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ORIGINAL RESEARCH

Inventory model using Machine Learning for demand forecast with imperfect deteriorating products and partial backlogging under carbon emissions

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Abstract

In today's environment, organizations utilise Machine Learning based-models to keep stocks depending on the demand for a particular type of product. This article develops an inventory model considering the imperfect deteriorating product in a fuzzy environment. The shortages are allowed and partially backlogged. Since the deterioration rate and defective percentage in quantity in the received lot may not be predicted precisely because it depends on many uncertain situations, therefore both are considered fuzzy variables. This study aims to determine the optimal ordering quantity and replenishment period to optimize (minimize) the average overall cost with carbon emission cost. The defuzzification process is done using the sign distance approach method. A methodology based on Machine Learning is used to demand forecast seasonally. Some numerical examples are taken to validate the proposed mathematical model. The findings demonstrate the generation of direct month-wise predicted demand for deteriorating products based on the input of the month value, enabling organizations to optimize their inventory management according to forecasted demand. A comparative analysis is conducted between fixed and month-wise forecasted demand by highlighting the advantages of machine learning-based forecasting approaches. Sensitivity analysis performs to examine the behaviour of several parameters on an optimal solution and provides some managerial insights.

Keywords Inventory model · Demand forecast · Deterioration · Imperfect product · Machine learning · Carbon emission

1 Introduction

Seasonal and weather conditions influence global market demand. Demand is the essential key variable to managing inventory in any business since it causes several major problems.

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Seasonal demand may be a major concern for businesses in practically any sector. It can be influenced by festivals and events that impact consumers' purchasing activity, such as Christmas or Black Friday, when individuals tend to spend more money. It can also be more directly related to the weather, such as higher garden furniture sales in the spring. The majority of the study has been done with deterministic demand; however, in reality, product demand varies seasonally. Better demand prediction allows a company to effectively manage inventories, reduce extra costs, and enhance customer service. Machine learning (ML) is an innovation which can be used to improve the accuracy and reliability of demand forecasts. The use of ML to accurately forecast product demand (seasonally) is an emerging approach. ML applications are seen in retailing, economics, military, health care fields, etc., in order to accomplish their purpose. Researchers are designing numerous algorithms for data mining using knowledge from various disciplines of study (de Almeida Neto & Castro, 2017). ML approaches that include Decision Tree-based Algorithms are among the basic and most powerful approaches to forecasting demand. So, the authors have implemented the decision tree classifier algorithm for seasonally demand forecast in our study.

Most physical products undergo deteriorate or decay over time. Products such as flowers, vegetables, fruits, medicines, and foodstuffs are susceptible to direct deterioration during the in-transit and holding period. Deterioration is characterized as damage or decay such that the product may no longer be used for its intended purpose. According to (Sethi & Sethi, 2006a), approximately 20–40% of food goes to waste between picking and consumer use. As a result, the products deterioration cannot be overlooked. There is no guarantee that the quality of a product is 100% perfect always. These products may also be partially imperfect due to manufacturing defects, man-handling problems, and damage during in-transit. In the traditional inventory model, the deterioration rate is considered deterministic, while in reality, there is uncertainty in the deterioration rate. To deal this uncertainty deterioration rate can be taken as a fuzzy variable. Fuzzy inventory model assuming fuzzy random deterioration rate has been thoroughly addressed (Naserabadi et al., 2014). The number of defective products in the delivered lot cannot be accurately predicted; therefore, the quantity of defective percentage is also taken as a fuzzy variable. In today's modern era, the increase in carbon emissions is a major concern for our scholars and organizations. The amount of Green House Gases has increased dramatically due to industrialization. Climate change is caused by the emission of greenhouse gases such as methane, carbon dioxide, etc. Carbon is the most responsible for climate change of all greenhouse gas emissions since it makes up the biggest percentage of global greenhouse gas emissions. As a result of all these issues, academicians are now focusing their efforts in the inventory system on reducing the total cost with carbon emissions. The shortage is permissible and unmet demands are backlogged. Since not each consumer accepts late deliveries; therefore, partial backlogging is taken in the study.

As far as our knowledge from the literature, the impact of demand forecast in imperfect deteriorating products has rarely been addressed. Keeping this in mind, two research problems can arise: (a) How can machine learning-based demand forecasting techniques improve the accuracy and predictability of seasonal demand forecasts for deteriorating products? (b) What are the advantages of utilising machine learning-based monthly predicted demand over fixed demand in inventory management? To address these problems, this article develops a machine learning-based fuzzy inventory model by considering imperfect deteriorating items under carbon emissions. A Machine Learning technique (Decision tree classifier) is used for demand forecast. The objective of this ML technique is to determine the accurate seasonal demand of the deteriorating product. The aim of this study is to find the optimal ordering quantity and replenishment period to optimize the total average cost with carbon emission cost. A solution procedure is derived to acquire the value of optimal decision variables. A numerical experiment shows that using seasonal forecasted demand instead of fixed demand significantly reduces overall cost. The remaining paper is arranged as follows: Sect. 2 introduces a brief literature review. Notations and assumptions are listed in Sect. 3. Section 4 describes the mathematical formulation of the model. Section 5 is the methodology section in which the machine learning methodology for demand forecast is presented. In Sect. 6, the model is validated with a numerical example. Section 7.2 presents the sensitivity analysis by changing the key parameters. The last section ends with the study's conclusion and some future research directions.

2 Literature review

Demand forecasting is a crucial component of business strategy since it allows organizations to optimize operations, save costs, and fulfil consumer expectations. Researchers have been researching numerous ways to increase demand forecasting accuracy while the literature in the field of machine learning is continually expanding. (Persinger & Levesque, 1983) studied the relationship between humidity, temperature, air conditions, and people's moods, demonstrating that the weather impacts people's moods. The demand forecasting model helps to predict the overstocking and understocking situations and when demand rises or falls (Wright & Schultz, 2018). On festivals such as Christmas and Diwali, Amazon, Flipkart, and other e-commerce companies experience an increase in demand as people actively seek gifts and festive supplies. Because of the dramatic rise in sales at such peak periods, it is critical for these businesses to precisely forecast demand rather than fix demand. Different researchers used different forecasting methods over time in inventory management. Decision trees are one of the most powerful data mining techniques widely used in various sectors (Hastie et al., 2009). (Carbonneau et al., 2008) classified simple time series methods as traditional methods, whereas a variety of ML approaches are viewed. (Kirshners et al., 2010) studied the joint analysis of continuous and discrete data using inductive decision trees. (Ren et al., 2020) conducted a thorough literature analysis on demand forecasting methods for fashionable products in order to identify the drawbacks of standard forecasting methods. Then, using a real-world case study, they investigated how a retailer handles the future demand forecasting and inventory scheduling challenge in practice. Recently, (Zohdi et al., 2022) used different machine learning algorithms such as decision tree, K-nearest neighbours, and gradient boosting to forecast demand and examine its accuracy and performance compared to other approaches.

