5.1

In HIRE-ASSISTANT, assuming that the candidates are presented in a random order, what is the probability that you hire exactly one time? What is the probability that you hire exactly n times?

Solution:

The boss hires exactly n times to find the best person, which means that the order of previous $(n-1)$ people interviewed is fixed, saying that only the n th person selected is best with only 1 permutation. Since the number of the combination of selection is $n!$. Thus $P(\text{hire exact } n \text{ times}) = \frac{1}{n!}$.

5.2

Use indicator random variables to computer the expected value of the sum of n dice.

Solution:

Let $X_i = I\{\text{a dice rolling up } i\} \Rightarrow X = \sum_{i=1}^{6} X_i$

In terms of one dice,

$$E(X) = E\left(\sum_{i=1}^{6} X_i\right) = \sum_{i=1}^{6} E(X_i) = \frac{1}{6}\sum_{i=1}^{6} i$$

Thus, in terms of n dice,

$$E(nX) = nE(X) = \frac{n}{6}\sum_{i=1}^{6} i = \frac{7}{2}n$$

5.3

Consider the PERMUTE-WITH-ALL procedure on the facing page, which instead of swapping element $A[i]$ with a random element from the subarray $A[i:n]$, swaps it with a random element from anywhere in the array. Does PERMUTE-WITH-ALL produce a uniform random permutation? Why or why not?

```
PERMUTE-WITH-ALL(A,n){
    for i=1 to n
        swap A[i] with A[RANDOM(1,n)];
}
```

Solution:

It cannot produce a unique permutation. Only a fake example can prove this.

Let $n=3$, from math we can obtain that the total number of permutation is $A_3^3=6$.

For convenience, let the initial permutation be $P_0=\{A[1],A[2],A[3]\}$.

Running the procedure,

when $n=1$, swap($A[1],A[\text{Random}(1,3)]$), which produces 3 intermediate states.

Notice that after a 'swap' operation, the value of $A[1]$, $A[2]$, and $A[3]$ will change.

For example, let $A[\text{Random}(1,3)]=A[2]$ when $n=1$, thus the value of $A[1]$ and $A[2]$ will be swapped, which means a new permutation

$P_1=\{A[2],A[1],A[3]\}$,

where $A[2]=(A[1])_{\text{new}}$, $A[1]=(A[2])_{\text{new}}$ and $A[3]=A[3]_{\text{new}}$. Therefore, we can know that these steps are related each other, because each choice in the previous step will have effect on the next one, which satisfies the principle of multiplication in counting principles. Thus, we have $3^3=27$ ending states in the end.

In order to get unique permutation, we need to verify the divisibility. 6 does not divide 27, so this procedure cannot get a unique permutation.