Annals of Operations Research (2024) 339:741–764
https://doi.org/10.1007/s10479-023-05234-4

ORIGINAL RESEARCH

Data-driven optimization models for inventory and financing decisions in online retailing platforms

Bingnan Yang[1] · Xianhao Xu[1] · Yeming Gong[2] · Yacine Rekik[3]

Accepted: 27 January 2023 / Published online: 13 March 2023

Abstract
With data-driven optimization, this study investigates the sellers' inventory replenishment and financial decisions, and lenders' interest rate decisions in online retailing platforms. Moreover, we focus on the annual large-scale promotion, which requires massive capital in a short period. While scholars studying the data-driven inventory replenishment problem hardly consider capital-constrained sellers, these problems are important because the seller's capital level can significantly influence the order quantity and generate different effects on inventory management. Hence, we propose two novel data-driven game-theoretic approaches (including separated and integrated methods) using machine learning and deep learning methods to optimize inventory replenishment and financial decisions for the sellers who obtain financial support from the online platform. Moreover, we propose a data-driven game-theoretic model for the online platform to optimize their interest rate considering the market potential. We explore the real retailing transaction data containing 199,390 weekly sales records. We find that the seller and lender can benefit when the seller chooses integrated machine learning and quantile regression method. However, we find that only a low capital level can motivate the seller to choose to borrow from the lender. Interestingly, our results also suggest that the lender has the motivation to build a data-driven system that helps sellers optimize inventory decisions. Our work identifies the optimal interest rate and inventory decision under the data-driven method. We propose data-driven decision support tools by evaluating the values of both the lender's and the seller's profit and provide new management insights on joint inventory and financing decisions.

✉ Yacine Rekik
yrekik@escp.eu

Bingnan Yang
yangbingnan@hust.edu.cn

Xianhao Xu
xxhao@hust.edu.cn

Yeming Gong
gong@em-lyon.com

1 School of Management, Huazhong University of Science and Technology, Wuhan 430074, China

2 AIM Institute, Emlyon Business School, 23 avenue Guy de Collogue, 69130 Ecully Cedex, France

3 ESCP Business School, 11 av. de la République, 75011 Paris, France

Keywords Data-driven decision making and analytics · Capital constraints · Machine learning · Deep learning · Online retailing

1 Introduction

Our research is partially motivated by the new supply chain financial practices of Amazon, eBay, Alibaba, Jingdong, and other online retailing platforms with data processing capabilities and financial services. For example, in 2011, Amazon launched a program called “Amazon Lending" (see Fig.1): it allows the seller who sells the product on Amazon to obtain the lending funds from Amazon to replenish inventory after offering their sales history, total sales, and other operational data. In this program, Amazon can analyze and mine authorized data to explore the market potential for optimal interest rates. Then sellers optimize their inventory management by optimal ordering quantity with a given interest rate financing support from Amazon. Stephan Aarstol, chief executive of Tower Paddle Boards, an Amazon seller, said he had taken eight loans totaling $1.5 million over three years starting in March 2014 from Amazon Lending. With the help of Amazon Lending, Tower Paddle Boards optimized its inventory replenishment and became one of the fast-growing companies in San Diego, U.S. More than $863 million sellers who sell the product on the Amazon platform have benefited from Amazon Lending at the end of 2019 (see markets.businessinsider.com, 2020). Similarly, in many online retailing platforms, sellers can acquire capital support from the platform’s lender through data authorizing and reviewing, which assists capital-constrained sellers in having additional funds that can be used to optimize inventory management.

Moreover, in recent years, large-scale seasonal promotions have become more prevalent. Annual promotional activities (e.g., Black Friday, Cyber Monday, Double Eleven) have repeatedly created new records in sales. The surging demand in the promotion events of sellers leads to considerable order quantity in the short term. Therefore, the sellers need to optimize the inventory and financial decisions based on highly volatile demand. Correspondingly, the platform needs to optimize its optimal interest rate decisions based on the dynamic market environment. For example, in 2021’s Black Friday shoppers’ store visits rose 48% from 2020 (see bloomberg.com). Walmart.com sales were about $76 million on the 2020 Friday after Thanksgiving, compared with $38 million in 2019, showing a 100% year-over-

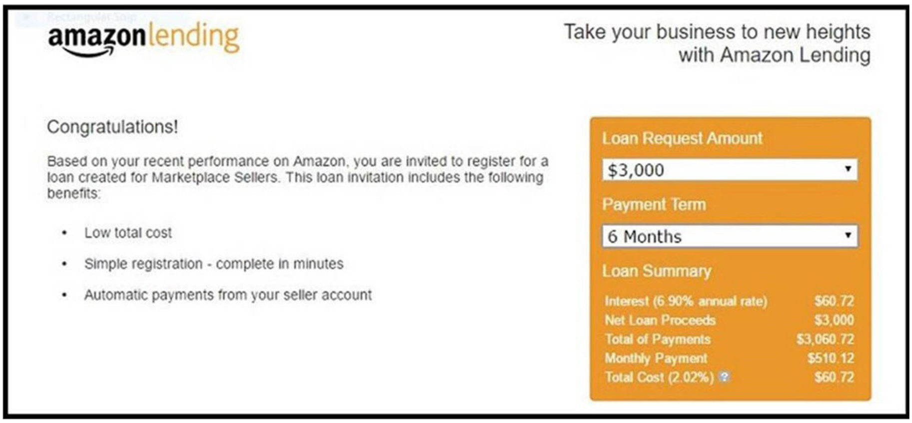


Fig. 1 An example of Amazon lending

year increase for the day's e-commerce sales (see junglescout.com). To cope with the soaring demand brought by the promotion season, sellers have to prepare considerable cash flow to order more products than regular replenishment, creating new challenges for inventory and financial decisions. With the new big data processing technology, the surging demand can be studied more accurately by data than before. Therefore, our paper focuses on optimizing inventory and financial decisions for financial-constrained sellers and also optimizing the interest rate for the platform by the data-driven approach.

The inventory and financing decisions have been investigated from the different financing topics, such as loans from bank credit (Buzacott & Zhang, 2004; Alan & Gaur, 2018; Raghavan & Mishra, 2011; Kouvelis & Zhao, 2016; Lai et al., 2021), trade credit (Lai et al., 2009; Yang, 2013; Gao et al., 2014), and the combination of bank credit and trade credit (Chen, 2015; Yang & Birge, 2018; Jing & Seidmann, 2014; Dong et al., 2021). Game-based methods have been widely used in these research problems. However, many studies assume demand distribution or even consider a specific demand distribution for optimizing the operational decision. Although several approaches are adopted to relax this strong assumption, demand is difficult to be accurately described by the demand distribution function. Oroojlooyjadid et al. (2020) find the resulting solution to the problem may not be optimal, especially during the significant promotion period. Instead, large datasets are available to help decision-makers understand the empirical distributions (Huber et al., 2019) and incorporate demand uncertainty in inventory models without demand distribution assumptions. Although several studies propose the data-driven model to study the demand or operational problems (Huber et al., 2019; Arunraj & Ahrens, 2015; Huber & Stuckenschmidt, 2020; Abbasi et al., 2020), the impact of sellers' transaction history on the inventory and the financing decisions has not been studied yet. Therefore, how to optimize the order quantity of the capital-constrained sellers with the data-driven game theory method is still an open question. Based on these practical motivations and literature gaps, our study answers the following research questions:
Research Question (1): For the capital-constrained seller in the online retailing platform, how to optimize its order quantity and financial decision through data-driven optimization?
Research Question (2): How does the online retailer platform optimize the interest rate using the data-driven method?

To answer these research questions, our research proposes two novel data-driven approaches to optimize order quantity and financial decisions for sellers. Furthermore, we propose a data-driven method for the platform's lender to optimize interest rates. These proposed approaches explore the sellers' real-world transaction behavior data containing 199,390 weekly sales records, including annual large-scale promotions. Moreover, due to the sellers' demand and capital level fluctuation, the online platform is required to adjust its interest rate, and the seller is required to modify the order quantity and financial decision at each promotion period stage. We can provide suggestions and solutions to optimize the interest rate, order quantity, and financial decision with our method for every contract period.

Instead of directly exploring the transaction data via data mining, our paper studies the impact of financial constraints on the efficiency of the supply chain based on data-driven. Our research makes the following contributions:

(1) We propose two novel game-theoretic data-driven methods for the seller that consider the effect of financial constraints on inventory management. The first method separates estimates demand using machine learning and deep learning approaches and optimizes inventory and financing decisions. The second method integrates machine learning and deep learning approaches with quantile regression to optimize inventory and financing decisions.

(2) Our proposed integrating model does not need the information of the demand's probability distributions. Instead, we extract valuable information from transaction behavior data, including historical transaction amount, holiday, promotion, and other information, to optimize sellers' order quantity and financial values. We explore the real dataset rather than the assumption of the demand distribution to understand the decision-making process of seller profit evaluation.
(3) Our experiments demonstrate how to make order quantity and financial decisions directly in the large-scale promotion. We provide a quantitative analysis of the relationship between capital level and sellers' profit.
(4) We propose a novel data-driven method for the platform to optimize its interest rate. Furthermore, our work explains why the platform lenders are willing to provide the capital for the seller. Moreover, we show how to make the financial service operate more reasonably.

