數位訊號處理．系統設計與模擬

P.1

主題：可調式分數延遲 FIR 數位濾波器之設計

引用論文相關內容：

A New Criterion for the Design of Variable Fractional-Delay FIR Digital Filters

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II. Review of Conventional LS Design of VFD FIR Digital Filters

For the purpose of comparison and the use of the following sections, the conventional LS design of VFD FIR digital filters is reviewed in this section. The desired response of a VFD FIR filter is given by

$$H_d(\omega,p) = e^{-j(I+p)\omega} = e^{-jI\omega}\left(\cos(\omega p) - j\sin(\omega p)\right), \quad |\omega| \le \omega_p; \quad -0.5 \le p \le 0.5 \tag{4}$$

where I is a prescribed mean group delay and p is the parameter used to adjust the group delay of a filter online. The used transfer function is characterized by

$$H(z,p) = \sum_{n=0}^{N} h_n(p) z^{-n} \tag{5}$$

where coefficients $h_n(p)$ are expressed as the polynomials of p by

$$h_n(p) = \sum_{m=0}^{M} h(n,m)p^m \tag{6}$$

hence

$$H(z,p) = \sum_{n=0}^{N}\sum_{m=0}^{M} h(n,m)p^m z^{-n} = \sum_{m=0}^{M} G_m(z)p^m \tag{7}$$

where subfilters $G_m(Z)$ are represented by

$$G_m(z) = \sum_{n=0}^{N} h(n,m) z^{-n}, \qquad 0 \le m \le M. \tag{8}$$

Obviously, (7) can be implemented by the Farrow structure [7], [15].

Equation (4) can be further represented by

$$H_d(\omega,p) = e^{-jI\omega}\sum_{m=0}^{\infty}\frac{(-jp\omega)^m}{m!} \cong \sum_{m=0}^{M}\left(\frac{(-j\omega)^m}{m!}e^{-jI\omega}\right)p^m \tag{9}$$

for sufficiently large M. Comparing (7) and (9), it can be found that the frequency response of $G_m(z)$ is used inherently to approximate $((-j\omega)^m/m!)e^{-jI\omega}$ for $0 \le m \le M$. Therefore, it is reasonable to choose the coefficients of $G_m(z)$ to be symmetric for even M and antisymmetric for odd M, and obviously, $I = N/2$. In this paper, only even N is used, and the case for odd N can be extended in a similar manner. Notice that the first subfilter $G_0(z)$ is designed to approximate $e^{-j(N/2)\omega}$, so $h(n,0) = \delta(n-(N/2))$. Hence, the frequency response of (7) can be written as

$$H(e^{j\omega},p) = e^{-j\frac{N}{2}\omega}\left[1 + \sum_{m=1}^{M_c}\sum_{n=0}^{N/2} a(n,m)p^{2m}\cos(n\omega) + j\sum_{m=1}^{M_s}\sum_{n=1}^{N/2} b(n,m)p^{2m-1}\sin(n\omega)\right] \tag{10}$$

where (11a)–(11c) are shown at the bottom of the page. Defining

$$\mathbf{a} = \left[a(0,1),\ldots,a\left(\frac{N}{2},1\right),\ldots, a(0,M_c),\ldots,a\left(\frac{N}{2},M_c\right)\right]^{\mathrm{T}} \tag{12a}$$

$$\mathbf{b} = \left[b(1,1),\ldots,b\left(\frac{N}{2},1\right),\ldots, b(1,M_s),\ldots,b\left(\frac{N}{2},M_s\right)\right]^{\mathrm{T}} \tag{12b}$$

$$\mathbf{c}(\omega,p) = \left[p^2,\ldots,p^2\cos\left(\frac{N}{2}\omega\right),\ldots,p^{2M_c},\ldots, p^{2M_c}\cos\left(\frac{N}{2}\omega\right)\right]^{\mathrm{T}} \tag{12c}$$

$$\mathbf{s}(\omega,p) = \left[p\sin(\omega),\ldots,p\sin\left(\frac{N}{2}\omega\right),\ldots, p^{2M_s-1}\sin(\omega),\ldots,p^{2M_s-1}\sin\left(\frac{N}{2}\omega\right)\right]^{\mathrm{T}}. \tag{12d}$$

Equation (10) can be represented by

$$H(e^{j\omega},p) = e^{-j\frac{N}{2}\omega}\left[1 + \mathbf{a}^{\mathrm{T}}\mathbf{c}(\omega,p) + j\mathbf{b}^{\mathrm{T}}\mathbf{s}(\omega,p)\right] \tag{13}$$

where the superscript $^{\mathrm{T}}$ denotes the transpose operator.

