WALRUS OPTIMIZER

A NOVEL NATURE-INSPIRED METAHEURISTIC ALGORITHM

A1105111 王騏

MENU

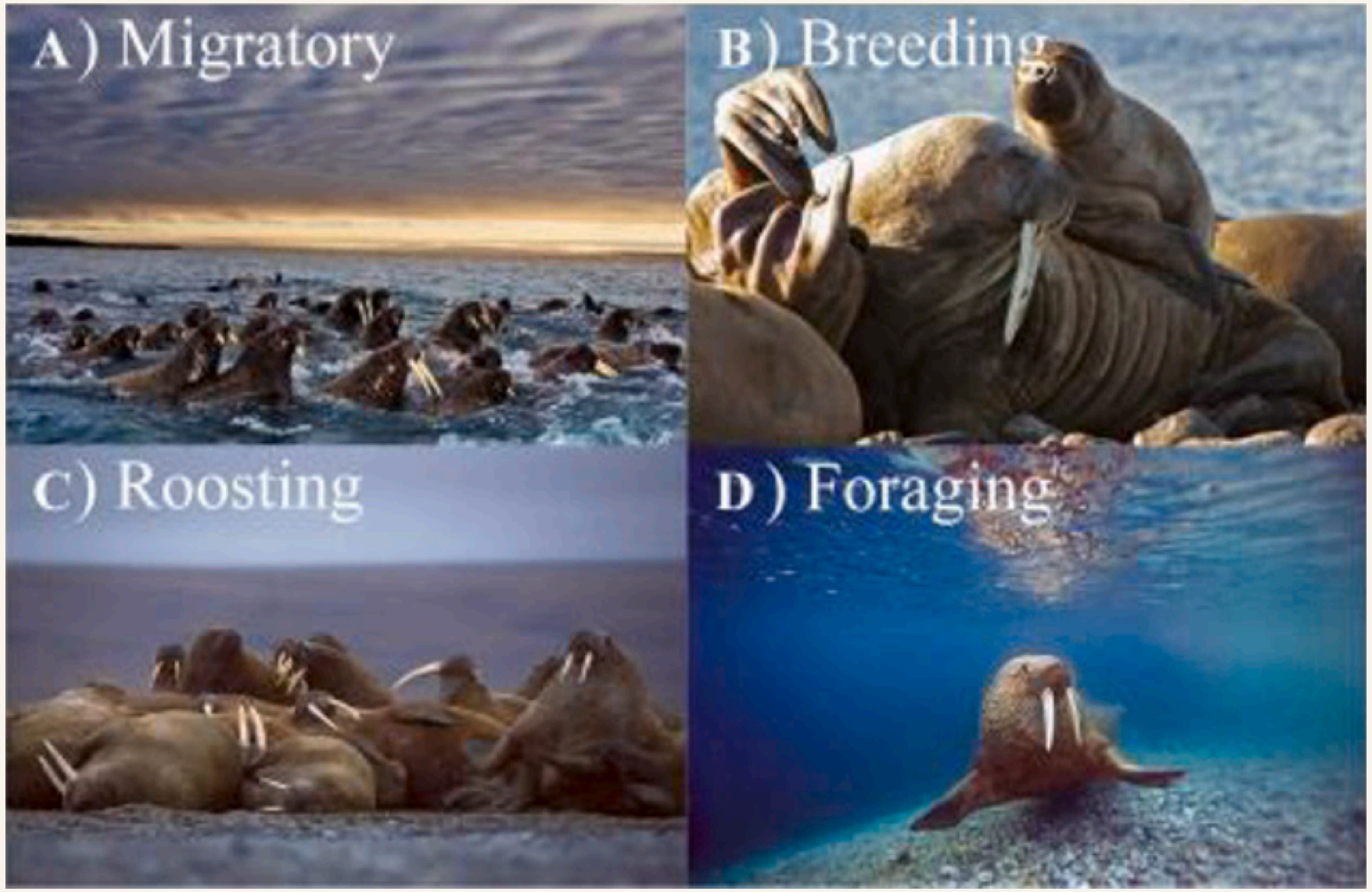
- Walrus and its biological fundamentals
- Overview
- Mathematical model and algorithm
- Procedure and flow
- Analysis
- Conclusion

WALRUS AND ITS BIOLOGICAL FUNDAMENTALS

WALRUS AND ITS BIOLOGICAL FUNDAMENTALS

- Establish territories
- Gergarious with strong social habits
- Foraging relies on sound and peers
- Defense strategy and aid counterparts

WALRUS AND ITS BIOLOGICAL FUNDAMENTALS



OVERVIEW

- Population based algorithm or Swarm intelligence
- Vigilantes and danger/safety signals are determining factors
- Assume social structure and interactions
- Fixed size population of walruses
- Tested on various benchmarks/CEC 2021/0-1 Knapsack

MATHEMATICAL MODEL AND ALGORITHM

Initilization

$$X = LB + rand(UB - LB)$$

- LB is the lower bound
- UB is the upper bound
- LB <= Search space <= Ub
- rand is a uniform random vector with values from 0 to 1

Initilization

$$X = \begin{bmatrix} X_{1,1} X_{1,2} \cdots X_{1,d} \\ X_{2,1} X_{2,2} \cdots X_{2,d} \\ \vdots \vdots \vdots \vdots \\ \vdots \vdots \vdots \vdots \\ X_{n,1} X_{n,2} \cdots X_{n,d} \end{bmatrix}_{n \times d}$$

Positions

$$F = \begin{bmatrix} (f_{1,1} f_{1,2} \cdots f_{1,d}) \\ (f_{2,1} f_{2,2} \cdots f_{2,d}) \\ \vdots \vdots \vdots \vdots \\ \vdots \vdots \vdots \vdots \\ (f_{n,1} f_{n,2} \cdots f_{n,d}) \end{bmatrix}_{n \times d}$$

Fitness

- n is population size
- d is dimension of the problem

MATHEMATICAL MODEL AND ALGORITHM

Initilization

- Adults account for 90% of population
- Juveniles are 10% of population
- Male to female ratio is 1 : 1

MATHEMATICAL MODEL AND ALGORITHM

Signals

- **Danger signals**

Determines whether to exploit or explore

Also determines whether to gather or flee

$$\text{Danger\_signal} = A^{*}R$$

$$\alpha = 1 - t/T$$

$$A = 2 \times \alpha$$

$$R = 2 \times r_1 - 1$$

r1 is a random number [0, 1]

