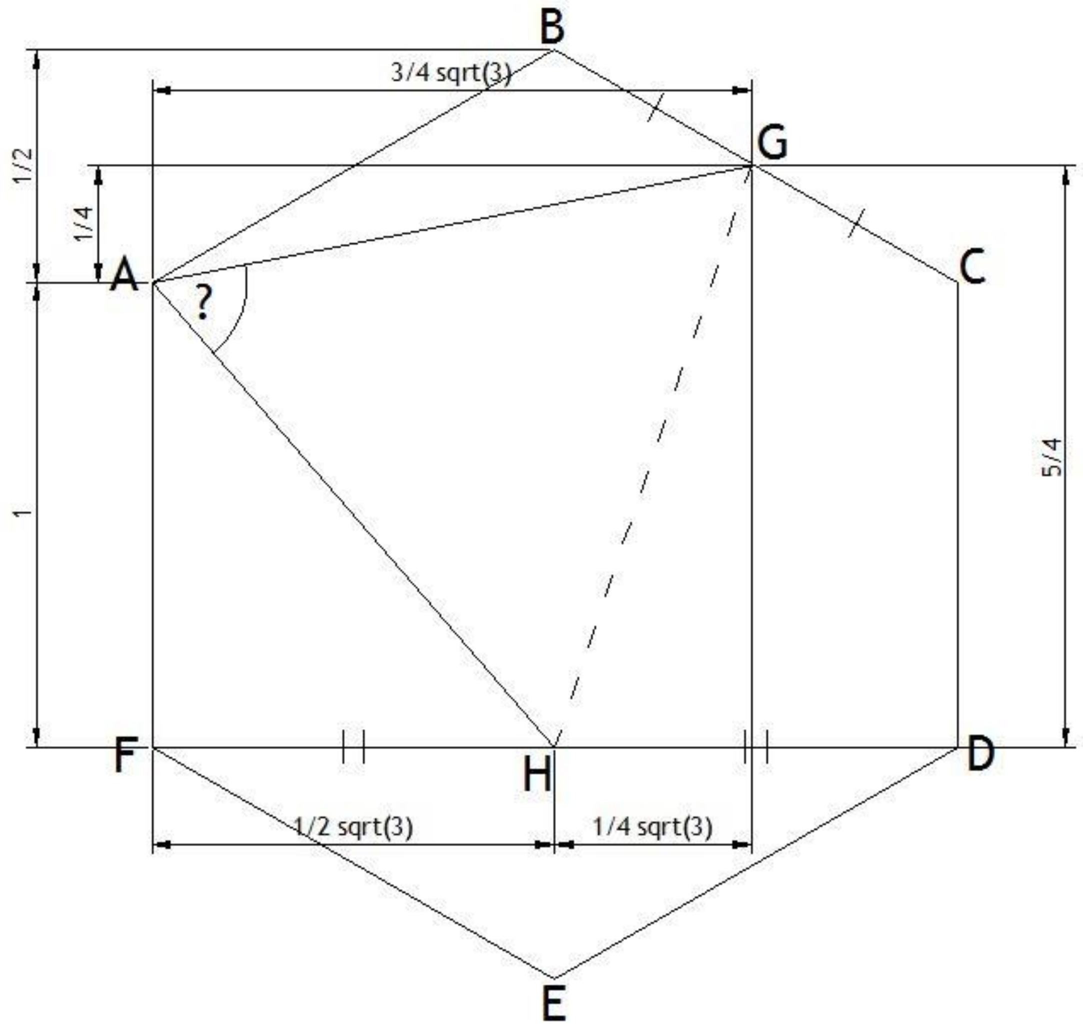
246.09 – Ghost Crossword – Elliott Line



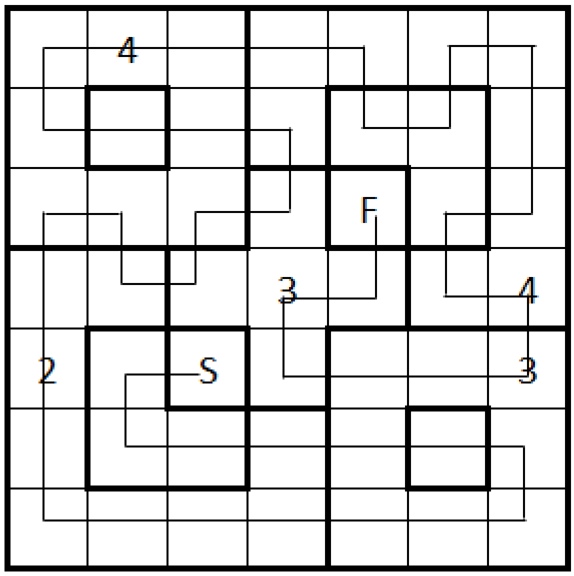
Method of solving: Numbers 1, 12 and 15 appear in both lists, so because of rules 1 and 2, they must either be on the top-left-bottom-right diagonal, or in pairs in corresponding position either side of it. Rule 6 means that 1 must be on the top row, so is either in top corner or in corresponding position in top row that 12 is in left-hand column. 1 cannot be in second column, as there are too many numbers for 12 to be in second row. If 1 is in third column or beyond, then either a 2x2 black square appears in the corner, or another number must lie on TLBR diagonal. Neither of these are allowed, so 1 must be in top left corner.
Now 4 across must also be on the top row to mirror 1 across (as 1 across is not allowed to fill the entire row). 4 must either be in 6th column or 7th column, depending on whether 1 across and 4 across are three or four 'letters' long. As 4 is across only, and not down, any attempt to make it three 'letters' long leads to a contradiction, therefore it is in the 6th column, and the square immediately to the left and below it are black. When you place a white or a black square, you can use rule 2 to transfer it around the grid, and very quickly the grid takes shape.

246.10 – Hexagon Angle – Elliott Line