The conventional objective error function for designing a VFD FIR filter is given by

$$e_c(\mathbf{a},\mathbf{b}) = \int_{-0.5}^{0.5}\int_{0}^{\omega_p} W(\omega)\left|H_d(\omega,p) - H(e^{j\omega},p)\right|^2 d\omega\, dp$$

$$\begin{cases} M_c = M_s = \frac{M}{2}, & M\text{: even} \\ M_c + 1 = M_s = \frac{M+1}{2}, & M\text{: odd} \end{cases} \tag{11a}$$

$$a(n,m) = \begin{cases} h\left(\frac{N}{2},2m\right), & n = 0; 1 \le m \le M_c \\ 2h\left(\frac{N}{2}-n,2m\right) = 2h\left(\frac{N}{2}+n,2m\right), & 1 \le n \le \frac{N}{2}; 1 \le m \le M_c \end{cases} \tag{11b}$$

$$b(n,m) = 2h\left(\frac{N}{2}-n,2m-1\right) = -2h\left(\frac{N}{2}+n,2m-1\right), \quad 1 \le n \le \frac{N}{2}; \quad 1 \le m \le M_s \tag{11c}$$

$$= \int_{-0.5}^{0.5}\int_{0}^{\omega_p} W(\omega)\left|\cos(p\omega) - j\sin(p\omega) - 1 - \mathbf{a}^{\mathrm{T}}\mathbf{c}(\omega,p) - j\mathbf{b}^{\mathrm{T}}\mathbf{s}(\omega,p)\right|^2 d\omega\, dp$$

$$= e_c(\mathbf{a}) + e_c(\mathbf{b}) \tag{14}$$

where $W(\omega)$ is a weighting function

$$e_c(\mathbf{a}) = \int_{-0.5}^{0.5}\int_{0}^{\omega_p} W(\omega)\left(\cos(p\omega) - 1 - \mathbf{a}^{\mathrm{T}}\mathbf{c}(\omega,p)\right)^2 d\omega\, dp$$

$$= s_a + \mathbf{r}_a^{\mathrm{T}}\mathbf{a} + \mathbf{a}^{\mathrm{T}}\mathbf{Q}_a\mathbf{a} \tag{15a}$$

$$e_c(\mathbf{b}) = \int_{-0.5}^{0.5}\int_{0}^{\omega_p} W(\omega)\left(\sin(p\omega) + \mathbf{b}^{\mathrm{T}}\mathbf{s}(\omega,p)\right)^2 d\omega\, dp$$

$$= s_b + \mathbf{r}_b^{\mathrm{T}}\mathbf{b} + \mathbf{b}^{\mathrm{T}}\mathbf{Q}_b\mathbf{b} \tag{15b}$$

in which

$$s_a = \int_{-0.5}^{0.5}\int_{0}^{\omega_p} W(\omega)\left(\cos(p\omega) - 1\right)^2 d\omega\, dp \tag{16a}$$

$$\mathbf{r}_a = -2\int_{-0.5}^{0.5}\int_{0}^{\omega_p} W(\omega)\left(\cos(p\omega) - 1\right)\mathbf{c}(\omega,p)\, d\omega\, dp \tag{16b}$$

$$\mathbf{Q}_a = \int_{-0.5}^{0.5}\int_{0}^{\omega_p} W(\omega)\mathbf{c}(\omega,p)\mathbf{c}^{\mathrm{T}}(\omega,p)\, d\omega\, dp \tag{16c}$$

$$s_b = \int_{-0.5}^{0.5}\int_{0}^{\omega_p} W(\omega)\sin^2(p\omega)\, d\omega\, dp \tag{16d}$$

$$\mathbf{r}_b = 2\int_{-0.5}^{0.5}\int_{0}^{\omega_p} W(\omega)\sin(p\omega)\mathbf{s}(\omega,p)\, d\omega\, dp \tag{16e}$$

$$\mathbf{Q}_b = \int_{-0.5}^{0.5}\int_{0}^{\omega_p} W(\omega)\mathbf{s}(\omega,p)\mathbf{s}^{\mathrm{T}}(\omega,p)\, d\omega\, dp. \tag{16f}$$

For LS design $W(\omega) = 1$ and by applying the technique in [22], the elements in $\mathbf{r}_a$, $\mathbf{Q}_a$, $\mathbf{r}_b$, and $\mathbf{Q}_b$ can be represented in closed form as (17a) and (17b), shown at the bottom of the page, where $(x \bmod y)$ denotes the remainder when integer x is divided by integer y, and $\lfloor u \rfloor$ denotes the largest integer that is less than or equal to real number u. In (17), K must be chosen large enough as in [22], and $K = 10$ is used in this paper.

