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Letter

Hydrogen atom in a laser-plasma

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Abstract
We scrutinize the behaviour of the eigenvalues of a hydrogen atom in a quantum plasma as it interacts with an electric field directed along $\theta = \pi$ and is exposed to linearly polarized intense laser field radiation. We refer to the interaction of the plasma with the laser light as laser-plasma. Using the Kramers–Henneberger (KH) unitary transformation, which is the semiclassical counterpart of the Block–Nordsieck transformation in the quantized field formalism, the squared vector potential that appears in the equation of motion is eliminated and the resultant equation is expressed in the KH frame. Within this frame, the resulting potential and the corresponding wavefunction have been expanded in Fourier series, and using Ehlotzky's approximation we obtain a laser-dressed potential to simulate an intense laser field. By fitting the exponential-cosine-screened Coulomb potential into the laser-dressed potential, and then expanding it in Taylor series up to $\mathcal{O}(r^4, \alpha_0^9)$, we obtain the eigensolution (eigenvalues and wavefunction) of the hydrogen atom in laser-plasma encircled by an electric field, within the framework of perturbation theory formalism. Our numerical results show that for a weak external electric field and a very large Debye screening parameter length, the system is strongly repulsive, in contrast with the case for a strong external electric field and a small Debye screening parameter length, when the system is very attractive. This work has potential applications in the areas of atomic and molecular processes in external fields, including interactions with strong fields and short pulses.

Keywords: perturbation technique, hydrogen atom, laser field radiation, quantum plasmas

(Some figures may appear in colour only in the online journal)

1. Introduction

Lasers have emerged as one of the world's indispensable technologies, employed in telecommunications, law enforcement, military equipment, etc. Recent progress in laser technology has aroused the interest of many researchers in investigating new sources of lasers in order to probe and control molecular structure, function and dynamics on the natural timescale of atomic motion, and femtosecond and electron motion on the attosecond timescale [1]. To obtain intense laser fields, it

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is necessary to concentrate large amounts of energy within a short period of time, and then focus the laser light onto a small area. In an intense laser system, a train of pulses of short duration is created by the oscillator. The energy of the pulses is then proliferated by the amplifier, which is eventually focused.

Atoms in intense laser fields have been a subject of active research for more than three decades due to their salient application in the invention of high-power short-pulse laser technologies. These atoms exhibit new properties that have been discovered via the study of multiphoton processes. When a high-power laser is directed into a gas of atoms, the magnitude of the electromagnetic field is found to be consistent with the Coulomb field, which binds a 1s electron in a hydrogen atom [2]. Within this context, many outstanding results have been reported so far (see [3–7] and references therein). It was shown in [8] that in the presence of an oscillating magnetic field, the ionization rate due to the laser field dwindles, and the electron density becomes ionized with a lower rate by keeping the magnetic field strength constant and increasing the intensity of the laser.

There has been renewed interest (see [9–13] and references therein) in studying atomic and molecular processes in the quantum plasma environment due to their applications in distinguishing various plasmas and also providing passable knowledge of collision dynamics [9]. The role that ionization processes and atomic excitation play in the conceptual understanding of various phenomena related to hot plasma physics and astrophysics is preeminent. The effects of quantum plasma environment atoms can be modelled by a screened potential which accounts for pair correlations. In accord with this, an enormous number of studies have investigated the influence of external fields on hydrogen atoms in quantum plasma (see [9, 10, 14] and references therein). Very recently, Falaye *et al* [9] found that to perpetuate a low-energy medium for the hydrogen atom in quantum plasmas, a strong electric field and a weak magnetic field are required, whereas the Aharonov–Bohm (AB) flux field can be used as a regulator.

Researchers have recently developed a keen interest in scrutinizing atomic processes in laser-plasma. Within this context, Idris *et al* [15] and Kurniawana and Kagawab [16] examined hydrogen emission in laser plasma via focusing a TEA CO_2 laser and a Nd-YAG laser on various types of samples doped with hydrogen. Some other outstanding reports can be found in [17–20] and references therein. However, it is worth mentioning that most of these worthy attempts were experimentally based. In the present work, our objective is to scrutinize the behaviour of the eigenvalues of a hydrogen atom in a quantum plasma as it interacts with an electric field, and is exposed to linearly polarized intense laser field radiation. To our best knowledge, a study like this has not been reported yet and in fact it represents a significant furtherance of [6, 9]. Consequently, we feel this work will be of interest in the areas of atomic structure and collisions in plasmas.

2. Formulation of the problem

In this section, we derive the equation of motion for a spherically confined hydrogen atom in a dense quantum plasma under an electric field, and exposed to linearly polarized intense laser field radiation. In order to achieve the goal of this section, we start with the following time-dependent Schrödinger wave equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t)=\left[-\frac{\hbar^2}{2\mu}\nabla^2-i\hbar\frac{e}{2\mu}[\mathbf{A}(\boldsymbol{r},t)\cdot\nabla+\nabla\cdot\mathbf{A}(\boldsymbol{r},t)]\right.\\ \left.+\frac{e^2}{2\mu}\mathbf{A}(\boldsymbol{r},t)^2-e\phi+V(\boldsymbol{r})+Fr\right]\Psi(\boldsymbol{r},t),\qquad(1)$$

with the scalar potential $\phi(\mathbf{r},t)$ and the vector potential $\boldsymbol{A}(\boldsymbol{r},t)$ which is invariant under the gauge transformation. μ is the effective mass of the electron. Furthermore, F denotes an electric field strength with an angle θ between F and r. With $\theta=\pi$, $Fr\cos(\theta)$ becomes Fr as shown in equation (1). We consider the Coulomb gauge, such that $\nabla\cdot\mathbf{A}(\boldsymbol{r},t)=0$ with $\phi=0$ in empty space, and then simplify the interaction term in equation (1) by performing gauge transformations within the framework of the dipole approximation. In this approximation, for an atom whose nucleus is located at the position r_0, the vector potential is spatially homogeneous $\mathbf{A}(\boldsymbol{r},t)\approx\boldsymbol{A}(t)$. Moreover, term $\mathbf{A}(\mathbf{r},t)^2$ appearing in equation (1) is considered for extremely high field strength. It is usually small and can be eliminated by extracting a time-dependent phase factor from the wave function via [21]