A and R are danger factors

t $\longrightarrow$ T α $\longrightarrow$ 0 A $\longrightarrow$ 0

MATHEMATICAL MODEL AND ALGORITHM

Signals

- **Safety signals**

Determines whether to roost or forage

$$\text{Safety\_signal} = r_2$$

r2 is a random number [0, 1]

Migration (Exploration)

When factors are too high abs(Danger signal) >= 1, the herd will migrate to safer areas

$$X_{i,j}^{t+1} = X_{i,j}^{t} + \text{Migration\_step}$$

$$\text{Migration\_step} = (X_m^t - X_n^t) \bullet \beta \bullet {r_3}^2$$

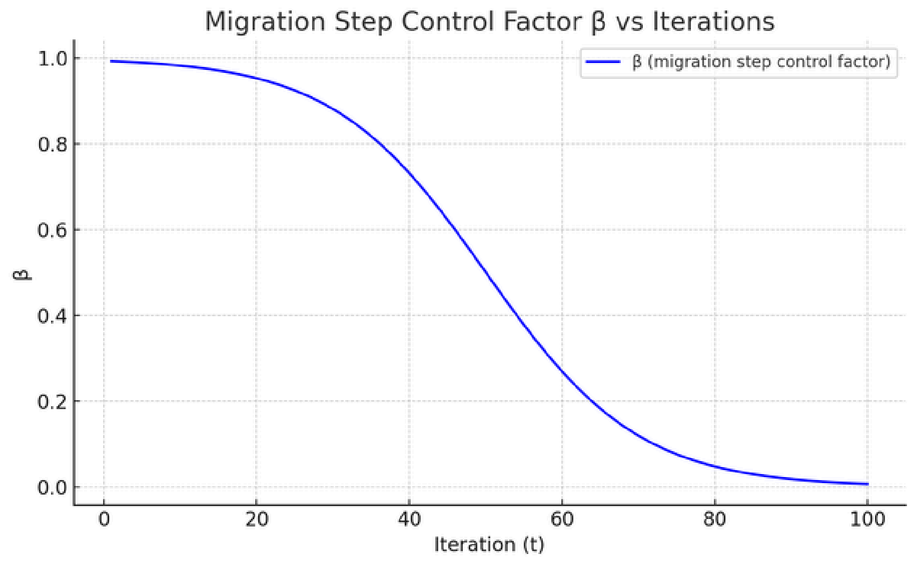
$$\beta = 1 - \frac{1}{1 + \exp(-\frac{t - \frac{T}{2}}{T} \times 10)}$$

$X_{i,j}^{t+1}$ and $X_{i,j}^{t}$ are next and current pos

X_m^t and X_n^t are two random vigilantes

β is the migration step control factor

r3 is a random number [0, 1]



Reproduction (Exploitation)

When factors are low enough

abs(Danger signal) < 1, the herd

will choose to roost or forage

MATHEMATICAL MODEL AND ALGORITHM

Roosting (Exploitation)

When the safety signal >= 0.5,

given that the abs(danger signal) < 1,

the walruses will choose to roost

The positions of male and juvenile walruses are crucial. Since the males protect juveniles from getting attacked

MATHEMATICAL MODEL AND ALGORITHM

Roosting (Exploitation)

- **Position update of female walruses**

The female is gradually influenced less by the male and more by the leader

$$Female_{i,j}^{t+1} = Female_{i,j}^{t} + \alpha \bullet \left(Male_{i,j}^{t} - Female_{i,j}^{t}\right) + (1-\alpha) \bullet \left(X_{best}^{t} - Female_{i,j}^{t}\right)$$

X_{best}^{t} is the lead walrus

$Male_{i,j}^{t}$ is the male walrus

$Female_{i,j}^{t}$ is the female walrus

Roosting (Exploitation)

- **Redistribution of male walruses**

The Halton sequence is employed,
to ensure randomness and uniformity

Essentially dividing search area into even several parts, then assigning random walruses at random points of the parts

Roosting (Exploitation)

- **Redistribution of male walruses**

- **Halton sequence**

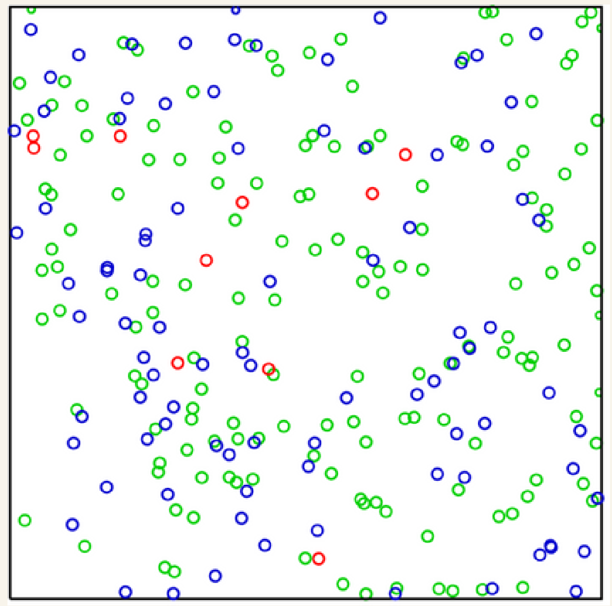
Halton sequences are deterministic but they are of low discrepancy. In other words they appear to be random

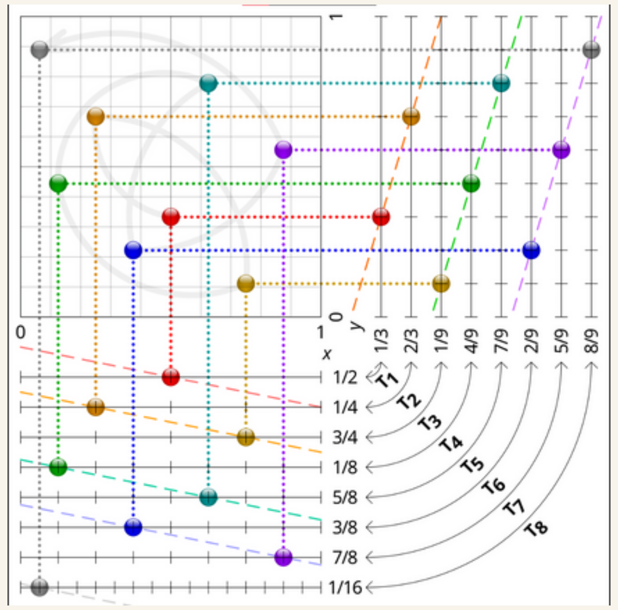
```
algorithm Halton-Sequence is
    inputs: index i
            base b
    output: result r

    f ← 1
    r ← 0

    while i > 0 do
        f ← f/b
        r ← r + f * (i mod b)
        i ← ⌊i/b⌋

    return r
```