Once $\mathbf{r}_a$, $\mathbf{Q}_a$, $\mathbf{r}_b$, and $\mathbf{Q}_b$ are obtained, the optimal solutions in the LS sense can be achieved by differentiating (14) with respect to $\mathbf{a}$ and $\mathbf{b}$, respectively, and then setting the results to zero as follows:

$$\frac{\partial e_c(\mathbf{a},\mathbf{b})}{\partial \mathbf{a}} = \frac{\partial e_c(\mathbf{a})}{\partial \mathbf{a}} = \mathbf{r}_a + 2\mathbf{Q}_a\mathbf{a} = 0 \tag{18a}$$

$$\frac{\partial e_c(\mathbf{a},\mathbf{b})}{\partial \mathbf{b}} = \frac{\partial e_c(\mathbf{b})}{\partial \mathbf{b}} = \mathbf{r}_b + 2\mathbf{Q}_b\mathbf{b} = 0 \tag{18b}$$

which yield

$$\mathbf{a} = -\frac{1}{2}\mathbf{Q}_a^{-1}\mathbf{r}_a \tag{19a}$$

$$\mathbf{b} = -\frac{1}{2}\mathbf{Q}_b^{-1}\mathbf{r}_b. \tag{19b}$$

$$\begin{cases} \mathbf{r}_a(i) = -4\sum_{k=1}^{K}\frac{(-1)^k}{(2k)!}\frac{0.5^{2m+2k+1}}{2m+2k+1}\int_0^{\omega_p}\omega^{2k}\cos(n\omega)d\omega \\ \mathbf{Q}_a(i,l) = \frac{0.5^{2m+2\hat{m}+1}}{2m+2\hat{m}+1}\left(\frac{\sin((n-\hat{n})\omega_p)}{n-\hat{n}} + \frac{\sin((n+\hat{n})\omega_p)}{n+\hat{n}}\right) \\ 0 \le i,l \le \left(\frac{N}{2}+1\right)M_c - 1, \quad n = \left(i \bmod \frac{N}{2}+1\right), \quad m = \left\lfloor \frac{i}{N/2+1} \right\rfloor + 1 \\ \hat{n} = \left(l \bmod \frac{N}{2}+1\right), \quad \hat{m} = \left\lfloor \frac{l}{N/2+1} \right\rfloor + 1 \end{cases} \tag{17a}$$

$$\begin{cases} \mathbf{r}_b(i) = 4\sum_{k=0}^{K}\frac{(-1)^k}{(2k+1)!}\frac{0.5^{2m+2k+1}}{2m+2k+1}\int_0^{\omega_p}\omega^{2k+1}\sin(n\omega)d\omega \\ \mathbf{Q}_b(i,l) = \frac{0.5^{2m+2\hat{m}-1}}{2m+2\hat{m}-1}\left(\frac{\sin((n-\hat{n})\omega_p)}{n-\hat{n}} - \frac{\sin((n+\hat{n})\omega_p)}{n+\hat{n}}\right) \\ 0 \le i,l \le \frac{NM_s}{2} - 1, \quad n = \left(i \bmod \frac{N}{2}\right) + 1, \quad m = \left\lfloor \frac{i}{N/2} \right\rfloor + 1 \\ \hat{n} = \left(l \bmod \frac{N}{2}\right) + 1, \quad \hat{m} = \left\lfloor \frac{l}{N/2} \right\rfloor + 1 \end{cases} \tag{17b}$$

目的說明：課堂上所提及系統其延遲時間大部為整數單位(如Type I及Type III linear-phase FIR filters)或整數加$\frac{1}{2}$單位(如Type II及Type IV linear-phase FIR filters)之系統，本單元所要設計之標的為延遲時間為任意值之FIR濾波器，且為可調式。該系統其應用主要之精神為可以提取離散信號間之任何位置信號值。

實驗內容原理概要及說明：

理想系統之頻率響應：

$$H_d(\omega,p)=e^{-j(I+p)\omega} \qquad |\omega|\le\omega_p,\ -0.5\le p\le 0.5$$

