Algorithms HW

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- Consider the map $M \times N \to N \otimes_R M$ given by $((m,n) \mapsto n \otimes m)$. Notice that this map is R-bilinear on $M \times N$ since $\otimes : M \times N \to M \otimes_R N$ is. Then by the universal property of tensor products we have a unique R-linear map $f : M \otimes_R N \to N \otimes_R M$ such that $f \circ \otimes = (m,n) \mapsto n \otimes m$. The same argument on the R-bilinear map $N \times M \to N \otimes_R M$ given by $(n,m) \mapsto (m \otimes n)$ gives a unique R-linear map $\tilde{f} : N \otimes_R M \to M \otimes_R N$ such that[1] $\tilde{f} \circ \times = (n,m) \mapsto m \otimes n$.

  Notice now that $f \circ \tilde{f} = id_{N \otimes_R M}$ and $\tilde{f} \circ f = id_{M \otimes_R N}$. Indeed $(f \circ \tilde{f})(m \otimes n) = f(n \otimes m) = m \otimes n$. And likewise for the other direction.

  That is, we have found a bijective R-linear map $M \otimes_R N \to N \otimes_R M$ and so in fact $M \otimes_R N$ is isomorphic to $N \otimes_R M$.

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[1] The "$\otimes$" in the following phrase now refers to the R-bilinear map $N \times M \to N \otimes_R M$, whereas earlier it referred to the R-bilinear map $M \times N \to M \otimes_R N$.

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