2 Literature review

Our study is relevant to two fields of literature: interface of operations and finance and data-driven inventory models.

2.1 Interface of operations and finance

As one of the first studies to consider capital constraints in the newsvendor model, Xu and Birge (2004) reveal how a firm's production decisions are affected by financial constraints. And Buzacott and Zhang (2004) incorporate asset-based financing into production decisions. They demonstrate the decision-making of a bank and retailers considering capital constraints in the newsvendor model. Cheng et al. (2021) investigate impacts of bank financing and trade credit on retailers' inventory decisions and demonstrate the impact of different financing ratios. Dada and Hu (2008) illustrate if the cost of borrowing is not too high, the capital-constrained newsvendor borrows funds to procure an amount that is less than would be ideal. Alan and Gaur (2018) point out that the commonly used method to lend money by banks is asset-based lending (ABL), in which borrowers need to offer their current assets, including their inventory, cash, and account receivables, as collateral for a secured loan.

However, many retailers are unable to make mortgage loans due to the lack of collateral. Hence, trade credit borrowing is a popular research area in the supply chain studied by many scholars (Lai et al., 2009; Yang, 2013; Gao et al., 2014). Moreover, it is the most critical short-term financial channel for companies, proven by theoretical and empirical evidence (Gao et al., 2014). Usually, upstream suppliers will provide deferred payments to retailers until the end of the sales season and after the revenue is received (Lai et al., 2009; Yang & Birge, 2018). As an extension, some studies focus on the operation considering credit guarantee, Yan et al. (2016) perform a comparative analysis of the optimal strategies among the various financing scenarios to examine the impact of the credit guarantee coefficient.

Moreover, some studies notice the gains brought about by the demand-related information, Yan and Wang (2014) propose an uncertain demand newsvendor model with capital constraint and demand forecast update, and the forecast can be improved by market signal. Zhao et al. (2017) study the capital constraint newsvendor model that is allowed to make an emergency order when gaining the demand information. Zheng et al. (2015) analyses a two-stage newsvendor system with a regular and emergency order. However, the emergency order costs more than the regular order, and the quantity is limited (Zheng et al., 2015). Unlike ordering emergency orders through more accurate demand information closer to the selling season, we explore the demand information directly from the data. Hence, we study the relationship between order quantities and finance decisions using the newsvendor model with capital constraints and innovation to optimize them with big data analytics.

2.2 Data-driven inventory models

After uncovering the usefulness of data, many studies begin to use data to study operational problems.

Huber et al. (2019) optimize order quantity from historical demand data and other feature data. They present a method that compares the optimized inventory decision based on the demand forecast and the historical forecast errors and integrates the forecasting model into the optimization problem. They also find that machine learning approaches substantially outperform traditional methods. Oroojlooyjadid et al. (2020) and Zhang and Gao (2017) propose the algorithm based on deep learning that optimizes the order quantities based on features of the demand data. Cao and Shen (2019) innovate a data-driven approach to determine stock levels via exploring the time-correlated or even nonstationary demand data. Moreover, Ban and Rudin (2019) propose algorithms based on the empirical risk minimization (ERM) principle for the data-driven newsvendor problem. Saghafian and Tomlin (2016) present a maximum-entropy-based technique, allowing the manager to effectively combine demand observations with distributional information in the form of bounds on the moments or tails for the repeated newsvendor problem.

Moreover, Oroojlooyjadid et al. (2020) summarize five main approaches in the literature. The first category is called the Estimate-As-Solution (EAS) approach. It forecasts the demand and then simply uses it as the order quantity. The second approach is called Separated Estimation and Optimization (SEO). It first estimates (forecasting) the demand distribution and then applies the estimation to an optimization problem. The third approach is called Empirical Quantile (EQ) method. It involves sorting the demand observations in ascending order and then estimating the quantile of the demand distribution. Demands are sorted in each Cluster, with the solution set equal to the quantile of the resulting implied demand distribution. A fourth approach applies several machine learning methods to a general optimization problem. It assigns weights for machine learning methods. The fifth approach is called Linear Machine Learning (LML) method. It postulates that the optimal order quantity is related to the demand features via a linear function that is closest to our proposed integrated approach for sellers.

However, the existing research mainly focuses on data-driven operations but neglects financial problems that the sellers may encounter. Many businesses are constrained by capital, especially during significant promotions. Different from the studies above, we investigate the benefits of transaction data and analyze them from a capital-constrained perspective and a game theory approach.

Table 1 Summary of notations

Notation	Description
p	Unit retail price
w	Unit wholesale price
s	Unit salvage value
B	Seller's initial capital level
r	Rate of interest
D	Market demand of seller
Q_N	Order quantity of the seller in the Not-borrowing scenario
Q_B	Order quantity of the seller in the Borrowing scenario

3 Models

3.1 Problem description

We consider a supply chain consisting of a lender from the online retailing platform (referred to as "he"), and one capital-constrained seller (referred to as "she") who sells the product on the online retailing platform. At the beginning of a new promotion selling season, the seller with limited initial capital B must choose the number of products to order. If the initial capital is insufficient, the seller can obtain financing support from the platform's lender after offering her transaction data to the lender. The lender decides its interest rate according to the seller's previous sales record by data-driven. Then, before the selling season, the seller starts to order products from the supplier at unit wholesale price w. During the selling period, the seller sells the products at unit price p, and the unsold products own unit salvage value s. Before presenting the optimization model, we summarize the notations in Table 1 for reference.

In the following sections, we present methods that use the data on both optimizations for the seller and the lender. In our methods, the lender and the seller can apply the data to reduce the risk of demand uncertainty and increase profits. The data used by the seller for optimizing order quantities and the lender for optimizing interest rates contain annual promotion transaction data and daily transaction data. We simultaneously optimize both seller and lender models with the same data.

3.2 Optimization for the seller

The seller has to determine optimal ordering quantities in every annual promotion season. Hence we propose methods that repeatedly solve the problem of optimizing ordering quantities and financial values for each annual promotion which are shown in Fig.2.

We first propose the separated optimal methods for the seller as shown in Fig.2a. This method separates demand estimation and ordering optimization. It assumes that the demand distribution (normal, exponential,...) is known and the data-driven approach permits to predict the demands moments (mean and variance). The predicted moments and the assumed demand distribution are then plugged into the optimisation problem. However, demand distribution and the associated expression of the cumulative demand distribution $F(\cdot)$ are unknown to the decision-maker in most real-world cases. Hence, we further propose an innovative integrated quantile regression with the optimization inventory decision model shown in Fig.2b. This integrated method does not require an assumption on the demand distribution. We rather consider in this integrated method that the seller's prediction of the demand is $q(\cdot)$, which can be acquired by data-driven methods from the historical data. Historical data $H = (d_{1,1}, X_{1,1,1}), ..., (d_{k,i}, X_{k,i,j}), ..., (d_{K,m}, X_{K,m,n})$ (k is the period of the data; i is the number of the records and feature denoted by index j) are collected from the seller, where $d_{k,i}$ is the historical demand record and $X_{k,i,j}$ is the vector of features (e.g. promotion, holiday, CPI, Unemployment rate).

When taking the decision on the replenishment, the seller faces three options in both separated and integrated methods: the seller could either (1) in case of shortage in the initial capital, introduce external capital from the lender for ordering (order quantity Q_B in this case) based on the given interest rate decided by the lender, or (2) order products using her initial capital (order quantity Q_N in this case) if the initial capital covers the total purchasing cost, or (3) use the whole initial capital to buy what is possible to buy (order quantity Q_C in this case).

3.2.1 Separated estimation and optimization methods

The separated optimization first predicts the demand moments using a data-driven approach, then assumes a distribution for the demand and finally involves the assumed demand distribution with the predicted moments into the inventory optimisation problem. This approach permits decision-makers to derive the inventory decisions: the optimal order quantities Q_B and Q_N of the capital-constrained newsvendor model.

Before applying this approach, we formulate the two inventory optimization models under the borrowing and not-borrowing scenarios with the train data and assess them with the test data.

We assume in this section that the seller determines her order quantity according to the demand distribution assumption and the moments prediction for a given interest rate fixed by the lender. She can choose to borrow capital from the lender to replenish the inventory or rely on her initial capital if the latter is enough to cover the purchasing costs.

Our proposed methods build the demand function $F(\cdot)$ (and the demand density function $f(\cdot)$) based on forecasts from historical sales data by assuming a given demand distribution for the demand. It first estimates the demand moments by forecasting the annual promotion demand through historical sales data and then plugs the estimate into the borrowing and not-borrowing optimization problems.