To start with, work out the lengths of the lines AG and AH using Pythagoras (having first worked out the horizontal and vertical components of each). It transpires that both AG and AH are equal to sqrt(7)/2 (~1.323). This alone is not of much use in working out the angle in question, however if we now calculate the length of the dashed line GH we discover that this too has a length of sqrt(7)/2. Therefore AGH is an equilateral triangle, and the angle we seek is 60 degrees.



246.11 – Haisu – Agnijo Banerjee



246.12 – Barber Shop Quintet – John Causer

Since they presumably cut each other’s hair, choose the one with the worst haircut

246.13 – Till when? – John Causer

No, they increase it. When there are no 10-item shoppers the express till and its operator will be idle. The most efficient way of reducing AVERAGE waiting time is to allow customers to insert themselves in a single queue in accordance with how many items they have – one item go to the front, 20 items join behind those with 19. If this results in big shoppers waiting a long time then that is because the supermarket is under-staffed and they would wait a long time come what may.

246.14 – Find the Link – John Causer

The Mask, a film starring Jim Carrey (1994)
Un Ballo in Maschera (a Masked Ball), an opera by Verdi (1859)
The full title of Comus, by John Mliton, is A Mask presented at Ludlow Castle, 1634
The Man in the Iron Mask, by Alexander Dumas, is a part of The Vicomte of Bragelonne (1847-50)
The Man in the Iron Mask is a track on Billy Bragg’s first album Life's a Riot with Spy vs Spy (1983)

246.15 – Philosophers – John Causer

Ludwig Wittgenstein
Bertrand Russell
Baruch Spinoza
John Stuart Mill
Ralph Waldo Emerson
Jean-Paul Sartre
Niccolo Machiavelli
Thomas Aquinas
René Descartes
Blaise Pascal
Simone de Beauvoir Boo!