MATHEMATICAL MODEL AND ALGORITHM

Roosting (Exploitation)

- **Position update of juvenile walruses**

$Juvenile_{i,j}^{t+1}$ and $Juvenile_{i,j}^{t}$ are next and current pos

P is the distress coefficient , random number [0, 1]

O is the reference safety position

$$Juvenile_{i,j}^{t+1} = \left(O - Juvenile_{i,j}^{t}\right) \bullet P$$

$$O = X_{best}^{t} + Juvenile_{i,j}^{t} \bullet LF$$

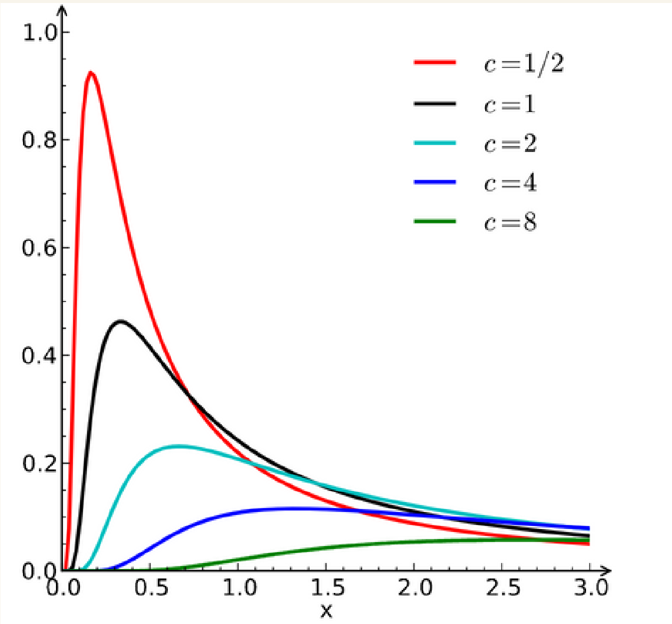
LF is a vector of random numbers based on Lévy distribution

Roosting (Exploitation)

- **Position update of juvenile walruses**

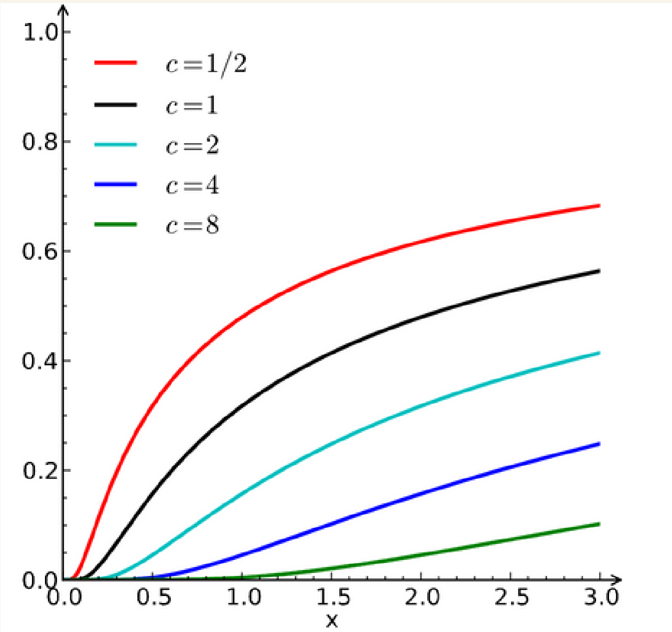
- **Lévy distribution**

$$f(x;\mu,c)=\sqrt{\frac{c}{2\pi}}\,\frac{e^{-\frac{c}{2(x-\mu)}}}{(x-\mu)^{3/2}}$$



The Lévy distribution is a continuous probability distribution for a non-negative random variable. It's a special case of the inverse-gamma distribution and is stable, making it ideal for exploration in metaheuristic algorithms.

$$F(x;\mu,c)=\operatorname{erfc}\left(\sqrt{\frac{c}{2(x-\mu)}}\right)=2-2\Phi\left(\sqrt{\frac{c}{(x-\mu)}}\right)$$



Roosting (Exploitation)

- **Position update of juvenile walruses**

- **Lévy distribution**

$$\text{Lévy}(a) = 0.05 \times \frac{x}{|y|^{\frac{1}{a}}}$$

x and y are normally distributed variables $x\ \mathrm{N}(0, \sigma_x^2)$

σ_x and σ_y are standard deviation $\Gamma(x) = (x+1)!$

$$\sigma_x = \left[\frac{\Gamma(1+\alpha)\sin\left(\frac{\pi\alpha}{2}\right)}{\Gamma\left(\frac{1+\alpha}{2}\right)\alpha 2^{\frac{(\alpha-1)}{2}}}\right]^{\frac{1}{\alpha}}, \sigma_y = 1, \alpha = 1.5$$

MATHEMATICAL MODEL AND ALGORITHM

Foraging (Exploitation)

- **Fleeing**

When abs(Danger signal) < 0.5, the herd will choose flee from current position to a new one

$$X_{i,j}^{t+1} = X_{i,j}^{t} \cdot R - \left| X_{best}^{t} - X_{i,j}^{t} \right| \cdot {r_4}^2$$

$\left| X_{best}^{t} - X_{i,j}^{t} \right|$ denotes the distance between the current walrus and the leader

r4 is a random number [0, 1]

Foraging (Exploitation)