Deterioration is a common phenomenon which is observed in most products. (Ghare & Schrader, 1963) was the first to work on the deteriorating item in the inventory systems with a constant decay rate. (Wang & Huang, 2014) developed a production inventory model for deteriorating seasonal items. (Lashgari et al., 2018) introduced a non-instantaneous decaying inventory model with partial payment and trade credit policy, and then found an optimal solution for the model. They included inventory shortages into consideration, which may occur as backorders, lost sales, or both. It is not possible that the quality of the product is perfect always, in general, some products are partially defective as a result of manufacturer error or deterioration. (Salameh & Jaber, 2000) suggested an economic production quantity (EPQ) model for imperfect quality items. The imperfect products are screened out and offered as a single batch at a discounted price. (Papachristos & Konstantaras, 2006) investigated the imperfect quality inventory system with defective products functioning as a stochastic process. Furthermore, many research on imperfect quality products has been done by (Hauck et al., 2021; Jaggi et al., 2013; Lin, 2010; Taleizadeh et al., 2016). (Wang et al., 2015) introduced an economic order quantity model with screening constraint of defective products. (Mohammadi et al., 2015) designed a model to optimise integrated production and product inspection policies in a deteriorating production system with imperfect quality control. In the retail sector, item shortages are a frequent occurrence and can lead to backlogs. Retailers frequently use tactics like backorders and lost sales to handle the problem when there is a shortage. When this situation happens, some consumers decide to wait patiently for the subsequent order, while others decide to buy the item from a different seller. When the demand for the product is partially met by alternate sources, this situation is referred to as a partial backlogging. (Wee et al., 2006) suggested a fully backordered integrated model for deteriorating products. Further, (Wee et al., 2007) expanded (Salameh & Jaber, 2000) model to include backorders. Partial backorder is allowed if lost sales are permitted. (Pentico et al., 2009) studied an EPQ model with partial backorder using a novel technique. (Khurana et al., 2015) presented a partially backlogging shortage production model having deterioration and inflation. (Diabat et al., 2017) developed a deteriorating-inventory model that takes into account partial payment systems in three distinct scenarios of shortages, namely shortages that are not permitted, shortages that are allowed with partial backordering, and shortages that are allowed with complete backordering. Finally, they discovered the best solution strategy for the suggested model. Recently, (Bishi & Sahu, 2018) developed deteriorating inventory model having quadratic demand and partial backlogging. (Sharifi et al., 2021) designed an EOQ lot size model with shortages for deteriorating poor quality items with damaging testing and inspection faults. The major sources of carbon emissions are transportation, storing and deterioration of products. (Hua et al., 2011) established an economic order quantity (EOQ) model that incorporates the impact of carbon costs associated with transportation and keeping stock. They implemented a carbon cap and trade policy to decrease carbon emissions. Generally, a carbon tax is intended to operate as a financial punishment for industries that emit emissions from their operations (Benjaafar et al., 2012). (Chen et al., 2013) designed an EOQ model to investigate the impacts of multiple carbon emission strategies. (Taleizadeh et al., 2020) took into account carbon tax policy. (Hasan et al., 2021) investigated a green inventory model under the conditions of a carbon emission penalty, cap-and-trade, and strict emission limit regulations. Transportation and warehousing emissions were added by (Battini et al., 2014). (Kazemi et al., 2018) investigated the influence of emission costs on a buyer's overall profit in an imperfect logistics system. (Mishra et al., 2021) proposed an optimal green inventory system with backorder and deterioration in the presence of controlled carbon emissions. (Mashud et al., 2021) developed a green inventory model with controllable carbon emission, deterioration, and advance payments.

Rather than implementing probability systems to solve inventory problems, fuzzy methods yield more appropriate solutions. The fuzzy set theory concept attracts numerous academicians' attention to inventory management problems. (Park, 1987) developed an EOQ model for analysing fuzzy set theory. (Chang et al., 1998) created an EOQ inventory model that treated backorders as interval fuzzy numbers. (Yao et al., 2000) used the extension principle to derive the fuzzy overall cost function after extending the standard EOQ model by assuming that the demand rate is a fuzzy number. (Chang, 2004) expanded the previously developed model in a fuzzy scenario by treating the demand rate and proportion of defective quality products as fuzzy integers. (Liu & Zheng, 2012) expanded (Wee et al., 2007) by taking into account fuzzy demand and defective item quality, and then integrated the model using centroid and graded mean integration representation approaches. (Kumar & Rajput, 2015) proposed a fuzzy inventory model for deteriorating products with time-dependent demand in which the backlogging rate is handled as a fuzzy number. (Jaggi et al., 2016) developed an optimum replenishment policy for a fuzzy inventory system with fuzzy type deterioration rate and shortages. (Shaikh et al., 2018) investigated a fuzzy inventory model for a deteriorating product with variable demand, allowable payment delays, and partially backlog under shortages. (Agarwal et al., 2022) developed a fuzzy EOQ model using Genetic Algorithm to optimize the total cost value.

Based on the literature mentioned earlier, none of the authors have presented the concept of machine learning for demand forecasts, imperfect products, partial backlogging, and carbon emissions. Therefore, we tried to fill the gap in the literature. The present article extends (Tiwari et al., 2018) model for only the retailer's perspective by using machine learning concepts and a fuzzy environment. Table 1 presents the relevant literature survey and research gap between previous and current study.

2.1 Research contribution

The main contribution of this work is the development of a machine learning-based fuzzy inventory model that considers imperfect deteriorating items under carbon emissions. The study addresses the limitations of traditional inventory models by incorporating uncertainties in deterioration rates and defective percentages as fuzzy variables. By utilizing a machine learning technique, specifically a decision tree classifier, the model aims to accurately forecast seasonal demand for deteriorating products. The significance of this contribution lies in its potential to improve the accuracy and predictability of demand forecasts for businesses. By incorporating machine learning-based monthly predicted demand instead of relying on fixed demand assumptions, companies can benefit from more efficient inventory management. The study demonstrates through a numerical experiment that using seasonal forecasted demand leads to a significant reduction in overall costs. Furthermore, the inclusion of carbon emissions costs in the inventory model showcases a focus on sustainability and environmental impact. By optimizing the ordering quantity and replenishment period to minimize total average cost with carbon emission costs, businesses can contribute to reducing their ecological footprint. Overall, the main contribution of this research lies in its integration of machine learning techniques, fuzzy variables, and carbon emissions considerations to enhance demand forecasting accuracy and provide a comprehensive approach to inventory management for deteriorating products. Therefore, the objective is to find the optimal policies that minimize the entire cost. Table 1 summarises the contributions of many researchers.