246.16 – Thinking About It – John Causer
SYMPOSIUM, by Plato (385-370 BC) or SYMPOSIUM, by Xenophon (late 360's BC)
KRITIK DER REINEN VERNUNFT (Critique of Pure Reason), by Immanuel Kant (1781)
LEVIATHAN, by Thomas Hobbes (1651)
TRACTATUS LOGICO-PHILOSOPHICUS, by Ludwig Wittgenstein (1921)

246.17 – Quartet – Elliott Line
Call the four numbers we are looking for, in ascending order, a, b, c and d. We know they are strictly ascending because if any were the same, some of the product pairs would be the same too.
ab must be the lowest product 6.
The second lowest product has to be ac = 8.
cd must be the highest product 32.
The second highest product has to be bd = 24.
However for the other two products we have a choice. In the original quartet of 2,3,4 and 8, ad = 16 and bc = 12. If we now assign those products the other way round we should end up with a different quartet of numbers. So:

ab=6
ac=8
ad=12
bc=16
bd=24
cd=32

If we multiply ab by ac and then divide by bc we will have a^2 = 3.
Therefore a = sqrt(3) = 1.732
We can easily work out the others now:
b = 6/sqrt(3) = (2)sqrt(3) = 3.464
c = 8/sqrt(3) = (8/3)sqrt(3) = 4.619
d = 12/sqrt(3) = (4)sqrt(3) = 6.928

The sum of this quartet is 16.743, so is indeed less than the 17 total of the original quartet.

246.18 – Riddle – Rosemary Hodgson
FACE MASK

246.19 – Poem – Rosemary Hodgson
If today you planted hope
In any hopeless heart;
If someone's burden's lighter
Because you played your part;
If you caused a laugh
Which chased some tears away;
If tonight your name is named
When someone kneels to pray;
Then your day has been well spent.

246.20 – Cryptograms – Rosemary Hodgson
A bird sitting in a tree is never afraid of the branch breaking, because her trust is not in the branch but in her own wings.
Always believe in yourself.
Be kinder than necessary, because everyone is fighting some kind of battle unseen.
Be a first rate version of yourself, not a second rate version of someone else.

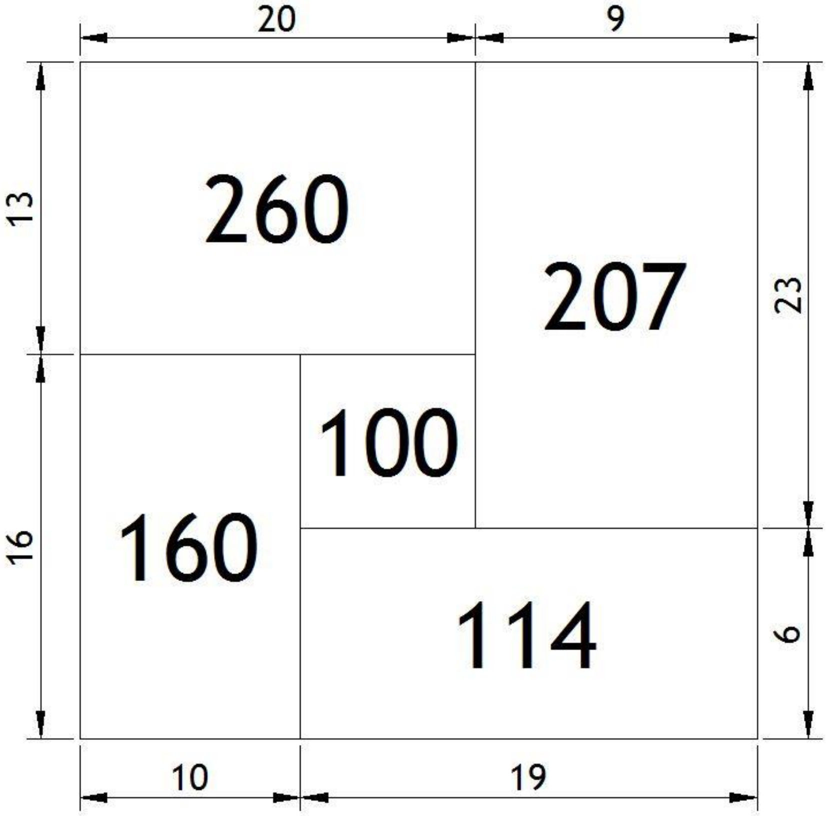
246.21 – Aphorisms – Rosemary Hodgson
The happiness in your life depends upon the quality of your thoughts.
Work hard in silence; let success be your noise.
Success occurs when opportunity meets preparation.
Respect is earned; honesty is appreciated; trust is gained; loyalty is returned.