- **Gathering**

Conversly, when abs(Danger signal) >= 0.5, the herd will find an area with higher food abundance

$$X_{i,j}^{t+1} = (X_1 + X_2)/2$$

$$\begin{cases} X_1 = X_{best}^t - a_1 \times b_1 \times \left| X_{best}^t - X_{i,j}^t \right| \\ X_2 = X_{second}^t - a_2 \times b_2 \times \left| X_{second}^t - X_{i,j}^t \right| \end{cases}$$

$$a = \beta \times r_5 - \beta$$

$$b = \tan(\theta)$$

X_{second}^t is the second walrus in the current iteration

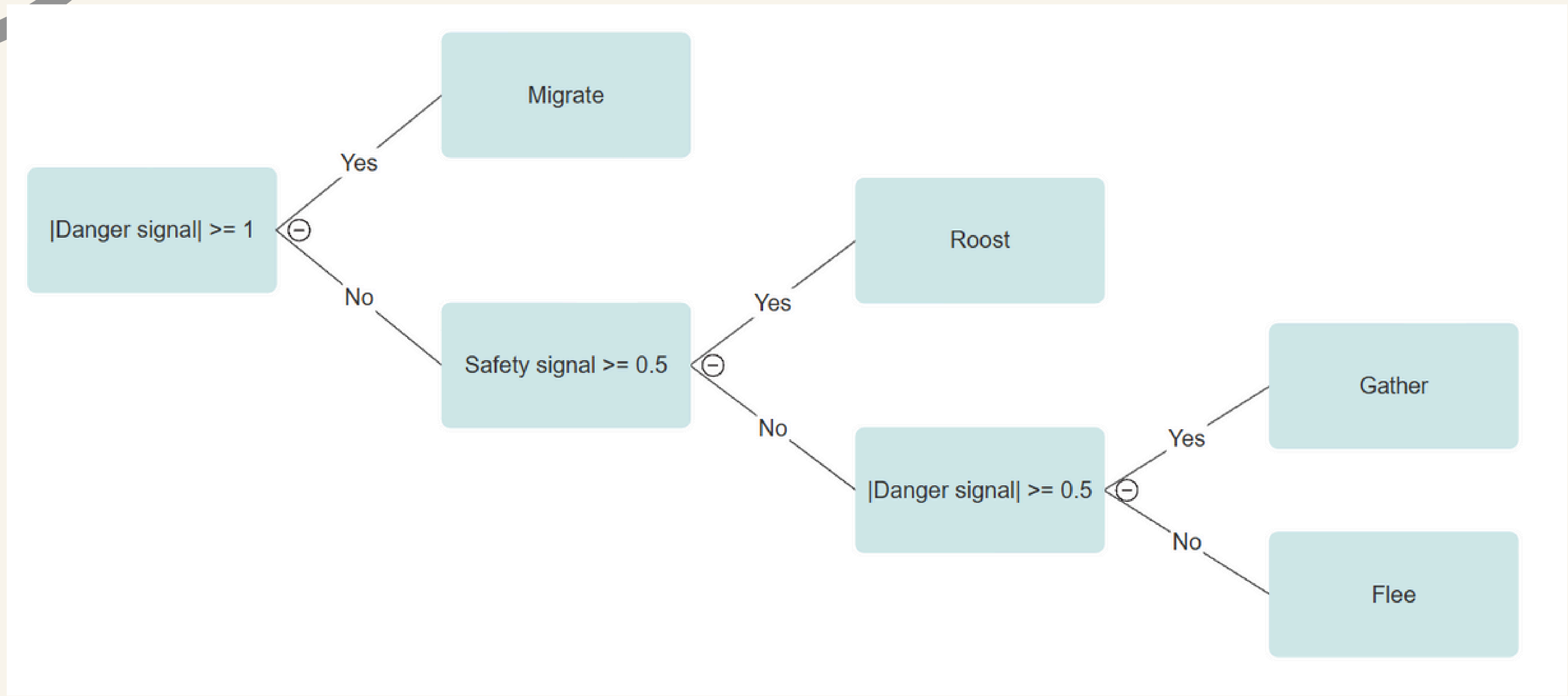
$\left| X_{best}^t - X_{i,j}^t \right|$ denotes the distance between the current walrus and the leader

$\left| X_{second}^t - X_{i,j}^t \right|$ denotes the distance between the current walrus and the second walrus

a and b are gathering coefficients, r5 is a random number [0, 1], θ [0, π]