$$\Psi^{v}(\boldsymbol{r},t)=\exp\left[\frac{ie^2}{2\mu\hbar}\int_{-\infty}^{t}\mathbf{A}(t')^2\mathrm{d}t'\right]\Psi(\boldsymbol{r},t),\qquad(2)$$

to obtain velocity gauge[7]

$$i\hbar\frac{\partial}{\partial t}\Psi^{v}(\mathbf{r},t)=\left[-\frac{\hbar^2}{2\mu}\nabla^2-i\hbar\frac{e}{\mu}\mathbf{A}(t)\cdot\nabla+V(r)+Fr\right]\Psi^{v}(\mathbf{r},t).\qquad(3)$$

A prerequisite to studying hydrogen atoms in intense high-frequency laser fields is transforming equation (3) to the Kramers–Henneberger (KH) accelerated frame. Now, with the introduction of the following unitary KH transformation

$$\Psi^{A}(\mathbf{r},t)=U^{\dagger}\Psi^{v}(\mathbf{r},t)\ \text{with}\quad U=\exp\left[-\frac{i}{\hbar}\boldsymbol{\alpha}(t)\,.\,\boldsymbol{p}\right],\\ \text{and}\ \boldsymbol{\alpha}(t)=\frac{e}{\mu}\int^{t}\mathbf{A}(t')\mathrm{d}t',\qquad(4)$$

which is a semiclassical counterpart of the Block–Nordsieck transformation in the quantized field formalism, the coupling term $\boldsymbol{A}(t)\cdot\boldsymbol{p}$ in the velocity guauge (i.e. equation (3)) is eliminated. More explicitly, this can be done via

$$i\hbar U^{\dagger}\frac{\partial}{\partial t}U\Psi^{A}(\boldsymbol{r},t)=U^{\dagger}\left[-\frac{\hbar^2}{2\mu}\nabla^2-i\hbar\frac{e}{\mu}\mathbf{A}(t)\cdot\nabla+V(\boldsymbol{r})+Fr\right]\\ \times U\Psi^{A}(\mathbf{r},t).\qquad(5)$$

Evaluation of the terms in equation (5) is straightforward and easy. However, let us try to be more explicit in evaluating the term $U^{\dagger}V(r)U$. This can be done via the Campbell–Baker–Hausdorff

[7] Because the vector potential $\mathbf{A}(t)$ is being coupled to the operator $\boldsymbol{p}/m$ via the Hamiltonian interaction. $\boldsymbol{p}=-i\hbar\nabla$.

identity: $e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + [\hat{A},\hat{B}] + [\hat{A},[\hat{A},\hat{B}]]/2\,! + \ldots$. Thus, we have

$$\begin{aligned} U^{\dagger}V(r)U &= \exp\left[\frac{\mathrm{i}}{\hbar}\boldsymbol{\alpha}(t)\cdot\boldsymbol{p}\right]V(\boldsymbol{r})\exp\left[-\frac{\mathrm{i}}{\hbar}\boldsymbol{\alpha}(t)\cdot\boldsymbol{p}\right] \\ &= V(\boldsymbol{r}) + [\boldsymbol{\alpha}(t)\cdot\nabla]V(\boldsymbol{r}) + \frac{1}{2!}[\boldsymbol{\alpha}(t)\cdot\nabla]^2V(\boldsymbol{r}) + \ldots \\ &= V[r+\boldsymbol{\alpha}(t)], \end{aligned} \tag{6}$$

where $\boldsymbol{\alpha}(t)$ denotes the displacement of a free electron in the incident laser field. Ergo, equation (5) becomes

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi^A(\boldsymbol{r},t) = -\frac{\hbar^2}{2\mu}\nabla^2\Psi^A(\boldsymbol{r},t) + V[\boldsymbol{r}+\boldsymbol{\alpha}(t)]\Psi^A(\boldsymbol{r},t) + Fr\Psi^A(\boldsymbol{r},t). \tag{7}$$

Equation (7) represents a space-translated version of the time-dependent Schrödinger wave equation with incorporation of $\boldsymbol{\alpha}(t)$ into the potential in order to simulate the interaction of the atomic system with the laser field. Three decades after its discovery by Pauli and Fierz [22], it was applied to study the renormalization of quantum electrodynamics by Kramers [23] and was later used to study interactions of atoms with lasers by Henneberger [24]. Within this framework, many outstanding works have been reported by a great number of erudite scholars (see [4, 25–27], and references therein).

