Assignment 1

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(All the results that are numerical and **imprecise** are expressed with only 2 digits in floating part.)

Exercise 1.3

Consider three different processors P1, P2, and P3 executing the same instruction set with the clock rates and CPIs given in the following table.

	Processor	Clock Rate	CPI
a.	P1	3 GHz	1.5
	P2	2.5 GHz	1.0
	P3	4 GHz	2.2
b.	P1	2 GHz	1.2
	P2	3 GHz	0.8
	P3	4 GHz	2.0

Figure 1(1.3.1-1.3.3)

(1) Which processor has the highest performance expressed instructions per second?

Solution:

In the situation where three processors execute the same instruction set, the total instruction set is the same, denoted by I, in group **a**:

P1:

$T_1 = \frac{1}{f_1} = \frac{1}{3GHz} = \frac{1}{3\times10^9}\text{s}$, use CPI to calculate the time needed to execute a instruction.

$$t_1 = CPI \cdot T_1 = 1.5 \cdot \frac{1}{3\times10^9} = 5\times10^{-10}\text{s} ,$$

$$p_1 = \frac{I}{t_1} = \frac{I}{5\times10^{-10}\text{s}} = 2\times10^9 I \text{ instruction/s}$$

P2:

$$T_2 = \frac{1}{f_2} = \frac{1}{2.5GHz} = \frac{1}{2.5\times10^9}\text{s}, \ t_2 = CPI \cdot T_2 = 4\times10^{-10}\text{s}$$

$$p_2 = \frac{I}{t_2} = \frac{I}{4\times10^{-10}}\text{s} = 2.5\times10^9 I \text{ instruction/s}$$

P3:

$$T_3 = \frac{1}{f_3} = \frac{1}{4GHz} = \frac{1}{4\times10^9}\text{s}, \ t_3 = CPI \cdot T_3 = 2.2 \cdot \frac{1}{4\times10^9} = 5.5\times10^{-10}\text{s}$$

$$p_3 = \frac{I}{t_3} = \frac{I}{5.5\times10^{-10}} = \frac{1}{5.5}\times10^{10} I \text{ instruction/s}$$

In group B:

P1:

$$T_1 = \frac{1}{f_1} = \frac{1}{2\times10^9}\text{s},\ t_1 = CPI\cdot T_1 = 1.2\times\frac{1}{2\times10^9} = 6\times10^{-10}\text{s}$$

$$p_1 = \frac{I}{t_1} = \frac{I}{6\times10^{-10}\text{s}} = \frac{1}{6}\times10^{10}I$$

P2:

$$T_2 = \frac{1}{f_2} = \frac{1}{3\times10^9}\text{s},\ t_2 = CPI\cdot T_2 = 0.8\times\frac{1}{3\times10^9} = 2.67\times10^{-10}\text{s}$$

$$p_2 = \frac{I}{t_2} = \frac{I}{2.67\times10^{-10}} = \frac{1}{2.67}\times10^{10}I$$

P3:

$$T_3 = \frac{1}{f_3} = \frac{1}{4GHz} = \frac{1}{4\times10^9}\text{s},\ t_3 = CPI\cdot T_3 = 2.0\cdot\frac{1}{4\times10^9} = 5\times10^{-10}\text{s}$$

$$p_3 = \frac{I}{t_3} = \frac{I}{5\times10^{-10}} = \frac{1}{5}\times10^{10}I$$

Thus, in both group **a** and **b**, the 2nd processor has the highest performance.

(2) If the processors each execute a program in 10 seconds, find the number of cycles and the number of instructions.

Solution:

(The number of cycles and instructions for each processor are denoted by c_i and I_i)

From the definition of CPU time, we can obtain that

$$time = clock\ cycle\cdot cycle\ time = \frac{clock\ cycle}{clock\ rate}$$

$$(\text{since } cycle\ time = \frac{1}{clock\ rate})$$

In group **a**:

P1:

$$c_1 = tf_1 = 10\times3\times10^9 = 3\times10^{10},\ I_1 = \frac{c_1}{CPI} = \frac{3\times10^{10}}{1.5} = 2\times10^{10}$$

P2:

$$c_2 = tf_2 = 10\times2.5\times10^9 = 2.5\times10^{10},\ I_2 = \frac{c_2}{CPI} = \frac{2.5\times10^{10}}{1} = 2.5\times10^{10}$$

P3:

$$c_3 = tf_3 = 10 \times 4 \times 10^9 = 4 \times 10^{10}, \ I_3 = \frac{c_3}{CPI} = \frac{4 \times 10^{10}}{2.2} = 1.82 \times 10^{10}$$

In group **b**:

P1:

$$c_1 = tf_1 = 10 \times 2 \times 10^9 = 2 \times 10^{10}, \ I_1 = \frac{c_1}{CPI} = \frac{2 \times 10^{10}}{1.2} = 1.67 \times 10^{10}$$

P2:

$$c_2 = tf_2 = 10 \times 3 \times 10^9 = 3 \times 10^{10}, \ I_2 = \frac{c_2}{CPI} = \frac{3 \times 10^{10}}{0.8} = 3.75 \times 10^{10}$$

P3:

$$c_3 = tf_3 = 10 \times 4 \times 10^9 = 4 \times 10^{10}, \ I_3 = \frac{c_3}{CPI} = \frac{4 \times 10^{10}}{2.0} = 2 \times 10^{10}$$

(3) We are trying to reduce the time(10s) by 30% but this leads to an increase of 20% in the CPI. What clock rate should we have to get this time reduction?

Solution:

Under this specified case($t_n = 7\text{s}$), using the formula obtained just now to find clock rate.(The new CPI for each processor is denoted by CPI_n, and the new clock rate for each processor is denoted by f_{in})

In group **a**:

P1: $CPI_n = 1.8, \ f_{1n} = \dfrac{c_{1n}}{t_n} = \dfrac{I_1 \cdot CPI_n}{t_n} = \dfrac{2 \times 10^{10} \times 1.8}{7} = 5.14GHz$

P2: $CPI_n = 1.2, \ f_{2n} = \dfrac{c_{2n}}{t_n} = \dfrac{I_2 \cdot CPI_n}{t_n} = \dfrac{2.5 \times 10^{10} \times 1.2}{7} = 4.29GHz$

P3: $CPI_n = 2.64, \ f_{3n} = \dfrac{c_{3n}}{t_n} = \dfrac{I_3 \cdot CPI_n}{t_n} = \dfrac{1.82 \times 10^{10} \times 2.64}{7} = 6.864GHz$

In group **b**:

P1: $CPI_n = 1.44, \ f_{1n} = \dfrac{c_{1n}}{t_n} = \dfrac{I_1 \cdot CPI_n}{t_n} = \dfrac{1.67 \times 10^{10} \times 1.44}{7} = 3.44GHz$

P2: $CPI_n = 0.96, \ f_{2n} = \dfrac{c_{2n}}{t_n} = \dfrac{I_2 \cdot CPI_n}{t_n} = \dfrac{3.75 \times 10^{10} \times 0.96}{7} = 5.14GHz$

P3: $CPI_n = 2.4, \ f_{3n} = \dfrac{c_{3n}}{t_n} = \dfrac{I_3 \cdot CPI_n}{t_n} = \dfrac{2 \times 10^{10} \times 2.4}{7} = 6.86GHz$

	Processor	Clock Rate	No. Instructions	Time
a.	P1	3 GHz	20.00E+09	7 s
	P2	2.5 GHz	30.00E+09	10 s
	P3	4 GHz	90.00E+09	9 s
b.	P1	2 GHz	20.00E+09	5 s
	P2	3 GHz	30.00E+09	8 s
	P3	4 GHz	25.00E+09	7 s

Figure 2(1.3.4-1.3.6)

(4) Find the IPC (instructions per cycle) for each processor.

