數位訊號處理、系統設計與模擬

系統設計名稱：可調式低通濾波器之設計

Problem formulation:

desired frequency response:

passband edge 變化的範圍

$$H_d(\omega,p)=\begin{cases} e^{-jI\omega} & 0\le\omega\le\omega_p \\ 0 & \omega_s\le\omega\le\pi \end{cases} \qquad \underbrace{\omega_{p1}\le\omega_p\le\omega_{p2}} \qquad \omega_s=\omega_p+\omega_t$$

$$\le (p+0.5)(\omega_{p2}-\omega_{p1})+\omega_{p1}$$

$$-0.5\le p\le 0.5$$

variable FIR digital filter:

令由調整參數 p 控制 ω_p

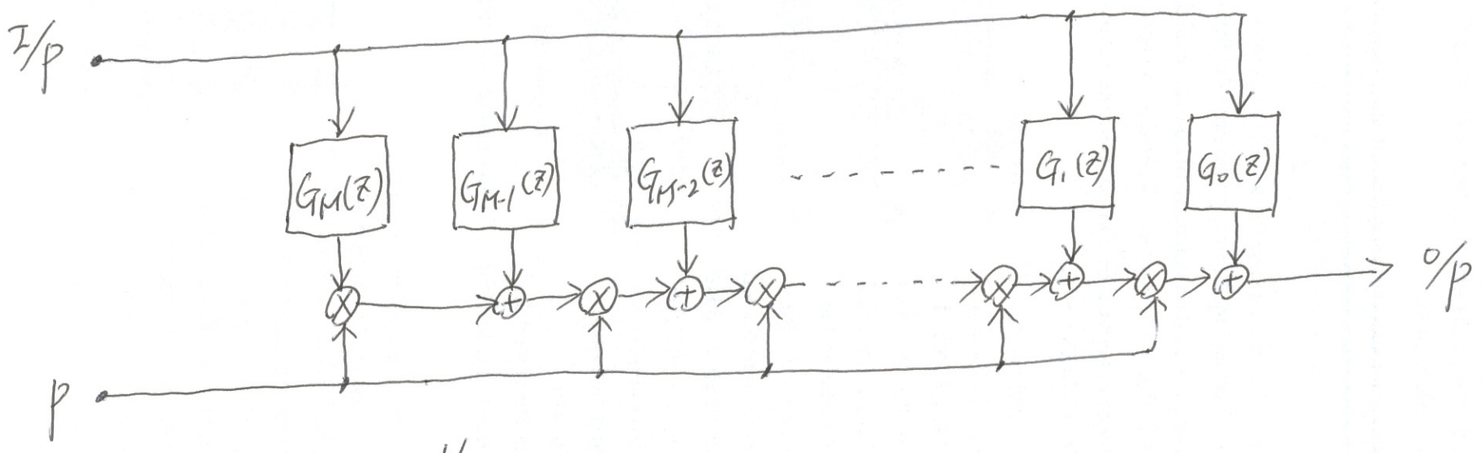
$$H(z,p)=\sum_{n=0}^{N} h_n(p)z^{-n} \qquad h_n(p)=\sum_{m=0}^{M} h(n,m)p^m$$

$$=\sum_{n=0}^{N}\sum_{m=0}^{M} h(n,m)p^m z^{-n}$$

$$=\sum_{m=0}^{M}\left(\sum_{n=0}^{N} h(n,m)z^{-n}\right)p^m$$

$$=\sum_{m=0}^{M} G_m(z)\,p^m$$

$$G_m(z)=\sum_{n=0}^{N} h(n,m)\,z^{-n}$$



for N: even, let $I=\frac{N}{2}$

$G_m(z)$: Type I FIR linear-phase filter

$$G_m(e^{j\omega})=e^{-j\frac{N}{2}\omega}\sum_{n=0}^{N/2} a(n,m)\cos(n\omega)$$

$$a(n,m)=\begin{cases} h\left(\frac{N}{2},m\right) & n=0 \\ 2h\left(\frac{N}{2}-n,m\right) & n=1,2,\cdots,\frac{N}{2} \end{cases}$$

$$H(e^{jw}, p) = e^{-j\frac{N}{2}w} \sum_{m=0}^{M} \sum_{n=0}^{N/2} a(n,m)\, p^m \cos(nw)$$

$$= e^{-j\frac{N}{2}w}\, a^T c(w,p)$$

$$a = [a(0,0)\ a(1,0) \cdots a(\tfrac{N}{2},0)\ a(0,1) \cdots a(\tfrac{N}{2},1) \cdots a(0,M) \cdots a(\tfrac{N}{2},M)]^T$$

