Derivative Pricing - Lecture 4

17 March 2018

1 Preliminaries

Open *Spider* or any other Python IDE and import the principal libraries to use (numpy, pylab,...)

2 Extracting ZC from a Market Curve

1) Load the IR Market Curve. To do so, define the path of the excel file "IRMarketCurve.xslx" and run the code below. Print "IRCurve" to ensure it is loaded correctly.

```
import xlrd

path = ...
book = xlrd.open_workbook(path)
sheet = book.sheet_by_index(0)

IRCurve = []
for row_index in range(sheet.nrows):
   row= ""
   Lign = []
   for col_index in range(sheet.ncols):
      value = sheet.cell(rowx=row_index,colx=col_index).value
      Lign.append(value)
   IRCurve.append(Lign)
```

2a) Build the Zero-Coupon value and rate curve from this IR Market Curve. To do so, fill the function "extractZCCurve" which is predefined then run it on IRCurve.

2b) Plot on the same graph both the Zero-Coupon and the Market rates (Eonia,Libor,Swaps) for different maturities. Also plot the Zero-Coupon values.

3a) Price a swap of maturity 10 years for a fixed rate equals to the swap rate of the same maturity. The notional is assumed to be 1 and the frequency of the fixed leg is 1 year. What can be said about the price and why ?

3b) Price a swap of maturity 20 years for a fixed rate equals to the swap rate + 50bps of the same maturity. The notional is assumed to be 1 and the frequency of the fixed leg is 1 year.

4) Compute the sensitivities of the 2 swaps defined above with respect to parallel shift of the IR Curve (1% shift).

3 Vasicek Model

In the Vasicek model, the dynamic of the instantaneous interest rate r_t under the risk-neutral probability is given by the SDE :

$$dr_t = a(\theta - r_t)dt + \sigma dW_t \text{ with } r_0 = r$$

The parameter a is the mean reversion speed, θ is the long term expectation of interest rate, σ is the volatility and r is the initial instantaneous rate.

3.1 Zero-Coupon Bond valuation within model

1) Implement the closed-form formula for Zero-Coupon provided below. Compute the price of a Zero-Coupon of 10 year maturity with the parameters $a = 0.5$, $\theta = 0.04$, $\sigma = 0.015$ and $r = 0.02$.

$$ZC(t,T) = A(t,T)\exp\left(-B(t,T)r_t\right)$$

where :

$$A(t,T) = \exp\left(\left(\theta - \frac{\sigma^2}{2a^2}\right)(B(t,T) - T + t) - \frac{\sigma^2}{4a}B(t,T)^2\right)$$

$$B(t,T) = \frac{1}{a}\left(1 - \exp(-a(T-t))\right)$$

2) Ensure the implementation is correct by computing the Zero-Coupon value using Monte-Carlo methods. Run the Monte-Carlo price for the same Zero-Coupon bond and the same parameters as for the previous question. Use 10'000 Monte-Carlo simulations and 100 time-steps.

3) Print both ZC extracted from Market and the value obtained from the model for all the different maturities.

3.2 Model Calibration

1) Load the function *root* from the scipy.optimize library.

2) Run the help for the root function to get more information regarding how it could/should be used.

3) Implement the following loss function :

$$L(a, \theta, \sigma, r) = \sum_{i=1}^{N} (ZC^{Mkt}(0, T_i) - ZC(0, T_i))^2$$

where T_i are all the available maturities of the Zero-Coupon rate curve extracted from the IR Market Curve and $ZC^{Mkt}(0, T_i)$ is the correspondong Zero-Coupon Bond value.

To avoid having a negative a parameter, perform a transformation by and exponential function.

4) Perform the calibration of the Vasicek model, *i.e.* find the set of parameters such as the function L is as close as possible to 0 using root and Loss function.

5) Plot ZC values for both Market and Model with the calibrated parameters. Plot ZC rates. What can you say about the quality of calibration ?

3.3 Swaption Pricing with Monte-Carlo

A swaption is a contract giving its buyer the right to enter, at a given date in the future, into a swap position with a given maturity, fixed rate (which is called the strike), *etc.*. The date at which the option can be exercised is called the *maturity* of the swaption. The maturity of the underlying swap is called the *tenor*.

1) Assuming the maturity of the swaption is T_0 and that the underlying swap exchange a fixed rate K against a floating rate at the same frequency with observation dates $T_0, ..., T_{N-1}$ and payment dates $T_1, ..., T_N$, what is the formula for the payoff of the swaption (assuming a notional of 1) ?

2) Write down the theoretical formula of the swaption price (risk-neutral expectation of discounted cash-flows).

3) Implement a Monte-Carlo method to obtain the swaption price.

4) Apply the algorithm to price a swaption of maturity 10 year, tenor 10 year and a strike of 2%.