Solution:

To find the number of instructions in each cycle, with the given time and clock rate, the clock period can be found so that the clock cycles can also be found as well.

$$cycle = \frac{time}{clock\ period} = time \cdot clock\ rate$$

$$IPC = \frac{I}{cycle} = \frac{I}{time \cdot clock\ rate}$$

In group **a**:

P1: $IPC = \dfrac{2 \times 10^{10}}{7 \times 3 \times 10^{9}} = 0.95$

P2: $IPC = \dfrac{3 \times 10^{10}}{10 \times 2.5 \times 10^{9}} = 1.2$

P3: $IPC = \dfrac{9 \times 10^{10}}{9 \times 4 \times 10^{9}} = 2.5$

In group **b**:

P1: $IPC = \dfrac{2 \times 10^{10}}{5 \times 2 \times 10^{9}} = 2$

P2: $IPC = \dfrac{3 \times 10^{10}}{8 \times 3 \times 10^{9}} = 1.25$

P3: $IPC = \dfrac{2.5 \times 10^{10}}{7 \times 4 \times 10^{9}} = 0.89$

(5) Find the clock rate for P2 that reduces its execution time to that of P1.

Solution:

Only for P2 in two groups individually. We can obtain that the total cycles for each processor is constant.

In group **a**:

$$c_2 = t_2 f_2 = 10 \times 2.5 \times 10^9 = 2.5 \times 10^{10},\ f_{2n} = \frac{c_2}{t_1} = \frac{2.5 \times 10^{10}}{7} = 3.57GHz$$

In group **b**:

$$c_2 = t_2 f_2 = 8 \times 3 \times 10^9 = 2.4 \times 10^{10}, \;\; f_{2n} = \frac{c_2}{t_1} = \frac{2.4 \times 10^{10}}{5} = 4.8GHz$$

(6) Find the number of instructions for P2 that reduces its execution time to that of P3.

In group **a**:

$$I_{2n} = \frac{t_3}{t_2} I_2 = \frac{9}{10} \times 3 \times 10^{10} = 2.7 \times 10^{10}$$

In group **b**:

$$I_{2n} = \frac{t_3}{t_2} I_2 = \frac{7}{8} \times 3 \times 10^{10} = 2.625 \times 10^{10}$$

Exercise 1.4

Consider two different implementations of the same instruction set architecture. There are four classes of instructions, A, B, C, and D. The clock rate and CPI of each implementation are given in the following table.

		Clock Rate	CPI Class A	CPI Class B	CPI Class C	CPI Class D
a.	P1	2.5 GHz	1	2	3	3
	P2	3 GHz	2	2	2	2
b.	P1	2.5 GHz	2	1.5	2	1
	P2	3 GHz	1	2	1	1

Figure 3(1.4.1-1.4.3)

(1) Given a program with 10^6 instructions divided into classes as follows: 10% class A, 20% class B, 50% class C, and 20% class D, which implementation is faster?

Solution:

In group **a**:

P1:

$$I_A = \frac{1}{10} I = 10^5, \;\; c_A = I_A \cdot CPI_A = 10^5,$$

$$I_B = \frac{1}{5} I = 2 \times 10^5, \;\; c_B = I_B \cdot CPI_B = 2 \times 10^5 \times 2 = 4 \times 10^5$$

$$I_C = \frac{1}{2} I = 5 \times 10^5, \;\; c_C = I_C \cdot CPI_C = 5 \times 10^5 \times 3 = 1.5 \times 10^6$$

$$I_D = \frac{1}{5} I = 2 \times 10^5, \;\; c_D = I_D \cdot CPI_D = 2 \times 10^5 \times 3 = 6 \times 10^5$$

$$
\begin{aligned}
t_1 &= t_{1A} + t_{1B} + t_{1C} + t_{1D} \\
&= T_1(c_A + c_B + c_C + c_D) \\
&= \frac{c_A + c_B + c_c + c_D}{f_1} \\
&= \frac{10^5 + 4\times 10^5 + 1.5\times 10^6 + 6\times 10^5}{2.5\times 10^9} \\
&= 1.04\times 10^{-3}\mathrm{s}
\end{aligned}
$$

P2:

$$I_A = \frac{1}{10} I = 10^5,\ c_A = I_A \cdot CPI_A = 2\times 10^5$$

$$I_B = \frac{1}{5} I = 2\times 10^5,\ c_B = I_B \cdot CPI_B = 2\times 10^5 \times 2 = 4\times 10^5$$

$$I_C = \frac{1}{2} I = 5\times 10^5,\ c_C = I_C \cdot CPI_C = 5\times 10^5 \times 2 = 10^6$$

$$I_D = \frac{1}{5} I = 2\times 10^5,\ c_D = I_D \cdot CPI_D = 2\times 10^5 \times 2 = 4\times 10^5$$

$$
\begin{aligned}
t_2 &= t_{2A} + t_{2B} + t_{2C} + t_{2D} \\
&= T_2(c_A + c_B + c_C + c_D) \\
&= \frac{c_A + c_B + c_c + c_D}{f_2} \\
&= \frac{2\times 10^5 + 4\times 10^5 + 10^6 + 4\times 10^5}{3\times 10^9} \\
&= 6.67\times 10^{-4}\mathrm{s}
\end{aligned}
$$

Thus, P2 is faster.