PROCEDURE AND FLOW

Pseudo code of the algorithm



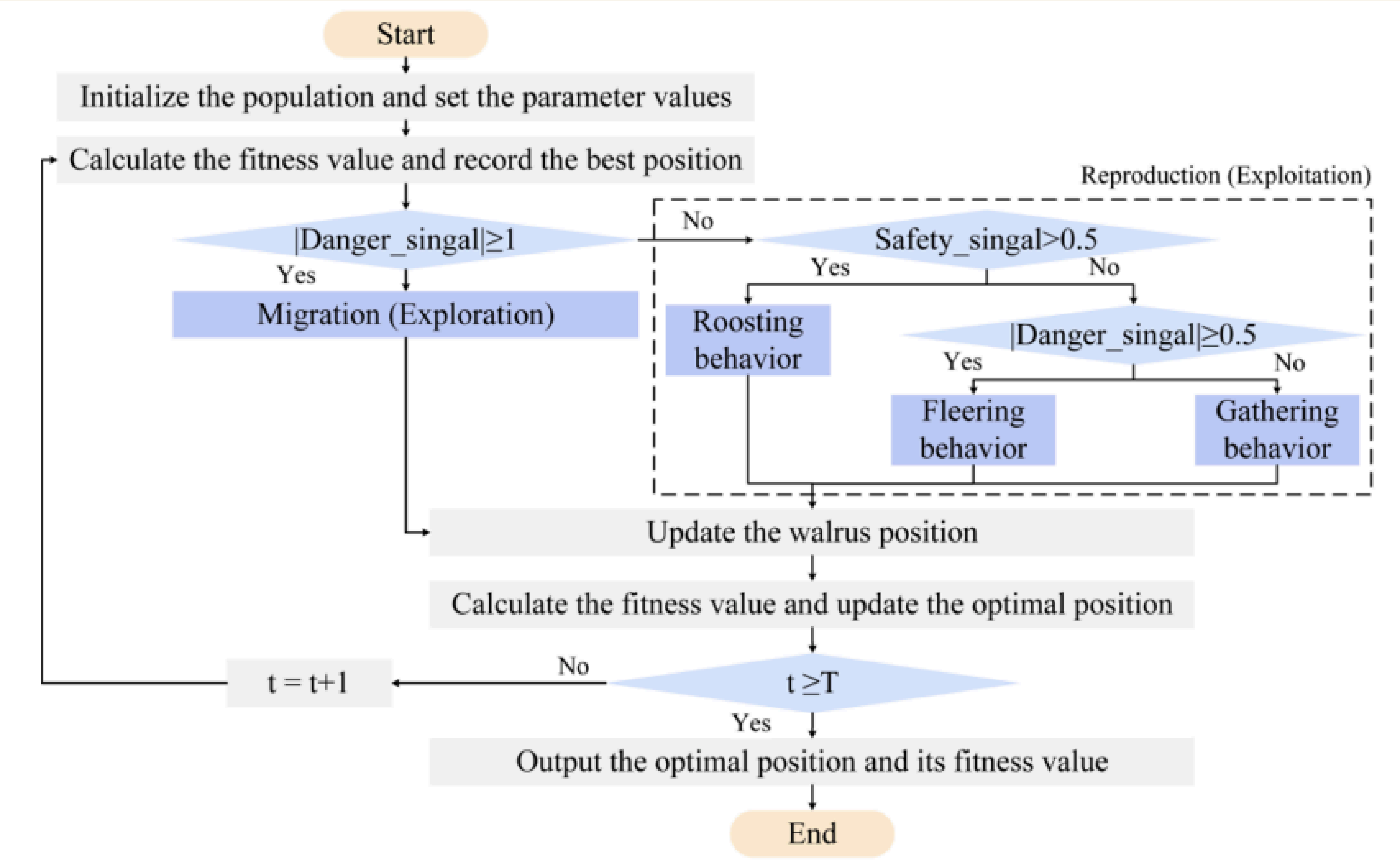
Algorithm 1: The pseudo code of WO

Input: Algorithm parameters (population size N, maximum iteration T)

```
Initialize the population and define the related parameters
Evaluate the fitness values and obtain the best solution
While t ≤ T
    If |Danger_signal| ≥ 1 {Exploration phase}
        Update new position of each walrus using Eq. (9)
    Else {Exploitation phase}
        If Safety_signal ≥ 0.5 // Breeding behavior //
        For each male walrus
            Update new position based on Halton sequence
        End For
        For each female walrus
            Update new position using Eq. (12)
        End For
        For each juvenile walrus
            Update new position using Eq. (13)
        End For
        Else // Foraging behavior //
        If |Danger_signal| ≥ 0.5 // Gathering behavior //
            Update new position of each walrus using Eq. (17)
        Else // Fleeing behavior //
            Update new position of each walrus using Eq. (18)
        End If
        End If
    End If
    Update the walrus position
    Calculate the fitness value and update the current best solution
    t = t + 1
End While
```

Output: the best solution

Flow chart


Start
Initialize the population and set the parameter values
Calculate the fitness value and record the best position
Reproduction (Exploitation)
|Danger_singal|≥1
Yes
No
Migration (Exploration)
Safety_singal>0.5
Yes
No
Roosting behavior
|Danger_singal|≥0.5
Yes
No
Fleering behavior
Gathering behavior
Update the walrus position
Calculate the fitness value and update the optimal position
t = t+1
No
t ≥T
Yes
Output the optimal position and its fitness value
End