For a steady field condition, the vector potential takes the form $\boldsymbol{A}(t) = (\mathcal{E}_0/\omega)\cos(\omega t)$ with $\boldsymbol{\alpha}(t) = \alpha_0\sin(\omega t)$, where $\alpha_0 = e\mathcal{E}_0/(\mu\omega^2)$ is the amplitude of oscillation of a free electron in the field (called the laser-dressing parameter), $\mathcal{E}_0$ denotes the amplitude of electromagnetic field strength and ω is the angular frequency. Now, considering a pulse where the electric field amplitude is steady, the wavefunction in the frame of KH takes the following Floquet form [21]:

$$\Psi^A(\boldsymbol{r},t) = \mathrm{e}^{-\frac{\mathrm{i}E_{\mathrm{KH}}}{\hbar}t}\sum_n \Psi_n^{E_{\mathrm{KH}}}(\boldsymbol{r})\mathrm{e}^{-\mathrm{i}n\omega t}, \tag{8}$$

where Floquet quasi-energy has been denoted by E_{KH}. The potential in the frame of KH can be expanded in Fourier series as [26]

$$V[\boldsymbol{r}+\boldsymbol{\alpha}(t)] = \sum_{m=-\infty}^{\infty} V_m(\alpha_0;\boldsymbol{r})\mathrm{e}^{-\mathrm{i}m\omega t} \quad \text{with}$$
$$V_m(\alpha_0;\boldsymbol{r}) = \frac{i^m}{\pi}\int_{-1}^{1} V(\boldsymbol{r}+\alpha_0\varrho)\frac{T_n(\varrho)}{\sqrt{1-\varrho^2}}\mathrm{d}\varrho, \tag{9}$$

where we have taken the period as $2\pi/\omega$ and introduced a new transformation of the form $\varrho = \sin(\omega t)$. Furthermore, $T_n(\varrho)$ are Chebyshev polynomials. Substituting equations (8) and (9) into equation (7) yields a set of coupled differential equations:

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_m(\alpha_0;\boldsymbol{r}) + Fr - (E_{\mathrm{KH}} + n\hbar\omega)\right]\Psi_n^{E_{\mathrm{KH}}}(\boldsymbol{r}) = -\sum_{m=-\infty}^{\infty,m\neq n} V_{n-m}\Psi_m^{E_{\mathrm{KH}}}(\boldsymbol{r}). \tag{10}$$

Considering $n = 0$ (which gives the lowest order approximation) and the high frequency limit (which made V_m with $m \neq 0$ vanish), equation (10) becomes

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(\alpha_0;\boldsymbol{r}) + Fr - E_{\mathrm{KH}}\right]\Psi_0^{E_{\mathrm{KH}}}(\boldsymbol{r}) = 0. \tag{11}$$

and the coefficient of the Fourier series for the potential becomes

$$\begin{aligned} V_0(\alpha_0;\boldsymbol{r}) &= \frac{1}{\pi}\int_{-1}^{1} V(\boldsymbol{r}+\alpha_0\varrho)\frac{\mathrm{d}\varrho}{\sqrt{1-\varrho^2}} \\ &= \frac{1}{\pi}\int_{0}^{1}[V(\boldsymbol{r}+\alpha_0\varrho) + V(\boldsymbol{r}-\alpha_0\varrho)]\frac{\mathrm{d}\varrho}{\sqrt{1-\varrho^2}}. \end{aligned} \tag{12}$$

Using the Ehlotzky approximation [28], one has $[V(\boldsymbol{r}+\alpha_0\varrho) + V(\boldsymbol{r}-\alpha_0\varrho)] \approx [V(\boldsymbol{r}+\alpha_0) + V(\boldsymbol{r}-\alpha_0)]$. Hence, by evaluating the integral, we obtain

$$V_0(\alpha_0;\boldsymbol{r}) = \frac{1}{2}[V(\boldsymbol{r}+\alpha_0) + V(\boldsymbol{r}-\alpha_0)]\,. \tag{13}$$

Equation (13) is the approximate expression to model the laser field. Now, we incorporate the model to simulate the behaviour of a hydrogen atom in dense quantum plasma [6, 9] into model potential (13). Then equation (11) becomes

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 - \frac{Ze^2}{r_{\alpha_0^+}}\exp\left(-\frac{r_{\alpha_0^+}}{\lambda_{\mathrm{D}}}\right)\cos\left(\frac{r_{\alpha_0^+}}{\lambda_{\mathrm{D}}}\right) - \frac{Ze^2}{r_{\alpha_0^-}}\exp\left(-\frac{r_{\alpha_0^-}}{\lambda_{\mathrm{D}}}\right)\cos\left(\frac{r_{\alpha_0^-}}{\lambda_{\mathrm{D}}}\right) + Fr - E_{\mathrm{KH}}\right]\Psi_0^{E_{\mathrm{KH}}}(\boldsymbol{r}) = 0, \tag{14}$$

where $r_{\alpha_0^{\pm}} = r \pm \alpha_0$, λ_{D} is the Debye screening length and Z denotes the atomic number that is found useful in describing the energy levels of light to heavy neutral atoms [9]. We have assumed the core of the hydrogenic system to be static which explains why a one-body system appears in equation (14) instead of a two-body one. The above equation (14) is the equation of motion for a spherically confined hydrogen atom in a dense quantum plasma under an electric field, and exposed to linearly polarized intense laser field radiation. To achieve our goal in this study, in the next section, we solve equation (14) using a perturbation formalism.

3. Eigenspectra calculation

Equation (14) is not solvable analytically. One can either use a numerical procedure or a perturbation formalism. Using the perturbation approach, we decompose the equation into two parts where the first part is exactly solvable and the other part is perturbation. Consequently, the eigenvalue solutions are represented in power series with the leading term corresponding to the solution of the exactly solvable part and the other part being a correction to the energy term which corresponds to the perturbation term. This approach has been used in numerous research reports (see [9, 29] and references therein). Now, we rewrite equation (14) as

$$\frac{\hbar^2}{2\mu}\left(\frac{\nabla^2\mathcal{X}_0(r)}{\mathcal{X}_0(r)} + \frac{\nabla^2\mathcal{Y}_0(r)}{\mathcal{Y}_0(r)} + 2\frac{\nabla\mathcal{X}_0(r)\nabla\mathcal{Y}_0(r)}{\mathcal{X}_0(r)\nabla\mathcal{Y}_0(r)}\right) = V_{\mathrm{eff.}}(r) - E_{\mathrm{KH}}, \tag{15}$$