In group **b**:
P1:

$$I_A = \frac{1}{10} I = 10^5,\ c_A = I_A \cdot CPI_A = 2\times 10^5$$

$$I_B = \frac{1}{5} I = 2\times 10^5,\ c_B = I_B \cdot CPI_B = 2\times 10^5 \times 1.5 = 3\times 10^5$$

$$I_C = \frac{1}{2} I = 5\times 10^5,\ c_C = I_C \cdot CPI_C = 5\times 10^5 \times 2 = 10^6$$

$$I_D = \frac{1}{5} I = 2\times 10^5,\ c_D = I_D \cdot CPI_D = 2\times 10^5$$

$$
\begin{aligned}
t_1 &= t_{1A} + t_{1B} + t_{1C} + t_{1D} \\
&= T_1(c_A + c_B + c_C + c_D) \\
&= \frac{c_A + c_B + c_c + c_D}{f_1} \\
&= \frac{2\times 10^5 + 3\times 10^5 + 10^6 + 2\times 10^5}{2.5\times 10^9} \\
&= 6.8\times 10^{-4}\mathrm{s}
\end{aligned}
$$

P2:

$$I_A = \frac{1}{10} I = 10^5,\ c_A = I_A \cdot CPI_A = 10^5$$

$$I_B = \frac{1}{5} I = 2\times 10^5,\ c_B = I_B \cdot CPI_B = 2\times 10^5 \times 2 = 4\times 10^5$$

$$I_C = \frac{1}{2} I = 5\times 10^5,\ c_C = I_C \cdot CPI_C = 5\times 10^5$$

$$I_D = \frac{1}{5} I = 2\times 10^5,\ c_D = I_D \cdot CPI_D = 2\times 10^5$$

$$
\begin{aligned}
t_2 &= t_{2A} + t_{2B} + t_{2C} + t_{2D} \\
&= T_2(c_A + c_B + c_C + c_D) \\
&= \frac{c_A + c_B + c_c + c_D}{f_2} \\
&= \frac{10^5 + 4\times 10^5 + 5\times 10^5 + 2\times 10^5}{3\times 10^9} \\
&= 4\times 10^{-4}\mathrm{s}
\end{aligned}
$$

Thus, P2 is faster.

(2) What is the global CPI for each implementation?

Solution:

$$CPI = \frac{c}{I} = \frac{tf}{I}$$

In group **a**:

P1: $CPI = \dfrac{t_1 f_1}{I} = \dfrac{1.04\times 10^{-3}\times 2.5\times 10^9}{10^6} = 2.6$

P2: $CPI = \dfrac{t_2 f_2}{I} = \dfrac{6.67\times 10^{-4}\times 3\times 10^9}{10^6} = 2.001$

In group **b**:

$$\text{P1:}\ \ CPI = \frac{t_1 f_1}{I} = \frac{6.8 \times 10^{-4} \times 2.5 \times 10^9}{10^6} = 1.7$$

$$\text{P2:}\ \ CPI = \frac{t_2 f_2}{I} = \frac{4 \times 10^{-4} \times 3 \times 10^9}{10^6} = 1.2$$

(3) Find the clock cycles required in both cases.

Solution: Since CPI has been obtained for each processor in the last question, we can use it to indirectly compute the clock cycles for them.

In group **a**:

$$\text{P1:}\ \ c_1 = CPI \cdot I = 2.6 \times 10^6$$

$$\text{P2:}\ \ c_2 = CPI \cdot I = 2.001 \times 10^6 \approx 2 \times 10^6$$

In group **b**:

$$\text{P1:}\ \ c_1 = CPI \cdot I = 1.7 \times 10^6$$

$$\text{P2:}\ \ c_1 = CPI \cdot I = 1.2 \times 10^6$$

	Arith	Store	Load	Branch	Total
a.	650	100	600	50	1400
b.	750	250	500	500	2000

Figure 4(1.4.4-1.4.6, shows the number of instructions)

(4) Assuming that arith instructions take 1 cycle, load and store 5 cycles, and branches 2 cycles, what is the execution time of the program in a 2 GHz processor?

Solution:

In group **a**:

$$\begin{aligned} c_a &= c_A + c_S + c_L + c_B \\ &= I_A + 5(I_S + I_L) + 2I_B \\ &= 650 + 5 \times 700 + 2 \times 50 \\ &= 4250 \end{aligned}$$

$$t_a = \frac{c_a}{f} = \frac{4250}{2 \times 10^9} = 2.125 \times 10^{-6}\text{s}$$

In group **b**:

$$\begin{aligned} c_b &= c_A + c_S + c_L + c_B \\ &= I_A + 5(I_S + I_L) + 2I_B \\ &= 750 + 5 \times 750 + 2 \times 500 \\ &= 5500 \end{aligned}$$

$$t_b = \frac{c_b}{f} = \frac{5500}{2 \times 10^9} = 2.75 \times 10^{-6}\text{s}$$

(5) Find the CPI for the program.

In group **a**:

$$CPI = \frac{t_a f}{I_a} = \frac{2.125 \times 10^{-6} \times 2 \times 10^9}{1400} = 3.04$$

In group **b**:

$$CPI = \frac{t_b f}{I_b} = \frac{2.75 \times 10^{-6} \times 2 \times 10^9}{2000} = 2.75$$

(6) If the number of load instructions can be reduced by one half, what is the speedup and the CPI?

Solution: In this specified condition, the total instructions will change, which will be:

In group **a**:

$$\begin{aligned} c_a &= c_A + c_S + c_L + c_B \\ &= I_A + 5(I_S + I_L) + 2I_B \\ &= 650 + 5 \times 400 + 2 \times 50 \\ &= 2750 \end{aligned}$$

$$t_a = \frac{c_a}{f} = \frac{2750}{2 \times 10^9} = 1.375 \times 10^{-6}\text{s}$$

$$speedup = \frac{2.125 \times 10^{-6}\text{s}}{1.375 \times 10^{-6}\text{s}} = 1.55$$

$$CPI = \frac{t_a f}{I_a} = \frac{1.375 \times 10^{-6} \times 2 \times 10^9}{1100} = 2.5$$

In group **b**:

$$
\begin{aligned}
c_b &= c_A + c_S + c_L + c_B \\
&= I_A + 5(I_S + I_L) + 2I_B \\
&= 750 + 5 \times 500 + 2 \times 500 \\
&= 4250
\end{aligned}
$$

$$t_b = \frac{c_b}{f} = \frac{4250}{2 \times 10^9} = 2.125 \times 10^{-6}\mathrm{s}$$

$$speedup = \frac{2.75 \times 10^{-6}\mathrm{s}}{2.125 \times 10^{-6}\mathrm{s}} = 1.29$$

$$CPI = \frac{t_b f}{I_b} = \frac{2.125 \times 10^{-6} \times 2 \times 10^9}{1750} = 2.43$$

Exercise 1.5

Consider two different implementations, P1 and P2, of the same instruction set. There are five classes of instructions (A, B, C, D, and E) in the instruction set. The clock rate and CPI of each class is given below.

		Clock Rate	CPI Class A	CPI Class B	CPI Class C	CPI Class D	CPI Class E
a.	P1	2.0GHz	1	2	3	4	3
	P2	4.0GHz	2	2	2	4	4
b.	P1	2.0GHz	1	1	2	3	2
	P2	3.0GHz	1	2	3	4	3

Figure 5(1.5.1-1.5.3)

(1) Assume that peak performance is defined as the fastest rate that a computer can execute any instruction sequence. What are the peak performances of P1 and P2 expressed in instructions per second?

Solution: From the information of the question, we can learn that not all the 5 classes of instructions are necessarily executed. Thus, we can find the peak performance for each processor assuming each of them only executes the class of instructions with least CPI. (Assume the total instructions for all processors are I.)