I : a prescribed mean group delay

可調式 FIR 數位濾波器：

$$H(Z,p)=\sum_{n=0}^{N} h_n(p)\, Z^{-n} \qquad p \text{ 為外部調整之參數}$$

$$h_n(p)=\sum_{m=0}^{M} h(n,m)\,p^m$$

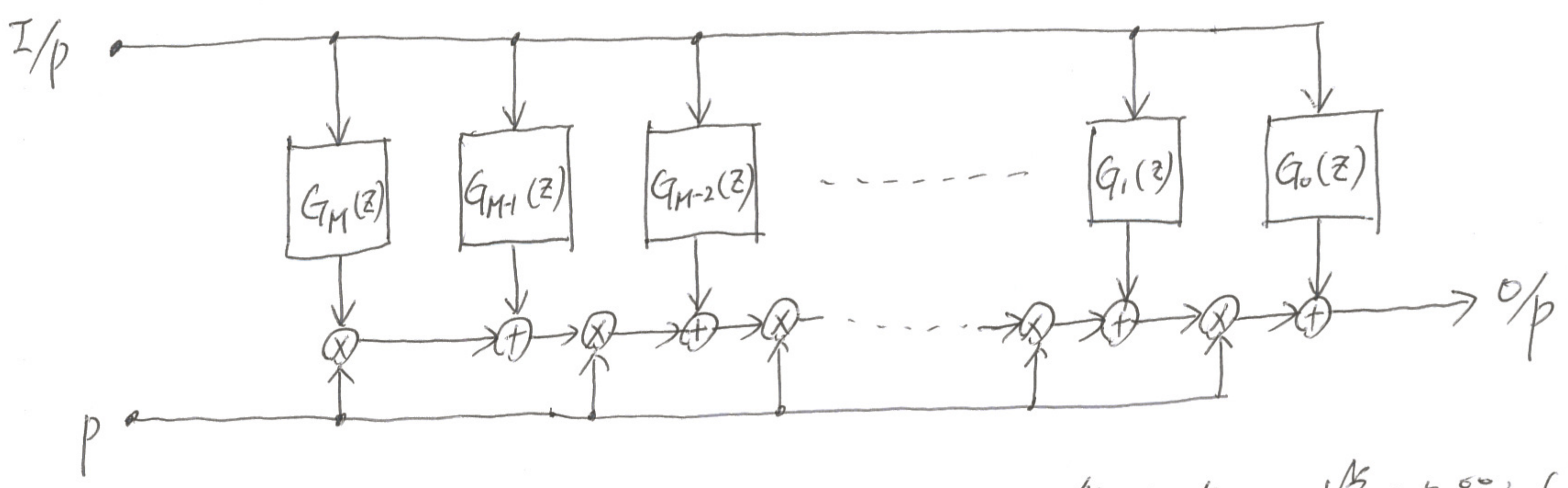
$$H(Z,p)=\sum_{n=0}^{N}\sum_{m=0}^{M} h(n,m)\,p^m Z^{-n}=\sum_{m=0}^{M}\left(\sum_{n=0}^{N} h(n,m)\,Z^{-n}\right)p^m$$

令 $G_m(Z)=\sum_{n=0}^{N} h(n,m)\,Z^{-n}$

$$\Rightarrow H(Z,p)=\sum_{m=0}^{N} G_m(Z)\,p^m$$

$$=G_0(Z)+G_1(Z)\,p+G_2(Z)\,p^2+\cdots+G_M(Z)\,p^M$$

結構圖：



$G_0(Z), G_1(Z), \ldots, G_M(Z)$ 一般稱之為子濾波器 (subfilter).

各子濾波器扮演之角色：

$$H_d(\omega,p)=e^{-jI\omega}\underbrace{\sum_{m=0}^{\infty}\frac{(-jp\omega)^m}{m!}}_{\text{↑ } e^{-jp\omega} \text{ 之泰勒展開式}} \cong \left[\sum_{m=0}^{M}\frac{(-j\omega)^m}{m!}\,e^{-jI\omega}\right]p^m$$

