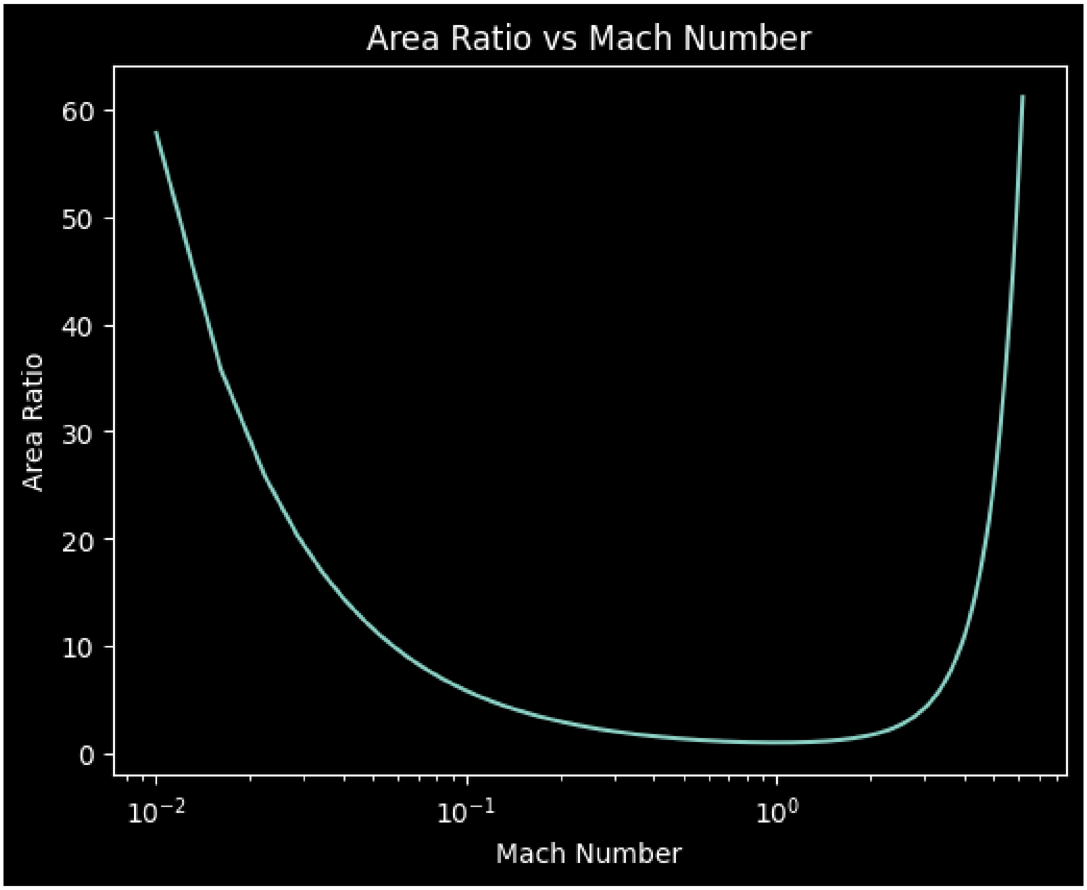
HW11

April 22, 2024

[1]:
```
%%capture
import __init__ as CFC

import math
import numpy as np
import matplotlib.pyplot as plt

import math
from scipy.optimize import root_scalar, root
```

0.1 11.1

[2]:
```
machs = np.linspace(0.01, 6.2, 1000)

ars = [CFC.ar_explicit(1.4, mach) for mach in machs]

plt.plot(machs, ars)
plt.xscale('log')
plt.title('Area Ratio vs Mach Number')
plt.xlabel('Mach Number')
plt.ylabel('Area Ratio')
plt.show()
```



0.2 11.2

0.2.1 11.2 a)

- $\dot{m}$ is constant by mass conservation
- γ & R are constants for a perfect gas
- and T_0 is constant for adiabatic flow, such as across a normal shock

Therefore, $p_0 A^* =$ constant

0.2.2 11.2 b)