ANALYSIS

Complexity

- **Time Complexity**

$$O(N \times D + 2 \times N \times D \times T)$$

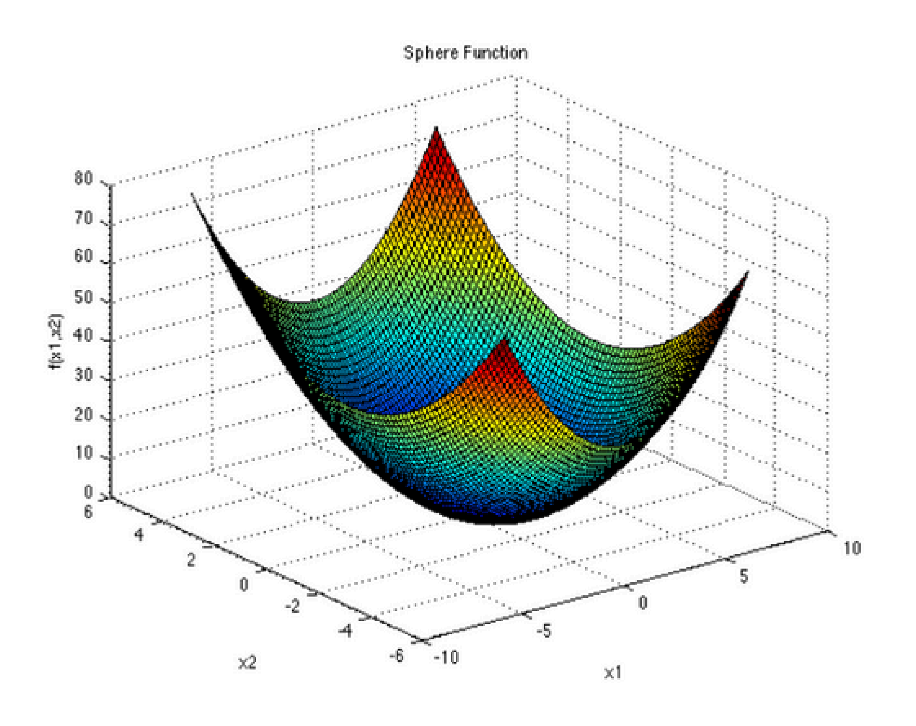
- **Space Complexity**

$$O(N \times D)$$

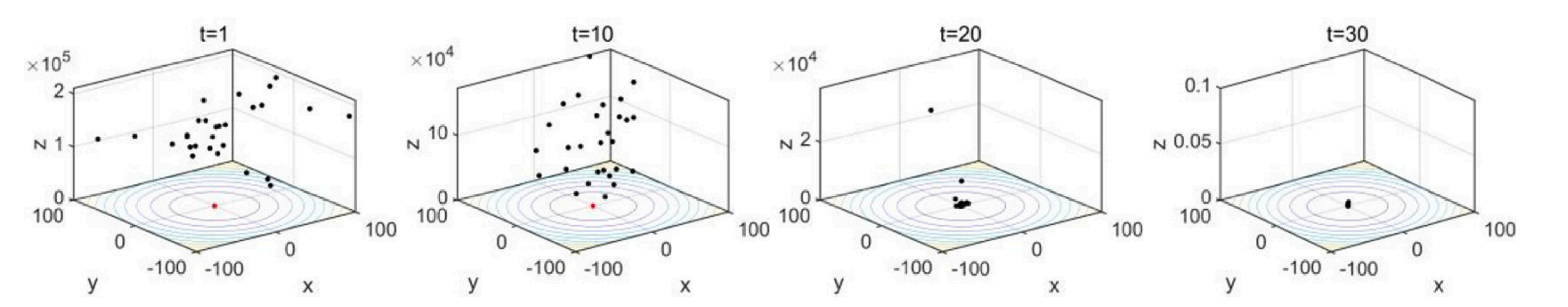
ANALYSIS

Unimodal and Multimodal

- **Sphere function**



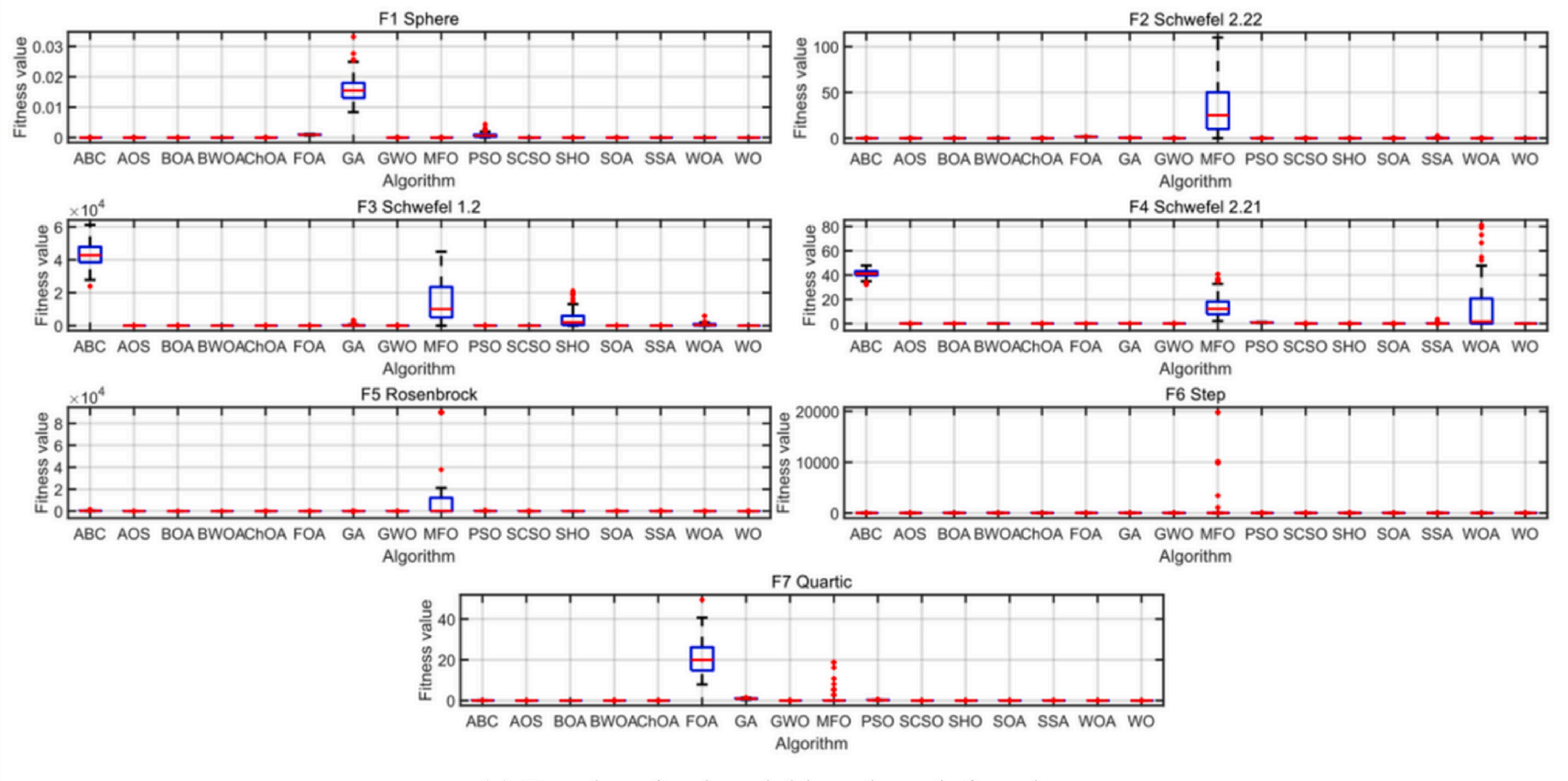
$$f(\mathbf{x}) = \sum_{i=1}^{d} x_i^2$$