$m=0$ $\quad G_0(e^{j\omega}) \xrightarrow{\text{approach}} e^{-jI\omega}$ ↑ 為一 pure delay I 之系統

$$G_0(e^{j\omega})=\sum_{n=0}^{N} h(n,0)\,e^{-jn\omega}$$

$$\Rightarrow h(n,0)=\delta\left(n-\frac{N}{2}\right)$$

m: even

$$G_m(e^{j\omega}) \xrightarrow{\text{approach}} \frac{(-j\omega)^m}{m!} e^{-jI\omega} = (-1)^{\frac{m}{2}} \frac{\omega^m}{m!} e^{-jI\omega}$$

$$\| $$

$$\sum_{n=0}^{N} h(n,m) e^{-jn\omega}$$

$$= e^{-j\frac{N}{2}\omega} \sum_{n=0}^{N/2} a(n,\tfrac{m}{2}) \cos(n\omega)$$

(Type I subfilter)

為理想之mth-order
線性相位微分器,
可用 Type I linear-phase
FIR filter 來實現

m: odd

$$G_m(e^{j\omega}) \xrightarrow{\text{approach}} \frac{(-j\omega)^m}{m!} e^{-jI\omega} = (-j)^m \frac{\omega^m}{m!} e^{-jI\omega}$$

$$\| $$

$$\sum_{n=0}^{N} h(n,m) e^{-jn\omega}$$

$$= e^{-j\frac{N}{2}\omega}\, j \sum_{n=1}^{N/2} b(n,\tfrac{m+1}{2}) \sin(n\omega)$$

(Type III subfilter)

為理想之mth-order
線性相位微分器,
可用 Type III linear-phase
FIR filter 來實現

整体系統之 frequency response:

$$H(e^{j\omega}, p) = e^{-j\frac{N}{2}\omega}\left[1 + \sum_{m=1}^{M_c}\sum_{n=0}^{N/2} a(n,m) p^{2m} \cos(n\omega) + j\sum_{m=1}^{M_s}\sum_{n=1}^{N/2} b(n,m) p^{2m-1} \sin(n\omega)\right]$$

$$= e^{-j\frac{N}{2}\omega}\left[1 + a^T c(\omega,p) + j\, b^T s(\omega,p)\right]$$

$$a = [a(0,1) \cdots a(\tfrac{N}{2},1) \cdots\cdots a(0,M_c) \cdots\cdots a(\tfrac{N}{2},M_c)]^T$$

$$b = [b(1,1) \cdots b(\tfrac{N}{2},1) \cdots\cdots b(1,M_s) \cdots\cdots b(\tfrac{N}{2},M_s)]^T$$

$$c(\omega,p) = [p^2 \cdots p^2\cos(\tfrac{N}{2}\omega) \cdots\cdots p^{2M_c} \cdots\cdots p^{2M_c}\cos(\tfrac{N}{2}\omega)]^T$$

$$s(\omega,p) = [p\sin(\omega) \cdots p\sin(\tfrac{N}{2}\omega) \cdots\cdots p^{2M_s-1}\sin(\omega) \cdots\cdots p^{2M_s-1}\sin(\tfrac{N}{2}\omega)]^T$$

$$\begin{cases} M_c = M_s = \frac{M}{2} & M:\text{even} \\ M_c + 1 = M_s = \frac{M+1}{2} & M:\text{odd} \end{cases}$$

$$a(n,m) = \begin{cases} h(\frac{N}{2}, 2m) & n=0 \quad 1 \le m \le M_c \\ 2h(\frac{N}{2}-n, 2m) = 2h(\frac{N}{2}+n, 2m) & 1 \le n \le \frac{N}{2} \quad 1 \le m \le M_c \end{cases}$$

$$b(n,m) = 2h(\tfrac{N}{2}-n, 2m-1) = -2h(\tfrac{N}{2}+n, 2m-1) \quad 1 \le n \le \tfrac{N}{2} \quad 1 \le m \le M_s$$

標的誤差函式：

$$e(a,b)=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\left|H_d(\omega,p)-H(e^{j\omega},p)\right|^2 d\omega dp$$

$$=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\left|\cos(p\omega)-j\sin(p\omega)-1-a^T c(\omega,p)-jb^T s(\omega,p)\right|^2 d\omega dp$$

$$=\underbrace{\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\left(\cos(p\omega)-1-a^T c(\omega,p)\right)^2 d\omega dp}_{\text{↑此項僅與a係數有關}}+\underbrace{\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\left(\sin(p\omega)+b^T s(\omega,p)\right)^2 d\omega dp}_{\text{↑此項僅與b係數有關}}$$