where $\Psi_0^{E_{\rm KH}}(\boldsymbol{r}) = \mathcal{X}_0(r)\mathcal{Y}_0(r)$ with $\mathcal{X}_0(r)$ as the wavefunction of the exactly solvable part and $\mathcal{Y}_0(r)$ as the moderating wavefunction. The effective potential $V_{\rm eff.}(r)$ represents the Taylor's series expansion of the potential terms in equation (14). This can be written as:

$$\begin{aligned}
V_{\rm eff.}(r) = &-\frac{2\mathcal{A}}{r}+\left(\frac{\mathcal{A}\alpha_0^8}{11340\lambda_{\rm D}^9}+\frac{\mathcal{A}\alpha_0^6}{315\lambda_{\rm D}^7}-\frac{\mathcal{A}\alpha_0^4}{15\lambda_{\rm D}^5}-\frac{2\mathcal{A}\alpha_0^2}{3\lambda_{\rm D}^3}+\frac{2\mathcal{A}}{\lambda_{\rm D}}\right)\\
&+r\left(F-\frac{\mathcal{A}\alpha_0^6}{180\lambda_{\rm D}^8}+\frac{\mathcal{A}\alpha_0^2}{\lambda_{\rm D}^4}\right)\\
&+r^2\left(-\frac{\mathcal{A}\alpha_0^8}{13860\lambda_{\rm D}^{11}}+\frac{\mathcal{A}\alpha_0^6}{405\lambda_{\rm D}^9}+\frac{\mathcal{A}\alpha_0^4}{21\lambda_{\rm D}^7}-\frac{2\mathcal{A}\alpha_0^2}{5\lambda_{\rm D}^5}-\frac{2\mathcal{A}}{3\lambda_{\rm D}^3}\right)\\
&+r^3\left(\frac{\mathcal{A}\alpha_0^8}{22680\lambda_{\rm D}^{12}}-\frac{\mathcal{A}\alpha_0^4}{36\lambda_{\rm D}^8}+\frac{\mathcal{A}}{3\lambda_{\rm D}^4}\right)+\mathcal{O}(r^4,\alpha_0^9),\quad \mathcal{A}=Ze^2.
\end{aligned}\tag{16}$$

The first term is the main part which corresponds to a shape invariant potential for which the superpotential is known analytically and the remaining part is taken as a perturbation, $\Delta V_{\rm eff.}(r)$. This approximation is only valid for $r/\lambda_{\rm D}\ll 1$. The effective potential and its approximate expansion have been delineated in figure 1. Now, taking the logarithmic derivatives of the perturbed and unperturbed wavefunctions as $W_0(r)=-(\hbar/\sqrt{2\mu})\mathcal{X}_0'/\mathcal{X}_0$ and $\Delta W_0(r)=-(\hbar/\sqrt{2\mu})\mathcal{Y}_0'/\mathcal{Y}_0$, and then substituting them into (15), yields the following equation

$$\frac{\hbar^2}{2\mu}\frac{\mathcal{X}_0''(r)}{\mathcal{X}_0(r)} = W_0^2(r)-\frac{\hbar}{\sqrt{2\mu}}W_0'(r) = -\frac{2\mathcal{A}}{r}-E_{\rm KH}^{(0)},\tag{17a}$$

$$\Delta W_0^2(r)-\frac{\hbar}{\sqrt{2\mu}}\Delta W_0'(r)+2W_0(r)\Delta W_0(r)=\Delta V_{\rm eff.}(r)-\Delta E_{\rm KH},\tag{17b}$$

where $E_{\rm KH}^{(0)}$ is the eigenvalue of the exactly solvable part and $\Delta E_{\rm KH}=E_{\rm KH}^{(1)}+E_{\rm KH}^{(2)}+E_{\rm KH}^{(3)}+\ldots$ is the correction to the energy which corresponds to the perturbation term. Equation (17a) is analytically solvable via formula method [30] to obtain

$$\mathcal{X}_0(r)=2\varsigma^{3/2}r{\rm e}^{-\varsigma r},\quad W_0(r)=-\frac{\hbar}{r\sqrt{2\mu}}+\frac{\mathcal{A}\sqrt{2\mu}}{\hbar},$$
$$E_{\rm KH}^{(0)}=-\varsigma\mathcal{A},\quad\text{where}\quad \varsigma=\frac{2\mu\mathcal{A}}{\hbar^2}.\tag{18}$$

By contrast, equation (17b) is not exactly solvable. It is therefore necessary to expand the related functions as $\Delta V_{\rm eff.}(r;\eta)=\sum_{i=1}^{\infty}\eta_iV_{\rm eff.}(r)^{(i)}$, $\Delta W_0(r;\eta)=\sum_{i=1}^{\infty}\eta_iW_0^{(i)}$, $\Delta E_0^{(i)}(\eta)=\sum_{i=1}^{\infty}\eta_iE_0^{(i)}$, where i represents the order of perturbation. We substitute these expressions into equation (17b) and then equate terms with the same power of η on both sides to obtain the following expressions

$$2W_0(r)W_0^{(1)}(r)-\frac{\hbar}{\sqrt{2\mu}}\frac{{\rm d}W_0^{(1)}(r)}{{\rm d}r}=V_{\rm eff.}^{(1)}(r)-E_{\rm KH}^{(1)},\tag{19a}$$

$$[W_0^{(1)}(r)]^2+2W_0(r)W_0^{(2)}(r)-\frac{\hbar}{\sqrt{2\mu}}\frac{{\rm d}W_0^{(2)}(r)}{{\rm d}r}=V_{\rm eff.}^{(2)}(r)-E_{\rm KH}^{(2)},\tag{19b}$$

$$\begin{aligned}&2\left[W_0(r)W_0^{(3)}(r)+W_0^{(1)}(r)W_0^{(2)}(r)\right]-\frac{\hbar}{\sqrt{2\mu}}\frac{{\rm d}W_0^{(3)}(r)}{{\rm d}r}\\&\quad=V_{\rm eff.}^{(3)}(r)-E_{\rm KH}^{(3)},\end{aligned}\tag{19c}$$

$$\begin{aligned}&2\left[W_0(r)W_0^{(4)}(r)+W_0^{(1)}(r)W_0^{(3)}(r)\right]+W_0^{(2)}(r)W_0^{(2)}(r)\\&\quad-\frac{\hbar}{\sqrt{2\mu}}\frac{{\rm d}W_0^{(4)}(r)}{{\rm d}r}=V_{\rm eff.}^{(4)}(r)-E_{\rm KH}^{(4)}.\end{aligned}\tag{19d}$$