246.22 – Missing Numbers – Rosemary Hodgson
(A+B) - (C+D) = 14
(AxB) x (C-D) = 77

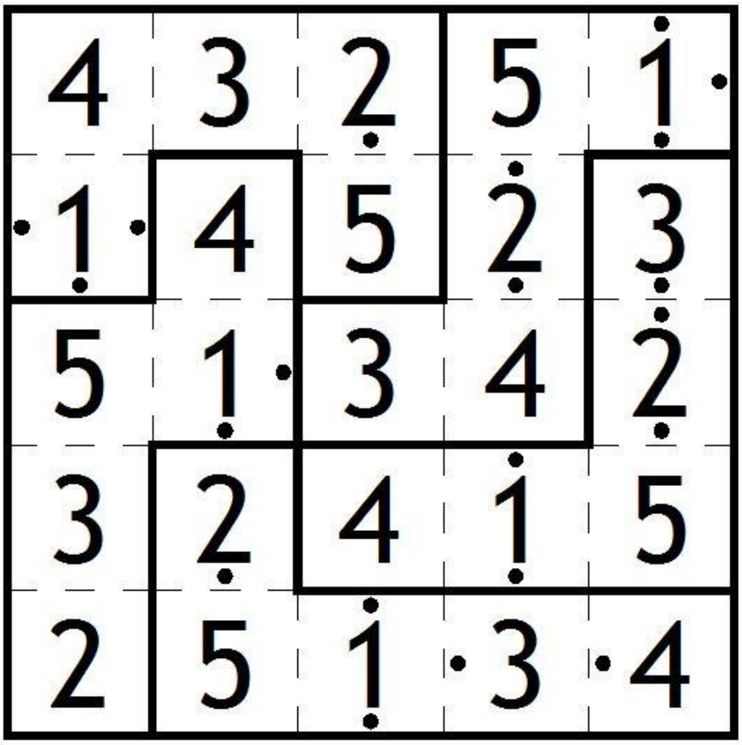
246.23 – Shortest Distances on a Cylinder – Paul Bostock
They are helices (spirals). If you were to unwrap the surface of the cylinder onto a flat plan, the path would become a straight line.

246.24 – Right Angled Triangle Spiral – Elliott Line
The angle between successive red spokes IS converging. The value it is converging towards is 2 radians, or 360/(pi) degrees, which is approximately 114.59 degrees.

246.25 – Squarespace – Elliott Line
There are just the right degrees of freedom to uniquely define this, however the equations are not easy so I was kind and made each length in the figure an integer, so that people could chance upon the solution by trying a few integer combinations. The area of the square space is 100.



246.26 – Su-Dot-U – Elliott Line



I start by using the dots at the edge of the grid to place the 1s in the first two rows and the bottom row, and the extra lines that that leads to. This fully determines the shape of the top left region, so I am also able to place the 2 in the top row. The 2 can also be placed in the second row, and the lower line from this fully determines the shape of the top right region. I can now use the dots to place the 2 in the third row, and the lower line from that. The remaining two 1s can be placed, along with the extra lines they tell us. It is now straightforward to place all of the other lines. This fixes the 2, 3 and 4 of the lower right shape (and so therefore also the 5). The final 2 can be placed in the bottom corner. The dot tells us the position of the 3 in the backwards L region, and the lack of dot tells us where the 3 goes in the S region. The last two 3s can now also be placed. From here on in, there is always a row, column or shape that only needs one more number to be complete.

246.27 – Tetrahedron Packing – Elliott Line

It's tempting to think that the answer is 125, as the space is the volume of 125 unit tetrahedrons, however tetrahedrons cannot pack without leaving gaps.