(1) Not-borrowing scenario

We first build the inventory replenishment problem for the not-borrowing scenario. The seller in this scenario prepares her inventory for upcoming annual promotions assuming that her initial capital is enough to cover the total purchasing cost. The objective of the model is to maximize the total expected profit.

$$\pi_{S_N} = p\min(D, Q_N) + s(Q_N - D)^+ - wQ_N \tag{1}$$

where $p\min(D, Q_N)$ is the sales revenue, $s(Q_N - D)^+$ is the salvage value of the surplus inventory, wQ_N is the purchase cost. Q_N should satisfy $Q_N \leq \frac{B}{w}$; otherwise, the seller can't order all products with the initial capital. The optimal decision is characterized in Proposition 1.

Proposition 1 *If the seller has sufficient capital, i.e., $Q_N^* \leq \frac{B}{w}$, then the seller's best response of optimal order quantity is $Q_N^* = F^{-1}(\theta, \frac{p-w}{p-s})$, where θ (e.g., mean and standard deviation)*

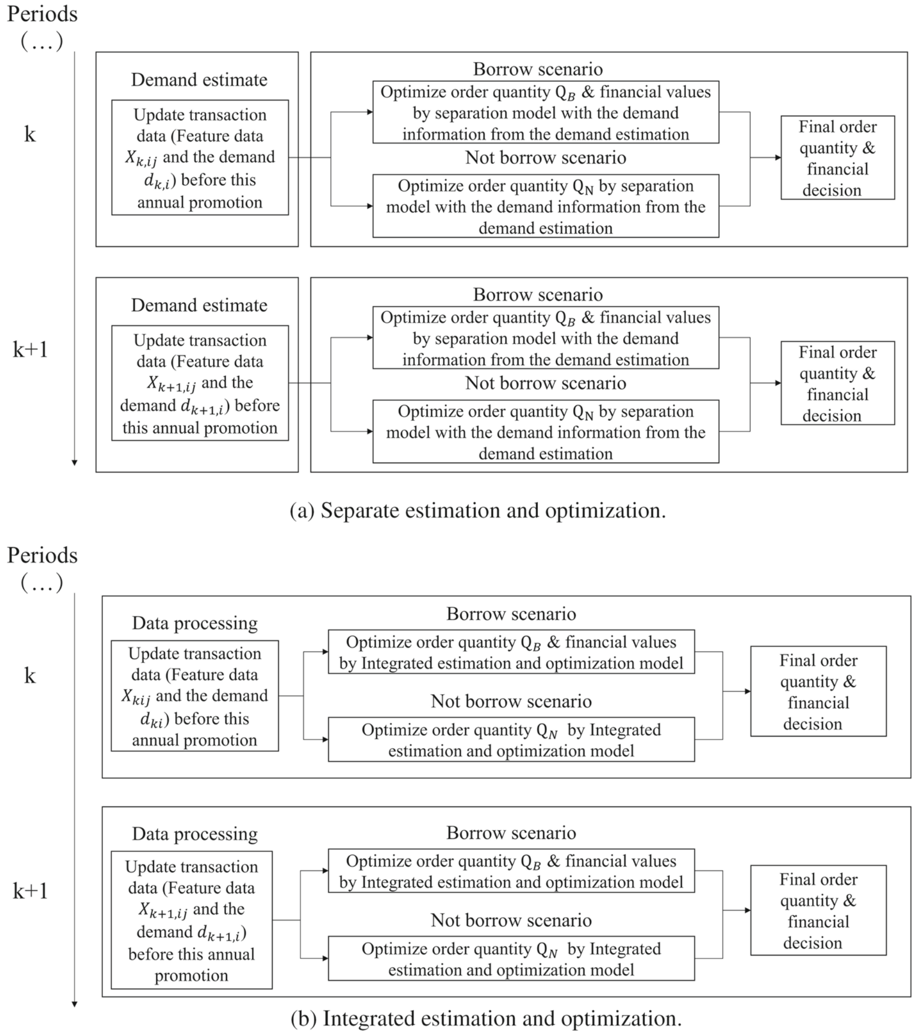


Fig. 2 Illustration of the methods framework for seller

are estimated based on historical transaction forecast. Then, the seller's expected profit in such a not-borrowing scenario is $\pi^*_{S_N} = (p-s)\int_0^{Q^*_N} xf(x)dx$. *Otherwise, the seller cannot order the optimal products with her initial capital.*

(2) Borrowing scenario

In case of a shortage in the initial capital, the seller decides to order the product by introducing external capital from the online retailing platform. Moreover, the seller needs to pay the interest fee to the lender. In such a case, the profit of the seller is as follows:

$$\pi_{S_B} = p\min(D, Q_B) + s(Q_B - D)^+ - wQ_B - r(wQ_B - B)^+ \tag{2}$$

where $p\min(D, Q_B)$ is the sales revenue, $s(Q_B - D)^+$ is the salvage value of the surplus inventory, wQ_B is the cost of replenishing inventory, and $r(wQ_B - B)^+$ is the interest of capital borrowing from the lender. Q_B should satisfy $Q_B \geq \frac{B}{w}$; otherwise, the seller does not need extra capital. The optimal decisions (ordered quantity and financing value) are characterized in Proposition 2.

Proposition 2 *If the seller does not have sufficient capital, i.e., $Q_B^* \geq \frac{B}{w}$, then the seller needs to borrow capital from the lender, and his best response of optimal operational decisions are $(Q_B^*, financing\ value) = (F^{-1}(\theta, \frac{p-w(1+r)}{p-s}), w * F^{-1}(\theta, \frac{p-w(1+r)}{p-s}) - B)$, where θ (e.g., mean and standard deviation) are estimated based on historical transaction forecast. Then, the seller's expected profit under this borrowing scenario is $\pi_{S_B}^* = (p-s)\int_0^{Q_B^*} xf(x)dx - rB$. Otherwise, the seller orders the optimal products with her initial capital.*

Between the case where the capital is enough (presented in Proposition 1) and the case with a need to borrow (presented in Proposition 2), there is a third scenario for the seller: use the available capital B to order what is possible to order without any overage and underage trade-offs. That is, the ordering quantity, in this case, is $Q_C^* = \frac{B}{w}$ and it corresponds to the case where the seller uses all her whole capital to purchase the products.

3.2.2 Integrated estimation and optimization with quantile regression methods

In this section, instead of estimating the demand distribution and optimizing order quantities, our methods directly optimize the order quantity by integrating the forecasting model into the optimization problem by trying diverse machine learning and deep learning models. After incorporating machine learning and deep learning models, our integrated methods allow for non-linear relationships, take more advantage of the transaction data, and decide order quantities by maximizing the profit function.

(1) Not-borrowing scenario

The model maximizing the profit of the seller under the not-borrowing scenario can be rewritten from Eq. (1) as follows:

$$\max \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n} -(p-w)(d_{ki} - q_k(\beta_j^N, X_{kij}))^+ - (w-s)(q_k(\beta_j^N, X_{kij}) - d_{ki})^+ \quad (3)$$

where $q_k(\beta_j^N, X_{kij})$ (k is the period of the data, i is the number of the records and j is the features of the data) is the output of the machine learning and deep learning methods, with input variables X_{kij} and the coefficient of feature β_j^N, we incorporate machine learning and deep learning models, which allows for non-linear relationships. The mathematical formulation of the not-borrowing method can be presented as follows:

$$\min_{\beta^N} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{(p-w)}{(p-s)}(d_{ki} - q_k(\beta_j^N, X_{kij}))^+ + \frac{(w-s)}{(p-s)}(q_k(\beta_j^N, X_{kij}) - d_{ki})^+ \quad (4)$$

$$\text{subject to:} \quad u_i^1 \geq d_{ki} - q_k(\beta_j^N, X_{kij}), \quad (5)$$

$$o_i^1 \geq q_k(\beta_j^N, X_{kij}) - d_{ki}, \quad (6)$$

$$u_i^1, o_i^1 \geq 0; i = 1, \ldots, m; j = 1, \ldots, n. \quad (7)$$

The objective function (4) minimizes the empirical underage and overage costs, while the constraints (5)-(7) ensure that deviations of the estimate from the actual demand are correctly assigned to underages and overages. By solving the problem for the empirical data $H = (d_{1,1}, X_{1,1,1}), ..., (d_{k,i}, X_{k,i,j}), ..., (d_{K,m}, X_{K,m,n})$, we obtain parameters β_j^{N*} for the machine learning and deep learning methods that maximum the empirical profits with respect to these data. Once the model has been trained, the resulting order quantities for period k is the quantile forecast with $q_k(\beta_j^{N*}, X_{kij})$.