Taking the superpotentials into account and then multiplying each term in equations (19a)–(19d) by $\mathcal{X}_0^2(r)$, we obtain first-, second- and third-order corrections to the energy and their superpotentials as follows:

$$\begin{aligned}E_{\rm KH}^{(1)}&=\int_0^\infty\mathcal{X}_0^2(r)r\left(F-\frac{\mathcal{A}\alpha_0^6}{180\lambda_{\rm D}^8}+\frac{\mathcal{A}\alpha_0^2}{\lambda_{\rm D}^4}\right){\rm d}r\\&=\frac{3}{2\varsigma}\left(F-\frac{\mathcal{A}\alpha_0^6}{180\lambda_{\rm D}^8}+\frac{\mathcal{A}\alpha_0^2}{\lambda_{\rm D}^4}\right),\end{aligned}\tag{20a}$$

$$\begin{aligned}W_0^{(1)}(r)&=\sqrt{\frac{2\mu}{\hbar^2}}\frac{1}{\mathcal{X}_0^2(r)}\int^r\mathcal{X}_0^2(\varrho)\left[E_{\rm KH}^{(1)}-\left(F-\frac{\mathcal{A}\alpha_0^6}{180\lambda_{\rm D}^8}+\frac{\mathcal{A}\alpha_0^2}{\lambda_{\rm D}^4}\right)\varrho\right]{\rm d}\varrho\\&=\frac{r}{\hbar\varsigma}\sqrt{\frac{\mu}{2}}\left(F-\frac{\mathcal{A}\alpha_0^6}{180\lambda_{\rm D}^8}+\frac{\mathcal{A}\alpha_0^2}{\lambda_{\rm D}^4}\right)\end{aligned}\tag{20b}$$

$$\begin{aligned}E_{\rm KH}^{(2)}&=\int_0^\infty\mathcal{X}_0^2(r)\\&\quad\times\left[r^2\left(-\frac{\mathcal{A}\alpha_0^8}{13860\lambda_{\rm D}^{11}}+\frac{\mathcal{A}\alpha_0^6}{405\lambda_{\rm D}^9}+\frac{\mathcal{A}\alpha_0^4}{21\lambda_{\rm D}^7}-\frac{2\mathcal{A}\alpha_0^2}{5\lambda_{\rm D}^5}-\frac{2\mathcal{A}}{3\lambda_{\rm D}^3}\right)\right.\\&\qquad\left.-W_0^{(1)2}(r)\right]{\rm d}r\\&=\left[\frac{\hbar^4}{4\mu^2\mathcal{A}}\left(-\frac{\alpha_0^8}{4620\lambda_{\rm D}^{11}}+\frac{\alpha_0^6}{135\lambda_{\rm D}^9}+\frac{\alpha_0^4}{7\lambda_{\rm D}^7}-\frac{6\alpha_0^2}{5\lambda_{\rm D}^5}-\frac{2}{\lambda_{\rm D}^3}\right)\right.\\&\qquad\left.-\frac{3\hbar^6}{32\mu^3\mathcal{A}^2}\left(\frac{F}{\mathcal{A}}-\frac{\alpha_0^6}{180\lambda_{\rm D}^8}+\frac{\alpha_0^2}{\lambda_{\rm D}^4}\right)^2\right]\end{aligned}\tag{20c}$$

$$\begin{aligned}W_0^{(2)}(r)&=\sqrt{\frac{2\mu}{\hbar^2}}\frac{1}{\mathcal{X}_0^2(r)}\left[\int^r\mathcal{X}_0^2(\varrho)\left(E_{\rm KH}^{(2)}+W_0^{(1)2}(\varrho)\right){\rm d}\varrho\right.\\&\quad\left.-\int^r\mathcal{X}_0^2(\varrho)\left(-\frac{\mathcal{A}\alpha_0^8}{13860\lambda_{\rm D}^{11}}+\frac{\mathcal{A}\alpha_0^6}{405\lambda_{\rm D}^9}+\frac{\mathcal{A}\alpha_0^4}{21\lambda_{\rm D}^7}-\frac{2\mathcal{A}\alpha_0^2}{5\lambda_{\rm D}^5}-\frac{2\mathcal{A}}{3\lambda_{\rm D}^3}\right)\varrho^2{\rm d}\varrho\right]\\&=\frac{r}{\varsigma\hbar}\sqrt{\frac{\mu}{2}}\left(r+\frac{2}{\varsigma}\right)\left(\frac{\mathcal{A}\alpha_0^8}{13860\lambda_{\rm D}^{11}}+\frac{\mathcal{A}\alpha_0^6}{405\lambda_{\rm D}^9}+\frac{\mathcal{A}\alpha_0^4}{21\lambda_{\rm D}^7}-\frac{2\mathcal{A}\alpha_0^2}{5\lambda_{\rm D}^5}-\frac{2\mathcal{A}}{3\lambda_{\rm D}^3}\right)\\&\quad-\frac{r}{\varsigma^3\hbar^3}\left(\frac{\mu}{2}\right)^{3/2}\left(r+\frac{2}{\varsigma}\right)\left(F-\frac{\mathcal{A}\alpha_0^6}{180\lambda_{\rm D}^8}+\frac{\mathcal{A}\alpha_0^2}{\lambda_{\rm D}^4}\right)^2\end{aligned}\tag{20d}$$