In group **a**:

P1:

$$c_{1\min} = I$$

$$t_{1\min} = c_{1\min} T_1 = \frac{c_{1\min}}{f_1} = \frac{I}{2\times 10^9}\text{s}$$,

$$p_{1\max} = \frac{I}{t_{1\min}} = \frac{I}{\frac{I}{2\times 10^9}} = 2\times 10^9 \ \text{instruction/s}$$

P2:

$$c_{2\min} = 2I$$

$$t_{2\min} = c_{2\min} T_2 = \frac{c_{2\min}}{f_2} = \frac{I}{2\times 10^9}\text{s}$$

$$p_{2\max} = \frac{I}{t_{2\min}} = \frac{I}{\frac{I}{2\times 10^9}} = 2\times 10^9 \ \text{instruction/s}$$

In group **b**:

P1:

$$c_{1\min} = I$$

$$t_{1\min} = c_{1\min} T_1 = \frac{c_{1\min}}{f_1} = \frac{I}{2\times 10^9}\text{s}$$

$$p_{1\max} = \frac{I}{t_{1\min}} = \frac{I}{\frac{I}{2\times10^9}} = 2\times10^9 \text{ instruction/s}$$

P2:

$$c_{2\min} = I$$

$$t_{2\min} = c_{2\min} T_2 = \frac{c_{2\min}}{f_2} = \frac{I}{3\times10^9}\text{s}$$

$$p_{2\max} = \frac{I}{t_{2\min}} = \frac{I}{\frac{I}{3\times10^9}} = 3\times10^9 \text{ instruction/s}$$

(2) If the number of instructions executed in a certain program is divided equally among the classes of instructions except for class A, which occurs twice as often as each of the others, which computer is faster? How much faster is it?

Solution: In this case, we can learn that all the classes of instructions are executed in this program. (Assume the total instructions for all processors are $6I$.)

In group **a**:

P1:

$$c_1 = 2I + 2I + 3I + 4I + 3I = 14I,\ t_1 = c_1 T_1 = \frac{c_1}{f_1} = \frac{14I}{2\times10^9} = \frac{7I}{10^9}\text{s}$$

P2:

$$c_2 = 4I + 2I + 2I + 4I + 4I = 16I,\ t_2 = c_2 T_2 = \frac{c_2}{f_2} = \frac{16I}{4\times10^9} = \frac{4I}{10^9}\text{s}$$

$$\Rightarrow \frac{p_2}{p_1} = \frac{t_1}{t_2} = \frac{7}{4} \Rightarrow \text{P2 is faster, which is } \frac{7}{4} \text{ times.}$$

In group **b**:

P1:

$$c_1 = 2I + I + 2I + 3I + 2I = 10I,\ t_1 = c_1 T_1 = \frac{c_1}{f_1} = \frac{10I}{2\times10^9} = \frac{5I}{10^9}\text{s}$$

P2:

$$c_2 = 2I + 2I + 3I + 4I + 3I = 14I,\ t_2 = c_2 T_2 = \frac{c_2}{f_2} = \frac{14I}{3\times10^9}\text{s}$$

$$\Rightarrow \frac{p_2}{p_1} = \frac{t_1}{t_2} = \frac{15}{14} \Rightarrow \text{P2 is faster, which is } \frac{15}{14} \text{ times.}$$

(3) If the number of instructions executed in a certain program is divided equally among the classes of instructions except for class E, which occurs twice as often as each of the others, which computer is faster? How much faster is it?

Solution: (Assume the total instructions for all processors are $6I$.)

In group **a**:

P1:

$$c_1 = I + 2I + 3I + 4I + 6I = 16I,\ t_1 = c_1 T_1 = \frac{c_1}{f_1} = \frac{16I}{2\times 10^9} = \frac{8I}{10^9}\mathrm{s}$$

P2:

$$c_2 = 2I + 2I + 2I + 4I + 8I = 18I,\ t_2 = c_2 T_2 = \frac{c_2}{f_2} = \frac{18I}{4\times 10^9} = \frac{9I}{2\times 10^9}\mathrm{s}$$

$$\Rightarrow \frac{p_2}{p_1} = \frac{t_1}{t_2} = \frac{16}{9} \Rightarrow \text{P2 is faster, which is } \frac{16}{9} \text{ times.}$$

In group **b**:

P1:

$$c_1 = I + I + 2I + 3I + 4I = 11I,\ t_1 = c_1 T_1 = \frac{c_1}{f_1} = \frac{11I}{2\times 10^9}\mathrm{s}$$

P2:

$$c_2 = I + 2I + 3I + 4I + 6I = 16I,\ t_2 = c_2 T_2 = \frac{c_2}{f_2} = \frac{16I}{3\times 10^9}\mathrm{s}$$

$$\Rightarrow \frac{p_2}{p_1} = \frac{t_1}{t_2} = \frac{33}{32} \Rightarrow \text{ P2 is faster, which is } \frac{33}{32} \text{ times.}$$

		No. Instructions				
		Compute	Load	Store	Branch	Total
a.	Program1	600	600	200	50	1450
b.	Program 2	900	500	100	200	1700

Figure 6(1.5.4-1.5.6)

(4) Assuming that computes take 1 cycle, loads and store instructions take 10 cycles, and branches take 3 cycles, find the execution time on a 3 GHz MIPS processor.

Solution:

In group **a**:

$$c_1 = 8750,\ t_1 = c_1 T = \frac{c_1}{f} = \frac{8750}{3\times 10^9} = 2.92\times 10^{-6}\mathrm{s}$$

In group **b**:

$$c_2 = 7500,\ t_2 = c_2 T = \frac{c_2}{f} = \frac{7500}{3\times 10^9} = 2.5\times 10^{-6}\mathrm{s}$$

(5) Assuming that computes take 1 cycle, loads and store instructions take 2 cycles, and branches take 3 cycles, find the execution time on a 3 GHz MIPS processor.

Solution:

In group **a**:

$$c_1 = 2350,\ t_1 = c_1 T = \frac{c_1}{f} = \frac{2350}{3 \times 10^9} = 7.83 \times 10^{-7}\text{s}$$

In group **b**:

$$c_2 = 2700,\ t_2 = c_2 T = \frac{c_2}{f} = \frac{2700}{3 \times 10^9} = 9 \times 10^{-7}\text{s}$$

(6) Assuming that computes take 1 cycle, loads and store instructions take 2 cycles, and branches take 3 cycles, what is the speedup if the number of compute instruction can be reduced by one-half?

Solution:

In group **a**:

$$c_1 = 2050,\ t_{1n} = c_1 T = \frac{c_1}{f} = \frac{2050}{3 \times 10^9} = 6.83 \times 10^{-7}\text{s}$$

$$speedup = \frac{t_1}{t_{1n}} = \frac{7.83 \times 10^{-7}}{6.83 \times 10^{-7}} = 1.15$$

In group **b**:

$$c_2 = 2250,\ t_{2n} = c_2 T = \frac{c_2}{f} = \frac{2250}{3 \times 10^9} = 7.5 \times 10^{-7}\text{s}$$

$$speedup = \frac{t_2}{t_{2n}} = \frac{9 \times 10^{-7}}{7.5 \times 10^{-7}} = 1.2$$

Exercise 1.6

Compilers can have a profound impact on the performance of an application on given a processor. This problem will explore the impact compilers have on execution time.