3 Notations and assumptions

3.1 Notations

Symbol	Description	Unit
D	Demand rate	Unit/year
θ	Deterioration rate $(0 \leq \theta \leq 1)$	–
$\widetilde{\theta}_i$	Deterioration rate in fuzzy sense, $i = 1, \ldots, 4$	
Q	Order quantity per cycle	Unit

Symbol	Description	Unit
T	Total replenishment period	Year
k	Defective percentage in Quantity	–
$\widetilde{k}_i$	Defective percentage in Quantity in fuzzy sense, $i = 1, \ldots, 4$	–
x	Screening rate	Unit/year
t_0	Quantity screening time ($t_0 = Q/x$)	Year
t_1	Time interval	–
O	Ordering cost	\$/order
C	Purchase cost	\$/unit
h	Holding cost	\$/unit/year
d_c	Deteriorating cost	\$/unit
s	Shortage cost	\$/unit
l	Lost sale cost	\$/unit
β	Backlogging rate ($0 \leq \beta \leq 1$)	–
S_c	Screening cost	\$/unit
T_f	Fixed transportation cost	\$
d	Distance between supplier to retailer	km
f_1	Fuel consumption when vehicle is empty	Litre/km
f_2	Fuel consumption per ton when vehicle is loaded	Litre/km/ton
w	Weight of product	Ton/unit
t_v	Variable transportation cost	\$/litre
C_d	Carbon emissions due to deterioration	Ton CO_2/unit
C_X	Carbon tax	\$/ton CO_2
A_{fe}	Carbon emissions due to fuel consumption	Ton CO_2/litre
g	Godown energy consumption per unit product	kWh/unit/year
E_e	Carbon emissions from electricity generation	Ton CO_2/kWh
G_e	Carbon emissions due to Godown;$G_e = gE_eC_X$	
TC	Total avarege cost per unit time	
$\widetilde{TC}$	Total avarege cost per unit time in fuzzy sense	
TC_d	Total average cost after defuzzification	

3.2 Assumptions

(i) A single kind of deteriorating product is taken, and the replenishment rate is infinite.

Table 1 Summary of literature review and research gap

Studies	Imperfect products	Deterioration	Screening cost	Shortages		Fuzzy concept	Carbon emission Cost	Forecasted demand
				Partial backlogging	Lost sales			
Mishra et al. (2013)		✓		✓	✓			
Mohammadi et al. (2015)	✓	✓	✓					
Jaggi et al. (2016)		✓		✓		✓		
Diabat et al. (2017)		✓		✓	✓			
Shaikh et al. (2018)		✓		✓		✓		
Tiwari et al. (2018)		✓	✓				✓	
Taleizadeh (2018)	✓			✓				
Wright and Schultz (2018)								✓
Rout et al. (2019)	✓	✓				✓		
Ren et al. (2020)								✓
Hauck et al. (2021)	✓		✓					
Sharifi et al. (2021)	✓	✓						
Cardenas-Barrón et al. (2022)	✓		✓	✓				
Mishra and Mishra (2022)	✓	✓	✓				✓	
Taleizadeh et al. (2022)				✓			✓	
This article	✓	✓	✓	✓	✓	✓	✓	✓

(ii) The demand pattern is forecasted demand.
(iii) Defective products come as a result of imperfect manufacturing and worker handling issues, and k is considered an interval trapezoidal fuzzy number.
(iv) The retailers perform a 100% screening process to detect defective products, and after the complete screening, collect and remove the imperfect products.
(v) Both the screening and demand rate occur simultaneously, and the screening rate is larger than the demand rate ($x > D$).
(vi) The deterioration rate is also taken as a trapezoidal fuzzy number.
(vii) The carbon emission due to transportation, godown/store, and deterioration are considered.
(viii) The shortage is permitted and partially backlogged.

4 Mathematical formulation

The sustainable inventory model of the imperfect deteriorating products for the retailer is discussed in this article. At the start of the time, the retailers receive Q units of products. From the initial time $t = 0$, the retailer conducts the screening process at the screening rate x. The stock level decreases due to deterioration and demand for period $[0, t_0]$, where $t_0 = Q/x$. A 100% screening process is done at the time t_0, and after that, non-perfect products are instantly removed from the stock. For the period $[t_0, t_1]$, the stock level depletes again due to both deterioration and demand and finishes at the time t_1. At $t = t_1$, there is a shortage that is partially backlogged at the rate β and continues up to T. At the end of the cycle, the stock reaches its highest shortage level, so to clear the backlog, the retailer again replenishes the products.

The rate of change of stock level at any time t during the period $[0, t_0]$, $[t_0, t_1]$ are described by the below differential equations:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -(1-k)D \tag{1}$$

With condition $I_1(0) = Q$.

$$I_1(t) = \frac{(1-k)}{\theta} D\left(e^{\theta(t_1-t)} - 1\right), \text{ for } t_0 \le t \le t_1 \tag{2}$$

$$I_1(t) = \frac{(1-k)}{\theta} D\left(e^{\theta(t_1-t)} - 1\right) + \frac{kD}{\theta}\left(e^{\theta t_1} - 1\right) \text{ for } 0 \le t \le t_0 \tag{3}$$

And the rate of change of the stock level at any time t during shortage period $[t_1, T]$:

$$\frac{dI_2(t)}{dt} = -\beta D, \text{ for } t_1 \le t \le T \tag{4}$$

With condition $I_2(t_1) = 0$.
The solution of Eq. (4) is

$$I_2(t) = \beta D(t_1 - t), \text{ for } t_1 \le t \le T \tag{5}$$

From Eq. (3), initial point $t = 0 \Rightarrow k = 0$

$$Q = I_1(0) = \frac{D}{\theta}\left(e^{\theta t_1} - 1\right) \tag{6}$$

The stock is present in the system during the interval $[0, t_1]$. Therefore, the cost of holding stock in storage is only determined for the interval $[0, t_1]$ only.

Holding cost $(HC) = h\left(\int_0^{t_0} I_1(t)dt + \int_{t_0}^{t_1} I_1(t)dt\right)$

$$HC = h\left(\frac{(1-k)}{\theta^2}D\left(e^{\theta t_1}-1\right) + \frac{kD}{\theta}t_0e^{\theta t_1} - \frac{Dt_1}{\theta} + \frac{kD}{\theta}(t_1 - t_0)\right) \tag{7}$$

The deteriorating cost due to deterioration of product during the period $[0, t_1]$ is:

$$\text{Deterioratingcost}(DC) = d_c(Q - Dt_1) \tag{8}$$

Shortage cost arises due to stock-out of products during the period $[t_1, T]$. The maximum quantity of shortage exists at time $t = T$.

Hence, the shortage cost $(SC) = s\int_{t_1}^{T} -I_2(t)dt$

$$SC = \frac{-s\beta D}{2}\left(2t_1T - T^2 - t_1^2\right) \tag{9}$$

Lost sale costs arise because not all consumers are ready to wait for the upcoming lot size.

$$\text{Lost sale cost}(LC) = l\int_{t_1}^{T}(1-\beta)Ddt \tag{10}$$

$$\text{Screening cost}(ScC) = Sc.Q \tag{11}$$

$$\text{Purchase cost}(PC) = c.Q \tag{12}$$

$$\begin{aligned}&\text{Tansportation cost } (TC) = \text{fixed transportation cost} + \text{fixed transportation cost}\\ &TC = T_f + (df_1t_v + df_2wQt_v)\end{aligned} \tag{13}$$

Carbon emission costs: The total carbon emission cost comes from deterioration of products, transportation, and store.