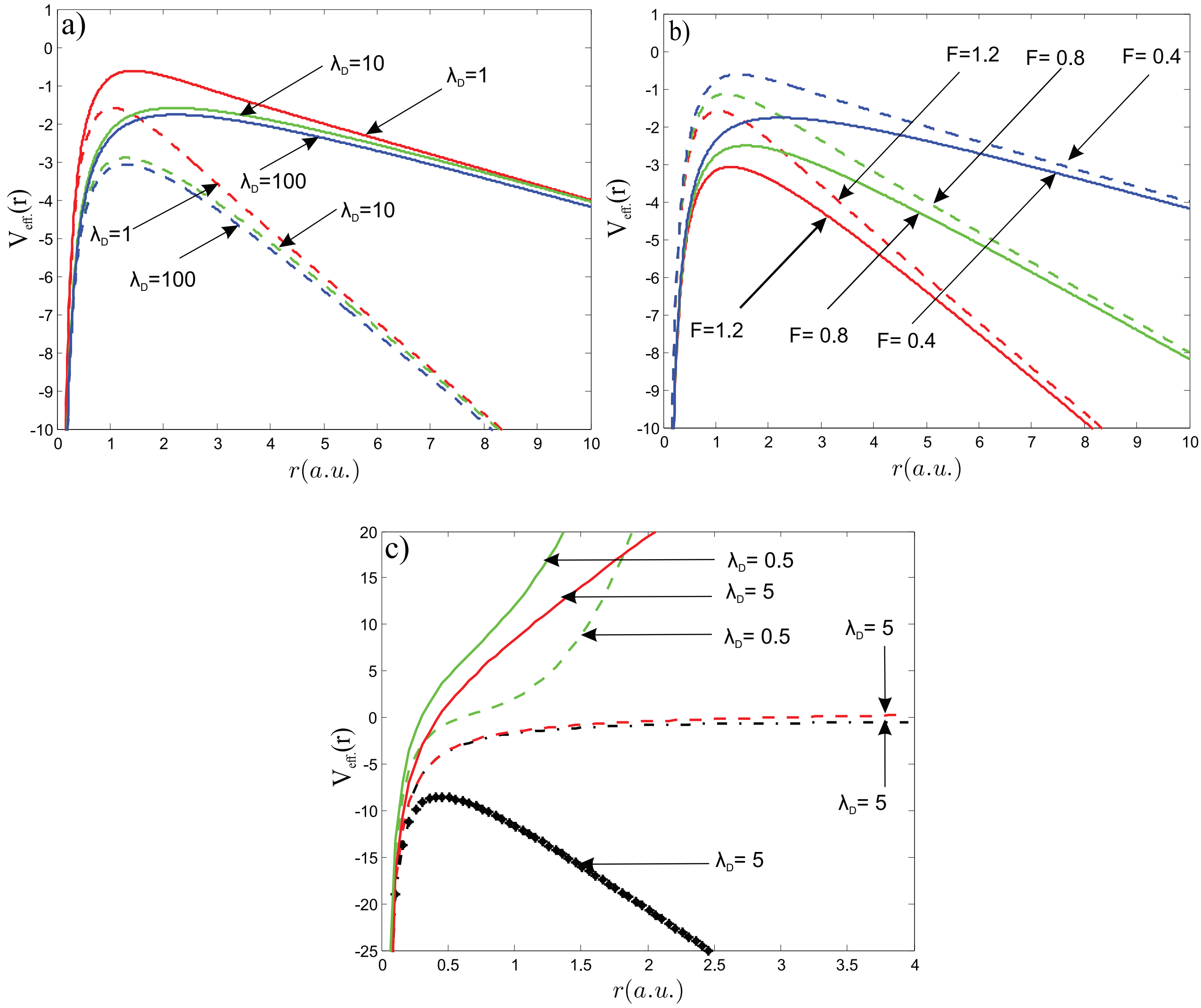

Figure 1. Plots of the model potential as a function of internuclear distance. In (a), we depict the shape of the model by considering a weak and strong external electric field via taking F as 0.4 and 1.2 respectively for various values of Debye screening lengths λ_D. The dashed line '- -' represents the context of weak F while the thick line represents strong F. Furthermore, in (b), we depict the shape of the model potential by taking the Debye length λ_D as 1 (dashed lines) and 100 (thick lines) for various values of electric field strengths. The figures (a) and (b) show that increasing the electric field strength will reduce the attractiveness of the effective potential and ergo the system becomes more repulsive. In figure (c), we show the accuracy of the approximate expression for the potential model (i.e. the Taylor series expansion of the effective potential) given in equation (16) for various values of λ_D and F. The line with marker type '$_*$' represents the effective potential for $F = 10$ while the dot-dashed line '. -' is for $F = 0.1$. The thick lines represent the approximate expression for the effective potential with $F = 10$ while the dashed line is for $F = 0.1$. This figure shows that the approximation is only valid for low F and high λ_D. It can also be inferred that, for the validity of the approximation to be maintained, r/λ_D must be $\ll 1$. We have taken $\alpha_0 = 0.001$ and all our numerical computations are in atomic units (a.u.).