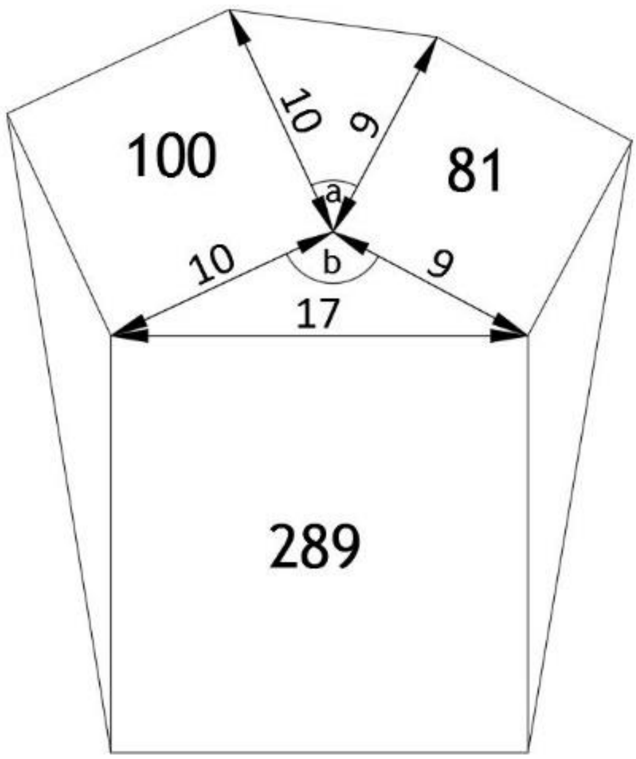
To the best of my knowledge the most you can fit in is 65:

On the bottom level you can fit 25 pointing upwards and 6 pointing downwards; on the next level up you can fit 16 pointing upwards and 3 pointing downwards; on the middle level you can fit 9 pointing upwards and 1 pointing downwards; on the next level you can fit 4, and a further 1 at the very top, giving a total of 65.

246.28 – Three Digits – Elliott Line
ABC = 239.

239 and 293 are prime.
329 = 7 x 47.
923 = 13 x 71.
392 = 2^3 x 7^2. Its proper divisors are: 1, 2, 4, 7, 8, 14, 28, 49, 56, 98 and 196. Their sum is 463, which is 71 greater than 392.
932 = 2^2 x 233. Its proper divisors are: 1, 2, 4, 233 and 466. Their sum is 706, which is 226 less than 932.

246.29 – Three Squares in a Hexagon – Elliott Line
To make life easier we can divide each of the areas by two, (as long as we remember to multiply the final answer by two to compensate).
The reason this makes things easier is that the areas will then be square numbers, and therefore the lengths of the edges of the squares will be whole numbers, namely 10, 9 and 17.
Consider the area of the triangle in the very centre. Knowing the lengths we can find the area using Heron's formula:
A^2 = s(s-a)(s-b)(s-c), where s is the semiperimeter (a+b+c)/2
The area works out to be 36.
Now consider the triangle between the top two squares. The area sine rule states that the area of a triangle is half the product of two adjacent sides, multiplies by the sine of the angle in between. At the moment we don't know the sine of angle a. In fact I have no intention of calculating it!
I'm going to use the fact that the sine of an angle (a) is the same as the sine of 180-a. This is because the sine graph between 0 and 180 forms a symmetrical curve centred on 90. Any two angles adding up to 180 (or to put it another way, averaging 90) will have the same sine.
Now if we look at angles a and b, they are a full 360 less two right angles (the corners of the squares), and so therefore will add up to 180. Sin a = sin b. Angle b is also between two lengths of 10 and 9. We can therefore see that the area of the top triangle is identical to the area of the middle triangle, which we already know to be 36.
We can use the exact same reasoning to show that the other two triangles are also 36.
Therefore the area of the whole hexagon is 100+81+289+(4x36), and not forgetting to double the answer to get the final answer of 1228.