$$=e(a)+e(b)$$

$$e(a)=s_a+r_a^T a+a^T Q_a a$$

$$e(b)=s_b+r_b^T b+b^T Q_b b$$

$$s_a=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\left(\cos(p\omega)-1\right)^2 d\omega dp \qquad s_b=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\sin^2(p\omega)d\omega dp$$

$$r_a=-2\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\left(\cos(p\omega)-1\right)c(\omega,p)d\omega dp \qquad r_b=2\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\sin(p\omega)\,s(\omega,p)d\omega dp$$

$$Q_a=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}c(\omega,p)\,c^T(\omega,p)d\omega dp \qquad Q_b=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}s(\omega,p)\,s^T(\omega,p)d\omega dp$$

$$\frac{\partial e(a,b)}{\partial a}=\frac{\partial e(a)}{\partial a}=r_a+2Q_a a=0 \Rightarrow a=-\frac{1}{2}Q_a^{-1}r_a$$

$$\frac{\partial e(a,b)}{\partial b}=\frac{\partial e(b)}{\partial b}=r_b+2Q_b b=0 \Rightarrow b=-\frac{1}{2}Q_b^{-1}r_b$$

鍵入並執行下列程式：

```
%
% Design of variable fractional-delay FIR digital filters
%
clear all;
clc;
N=50;
M=7;
wp=0.9*pi;
pointw=200;
```

w=0 ~ w=wp 之取樣點數

```
pointp=60;
```

p=-0.5 ~ p=0.5 之取樣點數

```
%
%
j=0+i;
NH=N/2;
if mod(M,2)==0
    Mc=M/2;
    Ms=M/2;
else
    Mc=(M-1)/2;
    Ms=(M+1)/2;
end
nma=(NH+1)*Mc;
```