(a) Sphere function

Unimodal and Multimodal

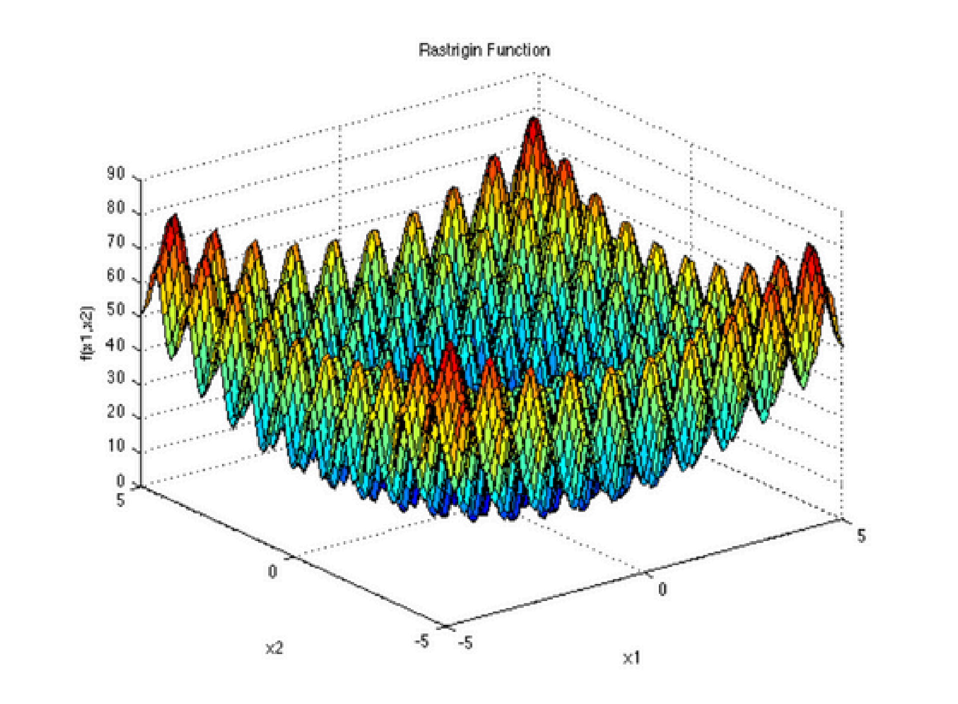
- **Unimodal WO vs others**



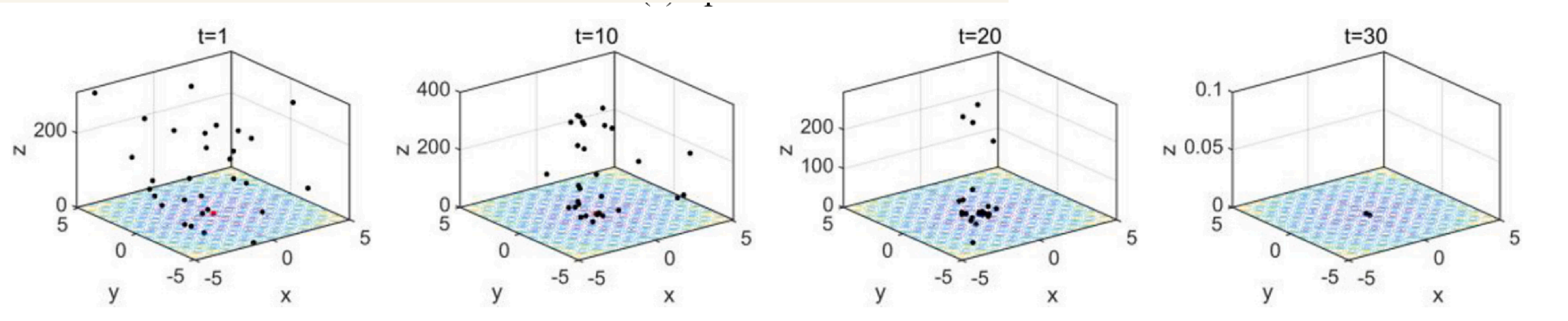
(a) Results of unimodal benchmark functions

Unimodal and Multimodal

- **Rastrigin function**



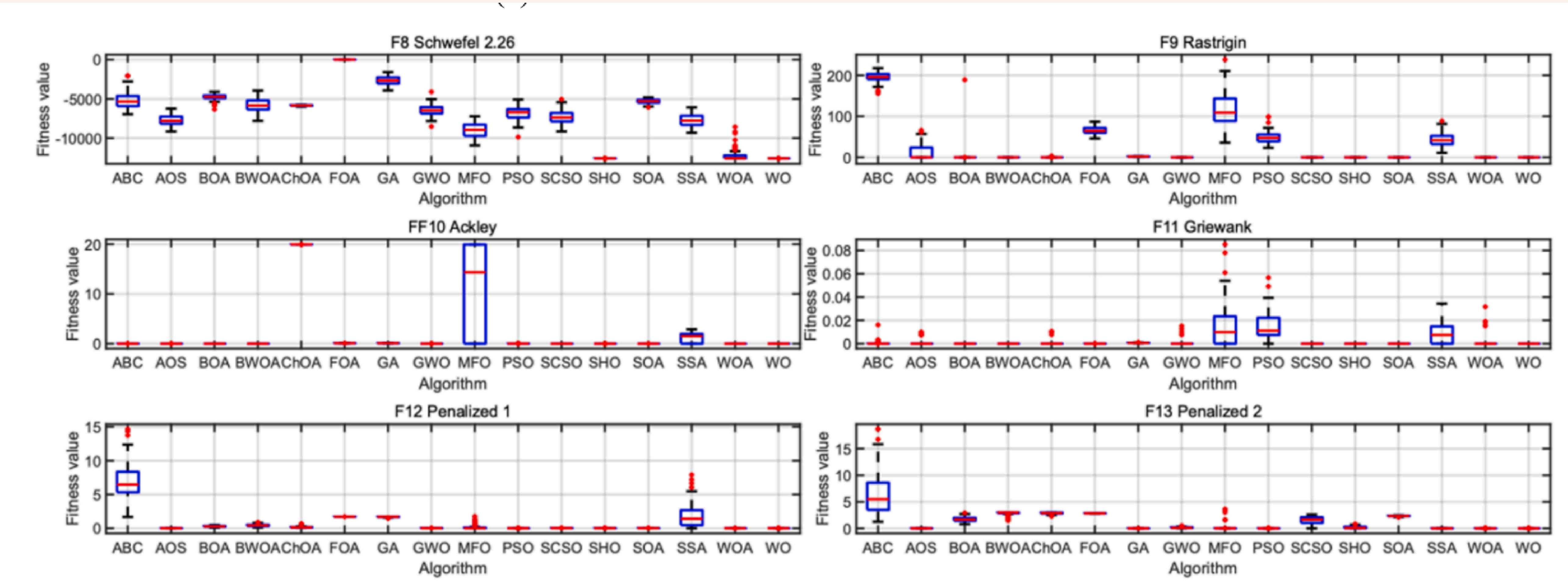
$$f(\mathbf{x}) = 10d + \sum_{i=1}^{d} \left[x_i^2 - 10\cos(2\pi x_i)\right]$$