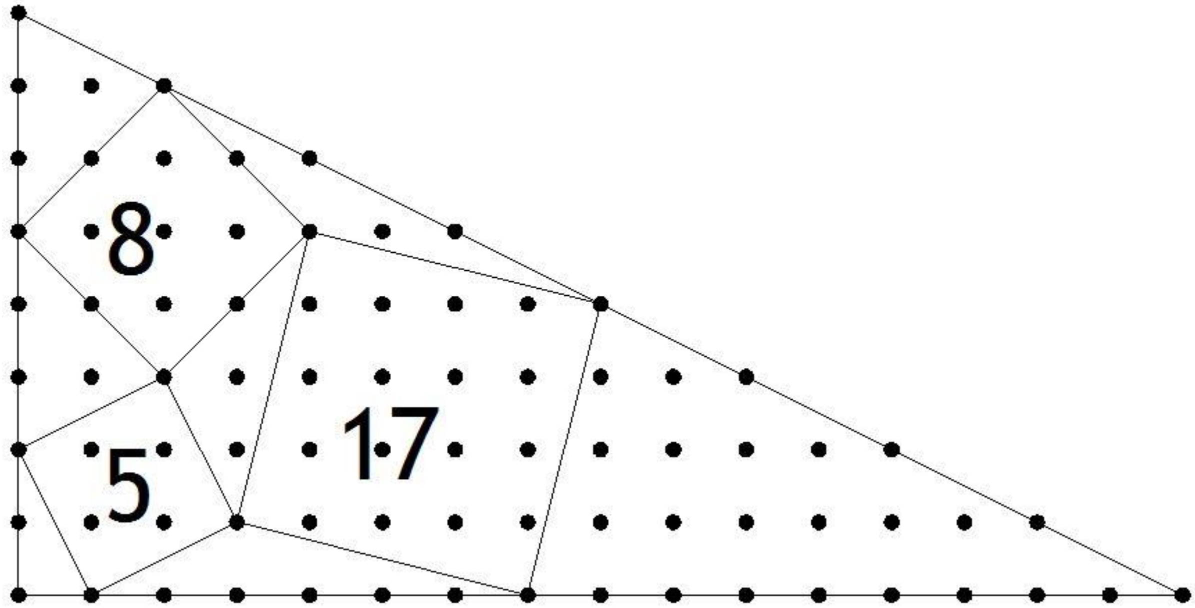


246.30 – Three Squares in a Triangle – Elliott Line

I don't know if having a related puzzle last week is a help or a hindrance, as my approach to solving, for what it's worth, is not at all similar to last week.

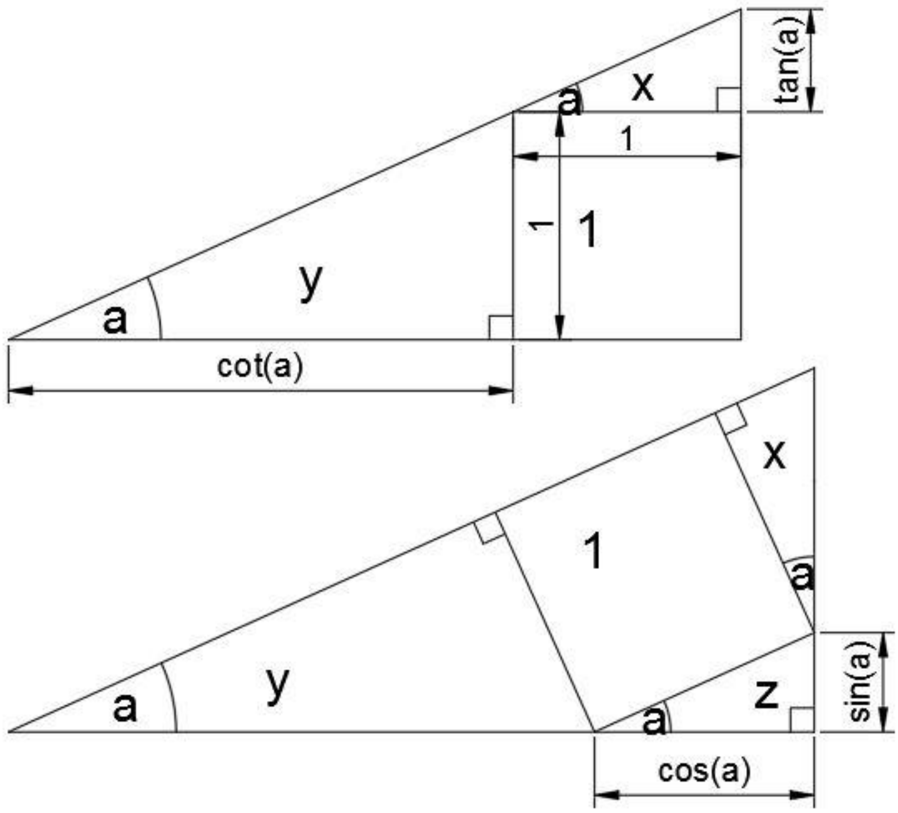
I notice that the given areas are each the sum of two squares (5=4+1, 8=4+4, 17=16+1). In general, if the area is a^2+b^2, then it's at least possible that each side of the square is a units in one direction, and b units in a perpendicular direction.