	Compiler A		Compiler B	
	No. Instructions	Execution Time	No. Instructions	Execution Time
a.	1.00E+09	1.8 s	1.20E+09	1.8 s
b.	1.00E+09	1.1 s	1.20E+09	1.5 s

Figure 7(1.6.1-1.6.3)

(1) For the same program, two different compilers are used. The table above shows the execution time of the two different compiled programs. Find the average CPI for each program given that the processor has a clock cycle time of 1 ns.

Solution:

In group **a**, for complier A:

$$c_A = \frac{t_A}{T} = \frac{1.8\text{s}}{10^{-9}\text{s}} = 1.8 \times 10^9,\ CPI = \frac{c_A}{I_A} = \frac{1.8 \times 10^9}{10^9} = 1.8$$

For compiler B:

$$c_B = \frac{t_B}{T} = \frac{1.8\text{s}}{10^{-9}\text{s}} = 1.8 \times 10^9,\ CPI = \frac{c_B}{I_B} = \frac{1.8 \times 10^9}{1.2 \times 10^9} = 1.5$$

In group **b**, for complier A:

$$c_A = \frac{t_A}{T} = \frac{1.1\text{s}}{10^{-9}\text{s}} = 1.1 \times 10^9,\ CPI = \frac{c_A}{I_A} = \frac{1.1 \times 10^9}{10^9} = 1.1$$

For compiler B:

$$c_B = \frac{t_B}{T} = \frac{1.5\text{s}}{10^{-9}\text{s}} = 1.5 \times 10^9,\ CPI = \frac{c_B}{I_B} = \frac{1.5 \times 10^9}{1.2 \times 10^9} = 1.25$$

(2) Assume the average CPIs found in 1.6.1, but that the compiled programs run on two different processors. If the execution times on the two processors are the same, how much faster is the clock of the processor running compiler A's code versus the clock of the processor running compiler B's code?

Solution:

In group **a**, for complier A:

$$T_A = \frac{t}{c_A} = \frac{t}{CPI_A \cdot I_A} \Rightarrow f_A = \frac{1}{T_A} = \frac{CPI_A \cdot I_A}{t}$$

For compiler B:

$$T_B = \frac{t}{c_B} = \frac{t}{CPI_B \cdot I_B} \Rightarrow f_B = \frac{1}{T_B} = \frac{CPI_B \cdot I_B}{t}$$

$\frac{f_A}{f_B} = \frac{CPI_A \cdot I_A}{CPI_B \cdot I_B} = \frac{1.8 \times 10^9}{1.5 \times 1.2 \times 10^9} = 1 \Rightarrow f_A = f_B$ (means compiler A and B are the same)

In group **b**, for compiler A and B:

$\frac{f_A}{f_B} = \frac{CPI_A \cdot I_A}{CPI_B \cdot I_B} = \frac{1.1 \times 10^9}{1.25 \times 1.2 \times 10^9} = 0.73 \Rightarrow f_A < f_B$ (means that the clock of compiler A is 0.73 times to that of compiler B)

(3) A new compiler is developed that uses only 600 million instructions and has an average CPI of 1.1. What is the speedup of using this new compiler versus using Compiler A or B on the original processor of 1.6.1?

Solution:

Using this new compiler:

$$c_n = CPI_n \cdot I_n = 1.1 \times 6 \times 10^8 = 6.6 \times 10^8$$

$$t_n = c_n T = 6.6 \times 10^8 \times 10^{-9}\text{s} = 0.66\text{s}$$

In group **a**:

Compared with compiler A:

$speedup = \frac{t_A}{t_n} = \frac{1.8\text{s}}{0.66\text{s}} = 2.73$, which indicates the new compiler is 2.73 times faster than A.

Compared with compiler B:

$speedup = \frac{t_B}{t_n} = \frac{1.8\text{s}}{0.66\text{s}} = 2.73$, which indicates the new compiler is 2.73 times faster than B.

In group **b**:

Compared with compiler A:

$speedup = \frac{t_A}{t_n} = \frac{1.1\text{s}}{0.66\text{s}} = 1.67$, which indicates the new compiler is 1.67 times faster than A.

Compared with compiler B:

$speedup = \frac{t_B}{t_n} = \frac{1.5\text{s}}{0.6\text{s}} = 2.27$, which indicates the new compiler is 2.27 times faster than B.

Consider two different implementations, P1 and P2, of the same instruction set. There are five classes of instructions (A, B, C, D, and E) in the instruction set. P1 has a clock rate of 4 GHz, and P2 has a clock rate of 6 GHz. The average number of cycles for each instruction class for P1 and P2 are listed in the following table.

		CPI Class A	CPI Class B	CPI Class C	CPI Class D	CPI Class E
a.	P1	1	2	3	4	5
	P2	3	3	3	5	5
b.	P1	1	2	3	4	5
	P2	2	2	2	2	6

Figure 8(1.6.4-1.6.6)

(4) Assume that peak performance is defined as the fastest rate that a computer can execute any instruction sequence. What are the peak performances of P1 and P2 expressed in instructions per second?

Solution:

Assume that two processors process the same amount of instructions, denoted by I.

In group **a**:

For processor P1, only Class A works:

$$c_{1\min} = I\,,\;\; t_{1\min} = c_{1\min} T_1 = \frac{c_{1\min}}{f_1} = \frac{I}{4\times 10^9}\,\mathrm{s}$$

$$p_{1\max} = \frac{I}{t_{1\min}} = \frac{I}{\frac{I}{4\times 10^9}\,\mathrm{s}} = 4\times 10^9 \;\; \text{instruction/s}$$

For processor P2, only either Class A or B or C works:

$$c_{2\min} = 3I\,,\;\; t_{2\min} = c_{2\min} T_2 = \frac{c_{2\min}}{f_2} = \frac{I}{2\times 10^9}\,\mathrm{s}$$

$$p_{2\max} = \frac{I}{t_{2\min}} = \frac{I}{\frac{I}{2\times 10^9}} = 2\times 10^9 \;\; \text{instruction/s}$$

In group **b**:

For processor P1, only Class A works:

$$c_{1\min} = I\,,\;\; t_{1\min} = c_{1\min} T_1 = \frac{c_{1\min}}{f_1} = \frac{I}{4\times 10^9}\,\mathrm{s}$$

$$p_{1\max} = \frac{I}{t_{1\min}} = \frac{I}{\frac{I}{4\times 10^9}\,\mathrm{s}} = 4\times 10^9 \;\; \text{instruction/s}$$

For processor P2, only either Class A or B or C or D works:

$$c_{2\min} = 2I\,,\;\; t_{2\min} = c_{2\min} T_2 = \frac{c_{2\min}}{f_2} = \frac{I}{3\times 10^9}\,\mathrm{s}$$

$$p_{2\max}=\frac{I}{t_{2\min}}=\frac{I}{\frac{I}{3\times10^9}}=3\times10^9 \text{ instruction/s}$$

(5) If the number of instructions executed in a certain program is divided equally among the five classes of instructions except for class A, which occurs twice as often as each of the others, how much faster is P2 than P1?