Proposition 3 *If the seller has sufficient capital, i.e., $\sum_{i=1}^m q_k(\beta_j^{N*}, X_{kij}) \leq \frac{B}{w}$, then the seller's best response of optimal order quantity is $Q_N^* = \sum_{i=1}^m q_k(\beta_j^{N*}, X_{kij})$. Then, the seller's expected profit without borrowing will be $\pi_{S_N}^* = p \min(\sum_{i=1}^m d_{ki}, \sum_{i=1}^m q_k(\beta_j^{N*}, X_{kij})) + s(\sum_{i=1}^m q_k(\beta_j^{N*}, X_{kij}) - \sum_{i=1}^m d_{ki})^+ - w \sum_{i=1}^m q_k(\beta_j^{N*}, X_{kij})$. Otherwise, the seller orders the optimal products with her initial capital.*

(2) Borrowing scenario

To maximize the profit of the seller under the borrowing model, we rewrite the model considering the demand estimate by the machine learning and deep learning as follows:

$$\max \frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} -(p - w(1+r))(d_{ki} - q_k(\beta_j^B, X_{kij}))^+ \\ -(w(1+r) - s)(q_k(\beta_j^B, X_{kij}) - d_{ki})^+ \tag{8}$$

The mathematical formulation of the borrowing order can be presented as follows:

$$\min_{\beta^B} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_u(d_{ki} - q_k(\beta_j^B, X_{kij}))^+ + c_o(q_k(\beta_j^B, X_{kij}) - d_{ki})^+ \tag{9}$$

$$\text{subject to:} \quad u_i^2 \geq d_{ki} - q_k(\beta_j^B, X_{kij}), \tag{10}$$

$$o_i^2 \geq q_k(\beta_j^B, X_{kij}) - d_{ki}, \tag{11}$$

$$u_i^2, o_i^2 \geq 0; i = 1, \ldots, m; j = 1, \ldots, n. \tag{12}$$

where $c_u = \frac{(p-w(1+r))}{(p-s)}$ and $c_o = \frac{(w(1+r)-s)}{(p-s)}$. The objective function (9) minimizes the empirical underage and overage costs, while the constraints (10)-(12) ensure that deviations of the estimate from the actual demand are correctly assigned to underages and overages. By solving the problem for the empirical data $H = (d_{1,1}, X_{1,1,1}), ..., (d_{k,i}, X_{k,i,j}), ..., (d_{K,m}, X_{K,m,n})$, we obtain parameters β_j^{B*} for the machine learning and deep learning methods that maximum the empirical profits with respect to these data. Once the model has been trained, the resulting order quantities for period k is the quantile forecast with $q_k(\beta_j^{B*}, X_{kij})$.

Proposition 4 *If the seller has sufficient capital, i.e., $\sum_{i=1}^m q_k(\beta_j^{B*}, X_{kij}) \geq \frac{B}{w}$, then the seller's best response of optimal order quantity is $Q_B^* = \sum_{i=1}^m q_k(\beta_j^{B*}, X_{kij})$. Then, the seller's expected profit without borrowing will be $\pi_{S_B}^* = p \min(\sum_{i=1}^m d_{ki}, \sum_{i=1}^m q_k(\beta_j^{B*}, X_{kij})) + s(\sum_{i=1}^m q_k(\beta_j^{B*}, X_{kij}) - \sum_{i=1}^m d_{ki})^+ - w \sum_{i=1}^m q_k(\beta_j^{B*}, X_{kij}) - r(w \sum_{i=1}^m q_k(\beta_j^{B*}, X_{kij}) - B)$. Otherwise, the seller cannot order the optimal products with her initial capital.*

However, these two optimal results are to be contrasted with the case the seller uses the whole capital to order what is possible to order with our underage and overage trade-offs, that is $Q_C^* = \frac{B}{w}$.

3.3 Problem optimization of the lender

In this section, instead of assuming a fixed interest rate, we introduce the lender as a new player of the game who is deciding the optimal interest rate based on the seller's transaction data for each annual promotion. We assume that the lender from the online retailing platform serves as a Stackelberg leader who determines the optimal interest rate based on the study of Tao et al. (2022).

In order to decide on the interest rate offered to the seller, we assume that the lender is given the data authorized from the seller, which helps him to analyze the potential of the seller's market. We assume that the lender, as a Stackelberg leader, is able to anticipate the reaction of the seller to his proposed interest rate (i.e., the lender is able to deduce the Q_B^* of the seller for a given interest rate r as presented in Proposition 2). Hence, the lender has access to the past transaction data of the seller to understand better the market potential and the seller's transaction situation. However, because of the potential business competitive advantages gained by analyzing interest rate information, online platforms hesitate to share their interest rate operation strategies, making it difficult for researchers and practitioners to build an integrated optimization as we did for the seller. We, therefore, use a data-driven approach to optimize the problem for the lender with the separated estimation and optimisation method. That is, the lender uses a data-driven approach to estimate the demand moments, then plugs these estimations in an optimisation framework enabling him to maximize his own expected profit.

We first note that the seller should avoid borrowing the capital from the lender while she is holding the cash because of the interest levied. Therefore, for the lender, we consider the profit as follows:

$$\pi_L = r(wQ_l - B)^+ \tag{13}$$

where Q_l is the order quantity of the seller. In the following proposition, we derive the optimal interest rate of the lender.

Proposition 5 *If the seller does not have sufficient capital, i.e., $Q_l^* \geq \frac{B}{w}$, then the lender would push the seller to order the optimal quantity maximising his own expected profit. If the demand has an Increasing General Failure Rate (IGFR), this optimal ordering quantity satisfies $F(\theta, Q_l^*) + Q_l^* f(\theta, Q_l^*) - \frac{B}{w} f(\theta, Q_l^*) = \frac{p-w}{p-s}$. Then, the lender's optimal interest rate proposal for the seller is $r^* = \frac{p-w}{w} - \frac{p-s}{w} F(\theta, Q_l)$*

Proof From the proposition 2, we find out that the reaction of the seller to a given interest rate is $F^{-1}(\theta, Q_l) = \frac{p-w(1+r)}{p-s}$. Then, we can derive the interest rate $r = \frac{p-w}{w} - \frac{p-s}{w} F(\theta, Q_l)$. The expected profit of the lender $\pi_l(Q_l) = r(wQ_l - B)$ can then be written as $\pi_l(Q_l) = (\frac{p-w}{w} - \frac{p-s}{w} F(\theta, Q_l))(wQ_l - B)$. The first and second derivatives of the lender's profit could be calculated as $\frac{\partial \pi_l(Q_l)}{\partial Q_l} = (p-w) - (p-s)F(\theta, Q_l) - (p-s)Q_l f(\theta, Q_l) + B\frac{p-s}{w} f(\theta, Q_l)$ and $\frac{\partial^2 \pi_l(Q_l)}{\partial Q_l^2} = -(p-s)[2f(\theta, Q_l) + \frac{\partial f(\theta, Q_l)}{\partial Q_l}](Q_l - \frac{B}{w})$. Using the IGFR condition and the fact that $(Q_l - \frac{B}{w})$ is necessarily positive in the borrowing scenario, it is straightforward to verify that the lender's expected profit function is concave. Setting the first derivative equal to zero permits to derive the optimal ordering quantity and associated optimal interest rate for the lender. □

This optimisation approach assumes that the demand distribution and associated cumulative and density functions are based on the estimation from a data-driven approach whose parameters θ (e.g., mean and standard deviation) are estimated based on historical transaction forecasts.

4 Optimisation methods

In this section, we present the data-driven methods used in the optimization for the seller and the lender.

4.1 Demand estimation

Recall that the realization of the demand $q(\cdot)$ is unknown to the decision-maker. Hence, this section introduces multiple machine learning and deep learning models to predict demand. Machine learning and deep learning methods have been applied to numerous forecasting tasks. In this section, we consider the most widely and successfully used machine learning and deep learning approaches to forecasting default risk probability.

Gradient Boosting (Friedman, 2001) is a method of Boosting which has received continuous attention since it was proposed. It is one of the most successful classifications and regression algorithms widely used in recent research. Gradient Boosting Decision Trees (GBDT) use the value of the negative gradient $r_t(Y, f(X)) = -[\frac{\partial L(Y, f(X))}{\partial f(X)}]_{f(X)=f_{t-1}(X)}$ as the pseudo-residuals of the boosting tree algorithm to fit base tree learner.

The GBDT is an additive model $\hat{Y} = F_T(X) = \sum_{i=1}^{T} f_t(X)$, it keeps generating the weak base tree learner through the gradient descent direction until the number of trees reaches the maximum limit T or the loss of the model no longer improves with iteration.

Light Gradient Boosting Machine (LGB) (Ke et al., 2017) is a framework that implements the GBDT algorithm and supports high-efficiency parallel training. LightGBM has the advantages of faster training speed, lower memory consumption, better accuracy, support for distribution, and fast processing of massive data. LightGBM proposed Gradient-based One-Side Sampling (GOSS), which obtains a smaller data size by excluding a significant proportion of data instances with small gradients. It also proposed Exclusive Feature Bundling (EFB), which reduces the number of features by bundling mutually exclusive features.

Artificial Neural Network (ANN) (Bishop et al., 1995) is one of the most widely used methods of deep learning. It is based on collections of connected units, or nodes, called artificial neurons, that loosely mimic neurons in a living brain. The neural network is an operational model consisting of numerous nodes connected. Each node represents a specific output function, called an activation function. Each connection between two nodes represents a weighted value for the signal passing through the connection, called weight, which is equivalent to the memory of an artificial neural network. When training the model with historical transaction data (feature data X_{ij} and the label of demand d_i), the output of the network varies according to the connection mode, weight value, and excitation function of the network. After the training step, the model can be used to estimate transaction data. The output of the network varies according to the connection mode, weight value, and excitation function of the network.