Carbon emission due to deterioration of the product: $C_d(Q - D(t_0 + t_1))C_X$

Carbon emissions due to transportation: $\left(df_1A_{fe} + df_2wA_{fe}Q\right)C_X$

$$\begin{aligned}\text{Carbon emission due store} : G_e\left(\int_0^{t_0} I_1(t)\,dt + \int_{t_0}^{t_1} I_1(t)\,dt\right)\\ = gE_eC_X\left(\frac{(1-k)}{\theta^2}D\left(e^{\theta t_1}-1\right) + \frac{kD}{\theta}t_0e^{\theta t_1} - \frac{Dt_1}{\theta}\right.\\ \left. + \frac{kD}{\theta}(t_1 - t_0)\right)\end{aligned}$$

Hence, total carbon emission (T_{ce}) is:

$$\begin{aligned}(T_{ce}) = C_X\Bigg(&C_d(Q - Dt_1) + \left(df_1A_{fe} + df_2wA_{fe}Q\right)\\ &+ gE_e\left(\frac{(1-k)}{\theta^2}D\left(e^{\theta t_1}-1\right) + \frac{kD}{\theta}t_0e^{\theta t_1} - \frac{Dt_1}{\theta} + \frac{kD}{\theta}(t_1 - t_0)\right)\Bigg)\end{aligned} \tag{14}$$

Using Eqs. (7)–(14), the total average cost per unit time is:

$$TC = \frac{1}{T}\left(O + h\left(\frac{(1-k)}{\theta^2}D\left(e^{\theta t_1}-1\right) + \frac{kD}{\theta}t_0e^{\theta t_1} - \frac{Dt_1}{\theta} + \frac{kD}{\theta}(t_1 - t_0)\right) + d_c(Q - Dt_1)\right.$$

$$+\frac{-s\beta D}{2}\left(2t_1T-T^2-t_1^2\right)+l(1-\beta)D(T-t_1)+Sc.Q+c.Q$$
$$+\left(T_f+(df_1t_v+df_2wQt_v)\right)+C_X\left(C_d(Q-Dt_1)+\left(df_1A_{fe}+df_2wA_{fe}Q\right)\right.$$
$$\left.\left.+\ gE_e\left(\frac{(1-k)}{\theta^2}D\left(e^{\theta t_1}-1\right)+\frac{kD}{\theta}t_0e^{\theta t_1}-\frac{Dt_1}{\theta}+\frac{kD}{\theta}(t_1-t_0)\right)\right)\right) \tag{15}$$

4.1 Fuzzy model

The basic definitions and algebraic operations with membership function of the fuzzy set theory proposed by (Zimmermann, 2011). In the inventory system, it is difficult for the decision-maker to find accurate values for known parameters. It means there is an uncertainty in some key parameters. Therefore, deterioration rate (θ) and defective percentage in quantity (k) are taken as trapezoidal fuzzy interval type. Fuzzy arithmetic operations for trapezoidal fuzzy number is briefly introduced by (Chen, 1985; Mahata & Goswami, 2013). Based on basic definitions and results, we have proceeded to fuzzify the proposed model.

Let $\theta=\left(\widetilde{\theta}_1,\widetilde{\theta}_2,\widetilde{\theta}_3,\widetilde{\theta}_4\right)$, $k=\left(\widetilde{k}_1,\widetilde{k}_2,\widetilde{k}_3,\widetilde{k}_4\right)$ are as a trapezoidal fuzzy number as shown in Fig. 1**.** Then the crisp total average cost function is changed into a fuzzy cost function. As the deterioration rate (θ) and defective percentage in quantity (k) is a trapezoidal fuzzy number, therefore $\widetilde{TC}$ is also taken as a trapezoidal fuzzy number.

$$\widetilde{TC}=\left(\widetilde{TC}_1,\widetilde{TC}_2,\widetilde{TC}_3,\widetilde{TC}_4\right) \tag{16}$$

where

$$\widetilde{TC}_i=\frac{1}{T}\left(O+h\left(\frac{\left(1-\widetilde{k}_{5-i}\right)}{\widetilde{\theta}_{5-i}^2}D\left(e^{\widetilde{\theta}_it_1}-1\right)+\frac{\widetilde{k}_iD}{\widetilde{\theta}_{5-i}}t_0e^{\widetilde{\theta}_it_1}-\frac{Dt_1}{\widetilde{\theta}_i}+\frac{\widetilde{k}_iD}{\widetilde{\theta}_i}(t_1-t_0)\right)\right.$$
$$+d_c\left(Q_i-Dt_1\right)+\frac{-s\beta D}{2}\left(2t_1T-T^2-t_1^2\right)+l\left(1-\beta\right)D\left(T-t_1\right)+Sc.Q_i+c.Q_i$$
$$+\left(T_f+(df_1t_v+df_2wQ_it_v)\right)+C_X\left(C_d\left(Q_i-Dt_1\right)+\left(df_1A_{fe}+df_2wA_{fe}Q_i\right)\right.$$

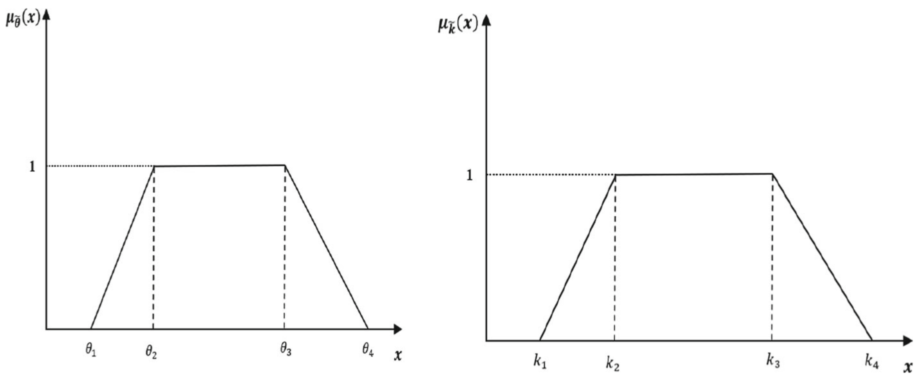

Fig. 1 Trapezoidal fuzzy number for deterioration rate (θ) and defective percentage in Quantity (k)

$$+gE_e\left(\frac{\left(1-\widetilde{k}_{5-i}\right)}{\widetilde{\theta}_{5-i}^2}D\left(e^{\widetilde{\theta}_i t_1}-1\right)+\frac{\widetilde{k}_i D}{\widetilde{\theta}_{5-i}}t_0 e^{\widetilde{\theta}_i t_1}-\frac{Dt_1}{\widetilde{\theta}_i}+\frac{\widetilde{k}_i D}{\widetilde{\theta}_{5-i}}(t_1-t_0)\right)\Bigg)\Bigg), i = 1, 2, 3, 4 \tag{17}$$

where

$$t_0 = Q_i/x \tag{18}$$

$$Q_i = \frac{D}{\widetilde{\theta}_{5-i}}\left(e^{\widetilde{\theta}_i t_1}-1\right), i = 1, 2, 3, 4 \tag{19}$$