Solution: (Assume the total instructions for all processors are $6I$.)

In group **a**:

P1: $c_1=2I+2I+3I+4I+5I=16I,\ t_1=c_1T_1=\frac{c_1}{f_1}=\frac{16I}{4\times10^9}=\frac{4I}{10^9}\text{s}$

P2: $c_2=6I+3I+3I+5I+5I=22I,\ t_2=c_2T_2=\frac{c_2}{f_2}=\frac{22I}{6\times10^9}=\frac{11I}{3\times10^9}\text{s}$

$\Rightarrow\frac{t_1}{t_2}=\frac{12}{11}\Rightarrow$ P2 is $\frac{12}{11}$ times faster than P1.

In group **b**:

P1: $c_1=2I+2I+3I+4I+5I=16I,\ t_1=c_1T_1=\frac{c_1}{f_1}=\frac{16I}{4\times10^9}=\frac{4I}{10^9}\text{s}$

P2: $c_2=4I+2I+2I+2I+6I=16I,\ t_2=c_2T_2=\frac{c_2}{f_2}=\frac{16I}{6\times10^9}=\frac{8I}{3\times10^9}\text{s}$

$\Rightarrow\frac{t_1}{t_2}=\frac{3}{2}\Rightarrow$ P2 is $\frac{3}{2}$ times faster than P1.

(6) At what frequency does P1 have the same performance of P2 for the instruction mix given in 1.6.5?

Solution: Let the frequency of P1 be f that enables P1 has the same performance of P2 in the case of the instruction mix given in question 5.

In group **a**:

P1: $t_1=\frac{c_1}{f}=\frac{16I}{f},\ p_1=\frac{6I}{t_1}=\frac{6I}{\frac{16I}{f}}=\frac{3f}{8}$

P2: $p_2=\frac{6I}{t_2}=\frac{6I}{\frac{11I}{3\times10^9}}=\frac{18}{11}\times10^9$

Since $p_1=p_2\Rightarrow f=4.36GHz$

In group **b**:

P1: $t_1=\frac{c_1}{f}=\frac{16I}{f},\ p_1=\frac{6I}{t_1}=\frac{6I}{\frac{16I}{f}}=\frac{3f}{8}$

P2: $p_2 = \frac{6I}{t_2} = \frac{6I}{\frac{8I}{3\times10^9}} = \frac{9}{4}\times10^9$

Since $p_1 = p_2 \Rightarrow f = 6GHz$

Exercise 1.9

Although the dynamic power is the primary source of power dissipation in CMOS, leakage current produces a static power dissipation VI_{leak}. The smaller the onchip dimensions, the more significant is the static power. Assume the figures shown in the following table for static and dynamic power dissipation for several generations of processors.

	Technology	Dynamic Power(W)	Static Power(W)	Voltage(V)
a.	180 nm	50	10	1.2
b.	70 nm	90	60	0.9

Figure 9(1.9.1-1.9.3)

(1) Find the percentage of the total dissipated power comprised by static power.

Solution:

In group **a**: $p_a = \frac{P_s}{P_{tot}} = \frac{1}{6} = 16.67\%$

In group **b**: $p_b = \frac{P_s}{P_{tot}} = \frac{2}{5} = 40\%$

(2) If the total dissipated power is reduced by 10% while maintaining the static to total power rate of problem 1.9.1, how much should the voltage be reduced to maintain the same leakage current?

Solution: From the formula given in the question, we can obtain that

$$0.9V_1 I_{\text{leak}} = V_2 I_{\text{leak}}$$

which means that $V_2 = 0.9V_1$

In group **a**: $V_2 = 0.9\times1.2\text{V} = 1.08\text{V}$

In group **b**: $V_2 = 0.9\times0.9\text{V} = 0.81\text{V}$

(3) Determine the ratio of static power to dynamic power for each technology.

Solution:

In group **a**: $r_1 = \frac{P_s}{P_d} = \frac{10}{50} = 0.2$

In group **b**: $r_2 = \frac{P_s}{P_d} = \frac{60}{90} = \frac{2}{3} = 0.67$

Consider now the dynamic power dissipation of different versions of a given processor for three different voltages given in the following table.

	1.2V	1.0V	0.8V
a.	75W	60W	35W
b.	62W	50W	30W

Figure 10(1.9.4-1.9.6)

(4) Determine the static power at 0.8V, assuming a static to dynamic power ratio of 0.6.

Solution:

In group **a**: $P_s = rP_d = 0.6 \times 35\,\mathrm{W} = 21\,\mathrm{W}$

In group **b**: $P_s = rP_d = 0.6 \times 30\,\mathrm{W} = 18\,\mathrm{W}$

(5) Determine the static and dynamic power dissipation assuming the rates obtained in problem 1.9.1.

Solution:

In group **a:** When the voltage is 1.2V,

$$P_d = r_1 P_{tot} = 75\,\mathrm{W} \times \frac{5}{6} = 62.5\,\mathrm{W}, \quad P_s = P_{tot} - P_d = 75\,\mathrm{W} - 62.5\,\mathrm{W} = 12.5\,\mathrm{W}$$

When the voltage is 1.0V,

$$P_d = r_1 P_{tot} = 60\,\mathrm{W} \times \frac{5}{6} = 50\,\mathrm{W}, \quad P_s = P_{tot} - P_d = 60\,\mathrm{W} - 50\,\mathrm{W} = 10\,\mathrm{W}$$

When the voltage is 0.8V,

$$P_d = r_1 P_{tot} = 35\,\mathrm{W} \times \frac{5}{6} = 29.17\,\mathrm{W}, \quad P_s = r_2 P_{tot} = \frac{1}{6} \times 35\,\mathrm{W} = 5.83\,\mathrm{W}$$

In group **b**: When the voltage is 1.2V,

$$P_d = r_1 P_{tot} = 62\,\mathrm{W} \times \frac{3}{5} = 37.2\,\mathrm{W}, \quad P_s = r_2 P_{tot} = \frac{2}{5} \times 62\,\mathrm{W} = 24.8\,\mathrm{W}$$

When the voltage is 1.0V,

$$P_d = r_1 P_{tot} = \frac{3}{5} \times 50\,\mathrm{W} = 30\,\mathrm{W}, \quad P_s = r_2 P_{tot} = \frac{2}{5} \times 50\,\mathrm{W} = 20\,\mathrm{W}$$

When the voltage is 0.8V,

$$P_d = r_1 P_{tot} = \frac{3}{5} \times 30\,\mathrm{W} = 18\,\mathrm{W}, \quad P_s = r_2 P_{tot} = \frac{2}{5} \times 30\,\mathrm{W} = 12\,\mathrm{W}$$

(6) Determine the geometric mean of the power variations between versions.

