HW10

April 15, 2024

[1]:
```
%%capture
import __init__ as CFC

import math
import numpy as np
import matplotlib.pyplot as plt

import math
from scipy.optimize import root_scalar, root
```

1 Problem Values

[2]:
```
gamma = 1.4

press_one = 56e3
temp_one = 260
mach_one = 4.0

theta_one = math.radians(8)
theta_two = math.radians(13)
```

1.1 First Oblique Shock (a)

[3]:
```
one_to_two = CFC.ObliqueShock(gamma, mach=mach_one, theta=theta_one)
one_to_two.tbm
```

[3]:
```
Theta: 0.139626
Beta:  0.357299
Mach:  4.000000
```

[4]:
```
press_two = press_one * one_to_two.shock_ratio.pressure_ratio
temp_two = temp_one * one_to_two.shock_ratio.temp_ratio
mach_two = one_to_two.mach_two

press_two, temp_two, mach_two
```

[4]:
```
(118533.73296473926, 326.0494342199998, 3.4272331299323535)
```

[5]:
```python
print(f"Shock angle (to Horizontal): {math.degrees(one_to_two.tbm.beta):.
↪4f}\nPressure in region 2: {press_two/1e3:.2f} KPa\nTemperature in region 2:␣
↪{temp_two:.2f} K\nMach number in region 2: {mach_two:.2f}")
```

```
Shock angle (to Horizontal): 20.4717
Pressure in region 2: 118.53 KPa
Temperature in region 2: 326.05 K
Mach number in region 2: 3.43
```

1.2 Second Oblique Shock (b)

[6]:
```python
two_to_three = CFC.ObliqueShock(gamma, mach=mach_two, theta=theta_two -␣
↪theta_one)
two_to_three.tbm
```

[6]:
```
Theta: 0.087266
Beta:  0.358711
Mach:  3.427233
```

[7]:
```python
press_three = press_two * two_to_three.shock_ratio.pressure_ratio
temp_three = temp_two * two_to_three.shock_ratio.temp_ratio
mach_three = two_to_three.mach_two

press_three, temp_three, mach_three
```

[7]:
```
(180441.39299916255, 368.4344244343824, 3.1336021012921016)
```

[8]:
```python
print(f"Shock angle (to Horizontal): {math.degrees(two_to_three.tbm.beta +␣
↪theta_one):.4f}\nPressure in region 3: {press_three/1e3:.2f}␣
↪KPa\nTemperature in region 3: {temp_three:.2f} K\nMach number in region 3:␣
↪{mach_three:.2f}")
```

```
Shock angle (to Horizontal): 28.5526
Pressure in region 3: 180.44 KPa
Temperature in region 3: 368.43 K
Mach number in region 3: 3.13
```

1.3 Wave Type Determination (c)

[9]:
```python
# Beta Values are in Radians in Reference frame of the flow

def potential_press_five(beta):
    one_to_five = CFC.ObliqueShock(gamma, mach=mach_one, beta=beta)
    return press_one * one_to_five.shock_ratio.pressure_ratio, one_to_five.theta

def potential_oblique_press_four(beta):
    # flow is at theta_two relative to horiz
    three_to_four = CFC.ObliqueShock(gamma, mach=mach_three, beta=beta)
```

```python
    return press_three * three_to_four.shock_ratio.pressure_ratio, theta_two -␣
↪three_to_four.theta

def potential_prandtl_press_four(far_side_mach):
    pm = CFC.PrandtlMeyer(gamma, mach_one=mach_three, mach_two=far_side_mach)

    press_ratio = CFC.IsentropicRatio(gamma, mach_three).pressure_ratio / CFC.
↪IsentropicRatio(gamma, far_side_mach).pressure_ratio
    return press_three * press_ratio, theta_two - pm.theta
```