[3]:
```
gamma = 1.4
exit_to_throat_area_ratio = 6.5

ar = CFC.AreaRatio(gamma, area_ratio=exit_to_throat_area_ratio,
    super_sonic=False)
ar
```

[3]:
```
Mach: 0.089460
Area Ratio:  6.500000
```

[4]:
```
isen = CFC.IsentropicRatio(gamma, mach=ar.mach)
isen
```

[4]:
```
Pressure Ratio: 1.00561
Temp Ratio: 1.00160
Density Ratio: 1.00401
```

[9]:
```
print(f"Exit Mach: {ar.mach:.5f}")
print(f"P_pe / P_0: {1 / isen.pressure_ratio:.5f}")
print(f"T_te / T_0: {1 / isen.temp_ratio:.5f}")
```

```
Exit Mach: 0.08946
P_pe / P_0: 0.99442
T_te / T_0: 0.99840
```

0.2.3 11.2 c)

[6]:
```
exit_over_inlet = 0.3

ar_super = CFC.AreaRatio(gamma, area_ratio=exit_to_throat_area_ratio,
 ↪super_sonic=True)
ar_super
```

[6]:
```
Mach: 3.453285
Area Ratio:  6.500000
```

[10]:
```
exit_shock = CFC.NormalShockRatio(gamma, mach=ar_super.mach)
exit_shock
```

[10]:
```
Pressure Ratio: 13.74604
Temp Ratio: 3.25158
Density Ratio: 4.22749
Stag Ratio: 0.22174
Exit Stag Ratio: 15.82403
Mach 2: 0.45300
```

[15]:
```
# PE / P1 / (P01 / P1) = PE / P01
exit_shock_press_ratio = exit_shock.pressure_ratio / CFC.IsentropicRatio(gamma,
 ↪mach=ar_super.mach).pressure_ratio
exit_shock_press_ratio
```

[15]:
```
0.1926222563600863
```

[17]:
```
print(f"Exit Mach: {exit_shock.mach_two:.5f}")
print(f"P_pe / P_0: {exit_shock_press_ratio:.5f}")
```

```
Exit Mach: 0.45300
P_pe / P_0: 0.19262
```

[20]:
```python
def between(bounds, val):
    small = min(bounds)
    large = max(bounds)

    return small <= val <= large

assert(between((1 / isen.pressure_ratio, exit_shock_press_ratio),␣
 ↪exit_over_inlet))
```

0.2.4 11.2 d)

$P_{0,1}A_t = P_{0,2}A_2^*$

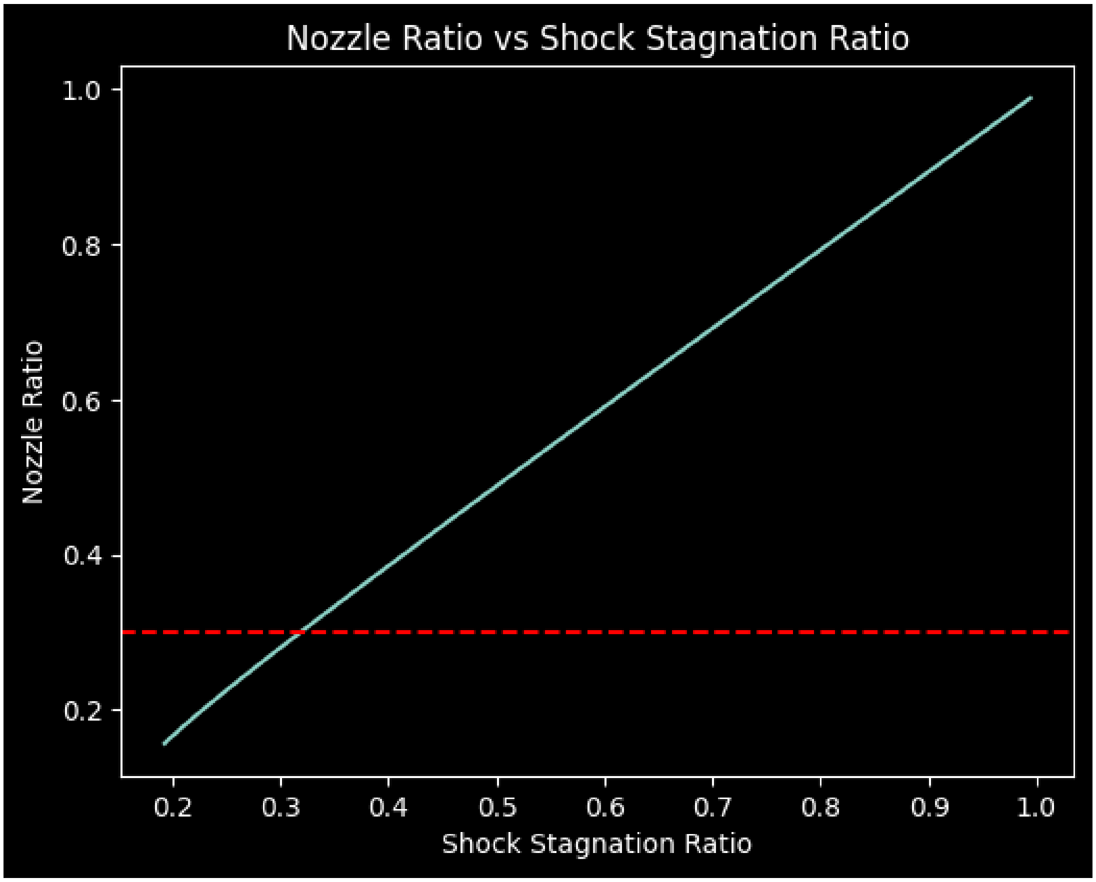
$\frac{A_t}{A_2^*} = \frac{P_{0,2}}{P_{0,1}}$