4.2 Integrated optimization with quantile regression

We apply the quantile regression algorithm to solve inventory models and implement this algorithm with machine learning and deep learning methods. The optimal no borrowing inventory decision and borrowing inventory decision in period i is given via the quantile regression algorithm which are denoted by $Q_N^* = QR_N(p, w, s, B, X_{kij}, d_{kij}, \beta_j^N)$ and $Q_B^* = QR_B(p, w, s, B, r, X_{kij}, d_{kij}, \beta_j^B)$ respectively.

The classical regression loss function relies on estimating the β by minimizing the sum of squares error Koenker and Hallock (2001) as shown in Eq. (14).

$$J(\beta) = \frac{1}{m}\sum_{i=1}^{m}(d_i - q(\beta_j, X_{ij}))^2 \tag{14}$$

A quantile regression, on the other hand, estimates $\hat{d}_{(\tau)}$ at the conditional of a quantile τ. In other words, quantile regression is able to obtain an estimated value $\hat{d}_{i_{(\tau)}}$ by giving an input X_{ij}, and it can be described as follow:

$$\hat{d}_{(\tau)} = q(\beta_{j_{(\tau)}}, X_{ij}) \tag{15}$$

where $\tau \epsilon (0, 1)$ is a fixed constant (Christmann & Steinwart, 2008). $\hat{d}_{(\tau)} = \left\{\hat{d}_{i_{(\tau)}} \mid i = 1, 2, ..., m\right\}$ is the estimate of demand based on τ quantiles and $\beta_{(\tau)} = \left\{\beta_{j_{(\tau)}} \mid j = 1, 2, ..., n\right\}$ is the estimated parameters of the quantiles model. Different from minimizing the sum of squares error, $\hat{d}_{(\tau)}$ can be estimated by minimizing the sum of pinball losses (Koenker & Bassett Jr, 1978) which can be defined as Eq. (16).

$$J(\beta_{(\tau)}) = \frac{1}{m}\sum_{i=1}^{m}\begin{cases} \tau(d_i - q(\beta_{j_{(\tau)}}, X_{ij})) & if \quad d_i - q(\beta_{j_{(\tau)}}, X_{ij}) \geq 0 \\ (1-\tau)(q(\beta_{j_{(\tau)}}, X_{ij}) - d_i) & else. \end{cases} \tag{16}$$

Therefore, the seller's inventory decision is reduced to the quantile regression. Then the seller optimizes her inventory (ordering quantity) and financing (amount to borrow) decisions for a given interest rate. Furthermore, the seller has to compare her financial performance under the two scenarios (borrowing and not-borrowing). As mentioned in the previous section, verification on the optimal quantities should be performed in order to validate the need to borrow or not ($Q_N \leq \frac{B}{w}$, $Q_B \geq \frac{B}{w}$). In summary, we develop an algorithm relying on the usage of integrated quantile regression in machine learning and deep learning methods. The calculation steps are shown in Algorithm 1.

5 Experiments

This section presents the optimization results and explicitly shows how to take advantage of the data-driven approaches to decide on the ordering quantities under the borrowing and no borrowing scenarios. Section 5.1 describes the data that we used in the experiments. Section 5.2 presents the results of the game-theoretic optimal method for the lender and the seller. Section 5.3 presents the results of our integrated method for the seller and highlights its added value compared to the separated method. All the analyses were operated in Python. Our empirical evaluation aims to assess the utilization and impact of the data-driven approaches on the two sides -seller and lender. To this end, we evaluate the economic performance of the different methods by using a real-world dataset.

Algorithm 1 Optimization algorithm

Input Transaction data (Feature data X_{kij} the demand d_{kij}; the retail price, wholesale price, salvage value, capital level, rate of interest

Output Q^{**}, financial values;

for each period $k \in K$ **do**
  $X_{k-1,ij} \leftarrow X_{kij}, d_{k-1,i} \leftarrow d_{ki}$
  $Q_B \leftarrow QR_B(p, w, s, B, r^*, X_{kij}, d_{kij}, \beta_j^B)$; financial values $\leftarrow B - w * Q_B$
  $Q_N \leftarrow QR_N(p, w, s, B, X_{kij}, d_{kij}, \beta_j^N)$; financial values $\leftarrow 0$
  $\hat{d_{ki}} \leftarrow ML(X_{kij}, d_{kij})$
  $\pi_{S_N}^{\hat{*}} = p\min(\sum_{i=1}^m \hat{d_{ki}}, \sum_{i=1}^m q_k(\beta_j^{N*}, X_{kij})) + s(\sum_{i=1}^m q_k(\beta_j^{N*}, X_{kij}) - \sum_{i=1}^m \hat{d_{ki}})^+ - w\sum_{i=1}^m q_k(\beta_j^{N*}, X_{kij})$
  $\pi_{S_B}^{\hat{*}} = p\min(\sum_{i=1}^m \hat{d_{ki}}, \sum_{i=1}^m q_k(\beta_j^{B*}, X_{kij})) + s(\sum_{i=1}^m q_k(\beta_j^{B*}, X_{kij}) - \sum_{i=1}^m \hat{d_{ki}})^+ - w\sum_{i=1}^m q_k(\beta_j^{B*}, X_{kij}) - r^*(w\sum_{i=1}^m q_k(\beta_j^{B*}, X_{kij}) - B)$
  if $Q_N^* \leq \frac{B}{w}$ **then**
    if $Q_B^* \geq \frac{B}{w}$ **then**
      if $\pi_{S_B}^{\hat{*}} > \pi_{S_N}^{\hat{*}}$ **then**
        $Q^{**} \leftarrow Q_B^*$, financial values $\leftarrow B - w * Q^{**}$
      else
        $Q^{**} \leftarrow Q_N^*$, financial values $\leftarrow 0$
      end if
    else
      $Q^{**} \leftarrow Q_N^*$, financial values $\leftarrow 0$
    end if
  else if $Q_B^* \geq \frac{B}{w}$ **then**
    $Q^{**} \leftarrow Q_B^*$, financial values $\leftarrow B - w * Q^{**}$
  else
    $Q^{**} \leftarrow \frac{B}{w}$, financial values $\leftarrow 0$
  end if
end for

5.1 Data of the experiment

This section deals with a real data set consisting of the seller's historical sales data provided by Kaggle and optimizes inventory and financing decisions based on these historical sales data. The data contain historical sales records of the seller containing multiple sub-stores including two years of annual promotions. Hence, we explore the transaction data set generated in nearly two years and involves 199,390 records (171,507 records for the training set and 27,883 records for the test set), including the 101 weeks of sales data from the seller spanning between 05-Feb-2010 to 06-Jan-2012. The corresponding weekly sales of the seller in the data set can indicate the demand of the seller. The strong annual seasonality of demand is shown in "Appendix A". The weeks including these holidays are weighted five times higher in the evaluation than non-holiday weeks.

We divide the data set into the training set and the test set. The data set provides the fuel price, holiday, temperature, and demand of each week. In addition, the seller runs promotional markdown events throughout the year. These markdowns precede significant holidays, the four most influential: the Super Bowl, Labour Day, Thanksgiving, and Christmas. After preprocessing the data set and feature engineering, we extract a feature set including thirty-six features, as described in Table 2.

Table 2 Feature set description

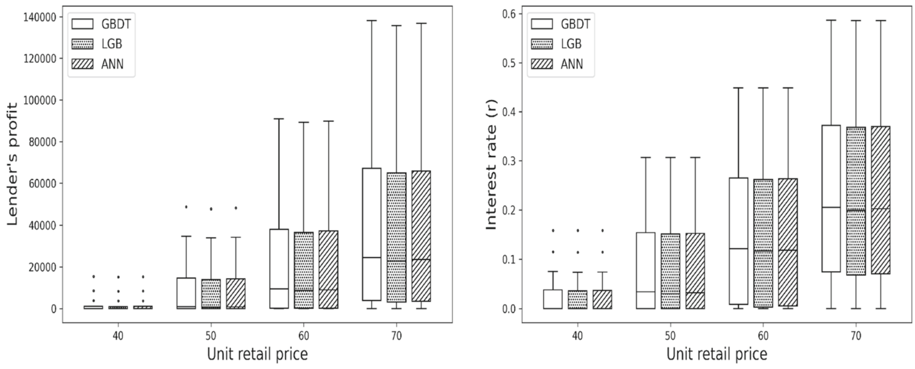
Features	Description
Store	The sub-store number
Size	Size of the sub-store
Dept	Department of the sub-store
Type	Type of the sub-store
Date	Specifying the Week (Friday of every Week)
Temperature	Average temperature in the region
FuelPrice	Cost of fuel in the region
MarkDown 1–5	Anonymized data related to promotional markdowns that Walmart is running
CPI	Consumer price index
Unemployment	Unemployment rate
IsHoliday	Whether the week is a special holiday week
Cluster	Cluster of the transaction sales record
Median Sales	Median Sales of different Type, Dept, Store IsHoliday and Month
Black Friday	Whether the week contains Black Friday
Christmas	Whether the week contains Black Friday
Count feature set	Total number of Type, Store, Dept, IsHoliday, Total number of different Cluster combine with different Type, Store, Dept, IsHoliday
Dept size	Dept divide by size

5.2 Numerical study of the game-theoretic optimal method for the lender and the seller

This section aims to gain some managerial insights by investigating numerically how the retail price and the seller's capital level influence the supply chain performance. Specifically, under the separated methods, we assume an exponential distribution for the demand as in Cai et al. (2014), Chen (2015) and Jin et al. (2019). The demand mean value is derived using the different data-driven techniques presented in the previous section and using the seller's historical sales data. We vary the value of the initial capital amount and derive the impact on the expected profit. As the Stackelberg leader, the lender will take the lead in deciding its interest rate to maximize its profit by setting the best interest rate.