a 之係數個數

```
nmb=NH*Ms;
```

b 〃 〃 〃

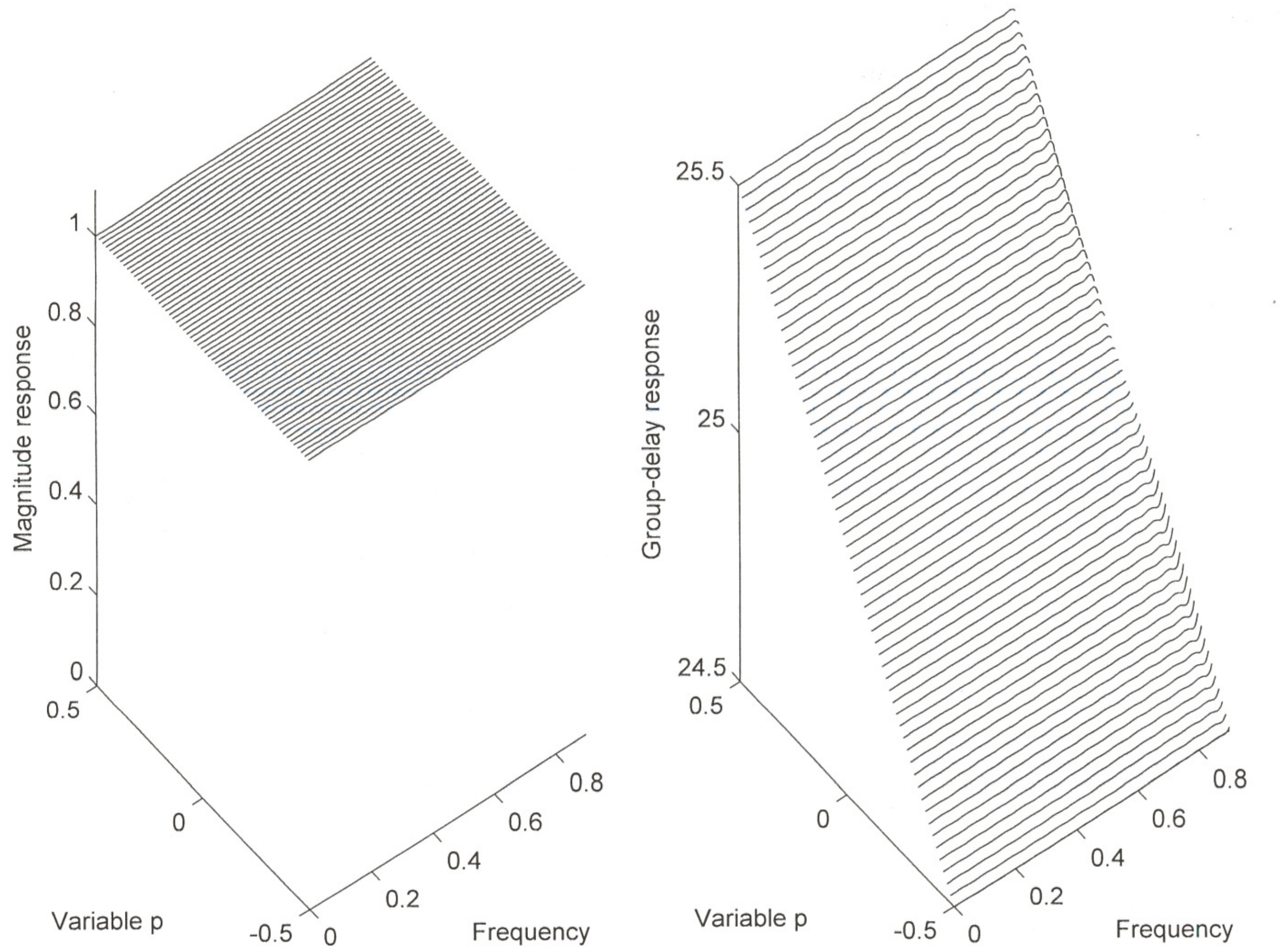
```
deltaw=wp/pointw;
deltap=1/pointp;
point=(pointw+1)*(pointp+1);
%
%
ra=zeros(nma,1);
Qa=zeros(nma,nma);
for iw=0:pointw
    w=iw*deltaw;
    for ip=0:pointp
        p=-0.5+ip*deltap;
        c=zeros(nma,1);
        for ic=0:nma-1
            n=mod(ic,NH+1);
            m=floor(ic/(NH+1))+1;
            c(ic+1)=p^(2*m)*cos(n*w);
        end
        ra=ra-2*(cos(p*w)-1)*c;
        Qa=Qa+c*c';
    end
end
ra=wp*ra/point;
Qa=wp*Qa/point;
a=-0.5*inv(Qa)*ra;
%

%
rb=zeros(nmb,1);
Qb=zeros(nmb,nmb);
for iw=0:pointw
    w=iw*deltaw;
    for ip=0:pointp
        p=-0.5+ip*deltap;
        s=zeros(nmb,1);
        for is=0:nmb-1
            n=mod(is,NH)+1;
            m=floor(is/NH)+1;
            s(is+1)=p^(2*m-1)*sin(n*w);
        end
        rb=rb+2*sin(p*w)*s;
        Qb=Qb+s*s';
    end
end
rb=wp*rb/point;
Qb=wp*Qb/point;
b=-0.5*inv(Qb)*rb;
%
%
```