Using the sign distance approach method to defuzzify the fuzzy cost function measured from $\widetilde{TC}$ to 0 (Sahoo et al., 2016).

$$TC_d = \frac{1}{4}\left(\widetilde{TC}_1+\widetilde{TC}_2+\widetilde{TC}_3+\widetilde{TC}_4\right) \tag{20}$$

Next, to acquire the optimum value, differentiate Eq. (20) partially with respect to T and t_1 and equating it to zero,

$$\frac{\partial\widetilde{TC}_d}{\partial T} = 0 \text{ and } \frac{\partial\widetilde{TC}_d}{\partial t_1} = 0 \tag{21}$$

To optimize the total cost ($\widetilde{TC}_d$), solving Eq. (21), we get the optimal value of T and t_1. To prove the sufficient condition, Eq. (20) satisfies the conditions:

$$\frac{\partial^2\widetilde{TC}_d}{\partial T^2} > 0, \frac{\partial^2\widetilde{TC}_d}{\partial t_1{}^2} > 0 \text{ and } \frac{\partial^2\widetilde{TC}_d}{\partial T^2}\frac{\partial^2\widetilde{TC}_d}{\partial t_1{}^2} - \frac{\partial^2\widetilde{TC}_d}{\partial T\partial t_1}\frac{\partial^2\widetilde{TC}_d}{\partial t_1\partial T} > 0 \tag{22}$$

The total cost function ($\widetilde{TC}_d$) is highly non-linear, so proving its convexity theoretically is difficult. Therefore, the convexity of the cost function is graphically shown with the help of the software MAPLE 18.

5 Methodology for demand forecast

In this study, the proposed model is developed by mainly concentrating on demand forecast using ML. Generally, most researchers consider the deterministic nature of demand, but there are random fluctuations. Due to these variations, the ML approach is better suited to forecast demand. A Decision Tree-based Classifier method is selected for this study as it is one of the simple and effective ML techniques. The objective of this approach is to find the accurate seasonal demand of the deteriorating product. The flowchart of the demand forecast methodology is depicted in Fig. 2.

To test the forecasted inventory model, PYTHON (3.10.0 version) code is used to predict the seasonal demand for deteriorating products. Some packages like Pandas for data analysis and import DecisionTreeClassifier, and train_test_split, from the sklearn library, are installed before running the PYTHON code. The data set is split into two parts to test the system: Training and testing data. We train 80% of the data and apply the Decision Tree Classifier algorithm for forecast output. After that, test the model with 20% of the data to find the forecasted demand. Finally, we insert the input parameter (month) and get the seasonally demand for deteriorating products.

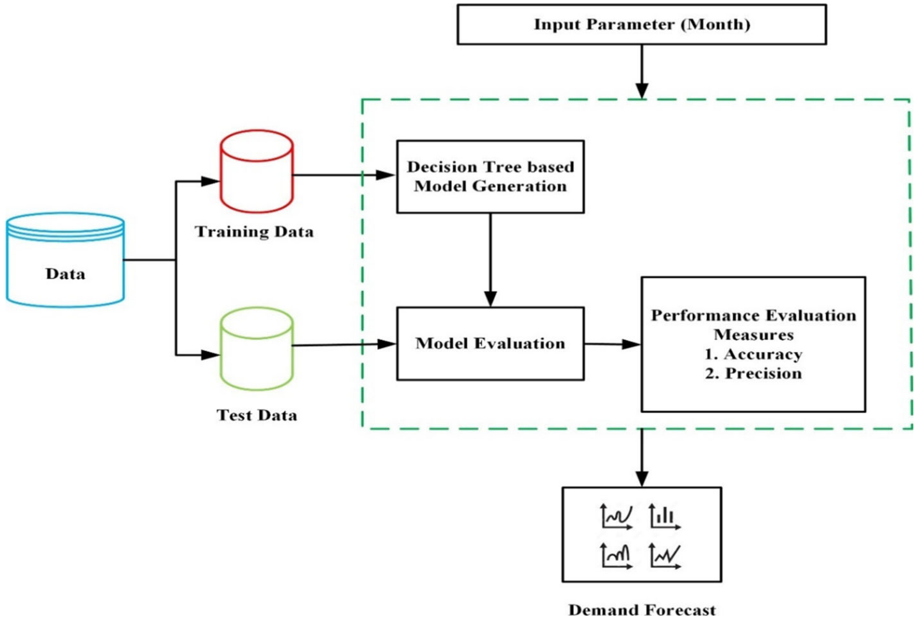

Fig. 2 An overview of demand forecast methodology

6 Numerical experiment

In this section, the numerical example is taken to validate the application of the model. The author estimates a deteriorating product seasonal demand based on data trends. The season-wise product demand data is shown in Table 2. A csv file "abc.csv" is created based on the data trend. In the next step, data is split into training and testing data. The Decision Tree-based Classifier method is used for forecasting the demand. 80% of the data is used for training purposes, and the rest 20% is used for testing. Finally, the direct month-wise forecasted demand is obtained by giving the month value as the input parameter, as shown in Fig. 3.

Example 1 The complete initial data for numerical example is as follows: $O = 2500, h = 5, C = 10, d_c = 2, S_c = 0.6, x = 100, T_f = 100, T_v = 0.01, s = 5, l = 1.5, \beta = 0.8, C_d = 0.0005, d = 10, (\widetilde{\theta}_1, \widetilde{\theta}_2, \widetilde{\theta}_3, \widetilde{\theta}_4) = (0.3, 0.31, 0.32, 0.34), (\widetilde{k}_1, \widetilde{k}_2, \widetilde{k}_3, \widetilde{k}_4) = (0.01, 0.02, 0.03, 0.04), C_X = 70, g = 15, E_e = 0.0006, w = 0.01, f_1 = 27, f_2 =$

Table 2 Season-wise product demand

Season name	Month	Product demand (D) (unit)
Winter	12, 1	85–95
Spring	2, 3	75–85
Summer	4, 5, 6	45–55
Monsoon	7, 8, 9	50–60
Autumn	10, 11	55–75

```
In [4]: # importing libraries
        import pandas as pd
        from sklearn.tree import DecisionTreeClassifier
        from sklearn.model_selection import train_test_split
        # from sklearn.externals import joblib
        from sklearn import tree

        # loading csv data
        hobby_data = pd.read_csv('abc.csv')
        X = hobby_data.drop(columns =['Demand'])
        y= hobby_data['Demand']
        X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.20)

        # modelling prediction
        model = DecisionTreeClassifier()
        model.fit(X_train, y_train)

        # make predictions
        X_test_new=[[1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12]]
        prediction = model.predict(X_test_new)
        prediction

Out[4]: array([86, 81, 83, 54, 48, 47, 56, 52, 54, 59, 57, 93], dtype=int64)
```

Fig. 3 Monthly demand output after tested with 20% data

0.57, $and A_{fe} = 0.003$. These numerical data is adopted form (Mishra & Mishra, 2022; Mishra et al., 2013) with some modification.

