HW12

April 30, 2024

[1]:
```
from scipy.optimize import root_scalar
import numpy as np, matplotlib.pyplot as plt
import math

import __init__ as CFC
```

1 12.1

[2]:
```
press_coeff_min = -0.41
```

1.1 12.1 a)

[3]:
```
mach_infin = 0.35
```

1.1.1 12.1 a) i)

[4]:
```
def pg_coeff(coeff, mach):
  beta = (1 - mach**2)**0.5
  return coeff / beta

cp_pg = pg_coeff(press_coeff_min, mach_infin)
print(f"P.G. Coefficient: {cp_pg:.5f}")
```

```
P.G. Coefficient: -0.43768
```

1.1.2 12.1 a) ii)

[5]:
```
def kt_coeff(coeff, mach):
  beta = (1 - mach**2)**0.5

  return coeff / (beta + (mach**2 * coeff / (2 * (1 + beta))))

cp_kt = kt_coeff(press_coeff_min, mach_infin)
print(f"K.T. Coefficient: {cp_kt:.5f}")
```

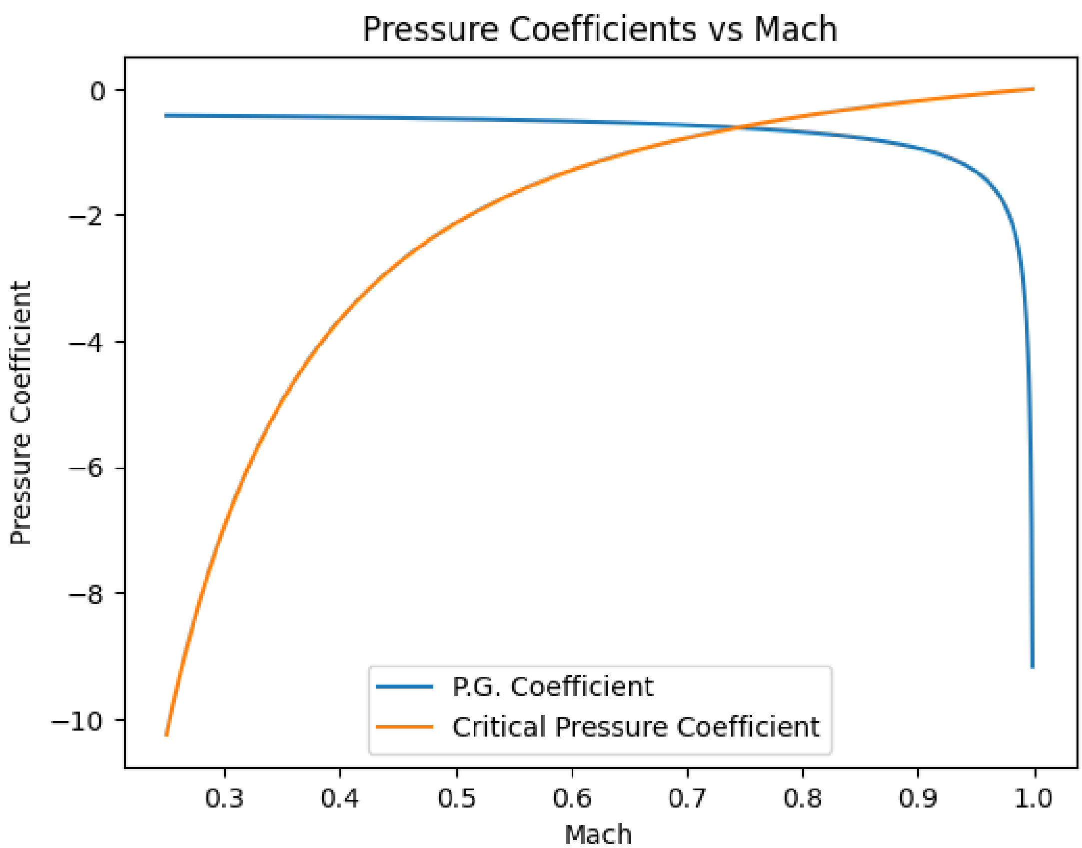
```
K.T. Coefficient: -0.44383
```

1.2 12.1 b)