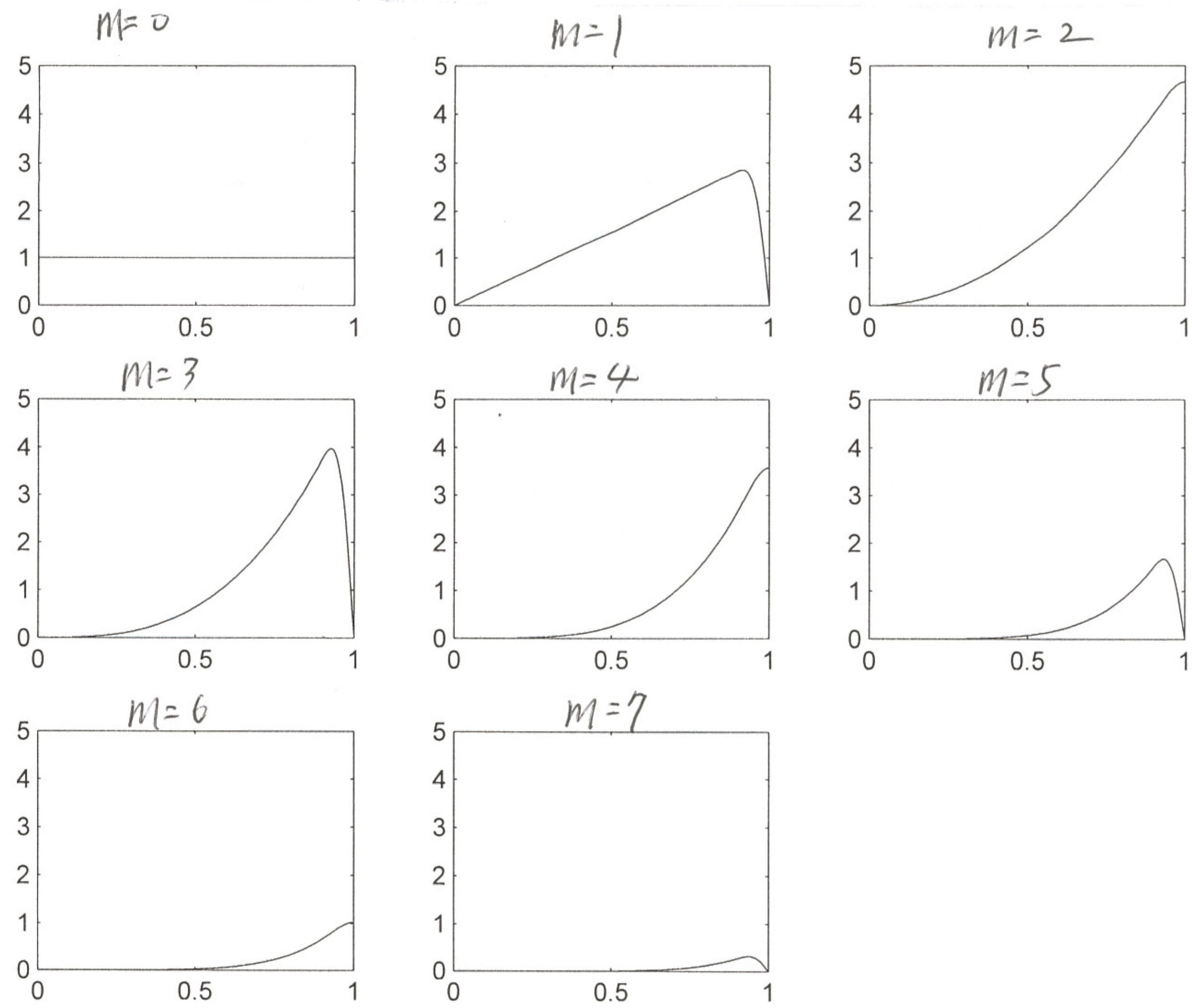
```
a2=reshape(a,NH+1,Mc);
b2=reshape(b,NH,Ms);
h=zeros(N+1,M+1);
h(NH+1,1)=1;
for im=1:Mc
    h(NH+1,2*im+1)=a2(1,im);
    h(1:NH,2*im+1)=0.5*a2(NH+1:-1:2,im);
    h(NH+2:N+1,2*im+1)=0.5*a2(2:NH+1,im);
end
for im=1:Ms
    h(1:NH,2*im)=0.5*b2(NH:-1:1,im);
    h(NH+2:N+1,2*im)=-0.5*b2(1:NH,im);
end
%
%
MR=zeros(pointw+1,pointp+1);
GD=zeros(pointw+1,pointp+1);
for ip=0:pointp
    p=-0.5+ip*deltap;
    hnp=h(:,1);
    for im=1:M
        hnp=hnp+h(:,im+1)*p^im;
    end
    MR(:,ip+1)=abs(freqz(hnp,1,0:deltaw:wp));
    GD(:,ip+1)=grpdelay(hnp,1,0:deltaw:wp);
end

%
%
XX=zeros(pointw+1,pointp+1);
YY=zeros(pointw+1,pointp+1);
for ip=0:pointp
    XX(:,ip+1)=(0:deltaw:wp)/pi';
end
for iw=0:pointw
    YY(iw+1,:)=-0.5:deltap:0.5;
end
%
subplot(1,2,1);
plot3(XX,YY,MR);
axis([0,wp/pi,-0.5,0.5,0,1.1]);
xlabel('Frequency');
ylabel('Variable p');
zlabel('Magnitude response');
subplot(1,2,2);
plot3(XX,YY,GD);
axis([0,wp/pi,-0.5,0.5,NH-0.5,NH+0.5]);
xlabel('Frequency');
ylabel('Variable p');
zlabel('Group-delay response');
pause;
%
% magnitude responses of subfilters
%
for im=0:M
    MRs=abs(freqz(h(:,im+1),1,0:pi/200:pi));
    subplot(3,3,im+1);
    plot(0:1/200:1,MRs);
    axis([0,1,0,5]);
end
```

執行結果如次頁上圖

執行結果如次頁下圖

執行結果：





第m個 subfilter 扮演 m-th order 之微分器