We first investigate how retailing price influence the lender's interest rate and profit. Figure 3 shows that the mean of lender's optimal interest rates and financing profit increase as the retailing price increase. This can be explained by the fact that the lender will set a higher interest rate and earn more profit if the seller's profit per unit of the product is higher. By comparing the different prediction techniques shown in Fig.3, we remark that the GBDT model ensures the highest benefit for the lender among the three models.

Figure 4 illustrates the optimal profit of the seller, which interest rate is set by the same estimation model. For example, for the LGB model, its interest rate is set according to the mean of the LGB model. The LGB seller's profit is given by the result of the LGB's separate estimation and optimization method. Here we find that the GBDT model also ensures the highest economic performance for the seller.



(a) Profit (b) Interest rate

Fig. 3 Lender's profit and interest rate among different models

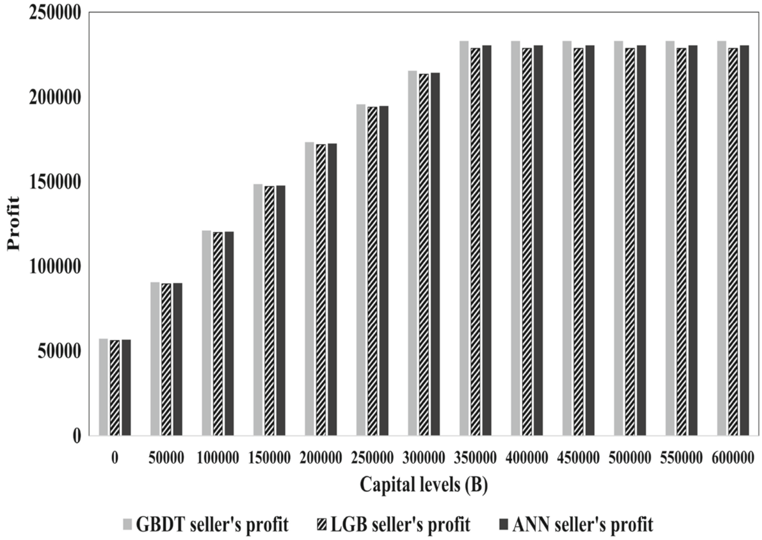


Fig. 4 Impact of the capital levels on the seller's profit

Therefore, we further analyze the seller's decision under the different model which interest rate set by the lender under the separate estimation and optimization method. Figure 5 shows that the lender's optimal interest rates and financing profit decrease as the seller's capital levels increase. This can be explained by the fact that the seller's sufficient capital will reduce the borrowing amount from the lender. There is a threshold above which the seller has enough capital and no longer needs to borrow from the lender, and the lender cannot earn interest from the seller starting from this threshold. For the GBDT separate approach (see in Fig.5), when the capital level is less than 350,000, the seller is facing a capital shortage. The lender's interest rate proposal depends on the initial capital and the lender has no financial incentive to offer the loan to the seller starting from a seller's initial capital equal to 300,000. From 300,000 to 350,000, the seller's best option is to use the whole capital to order what is possible to order with this amount without underage/overage trade-offs. Starting from an

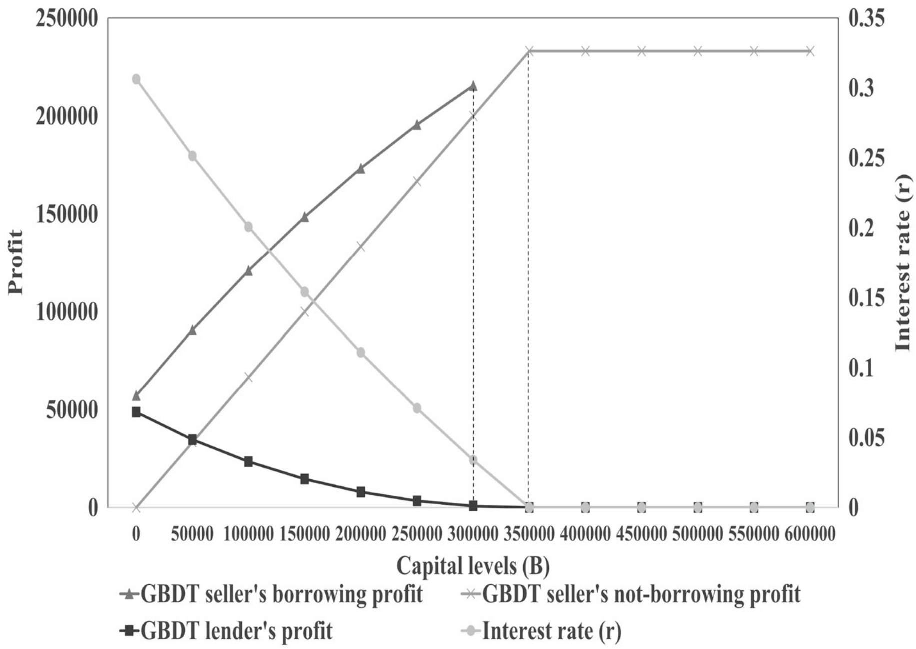


Fig. 5 Impact of the capital levels on the seller's decisions of GBDT model

initial capital equal to 350,000, the seller is self-sufficient from the capital point of view and has rather to optimise her ordering quantity.

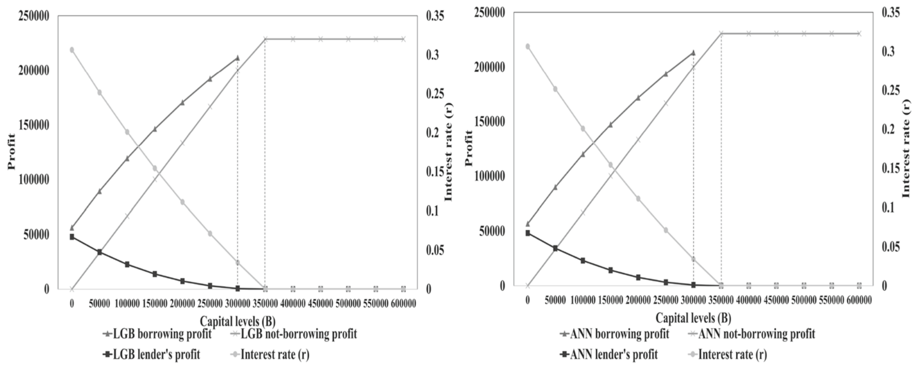
From Fig. 6, we find that although the LGB and ANN models are not as good as the GBDT model in terms of profit performance, the LGB and ANN models are similar to the GBDT model. The seller should face the three options (optimise the borrowing quantity, use whole initial capital, optimize the no-borrowing quantity). When the seller's capital level reaches a certain value, the lender will no longer provide loans.The seller then uses all of her initial capital to order the product (e.g., Fig. 6a and b when the capital level is between 300,000 to 350,000) or order the optimal order quantity calculated by the not-borrowing approach (e.g., Fig. 6a and b when the capital level exceeds 350,000).

From the experiments, we find out that the GBDT-based financial support encourages the capital-constrained seller to sell more products (see in Fig. 7) and gain more profit.

5.3 Numerical study of the integrated method for the seller

In this section, we employ the quantile regression approach that integrates demand estimation into the optimization model for the GBDT, LGB, and ANN. Moreover, we assume that the GBDT model is used by the lender in order to optimize the interest rate. That is, the interest rate is provided as the input for the seller after being optimized by the lender.

Compared to the separated method, when the lender provides the same interest rate, we can observe from Fig. 8 that the integrated method can significantly improve both the seller's and lender's profit. For example, when the capital level is equal to 0, the seller's profit under the integrated method is nearly three times her profit under the separated method. The average



(a) LGB (b) ANN

Fig. 6 Impact of the capital levels on the seller's decisions of LGB and ANN models

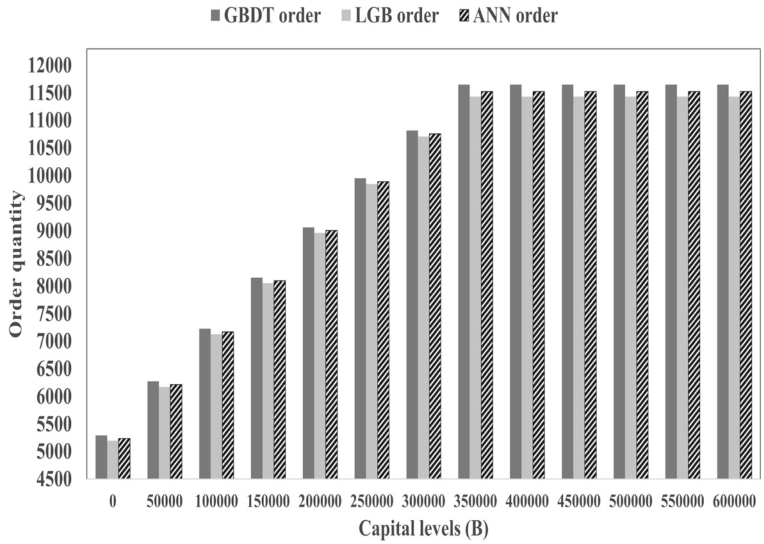


Fig. 7 Impact of the capital levels on the seller's order quantity

increase in profit enabled by the use of the integrated method instead of the separated one is over 55%.