$\frac{A_e}{A_2^*} = \frac{A_e}{A_t}\frac{A_t}{A_2^*} = \frac{A_e}{A_t}\frac{P_{0,2}}{P_{0,1}}$

$\frac{A_e}{A_2^*} = A_{ratio}(\gamma, M_3)$

0.2.5 11.2 e)

[45]:
```python
def nozzle_ratio(exit_over_throat, shock_stag_ratio):
    exit_over_two_star = exit_over_throat * shock_stag_ratio
    mach_three = CFC.AreaRatio(gamma, area_ratio=exit_over_two_star,␣
 ↪super_sonic=False).mach

    exit_over_post_shock_stag = 1 / CFC.IsentropicRatio(gamma, mach=mach_three).
 ↪pressure_ratio

    return exit_over_post_shock_stag * shock_stag_ratio

ratios = np.linspace(1 / isen.pressure_ratio, exit_shock_press_ratio, 1000)
nozzle_ratios = [nozzle_ratio(exit_to_throat_area_ratio, ratio) for ratio in␣
 ↪ratios]

plt.plot(ratios, nozzle_ratios)
plt.axhline(y=exit_over_inlet, color='r', linestyle='--')
plt.title('Nozzle Ratio vs Shock Stagnation Ratio')
plt.xlabel('Shock Stagnation Ratio')
plt.ylabel('Nozzle Ratio')
plt.show()

shock_stag_ratio = root_scalar( lambda x:␣
 ↪nozzle_ratio(exit_to_throat_area_ratio, x) - exit_over_inlet,
                                bracket=(1 / isen.pressure_ratio,␣
 ↪exit_shock_press_ratio)).root
print(f"Shock Stagnation Ratio: {shock_stag_ratio:.5f}")
```




```
Shock Stagnation Ratio: 0.31858
```

0.2.6 11.2 f)

[46]:
```
mach_one = root_scalar(lambda mach: CFC.NormalShockRatio(gamma, mach=mach).
    ↪stag_ratio - shock_stag_ratio, bracket=(1, 10)).root
print(f"Pre-Shock Mach: {mach_one:.5f}")
```

```
Pre-Shock Mach: 3.03507
```

0.2.7 11.2 g)

[47]:
```
shock_area_ratio = CFC.AreaRatio(gamma, mach=mach_one).area_ratio
print(f"Shock Area Ratio: {shock_area_ratio:.5f}")
```

```
Shock Area Ratio: 4.37834
```