[6]:
```
def critical_press_coeff(mach, gamma=1.4):
  coeff = 2 / (gamma * mach**2)
  arg = (2 + (gamma - 1) * mach**2) / (gamma + 1)

  coeff *= (arg**(gamma / (gamma - 1)) - 1)

  return coeff

cp_crit = critical_press_coeff(mach_infin)
print(f"Critical Pressure Coefficient @ M_infin: {cp_crit:.5f}")
```

```
Critical Pressure Coefficient @ M_infin: -4.95643
```

[7]:
```
machs = np.linspace(2.5e-1, 1 - 1e-3, 1000)

correcteds = pg_coeff(press_coeff_min, machs)
critical_presses = critical_press_coeff(machs)

plt.plot(machs, correcteds, label="P.G. Coefficient")
plt.plot(machs, critical_presses, label="Critical Pressure Coefficient")
plt.title("Pressure Coefficients vs Mach")
plt.xlabel("Mach")
plt.ylabel("Pressure Coefficient")
plt.legend()
plt.show()
```



[8]:
```
def implicit(mach):
  return critical_press_coeff(mach) - pg_coeff(press_coeff_min, mach)

critical_mach = root_scalar(implicit, bracket=[0.25, 1 - 1e-3]).root
print(f"Critical Mach: {critical_mach:.5f}")
```

```
Critical Mach: 0.74363
```

2 12.2

[9]:
```
foil_mach = 2.4
alpha = math.radians(7)
theta = math.radians(10)

def local_press_coeff(mach, relative_angle):
  beta = (mach**2 - 1)**0.5

  return 2 * relative_angle / beta
```

[10]:
```
lift_sum = 0
drag_sum = 0

for fore_aft in [1, -1]:
  for upper_lower in [1, -1]:
    # front upper is + theta, back lower is + theta, otherwise - theta
    relative_angle = -alpha + fore_aft * upper_lower * theta

    lift_comp = math.cos(relative_angle)
    drag_comp = -math.sin(relative_angle)

    local_press_coefficient = local_press_coeff(foil_mach, relative_angle)
    print("Local Pressure Coefficient ({}, {}): {:.5f}".format( "fore" if␣
 ↪fore_aft == 1 else "aft",
                                                              "upper" if␣
 ↪upper_lower == 1 else "lower",
                                                             ␣
 ↪local_press_coefficient))

    lift_sum += lift_comp * local_press_coefficient * -upper_lower
    drag_sum += drag_comp * local_press_coefficient * fore_aft

print(f"Lift: {lift_sum:.5f}")
print(f"Drag: {drag_sum:.5f}")
```

```
Local Pressure Coefficient (fore, upper): 0.04800
Local Pressure Coefficient (fore, lower): -0.27199
Local Pressure Coefficient (aft, upper): -0.27199
Local Pressure Coefficient (aft, lower): 0.04800
Lift: 0.00000
Drag: -0.00000
```

Given the symmetry of the pressure coefficients, the lift and drag forces cancel for this airfoil.

3 12.3

[11]:
```
drag_machs = np.linspace(1 + 1e-6, 5, 1000)

def drag_coeff(mach, alpha):
  beta = (mach**2 - 1)**0.5
  return 4 * alpha**2 / beta

rho_inf = 1.293
T_inf = 273.15 + 15

R = 287.05
```

```
def drag_per_unit_area(mach, alpha, gamma=1.4):
  T = T_inf
  a = (gamma * R * T)**0.5

  return drag_coeff(mach, alpha) * 0.5 * rho_inf * (mach * a)**2

drags = drag_per_unit_area(drag_machs, math.radians(7))

plt.plot(drag_machs, drags)
plt.title("Drag Force Per Unit Area vs Mach")
plt.xlabel("Mach")
plt.ylabel("Drag Force Per Unit Area")
plt.xscale("log")
plt.yscale("log")
plt.show()
```