Solution:

In group **a**: $G=\sqrt[3]{\prod_{i=1}^{3} P_{toti}}=\sqrt[3]{75\,\mathrm{W}\times 60\,\mathrm{W}\times 35\,\mathrm{W}}\approx 54\,\mathrm{W}$

In group **b**: $G=\sqrt[3]{\prod_{i=1}^{3} P_{toti}}=\sqrt[3]{62\,\mathrm{W}\times 50\,\mathrm{W}\times 30\,\mathrm{W}}=45.31\,\mathrm{W}$

Exercise 1.11

The following table shows manufacturing data for various processors.

	Wafer Diameter	Dies per Wafer	Defects per Unit Area	Cost per Wafer
a.	15 cm	84	0.020 defects/cm²	12
b.	20 cm	100	0.031 defects/cm²	15

Figure 11(1.11.1-1.11.3)

(1) Find the yield.

Solution: Let the wafer area be A_w, and the die area be A_D. Denote defects per Unit Area by D for short.(Let $\pi=3.14$) Let the yield be Y

In group **a**:

$$A_w=\pi\cdot\left(\frac{d}{2}\right)^2=\frac{1}{4}\pi d^2=176.63\,\mathrm{cm}^2,\ A_D=\frac{A_w}{n}=\frac{176.63\,\mathrm{cm}^2}{84}=2.1\,\mathrm{cm}^2$$

$$Y=\frac{1}{\left(1+\frac{1}{2}DA_D\right)^2}=\frac{1}{\left(1+\frac{1}{2}\times 0.02\,\mathrm{defects/cm}^2\times 2.1\,\mathrm{cm}^2\right)^2}=0.96$$

In group **b**:

$$A_w=\pi\cdot\left(\frac{d}{2}\right)^2=\frac{1}{4}\pi d^2=314\,\mathrm{cm}^2,\ A_D=\frac{A_w}{n}=\frac{314\,\mathrm{cm}^2}{100}=3.14\,\mathrm{cm}^2$$

$$Y=\frac{1}{\left(1+\frac{1}{2}DA_D\right)^2}=\frac{1}{\left(1+\frac{1}{2}\times 0.031\,\mathrm{defects/cm}^2\times 3.14\,\mathrm{cm}^2\right)^2}=0.91$$

(2) Find the cost per die.

Solution: Let the cost per die be C_d, $C_d=\frac{C_w}{nY}$

In group **a**: $C_{da}=\frac{C_{wa}}{n_a Y_a}=\frac{12}{84\times 0.96}=0.15$

In group **b**: $C_{db}=\frac{C_{wb}}{n_b Y_b}=\frac{15}{100\times 0.91}=0.16$

(3) If the number of dies per wafer is increased by 10% and the defects per area unit increases by 15%, find the die area and yield.

Solution:

In group **a**:

$$A_D = \frac{A_w}{n_{new}} = \frac{176.63\,\text{cm}^2}{92.4} = 1.91\,\text{cm}^2$$

$$Y = \left(1 + \frac{D_{new} A_D}{2}\right)^{-2} = \frac{1}{\left(1 + \frac{1}{2} D_{new} A_D\right)^2} = \frac{1}{\left(1 + \frac{1}{2} \times 0.023\,\text{defects/cm}^2 \times 1.91\,\text{cm}^2\right)^2} = 0.96$$

In group **b**:

$$A_D = \frac{A_w}{n_{new}} = \frac{314\,\text{cm}^2}{110} = 2.85\,\text{cm}^2$$

$$Y = \frac{1}{\left(1 + \frac{1}{2} D_{new} A_D\right)^2} = \frac{1}{\left(1 + \frac{1}{2} \times 0.03565\,\text{defects/cm}^2 \times 2.85\,\text{cm}^2\right)^2} = 0.91$$

Suppose that, with the evolution of the electronic devices manufacturing technology, the yield varies as shown in the following table.

	T1	T2	T3	T4
Yield	0.85	0.89	0.92	0.95

Figure 12(1.11.4-1.11.5)

(4) Find the defects per area unit for each technology given a die area of 200 mm^2.

Solution:

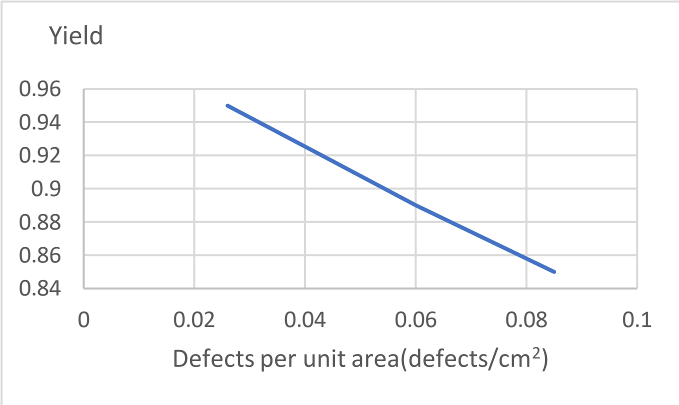
For T1: $Y_1 = \dfrac{1}{\left(1 + \frac{1}{2} D_1 A_D\right)^2} = 0.85 \Rightarrow \quad D_1 = 0.085\,\text{defects/cm}^2$

For T2: $Y_2 = \dfrac{1}{\left(1 + \frac{1}{2} D_2 A_D\right)^2} = 0.89 \Rightarrow \quad D_2 = 0.06\,\text{defects/cm}^2$

For T3: $Y_3 = \dfrac{1}{\left(1 + \frac{1}{2} D_3 A_D\right)^2} = 0.92 \Rightarrow \quad D_3 = 0.043\,\text{defects/cm}^2$

For T4: $Y_4 = \dfrac{1}{\left(1 + \frac{1}{2} D_4 A_D\right)^2} = 0.95 \Rightarrow \quad D_4 = 0.026\,\text{defects/cm}^2$

(5) Represent graphically the variation of the yield together with the variation of defects per unit area.



Exercise 1.12

The following table shows results for SPEC CPU2006 benchmark programs running on an AMD Barcelona.

	Name	Intr. Count(10^9)	Execution Time(seconds)	Reference Time(seconds)
a.	bzip2	2389	750	9650
b.	go	1658	700	10,490

Figure 13(1.12.1-1.12.3)

(1) Find the CPI if the clock cycle time is 0.333 ns.

Solution: Let the number of cycle in group a and b be c_a and c_b, respectively.

In group **a**:

$$c_a = \frac{t_a}{T} = \frac{750\text{s}}{3.33\times10^{-10}\text{s}} = 2.25\times10^{12}$$

$$CPI_a = \frac{c_a}{I_a} = \frac{2.25\times10^{12}}{2.389\times10^{12}} = 0.94$$

In group **b**:

$$c_b = \frac{t_a}{T} = \frac{700\text{s}}{3.33\times10^{-10}\text{s}} = 2.1\times10^{12}$$

$$CPI_b = \frac{c_b}{I_b} = \frac{2.1\times10^{12}}{1.658\times10^{12}} = 1.27$$

(2) Find the SPEC ratio.