(b) Rastrigin function

Unimodal and Multimodal

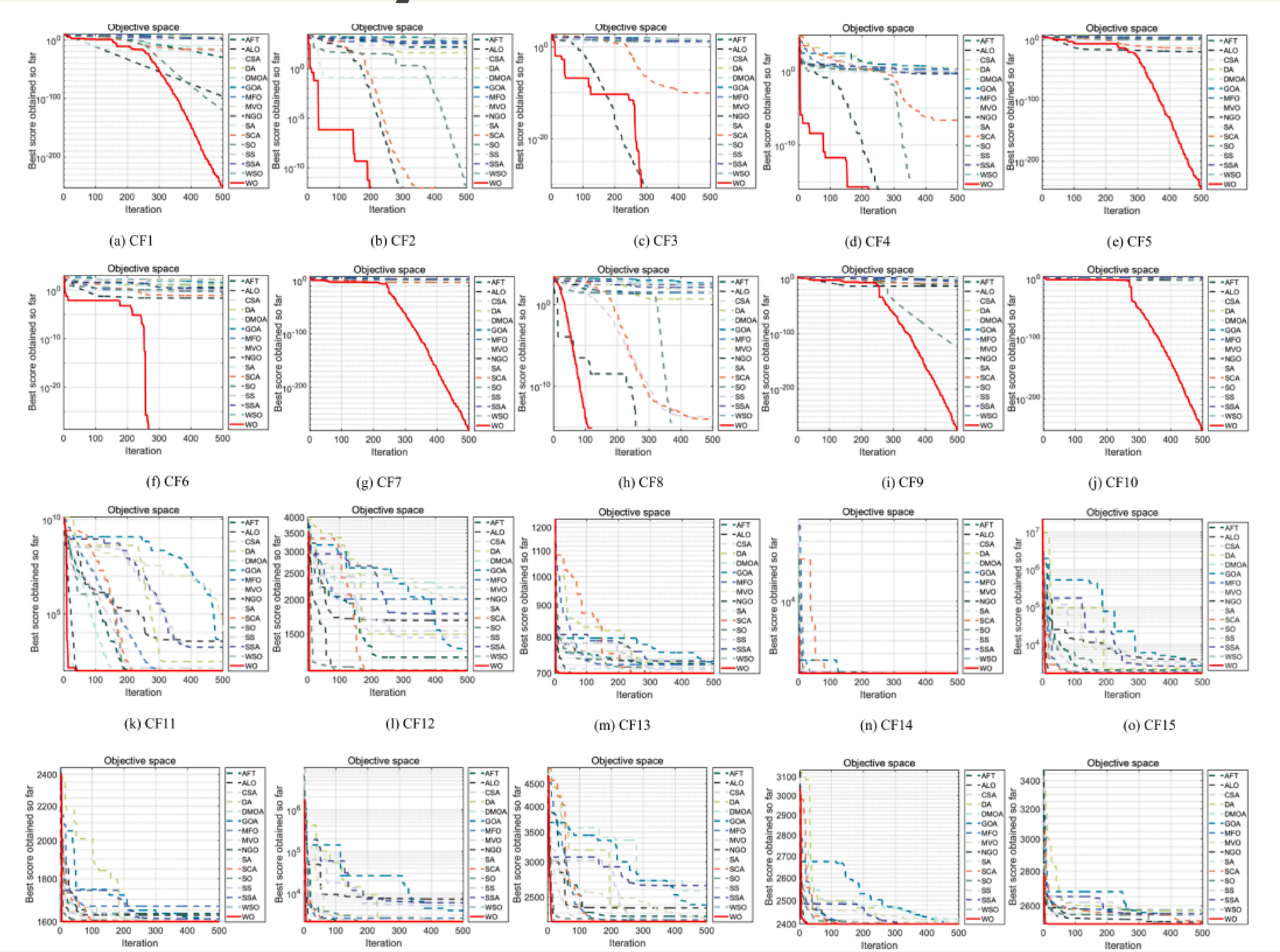
- **Multimodal WO vs others**



(b) Results of multimodal benchmark functions

ANALYSIS

CEC 2021 analysis



ANALYSIS

Friedman mean rank for scalibility

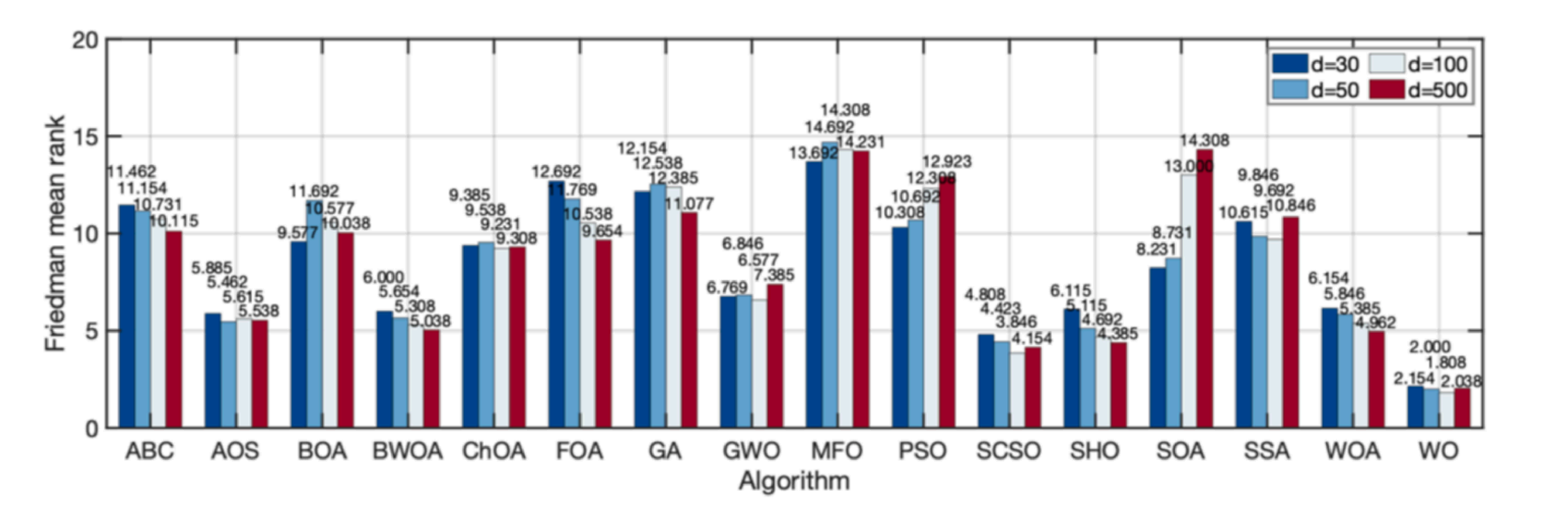


Fig. 8. Friedman mean rank results for different dimensions.

The lower the rank, the better the performance

CONCLUSION

From the experimental analysis, it is obvious that the performance of the Walrus optimizer is very competitive. Possibly due to the complex structure and randomness, not only allows the algorithm to search in a very wide direction, but also ultimately find the optimal solution.

THANKS

for

Listening