$$
\begin{aligned}
E_{\mathrm{KH}}^{(3)} &= \int_0^\infty \mathcal{X}_0^2(r)\left[r^3\left(\frac{\mathcal{A}\alpha_0^8}{22680\lambda_D^{12}} - \frac{\mathcal{A}\alpha_0^4}{36\lambda_D^8} + \frac{\mathcal{A}}{3\lambda_D^4}\right) - W_0^{(1)}(r)W_0^{(2)}(r)\right]\mathrm{d}r \\
&= \frac{1}{2\varsigma^3}\left[\left(\frac{\mathcal{A}\alpha_0^8}{1512\lambda_D^{12}} - \frac{5\mathcal{A}\alpha_0^4}{12\lambda_D^8} + \frac{5\mathcal{A}}{\lambda_D^4}\right) + \frac{27\mu^2}{4\hbar^4\varsigma^4}\left(F - \frac{\mathcal{A}\alpha_0^6}{180\lambda_D^8} + \frac{\mathcal{A}\alpha_0^2}{\lambda_D^4}\right)^2\right. \\
&\quad - \frac{9\mu\mathcal{A}}{2\hbar^2\varsigma^2}\left(F - \frac{\mathcal{A}\alpha_0^6}{180\lambda_D^8} + \frac{\mathcal{A}\alpha_0^2}{\lambda_D^4}\right) \\
&\quad \left.\times\left(-\frac{\alpha_0^8}{4620\lambda_D^{11}} + \frac{\alpha_0^6}{135\lambda_D^9} + \frac{\alpha_0^4}{7\lambda_D^7} - \frac{6\alpha_0^2}{5\lambda_D^5} - \frac{2}{\lambda_D^3}\right)\right].
\end{aligned} \tag{20e}
$$

With equations (20*a*)–(20*e*), we obtain the approximate energy eigenvalues and the wavefunction of the hydrogen atom in the laser-plasma encircled by an electric field as:

$$
\begin{aligned}
E_{\mathrm{KH}} \approx E_{\mathrm{KH}}^{(0)} &+ \left(\frac{\mathcal{A}\alpha_0^8}{11340\lambda_D^9} + \frac{\mathcal{A}\alpha_0^6}{315\lambda_D^7} - \frac{\mathcal{A}\alpha_0^4}{15\lambda_D^5} - \frac{2\mathcal{A}\alpha_0^2}{3\lambda_D^3} + \frac{2\mathcal{A}}{\lambda_D}\right) \\
&+ E_{\mathrm{KH}}^{(1)} + E_{\mathrm{KH}}^{(2)} + E_{\mathrm{KH}}^{(3)} + \ldots,
\end{aligned} \tag{21}
$$

Table 1. Energy eigenvalues (in a.u.) of the hydrogen atom in quantum plasma as it interacts with the electric field and is exposed to linearly polarized intense laser field radiation.

F	0.0001	0.0004	0.001	0.004	0.01	0.04
$E_{\mathrm{KH}}^{(\lambda_{\mathrm{D}}=100)}$	−1.979 9255	−1.979 7005	−1.979 2506	−1.977 0016	−1.972 5072	−1.950 1083
λ_{D}	5	10	20	40	80	100
$E_{\mathrm{KH}}^{(F=0.01)}$	−1.595 9955	−1.792 9741	−1.892 5671	−1.942 5144	−1.967 5077	−1.9725072