Solution: $r_{SPEC} = \dfrac{t_{ref}}{t_{exe}}$

In group **a:** $r_{SPEC} = \dfrac{t_{ref}}{t_{exe}} = \dfrac{9650\text{s}}{750\text{s}} = 12.87$

In group **b:** $r_{SPEC} = \dfrac{t_{ref}}{t_{exe}} = \dfrac{10490\text{s}}{700\text{s}} = 14.99$

(3) For these two benchmarks, find the geometric mean of SPEC ratio.

Solution: Denote the geometric mean of SPEC ratio by G.

$$G = \sqrt{\prod_{i=1}^{2} r_{SPECi}} = \sqrt{12.87 \times 14.99} = 13.89$$

The following table shows data for further benchmarks.

	Name	CPI	Clock Rate	SPEC ratio
a.	libquantum	1.61	4 GHz	19.8
b.	astar	1.79	4 GHz	9.1

Figure 14(1.12.4-1.12.6)

(4) Find the increase in CPU time if the number of instructions of the benchmark is increased by 10% without affecting the CPI.

Solution:

$$t_{CPU} = n_{cycle} T = \frac{n_{cycle}}{f} = \frac{CPI \cdot I}{f}$$

Thus, if only the number of instructions of the benchmark increases by 10% without affecting any other factors involved in the formula given above, the increase in CPU time is also 10%, which means $t_{CPUnew} = 1.1 t_{CPU}$.

(5) Find the increase in CPU time if the number of instructions of the benchmark is increased by 10% and the CPI is increased by 5%.

Solution: Find the new CPU time directly.

$$t_{CPUnew} = \frac{CPI_{new} \cdot I_{new}}{f} = \frac{1.05 CPI \cdot 1.1 I}{f} = \frac{1.155 CPI \cdot I}{f} = 1.155 t_{CPU}$$

Thus, the increase in CPU time is 15.5%.

(6) Find the change in SPEC ratio for the change described in 1.12.5.

Solution: Reference time is constant.

Since, $r_{SPEC} = \frac{t_{ref}}{t_{CPU}} \Rightarrow \frac{r_{SPECnew}}{r_{SPEC}} = \frac{t_{CPU}}{t_{CPUnew}} = \frac{t_{CPU}}{1.155 t_{CPU}} = \frac{1}{1.155} = 0.87$

Thus, the change in SPEC ratio is 13%, decreasing compared with old SPEC ratio.

Exercise 1.13

Suppose that we are developing a new version of the AMD Barcelona processor with a 4 GHz clock rate. We have added some additional instructions to the instruction set in such a way that the number of instructions has been reduced by 15% from the values shown for each benchmark in Exercise 1.12. The execution times obtained are shown in the following table.

	Name	Execution Time (seconds)	Reference Time (seconds)	SPECratio
a.	bzip2	700	9650	13.7
b.	go	620	10490	16.9

Figure 15(1.13.1-1.13.3)

(1) Find the new CPI.

Solution:

In group **a**:

$$c_a = \frac{t_a}{T} = t_a f = 700\text{s} \times 4 \times 10^9\text{Hz} = 2.8 \times 10^{12}$$

$$CPI = \frac{c_a}{I_a} = \frac{2.8 \times 10^{12}}{0.85 \times 2.389 \times 10^{12}} = 1.38$$

In group **b**:

$$c_b = \frac{t_b}{T} = t_b f = 620\text{s} \times 4 \times 10^9\text{GHz} = 2.48 \times 10^{12}$$

$$CPI = \frac{c_b}{I_b} = \frac{2.48 \times 10^{12}}{0.85 \times 1.658 \times 10^{12}} = 1.76$$

(2) In general, these CPI values are larger than those obtained in previous exercises for the same benchmarks. This is due mainly to the clock rate used in both cases, 3 GHz and 4 GHz. Determine whether the increase in the CPI is similar to that of the clock rate. If they are dissimilar, why?

Solution: In question (1), we have obtained the CPI when the clock rate is 3 GHz.

The increase of the clock rate is $\frac{f_{new}}{f_{old}} = \frac{4\ \text{GHz}}{3\ \text{GHz}} = 1.33$

In group **a**: $\frac{CPI_{new}}{CPI_{old}} = \frac{1.38}{0.94} = 1.47 > 1.33$

In group **b**: $\frac{CPI_{new}}{CPI_{old}} = \frac{1.76}{1.27} = 1.39 > 1.33$

Thus, the increase in the CPI is unsimilar to that of the clock rate since the CPU time has been reduced by a lower percentage, dominating the change of CPI more than the increase of the number of the instruction.

(3) How much has the CPU time been reduced?

Solution:

In group **a**: $\frac{t_{CPUnew}}{t_{CPU}} = \frac{700}{750} = 0.93$, which means the decrease of the CPU time is 7%.

In group **b**: $\frac{t_{CPUnew}}{t_{CPU}} = \frac{620}{700} = 0.89$, which means the decrease of the CPU time is 11%.

	Name	Execution Time (seconds)	CPI	Clock Rate
a.	libquantum	960	1.61	3 GHz
b.	astar	690	1.79	3 GHz

Figure 16(1.13.4-1.13.6)

(4) If the execution time is reduced by an additional 10% without affecting to the CPI and with a clock rate of 4 GHz, determine the number of instructions.

Solution:

In group **a**:

$t_{anew} = 0.9 \times 960\mathrm{s} = 864\mathrm{s},$

$c_{anew} = \dfrac{t_{anew}}{T_{anew}} = t_{anew} f_{anew} = 864\mathrm{s} \times 4 \times 10^{9}\mathrm{Hz} = 3.456 \times 10^{12}$

$I_{anew} = \dfrac{c_{anew}}{CPI_a} = \dfrac{3.456 \times 10^{12}}{1.61} = 2.14658 \times 10^{12}$

In group **b:**

$t_{bnew} = 0.9 \times 690\mathrm{s} = 621\mathrm{s}$

$c_{bnew} = \dfrac{t_{bnew}}{T_{bnew}} = t_{bnew} f_{bnew} = 621\mathrm{s} \times 4 \times 10^{9}\mathrm{Hz} = 2.484 \times 10^{12}$

$I_{bnew} = \dfrac{c_{bnew}}{CPI_b} = \dfrac{2.484 \times 10^{12}}{1.79} = 1.38771 \times 10^{12}$

(5) Determine the clock rate required to give a further 10% reduction in CPU time while maintaining the number of instructions and with the CPI unchanged.

Solution:

$$I = \frac{c}{CPI} = \frac{tf}{CPI} \Rightarrow f = \frac{I \cdot CPI}{t}$$

(Notice that the reference group is the one with the clock rate of 3 GHz.)

$f_{new} = \dfrac{I \cdot CPI}{0.9t} = \dfrac{1}{0.9} f = \dfrac{1}{0.9} \times 3\ \mathrm{GHz} = 3.33\mathrm{GHz}$

(6) Determine the clock rate if the CPI is reduced by 15% and the CPU time by 20% while the number of instructions is unchanged.

Solution:

$f_{new} = \dfrac{I \cdot 0.85CPI}{0.8t} = \dfrac{0.85}{0.8} f = 1.0625f = 1.0625 \times 3\ \mathrm{GHz} = 3.1875\mathrm{GHz}$