On a hunch I hopefully set out lattice points of a coordinate system, and sure enough, each of the corners of each of the squares lies exactly on a lattice point. It is then straightforward to work out what the coordinates of the triangle corners are and then calculate the area of the triangle is 64.



246.31 – Triangle Area (Tough) – Elliott Line

Imagine a couple of (differently) scaled down versions of the triangle, one where the corner square becomes area 1, and one where the sloping square becomes area 1. From using similar triangles and chasing the angle 'a' around the figure we can see that the areas of some of the other parts of the triangle will be the same across the two figures. We can call these triangular areas x, y and z.
(I should probably point out at this point, while I'm introducing lots of new variables, that I've no intention of ever calculating what a, x or y evaluate to. z on the other hand…).



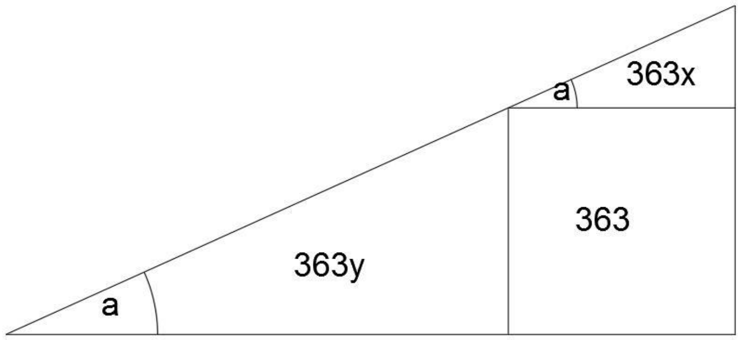
Each of the triangles with area x, y or z has one right angle, one angle 'a', and one side of length 1. You can express x y and z in terms of trig functions of a:

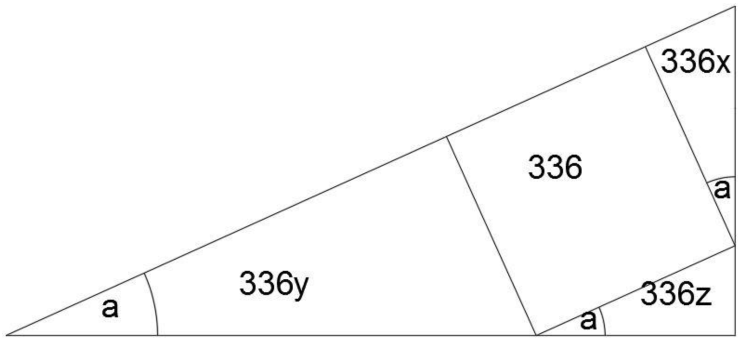
$x = \tan(a)/2$ ($= \sin(a)/2\cos(a)$)
$y = \cot(a)/2$ ($= \cos(a)/2\sin(a)$)
$z = \sin(a)\cos(a)/2$

If you multiply x and y together the tangent and cotangent cancel out and you get simply $xy = 1/4$

If you add the reciprocals of x and y:
$1/x + 1/y = 2\cos(a)/\sin(a) + 2\sin(a)/\cos(a) =$
$2((\cos(a))^2+(\sin(a))^2)/ \sin(a)\cos(a)$
But since $(\cos(a))^2+(\sin(a))^2 = 1$ by definition:
$1/x + 1/y = 1/z$
Putting these two facts together we get $(x+y)/(1/4)=1/z$
$4(x+y)=1/z$
$x+y = 1/4z$

Now scaling the two triangles back up so that the square areas are 363 and 336 respectively, and equating the areas of the two triangles we find that:





Area = 363(1+x+y) = 336(1+x+y+z)
Substituting x+y = 1/4z
Area = 363(1+1/4z) = 336(1+1/4z+z)

I'm now going to multiply each term by z and group terms on one side:
336z^2 - 27z - 27/4 = 0
Using the quadratic formula we see that:
z = (27 +/- sqrt(27^2 + 27x336))/(2x336)
z = (27+99)/672 (since z is an area and must be positive)
z = 3/16

Plugging that back into one of the area formulae:
Area = 363(1+1/4z) = 363+484 = 847, which is the final answer.