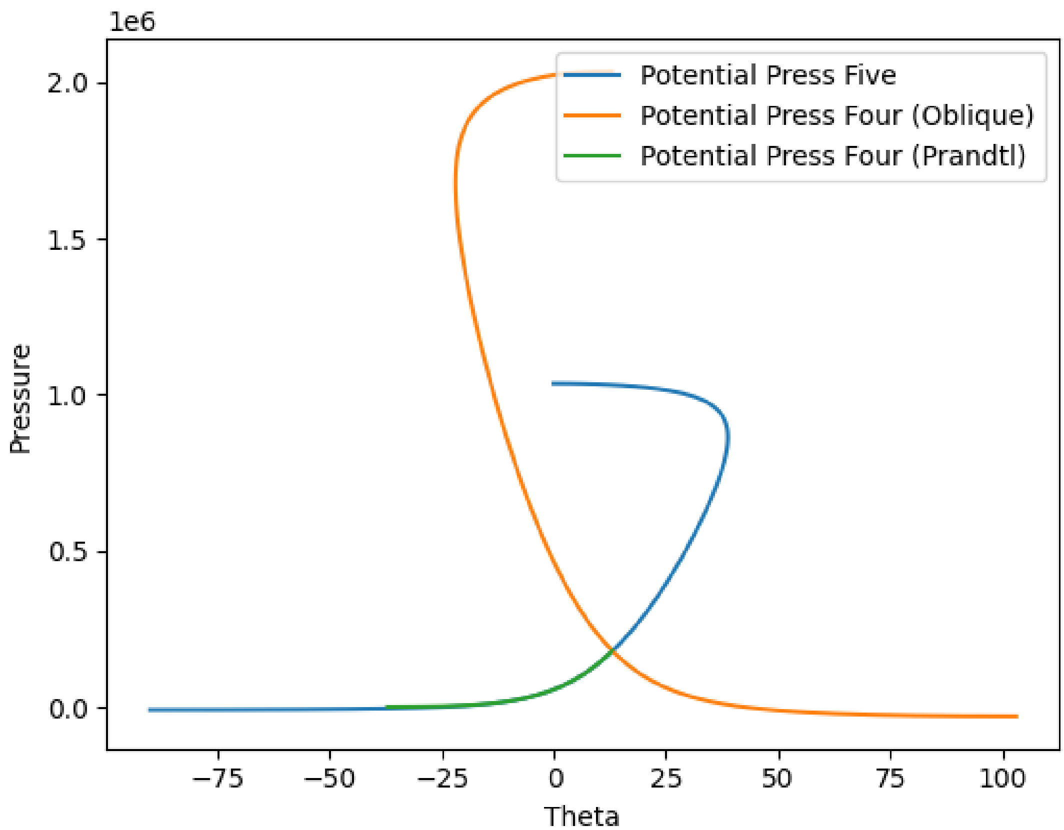
[10]:
```python
betas = np.linspace(1e-6, np.pi/2, 100)

press_fives, theta_fives = zip(*[potential_press_five(beta) for beta in betas])
press_four_obliques, theta_four_obliques =␣
↪zip(*[potential_oblique_press_four(beta) for beta in betas])


mach_fours = np.linspace(mach_three, 10.0, 100)
press_four_prandtls, theta_four_prandtls =␣
↪zip(*[potential_prandtl_press_four(mach) for mach in mach_fours])
```

[11]:
```python
plt.plot([math.degrees(t) for t in theta_fives], press_fives, label='Potential␣
↪Press Five')
plt.plot([math.degrees(t) for t in theta_four_obliques], press_four_obliques,␣
↪label='Potential Press Four (Oblique)')
plt.plot([math.degrees(t) for t in theta_four_prandtls], press_four_prandtls,␣
↪label='Potential Press Four (Prandtl)')

plt.xlabel('Theta')
plt.ylabel('Pressure')
plt.legend()
plt.show()
```



1.3.1 Compute the dual oblique shock intersection point

[12]:
```
def residuals(betas):
    beta_five, beta_four = betas
    press_five, theta_five = potential_press_five(beta_five)
    press_four, theta_four = potential_oblique_press_four(beta_four)

    return [press_five - press_four, theta_five - theta_four]
```

[13]:
```
sol = root(residuals, [0.1, 0.1])
sol.x
```

[13]: array([0.43873901, 0.32388911])

[14]:
```
press_five, theta_five = potential_press_five(sol.x[0])
press_four, theta_four = potential_oblique_press_four(sol.x[1])

print(f"Press 5: {press_five/1e3:.2f} KPa, Theta 5: {math.degrees(theta_five):.
↪4f}")
```

```
print(f"Press 4: {press_four/1e3:.2f} KPa, Theta 4: {math.degrees(theta_four):.
↪4f}")
```

```
Press 5: 179.30 KPa, Theta 5: 13.0784
Press 4: 179.30 KPa, Theta 4: 13.0784
```

1.4 Solve for the conditions in region 4 & 5 (d)

[15]:
```
one_to_five = CFC.ObliqueShock(gamma, mach=mach_one, beta=sol.x[0])
mach_five = one_to_five.mach_two
```

[16]:
```
three_to_four = CFC.ObliqueShock(gamma, mach=mach_three, beta=sol.x[1])
mach_four = three_to_four.mach_two
```

[17]:
```
print(f"Pressure @ 5: {press_five/1e3:.2f} KPa")
print(f"Mach 5: {mach_five:.4f}")
print(f"Mach 4: {mach_four:.4f}")
print(f"Theta 5: {math.degrees(theta_five):.4f}")
```

```
Pressure @ 5: 179.30 KPa
Mach 5: 3.0669
Mach 4: 3.1379
Theta 5: 13.0784
```

1.5 What happens when the wave between 3 & 4 reaches the wedge (e)

I believe there are an infinite cascade of oblique shocks as the flow from region 4 onwards becomes compressed into less and less space between the slipline of 4 & 5 and the wedge.

Probably something like the path of a ball bouncing between the slipline and the wedge