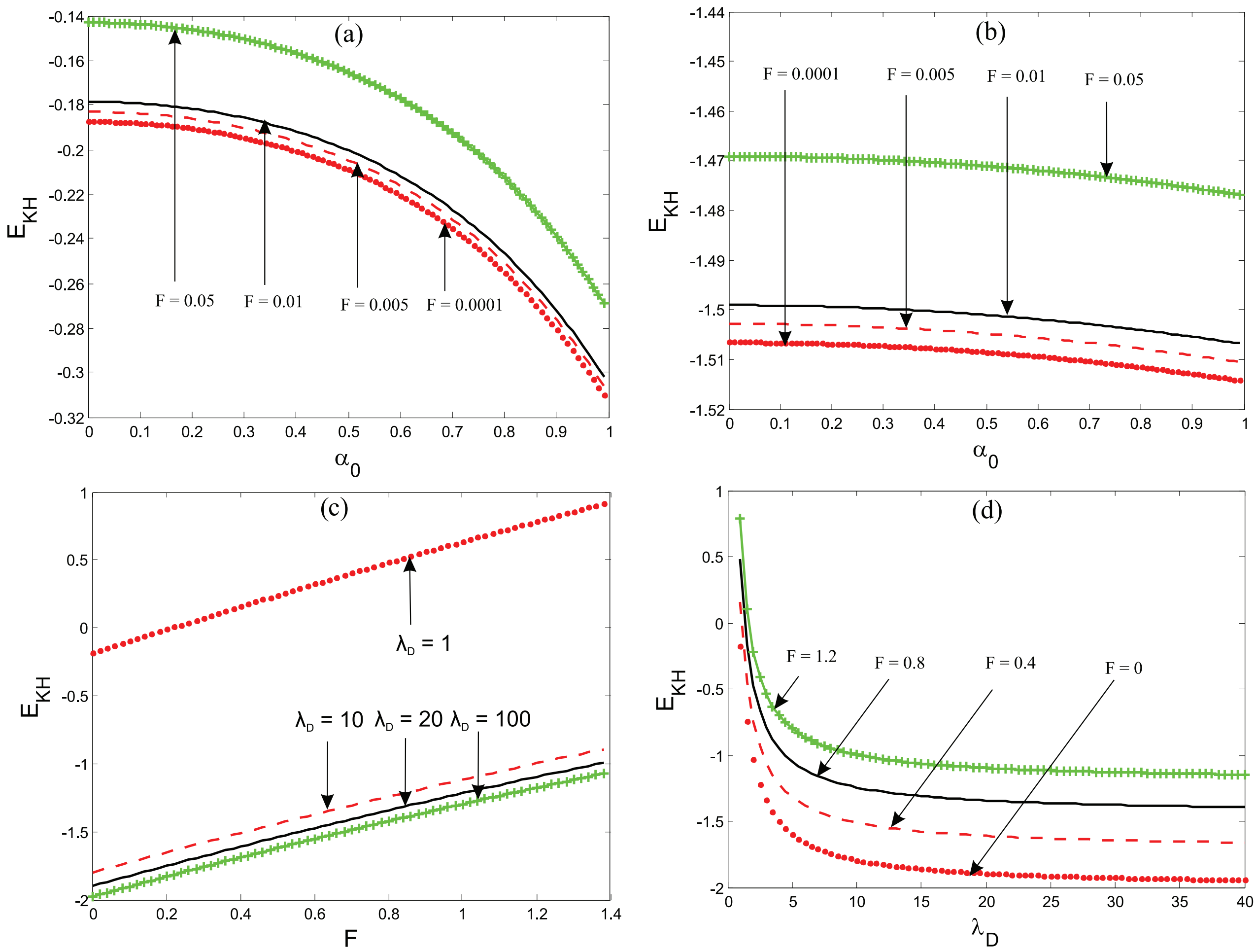

Figure 2. Plots of the energy eigenvalues of the hydrogen atom in quantum plasma as it interacts with the electric field and is exposed to linearly polarized intense laser field radiation, as a function of various model parameters. (a) Energy eigenvalues as a function of the laser-dressing parameter with $\lambda_{\mathrm{D}} = 1$ for various values of electric field intensities. (b) Same as (a) but for $\lambda_{\mathrm{D}} = 4$. (c) Eigenvalues as a function of electric field with $\alpha_0 = 0.0001$ and for various values of Debye screening lengths. (d) Eigenvalues as a function of Debye screening length with $\alpha_0 = 0.0001$ and for various values of electric field intensities. All our numerical computations are in a.u..

and

$$\Psi_0^{E_{\mathrm{KH}}}(\boldsymbol{r}) \approx 2\varsigma^{3/2} r \exp(-\varsigma r)\exp\left(-\sqrt{\frac{2\mu}{\hbar^2}}\int^r (W_0^{(1)}(\varrho)+W_0^{(2)}(\varrho))\mathrm{d}\varrho\right), \tag{22}$$

respectively. The behaviour of the energy eigenvalues of the hydrogen atom in the quantum plasma as it interacts with the electric field and is exposed to linearly polarized intense laser field radiation as a function of various model parameters is shown in table 1 and figure 2. As can be seen in table 1, increasing the intensity of the external electric field leads to a corresponding increment in the bound state energy of the hydrogen atom and the energy spacing becomes proliferated for a fixed Debye screening length. However, for a fixed weak external electric field, the scenario is quite different. The energy eigenvalues dwindle with increasing λ_{D} and the energy spacing increases. The results in this table show that for a very weak external electric field and a very large Debye screening length, the energy levels become more negative and the system becomes strongly repulsive. To substantiate the results in table 1, in figure 2(a), we scrutinize the behaviour of the eigenvalues of a hydrogen atom as a function of the laser-dressing parameter. For a particular external electric field, the energy shift is 0 for $\alpha_0 < 0.06$ with $\lambda_{\mathrm{D}} = 1$. But as α_0 proliferates, the energy level diminishes monotonically and becomes more negative. A significant change is seen in the localization of the bound state. However, when the Debye screening length is increased to 4 as we have in figure 2(b), a slight shift is seen in the bound state energies with pronounced characteristics even when the intensity of the external electric field changes. Figures 2(a) and (b) show the susceptibility of the eigenvalues of the hydrogen atom to λ_D. In fact, for the system to respond to variation in α_0 with various changes in electric field, a very small Debye screening parameter must be considered.

Furthermore, in figure 2(c), we study the behaviour of the eigenvalues of the hydrogen atom as a function of the external electric field for various lengths of Debye screening parameters. For various λ_D, the system becomes strongly attractive as the intensity of the electric field increases. In fact, this figure corroborates the result of figure 2(b). For a strong F and very small Debye screening length, the energy levels tend towards positivity and the system becomes strongly attractive. In figure 2(d), we elucidate figure 2(c) further. As can be seen, the energy levels under the influence of an external electric field decrease monotonically with increasing Debye screening length until $\lambda_D \approx 25$. For $\lambda_D > 25$, no distinct variation can be discerned irrespective of the variation in intensity of the external electric field. In general, our numerical results show that the repulsiveness of the system can be permuted via manipulation of the external electric field. For instance, with a weak external electric field and a very large length of Debye screening parameter, the system is strongly repulsive, whereas for a strong external electric field and small length of Debye screening parameter, the system is strongly attractive.

4. Concluding remarks

We scrutinize the behaviour of a hydrogen atom's eigenspectra in a quantum plasma as it interacts with an electric field and is exposed to linearly polarized intense laser field radiation. Using the KH unitary translation, which is the semiclassical counterpart of the Block–Nordsieck transformation in the quantized field formalism, the squared vector potential that appears in the equation of motion is eliminated and the resultant equation is represented in the KH frame. Within this frame, the resulting potential and the corresponding wavefunction have been expanded in Fourier series and using Ehlotzky's approximation, we obtain a laser-dressed potential to simulate an intense laser field. By fitting a more general exponential screened Coulomb potential into the laser-dressed potential, and then expanding in Taylor series up to $\mathcal{O}(r^4, \alpha_0^9)$, we obtain the eigenspectra of the hydrogen atom in laser-plasma encircled by an electric field directed along $\theta = \pi$, within the framework of a perturbation theory formalism. We have greatly simplified all mathematical expressions to the fewest possible terms so as to ensure that this Letter will not only be readable to the experts but will also be understood by graduate students. We hope that this study will inspire progress in the future by exploring the molecular system in laser-plasmas and also studying equation (10) for complex states (i.e. $n \geqslant 1$).

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