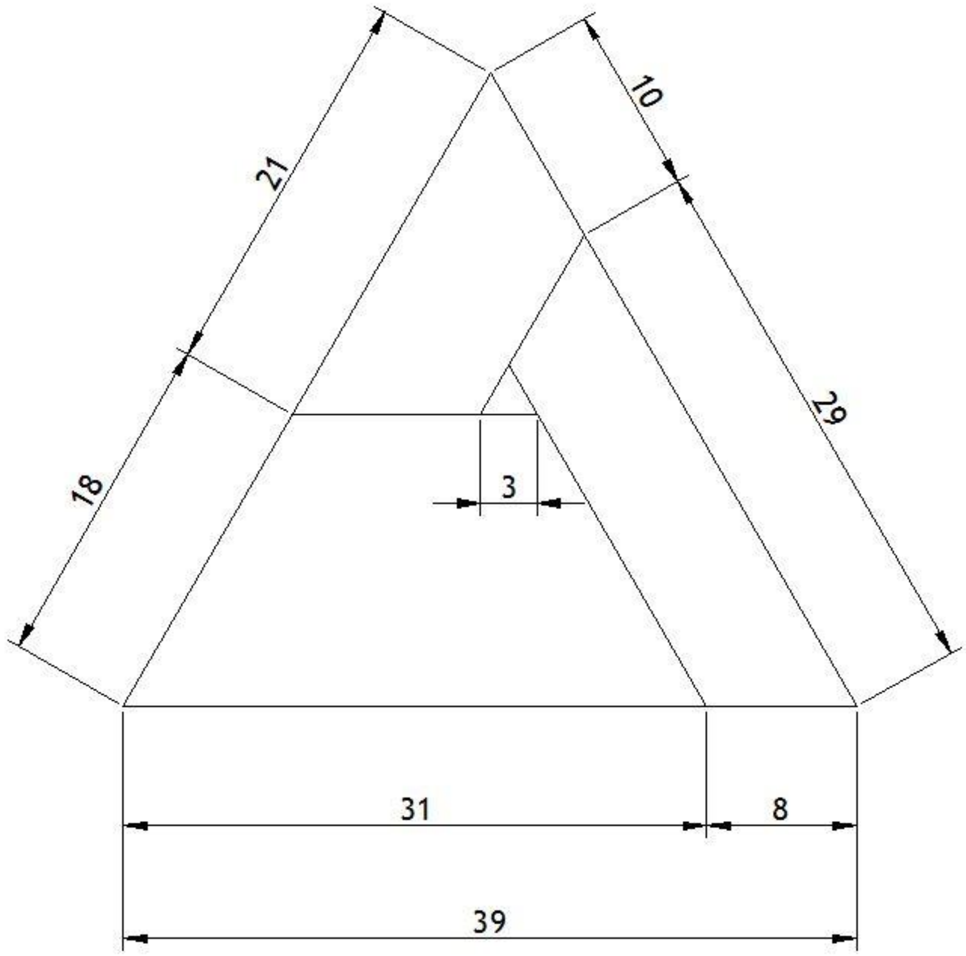
246.32 – Trispace – Elliott Line

a=39

Below is the figure fully dimensioned.

Quite interestingly, in all cases the total of the three trapezoid perimeters will equal 5a+b. Since in this case the total was 198, and a=13b, this provides a useful shortcut to the fact that b=3 and therefore a=39 (although of course deriving this fact is more work in the short term).

Despite the fact that you don't need to calculate the lengths of each of the trapezoids, there is only one figure which satisfies the conditions.



246.33 – Two Squares – Elliott Line

There are a few ways of solving this but the most intuitive is probably to dissect the figure into identical triangles and then simply count them. There are 18 in the tilted square and 16 in the lower square, so therefore their areas are in a ratio of 9:8.