After getting month-wise demand, the outcome of the optimal total cost for an individual month per unit time for example 1 is depicted in Table 3. Due to highly non-linear cost function, we have used MAPLE software to determine optimal values. The average of the optimal total cost, replenishment period and optimal ordering quantity are $TC_d = \$1145.2$,

Table 3 Optimal total cost for an individual month

Month	Cycle length (T)	Optimal ordering quantity (Q) (units)	Total carbon emission cost (T_{ce}) ($)	Optimal total cost (TC_d) ($)
1	4.31	42	64.30	1353.18
2	4.46	44	65.02	1309.70
3	4.40	43	64.73	1327.24
4	5.56	47	69.55	1049.85
5	5.90	47	70.74	984.69
6	5.97	47	70.94	973.49
7	5.44	47	69.17	1070.86
8	5.66	47	69.94	1028.50
9	5.56	47	69.55	1049.85
10	5.30	46	68.62	1101.75
11	5.39	47	68.99	1081.24
12	4.12	41	63.37	1412.10

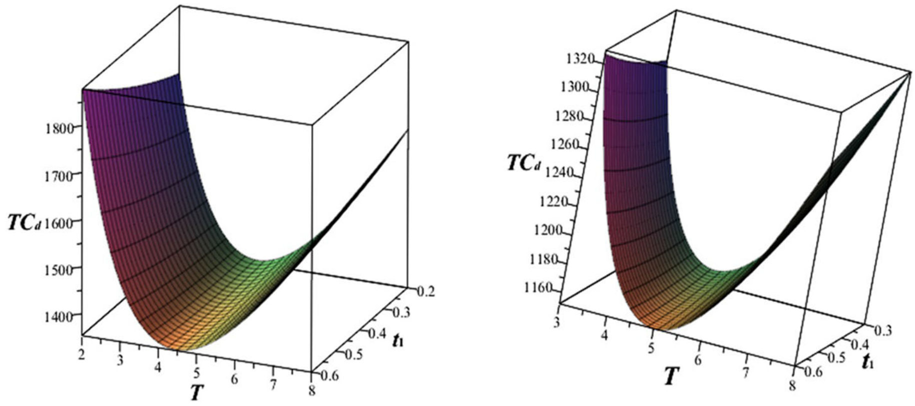

Fig. 4 Convexity of total cost function (TC_d) for month first ($D = 86$) and mean value of demand versus T and t_1

$T = 5.17$ and $Q = 45$ unit. The mean of seasonal product demand is $D = 64$ units. the optimal total cost, replenishment period and optimal ordering quantity at the mean value of demand are $TC_d = \$1152$, $T = 5.074$ and $Q = 46$ units. The total carbon emission cost at the mean value of demand is $T_{ce} = \$67.73$. Fig. 4

Example 2 The data for the numerical example is as follows: $O = 100, h = 5, C = 2, d_c = 2, S_c = 0.6, x = 200, T_f = 100, T_v = 0.01, s = 5, l = 2, \beta = 0.6, C_d = 0.0006, A_{fe} = 0.002, (\widetilde{\theta}_1, \widetilde{\theta}_2, \widetilde{\theta}_3, \widetilde{\theta}_4) = (0.05, 0.051, 0.052, 0.054), \left(\widetilde{k}_1, \widetilde{k}_2, \widetilde{k}_3, \widetilde{k}_4\right) = (0.01, 0.02, 0.03, 0.04), g = 15, E_e = 0.0006, w = 0.001, f_1 = 10, f_2 = 2.6, d = 6, and\ C_X = 10$. The following numerical example is taken from (Mishra & Mishra, 2022; Taleizadeh et al., 2020) with some modification.

The seasonal demand data for Example 2 is same as mention in Table 2 and the monthly demand output after tested with 20% of the data is shown in Fig. 3. Now, the average of the optimal total cost, replenishment period and optimal ordering quantity are $TC_d = \$312.19$, $T = 1.75$ and $Q = 22$ unit. The mean of seasonal product demand is $D = 64$ units. The optimal total cost, replenishment period and optimal ordering quantity at the mean value of demand are $TC_d = \$315$, $T = 1.72$ and $Q = 22$ units.

The above analysis (Examples 1 and 2) shows that when seasonal forecasted demand is used instead of fixed demand, there is a significant reduction in the overall cost. Therefore, seasonal demand will be more beneficial for our inventory system than fixed demand. In summary, these numerical examples demonstrate the practical applicability of the proposed model and its potential to improve stock management decisions for firms.

7 Sensitivity analysis and managerial insights

7.1 Sensitivity analysis

This section discusses the effect of changing the parameter's value on the model. In the sensitivity study, a set of parameters $\left\{O, h, C, d_c, S_c, x, T_f, T_v, s, l, \beta, C_d, A_{fe}, C_X\right\}$ decrease and increase by 20% are carried out. The outcomes of the analysis are depicted in Table 4

Table 4 Sensitivity of cost parameters $\left(O, h, C, d_c, S_c, x, T_f, T_v, s, l, \beta, C_d, A_{fe}, C_X\right)$ with 20% variation

Parameters	Value	% Change	T	t_1	Total cost (TC_d)	Parameters	Value	% Change	T	t_1	Total cost (TC_d)
O	1500	−40	3.27	0.18	1089.71	h	3	−40	4.41	0.57	1346.90
	2000	−20	3.83	0.33	1230.45		4	−20	4.35	0.50	1350.36
	2500	0	4.31	0.46	1353.18		5	0	4.31	0.46	1353.18
	3000	+20	4.74	0.56	1463.55		6	+20	4.28	0.41	1355.54
	3500	+40	5.13	0.66	1564.76		7	+40	4.25	0.38	1357.54
C	6	−40	4.52	0.83	1297.68	d_c	1.2	−40	4.32	0.47	1352.58
	8	−20	4.43	0.64	1329.55		1.6	−20	4.32	0.46	1352.88
	10	0	4.31	0.46	1353.18		2	0	4.31	0.46	1353.18
	12	+20	4.18	0.27	1368.98		2.4	+20	4.31	0.45	1353.47
	14	+40	4.03	0.10	1377.17		2.8	+40	4.30	0.44	1353.76
S_c	0.36	−40	4.33	0.48	1350.77	x	60	−40	4.31	0.45	1353.60
	0.48	−20	4.32	0.47	1351.99		80	−20	4.31	0.45	1353.35
	0.60	0	4.31	0.46	1353.18		100	0	4.31	0.46	1353.18
	0.72	+20	4.30	0.44	1354.35		120	+20	4.31	0.46	1353.08
	0.84	+40	4.30	0.43	1355.49		140	+40	4.31	0.46	1353.00
T_f	60	−40	4.28	0.45	1343.87	T_v	0.006	−40	4.31	0.45	1352.93
	80	−20	4.29	0.45	1348.54		0.008	−20	4.31	0.46	1353.06