Here again, we have the three options facing the seller: better to borrow initially, use the whole capital if it ranges from 350,000 to 500,000 and optimize the no-borrowing ordering quantity above 500,000.

Similarly, we compared the LGB and ANN models. After using the integrated method, we find that compared with the LGB model and the ANN model, after adopting the integrated model, the profit growth rate of the seller and lender is similar to the one obtained with the GBDT.

We now explore the impact of the product selling price. We experiment with the GBDT model by increasing the retail price of the product, and we find that when the retail price of the product increases, there is an incentive to order more products for the seller since she can

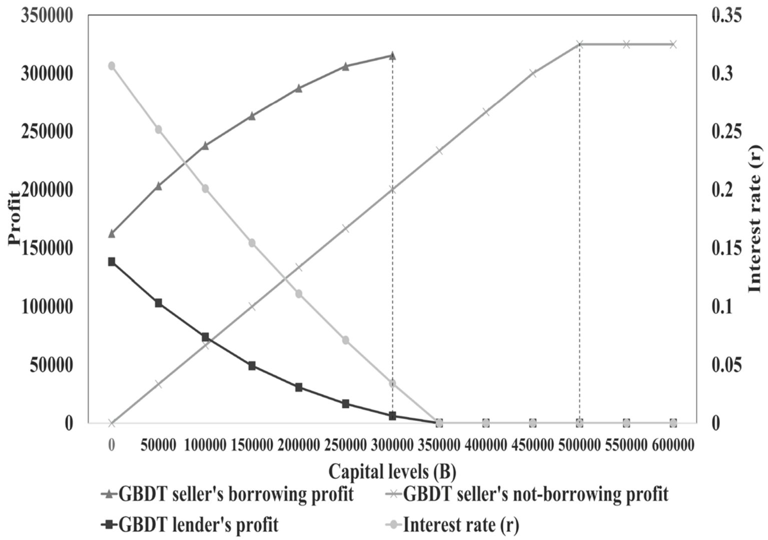
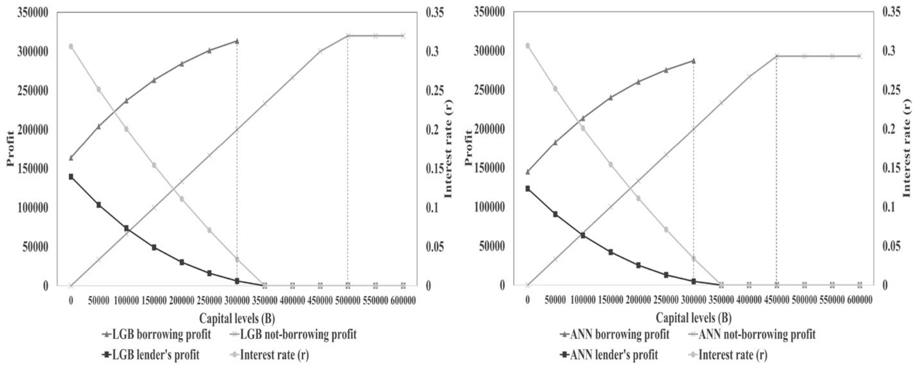


Fig. 8 Impact of the capital levels on the seller's decisions of GBDT model



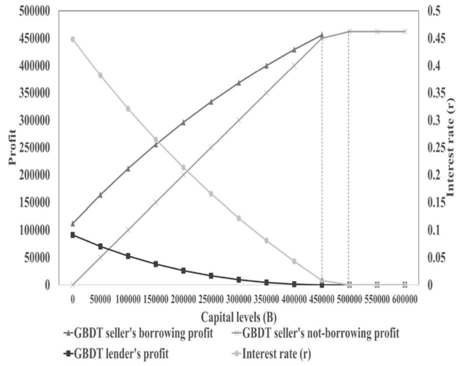
(a) LGB (b) ANN

Fig. 9 Impact of the capital levels on the seller's decisions of LGB and ANN models

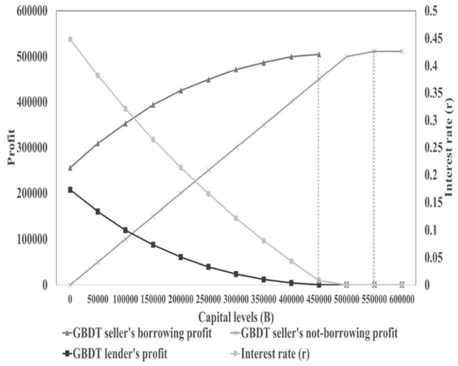
obtain a greater profit. For the lender, he will be able to charge higher interest. The retailing price change also impacts the thresholds between the borrowing and no-borrowing scenarios.

We also use a nonparametric test on profit values (lender's profit and seller's profit) to demonstrate how the integrated method outperforms the separated method. This test establishes the results of the profit given by two methods are significantly different. The test's p-values are displayed in Table 3. A p-value less than 1% implies that, at a 1% significance level, the mean profit by one integrated method is higher than that of the separated method.

From the above experimental results, we can see that after adopting the integrated model, the profit of the seller is improved, and the lender's profit is also significantly improved at the same time. The seller increases its order quantity to obtain a higher profit due to the improvement of the optimized inventory model. However, small and medium-sized enterprises with a low initial capital level do not have enough capital to build a data-driven ordering system to



(a) Separated method



(b) Integrated method

Fig. 10 Impact of the capital levels on the seller's decisions of GBDT model when p=60

Table 3 p-values of the nonparametric test between the integrated method and separated method

Model	p-values
(a) Results of lender's profit	
Gradient Boosting Decision Trees (GBDT)	0.000**
Light Gradient Boosting Machine (LGB)	0.000**
Artificial Neural Network (ANN)	0.000**
(b) Results of seller's profit	
Gradient Boosting Decision Trees (GBDT)	0.000**
Light Gradient Boosting Machine (LGB)	0.000**
Artificial Neural Network (ANN)	0.000**

$p < 0.05 * p < 0.01 **$

help them operate in the early stages of their business. In order to better help sellers to operate the business on the online platform and improve the platform's profit, the e-commerce platform with solid data processing capabilities and data authorization from the seller has been driving to build off an order optimization system as proposed in our study to guide sellers selling on the platform.

5.4 Discussion

The results of our experiments give us an important management implication on how the retailing price and capital level are important factors that affect inventory and financing decisions. Online retail platforms and sellers should pay more attention to these factors, and they can use the available data in order to jointly decide on the financial and inventory solutions. Hence, before large-scale e-commerce promotions, the online retail platform should comprehensively collect authorized transaction data and thoroughly mine and analyze. The seller should also analyze her transaction data and check her capital level before the upcoming large-scale e-commerce promotion season.

Our proposed methods reveal that in the borrowing scenario, the seller and the lender will obtain a higher average profit than in the not-borrowing scenario. These results have an important management implication that online retail platforms are motivated to establish data-driven financial support methods for themselves and sellers. Our study finds that the sellers will increase their order quantity due to the incentive of higher product profit, and the online retail platform will benefit more from the sellers' larger lack of capital. These results have an important management implication that online retail platforms are encouraged to offer more capital to sellers who sell more profitable products and provide a higher priority to them.

Overall, our proposed data-driven optimization model provides online retail platforms and sellers with a solution that can handle massive amounts of data according to sellers' initial capital level and historical transaction data. Our proposed optimization model will meet large-scale e-commerce promotions demand for sellers through efficiently data-driven methods. Moreover, our proposed data-driven optimization model can serve the online retail platform for the scenarios where considerable financing decisions are required in a short period.

6 Conclusions

This paper designs the inventory and financing decision optimization approaches for the sellers who sell the products on the online retailing platform, taking advantage of the seller's transaction data characteristics. We propose applying state-of-art machine learning and deep learning algorithms to use the abundant data in inventory and financing decisions on two levels: separate estimation and optimization and integrated estimation and optimization. We discuss the borrowing and not borrowing scenarios, and two multi-period optimization models are established for inventory replenishment. Then we present solutions for two models and give data-driven methods to ensure the demand information, special holidays, promotional information, and massive implicit information of transaction data can be studied by our proposed models. Next, we present the solution for the platform to optimize its interest rate. We provide the perspective to explore the motivations of online retail platforms to provide financial services. Finally, we present inventory and financing decisions under some conditions and make a quantitative analysis of the interest rate and benefit of the seller and the lender to reveal their relationship. We find that the e-commerce platform with solid data processing capabilities has the ability and drive to build an order optimization system as proposed in our study to support the seller's business.