$$c(w,p) = [1\ \cos w \cdots \cos(\tfrac{N}{2}w)\ p \cdots p\cos(\tfrac{N}{2}w) \cdots p^M \cdots p^M \cos(\tfrac{N}{2}w)]^T$$

objective error function:

$$e(a) = \int_{-0.5}^{0.5}\int_{0}^{w_p} [1 - a^T c(w,p)]^2 dw\,dp + \int_{-0.5}^{0.5}\int_{w_s}^{\pi} [a^T c(w,p)]^2 dw\,dp$$

$$= \int_{-0.5}^{0.5}\int_{0}^{w_p} [1 - 2a^T c(w,p) + a^T c(w,p) c^T(w,p) a]\, dw\,dp$$

$$+ \int_{-0.5}^{0.5}\int_{w_s}^{\pi} a^T c(w,p) c^T(w,p) a\, dw\,dp$$

$$= s + r^T a + a^T Q_p a + a^T Q_s a$$

$$s = \int_{-0.5}^{0.5}\int_{0}^{w_p} 1\, dw\,dp$$

$$r = -2\int_{-0.5}^{0.5}\int_{0}^{w_p} c(w,p)\, dw\,dp$$

$$Q_p = \int_{-0.5}^{0.5}\int_{0}^{w_p} c(w,p) c^T(w,p)\, dw\,dp$$

$$Q_s = \int_{-0.5}^{0.5}\int_{w_s}^{\pi} c(w,p) c^T(w,p)\, dw\,dp$$

$$\frac{\partial e(a)}{\partial a} = r + 2Q_p a + 2Q_s a = 0 \Rightarrow a = \frac{-1}{2}(Q_p + Q_s)^{-1} r$$

金建入下列程式並執行之：

```
%
% Design of variable fractional-delay (VFD) FIR digital filters
%
clear all;
clc;
N=50;
M=7;
wp1=0.3*pi;
wp2=0.6*pi;
wt=0.1*pi;
sampling_w=200;
sampling_p=60;
%
%
NH=N/2;
nma=(M+1)*(NH+1);
deltaw=pi/sampling_w;
deltap=1/sampling_p;
%
%
sampling_pass=0;
sampling_stop=0;
ra=zeros(nma,1);
Qp=zeros(nma,nma);
Qs=zeros(nma,nma);
for iw=0:sampling_w
    w=iw*deltaw;
    for ip=0:sampling_p
        p=-0.5+ip*deltap;
        cwp=zeros(nma,1);
        for inm=0:nma-1
            n=mod(inm,NH+1);
            m=floor(inm/(NH+1));
            cwp(inm+1)=p^(m)*cos(n*w);
        end
        if p >= (w-wp1)/(wp2-wp1)-0.5
            sampling_pass=sampling_pass+1;
            ra=ra-2*cwp;
            Qp=Qp+cwp*cwp';
        elseif p <= (w-wp1-wt)/(wp2-wp1)-0.5
            sampling_stop=sampling_stop+1;
            Qs=Qs+cwp*cwp';
        end
    end
end
ra=0.5*(wp1+wp2)*ra/sampling_pass;
Qp=0.5*(wp1+wp2)*Qp/sampling_pass;
Qs=0.5*(pi-wp1-wt+pi-wp2-wt)*Qs/sampling_stop;
a=-0.5*inv(Qp+Qs)*ra;
a2=reshape(a,NH+1,M+1);
%
%
h=zeros(N+1,M+1);
for im=0:M
    h(NH+1,im+1)=a2(1,im+1);
    h(1:NH,im+1)=0.5*a2(NH+1:-1:2,im+1);
    h(NH+2:N+1,im+1)=0.5*a2(2:NH+1,im+1);
end
%
%
MR=zeros(sampling_w+1,sampling_p+1);
for ip=0:sampling_p
    p=-0.5+ip*deltap;
    h1=h(:,1);
    for im=1:M
        h1=h1+p^im*h(:,im+1);
    end
    MR(:,ip+1)=abs(freqz(h1,1,0:deltaw:pi));
end
XX=zeros(sampling_w+1,sampling_p+1);
YY=zeros(sampling_w+1,sampling_p+1);
for nw=0:sampling_w
    w=nw*deltaw;
    XX(nw+1,:)=(w/pi)*ones(1,sampling_p+1);
end
for np=0:sampling_p
    p=-0.5+np*deltap;
    YY(:,np+1)=p*ones(sampling_w+1,1);
end
close all;
plot3(XX,YY,MR);
axis([0,1,-0.5,0.5,0,1.1]);
xlabel('Normalized frequency (\omega/\pi)');
ylabel('Variable p');
zlabel('Magnitude response');
```

執行結果：