Table 4 (continued)

Parameters	Value	% Change	T	t_1	Total cost (TC_d)	Parameters	Value	% Change	T	t_1	Total cost (TC_d)
	100	0	4.31	0.46	1353.18		0.010	0	4.31	0.46	1353.18
	120	+20	4.33	0.46	1357.81		0.012	+20	4.31	0.46	1353.31
	140	+40	4.35	0.46	1362.41		0.014	+40	4.31	0.46	1353.43
s	3	−40	5.22	0.15	1071.37	l	0.9	−40	4.31	0.45	1343.95
	4	−20	4.68	0.32	1224.64		1.2	−20	4.31	0.45	1348.57
	5	0	4.31	0.46	1353.18		1.5	0	4.31	0.46	1353.18
	6	+20	4.04	0.56	1463.74		1.8	+20	432	0.46	1357.79
	7	+40	3.84	0.65	1560.49		2.1	+40	4.32	0.46	1362.40
β	0.48	−40	5.26	0.20	1111.23	C_X	42	−40	4.30	0.45	1347.20
	0.64	−20	4.69	0.34	1243.80		56	−20	4.31	0.46	1350.20
	0.80	0	4.31	0.46	1353.18		70	0	4.31	0.46	1353.18
	0.96	+20	4.03	0.55	1445.95		84	+20	4.32	0.45	1356.16
							98	+40	4.32	0.45	1359.13
C_d	0.0003	−40	4.31	0.45	1353.17	A_{fe}	0.0018	−40	4.29	0.45	1347.87
	0.0004	−20	4.31	0.46	1353.18		0.0024	−20	4.30	0.45	1350.53
	0.0005	0	4.31	0.46	1353.18		0.0030	0	4.31	0.46	1353.18
	0.0006	+20	4.31	0.46	1353.19		0.0036	+20	4.32	0.46	1355.83
	0.0007	+40	4.31	0.46	1353.20		0.0042	+40	4.33	0.46	1358.47

and Figs. 5, 6, 7, 8, 9, 10 and 11. The numerical data of Example 1 presents the initial value of all the parameters for the sensitivity analysis.

The following highlights can be concluded from this sensitivity study.

(i) The sensitivity of cost parameters $(O, C), and(h, d_c)$ are shown in Table 4. It shows that when the percentage change in parameters (O, C) $and(h, d_c)$ is increased, the total cost (TC_d) increases while the replenishment period *(T)* increases in O and decreases in rest parameters. From Figs. 5 and 6, it can be concluded that the parameter O is highly sensitive than the rest.