Our research makes the following contributions: We extract useful information via machine learning and deep learning models from the seller's transaction data to optimize inventory and financing decisions for the seller. Also, we propose a method for the lender to optimize his interest rate. We are among the earliest to propose a data-driven game-theoretic method to determine inventory and financing decisions for the seller and the lender. Moreover, we explore a real data set and employ three models, including machine learning and deep learning to analyze the data. We find that the GBDT method performs best. Our experiments demonstrate how to explicitly order quantity and financing values before the promotion selling season and provide a new quantitative analysis of the relationship between capital level, interest rate, lender's profit, and seller's profit.

Our research has limitation since we mainly focus on the annual large-scale promotion, which requires enormous capital in a short period. In the future, we can consider the daily shortage of capital to build up the multi-cycle inventory and financing decisions model, which supports decision makers' daily decisions.

Acknowledgements This work was supported by the National Science Foundation of China (Grant No. 71620107002, 71821001 and 71971095).

Declarations

Conflict of interest No conflicts of interest in the development of this research.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the study.

Appendix A: The historical transaction data of the seller

See Fig.11.

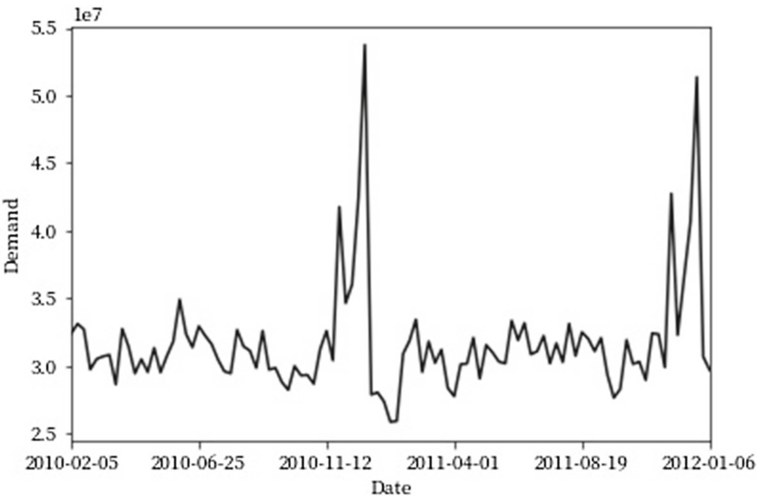


Fig. 11 Demand of seller

References

Abbasi, B., Babaei, T., Hosseinifard, Z., Smith-Miles, K., & Dehghani, M. (2020). Predicting solutions of large-scale optimization problems via machine learning: A case study in blood supply chain management. *Computers and Operations Research, 119*(104), 941.

Alan, Y., & Gaur, V. (2018). Operational investment and capital structure under asset-based lending. *Manufacturing and Service Operations Management, 20*(4), 637–654.

Arunraj, N. S., & Ahrens, D. (2015). A hybrid seasonal autoregressive integrated moving average and quantile regression for daily food sales forecasting. *International Journal of Production Economics, 170*, 321–335.

Ban, G. Y., & Rudin, C. (2019). The big data newsvendor: Practical insights from machine learning. *Operations Research, 67*(1), 90–108.

Bishop, C. M. (1995). *Neural networks for pattern recognition*. Oxford: Oxford University Press.

Buzacott, J. A., & Zhang, R. Q. (2004). Inventory management with asset-based financing. *Management Science, 50*(9), 1274–1292.

Cai, G., Chen, X., & Xiao, Z. (2014). The roles of bank and trade credits: Theoretical analysis and empirical evidence. *Production and Operations Management, 23*(4), 583–598.

Cao, Y., & Shen, Z. J. M. (2019). Quantile forecasting and data-driven inventory management under nonstationary demand. *Operations Research Letters, 47*(6), 465–472.

Chen, X. (2015). A model of trade credit in a capital-constrained distribution channel. *International Journal of Production Economics, 159*, 347–357.

Cheng, Y., Wu, D. D., Olson, D. L., & Dolgui, A. (2021). Financing the newsvendor with preferential credit. *International Journal of Production Research: Bank versus Manufacturer, 59*(14), 4228–4247.

Christmann, A., & Steinwart, I. (2008). Consistency of kernel-based quantile regression. *Applied Stochastic Models in Business and Industry, 24*(2), 171–183.

Dada, M., & Hu, Q. (2008). Financing newsvendor inventory. *Operations Research Letters, 36*(5), 569–573.

Dong, G., Liang, L., Wei, L., Xie, J., & Yang, G. (2021). Optimization model of trade credit and asset-based securitization financing in carbon emission reduction supply chain. Annals of Operations Research, 1–50.

Friedman, J. (2001). Greedy function approximation: A gradient boosting machine. *Annals of Statistics, 29*(5), 1189–1232.

Gao, D., Zhao, X., & Geng, W. (2014). A delay-in-payment contract for pareto improvement of a supply chain with stochastic demand. *Omega, 49*, 60–68.

Huber, J., & Stuckenschmidt, H. (2020). Intraday shelf replenishment decision support for perishable goods. *International Journal of Production Economics, 231*, 107828.

Huber, J., Müller, S., Fleischmann, M., & Stuckenschmidt, H. (2019). A data-driven newsvendor problem: From data to decision. *European Journal of Operational Research, 278*(3), 904–915.

Jin, W., Zhang, Q., & Luo, J. (2019). Non-collaborative and collaborative financing in a bilateral supply chain with capital constraints. *Omega, 88*, 210–222.

Jing, B., & Seidmann, A. (2014). Finance sourcing in a supply chain. *Decision Support Systems, 58*, 15–20.

Ke, G., Meng, Q., Finley, T., Wang, T., Chen, W., Ma, W., Ye, Q., & Liu, T. Y. (2017). Lightgbm: A highly efficient gradient boosting decision tree. *Advances in Neural Information Processing Systems, 30*, 3146–3154.

Koenker, R., & Bassett, Jr G. (1978). Regression quantiles. Econometrica: Journal of the Econometric Society, 33–50.

Koenker, R., & Hallock, K. F. (2001). Quantile regression. *Journal of Economic Perspectives, 15*(4), 143–156.

Kouvelis, P., & Zhao, W. (2016). Supply chain contract design under financial constraints and bankruptcy costs. *Management Science, 62*(8), 2341–2357.

Lai, G., Debo, L. G., & Sycara, K. (2009). Sharing inventory risk in supply chain: The implication of financial constraint. *Omega, 37*(4), 811–825.

Lai, Z., Lou, G., Zhang, T., & Fan, T. (2021). Financing and coordination strategies for a manufacturer with limited operating and green innovation capital: bank credit financing versus supplier green investment. Annals of Operations Research, 1–35

Oroojlooyjadid, A., Snyder, L. V., & Takáč, M. (2020). Applying deep learning to the newsvendor problem. *IISE Transactions, 52*(4), 444–463.

Raghavan, N. S., & Mishra, V. K. (2011). Short-term financing in a cash-constrained supply chain. *International Journal of Production Economics, 134*(2), 407–412.

Saghafian, S., & Tomlin, B. (2016). The newsvendor under demand ambiguity: Combining data with moment and tail information. *Operations Research, 64*(1), 167–185.

Tao, Y., Yang, R., Zhuo, X., Wang, F., & Yang, X. (2022). Financing the capital-constrained online retailer with risk aversion: Coordinating strategy analysis. Annals of Operations Research, 1–29

Xu, X., & Birge, J.R. (2004). Joint production and financing decisions: Modeling and analysis. Available at SSRN 652562

Yan, N., Sun, B., Zhang, H., & Liu, C. (2016). A partial credit guarantee contract in a capital-constrained supply chain: Financing equilibrium and coordinating strategy. *International Journal of Production Economics, 173*, 122–133.

Yan, X., & Wang, Y. (2014). A newsvendor model with capital constraint and demand forecast update. *International Journal of Production Research, 52*(17), 5021–5040.

Yang, M. (2013). Research on supply chain finance pricing problem under random demand and permissible delay in payment. *Procedia Computer Science, 17*, 245–257.

Yang, S. A., & Birge, J. R. (2018). Trade credit, risk sharing, and inventory financing portfolios. *Management Science, 64*(8), 3667–3689.

Zhang, Y., & Gao, J. (2017). Assessing the performance of deep learning algorithms for newsvendor problem. In International conference on neural information processing, pp. 912–921, Springer.

Zhao, D., Zhang, B., & Wang, Z. (2017). A bank credit model with capital-constrained newsvendor under two ordering opportunities. *Journal of Systems Science and Information, 4*(5), 408–418.

Zheng, M., Shu, Y., & Wu, K. (2015). On optimal emergency orders with updated demand forecast and limited supply. *International Journal of Production Research, 53*(12), 3692–3719.