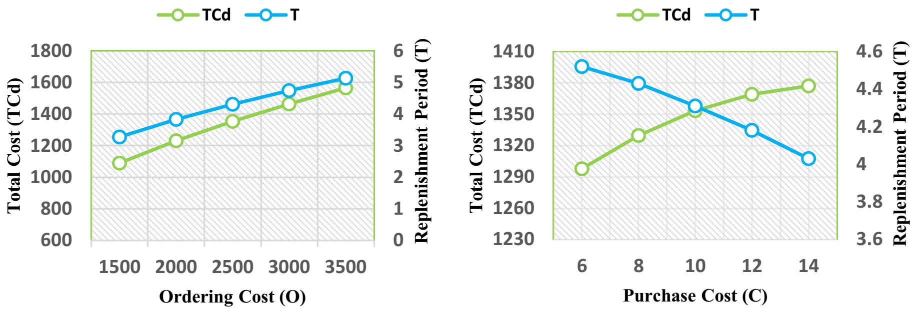

Fig. 5 Sensitivity study when ordering and purchase cost are changed

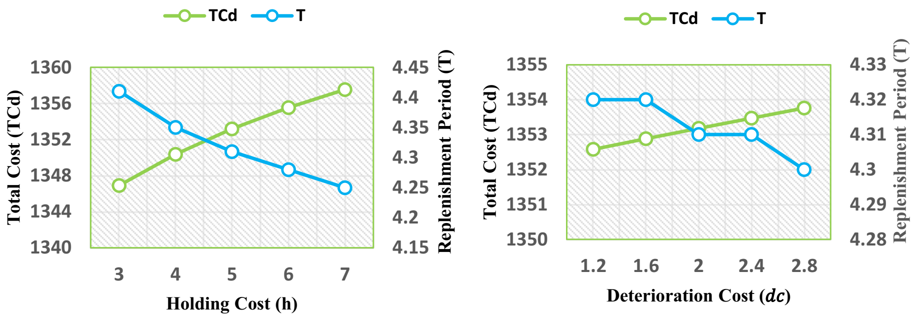

Fig. 6 Sensitivity study when holding and deterioration cost are changed

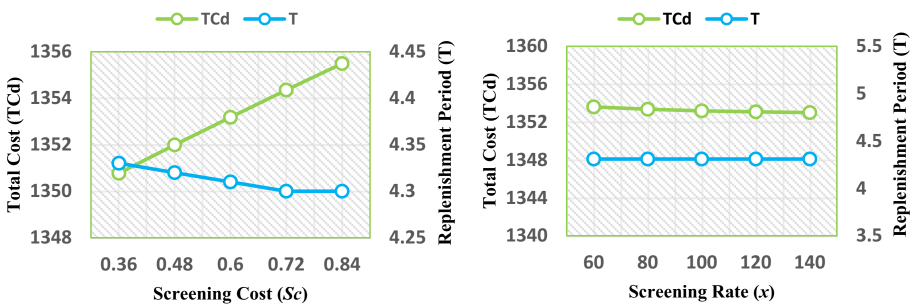

Fig. 7 Sensitivity study when screen cost and screening rate are changed

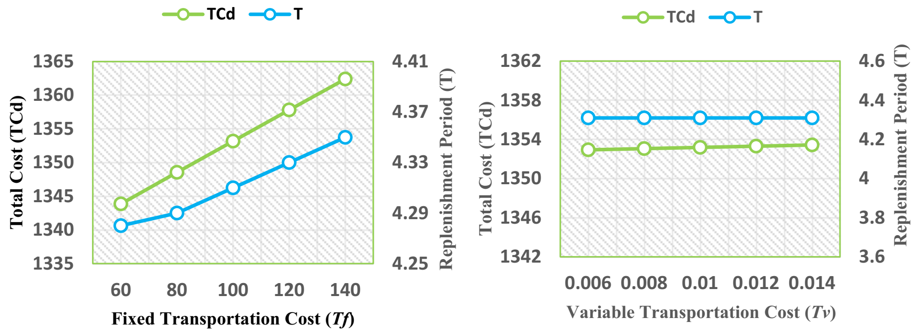

Fig. 8 Sensitivity study when fixed and variable transportation cost are changed

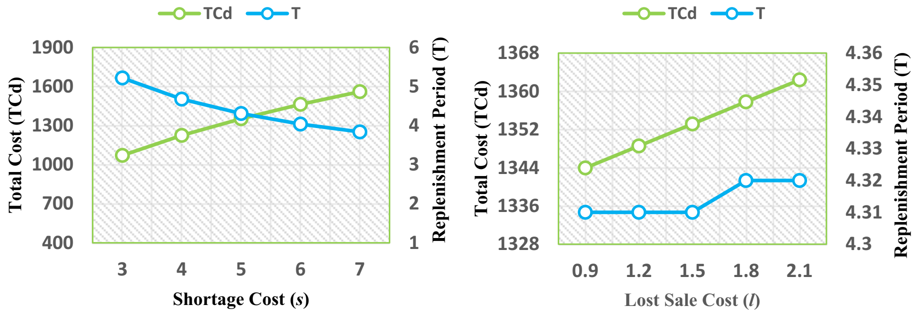

Fig. 9 Sensitivity study when shortage and lost sale cost are changed

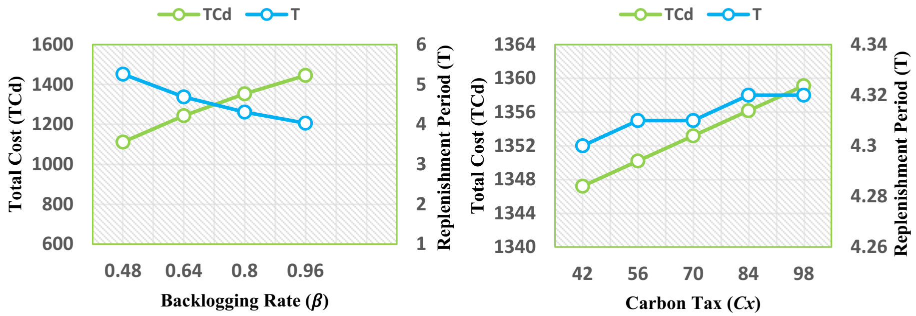

Fig. 10 Sensitivity study when backlogging rate and carbon tax are changed

(ii) The sensitivity table and graphical analysis of screening (S_c, x) and transportation $\left(T_f, T_v\right)$ parameters are shown in Table 4 and Figs. 7 and 8. On increasing the percentage change in parameters, the variation in the total cost (TC_d) and the replenishment period *(T)* at a meagre rate is observed. It means that the screening and transportation parameters are less sensitive.

(iii) On increasing the parameter values (s, l, β), a large variation in total cost (TC_d) is observed in s and β while less in l as shown in Table 4. From the graphical analysis

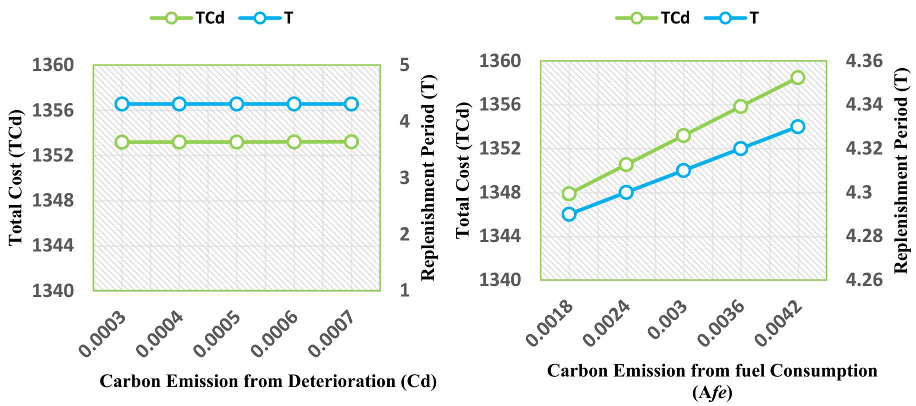

Fig. 11 Sensitivity study when carbon emission from deterioration and fuel consumption are changed

Figs. 9 and 10, it can be observed that parameters s and β are susceptible, whereas l is less sensitive.

(iv) All the carbon emission parameters are significantly less sensitive because when the percentage change in the parameter $\left(C_d, A_{fe}, C_X\right)$ is increased, a small change is observed in total cost (TC_d) and replenishment period (T) as depicted in Figs. 10 and 11.

In summary, this sensitivity analysis presents useful insights into the sensitivity of various parameters, highlighting which variables have a major impact on the total cost and replenishment period of the system under consideration.

7.2 Managerial insights

This research allows inventory managers to manage situations like seasonal demand fluctuations better. The article emphasises the necessity of precisely forecasting seasonal demand to efficiently manage inventories and optimize costs in various businesses. Machine learning approaches, such as decision tree classifiers, may considerably enhance the accuracy of demand forecasts. One of the important managerial insights from this study is that relying on fixed demand assumptions can be costly. Businesses can significantly reduce overall costs by incorporating seasonal forecasted demand instead of fixed demand. For example, consider a retail company that sells fruits and vegetables. Traditionally, they may order a fixed quantity of inventory throughout the year, which leads to surpluses during the off-season and shortages during high demand. However, the company's manager can modify their ordering amounts and replenishment periods appropriately by employing machine learning to precisely forecast seasonal demand trends. As a result, companies can optimize their inventory levels, reduce the expenses associated with surplus inventory, prevent stock-outs, and improve the customer experience. The results illustrate the optimal policy for organizations to handle inventory systems with imperfect deteriorating products. The policymaker would want to reduce the overall cost. As a result, they would have more control over the more sensitive parameters concerning total cost. Businesses may improve their operational efficiency by using seasonal demand estimates, optimising ordering amounts, and detecting sensitive cost factors. Therefore, implementing these findings in practical situations, such as the retail and pharmaceutical industries, can lead to tangible advantages in terms of improved management policy and profitability.

8 Conclusion

This article develops a more realistic inventory model for imperfect deteriorating products by considering a Machine Learning approach for seasonally demand forecast. This study also considers deterioration rate and defective percentage quantity as fuzzy variables due to uncertainty, wherein the shortages are permitted and are partially backlogged. A Screening process has been done to weed out defective products from perfect products. The product demand is a crucial parameter for any business, which varies seasonally. Therefore, seasonal forecasted demand is examined by the Decision Tree-based Classifier method. The outcome displays how the direct month-wise predicted demand for the deteriorating product is generated by making the month value an input parameter. This helps organizations/managers better manage their inventory according to forecasted demand. For defuzzification, the Sign Distance method is employed to find the optimal replenishment period and optimal ordering quantity to minimize the total average cost with emission cost. The numerical experiment has verified the mathematical model. The results show that the overall cost is significantly reduced when forecasted seasonal demand is implemented rather than fixed demand. The sensitivity study identifies which parameters are more sensitive, indicating that the management should pay more attention to these parameters. The convexity of the total cost function is shown graphically due to the highly non-linear cost function.

The present model is extended by considering a different approach to forecast demand like (Autoregressive integration moving average (ARIMA), XGBoost classifier method etc.). A comparison study can also be done between distinct forecasting methods. This model can be further extended by taking the type-2 fuzzy variable for uncertain parameters.

Declarations

Conflict of interest There is no known existing conflict of interest to the authors.

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