1. Check corner solution
(If WEA is on the boundry, sometimes price ratio $\neq$ MRS)

Example : ( corner is not possible )

Individuals with Cobb-Douglas utility, and their individual endowments are $\gg 0$

$\Rightarrow$ corner is impossible

$\because$ If $i$ chooses corner $\Rightarrow U_i = 0$

but $U_i(\underbrace{e_i}_{\gg 0}) > 0$

Example : ( WEA is a corner solution )

$U_1 = X_1 + 2 \ln y_1$ , $e_1 = (0,1)$

$U_2 = X_2^{\frac{1}{2}} y_2^{\frac{1}{2}}$ , $e_2 = (1,1)$

let $(P_1, P_2) = (P, 1)$ , $P > 0$.

for ①, $\dfrac{1}{2 \cdot \frac{1}{y_1}} = P \Rightarrow y_1 = 2P$

for ②, $y_2 = \dfrac{\frac{1}{2}(P+1)}{1} = \frac{1}{2}(P+1)$

For WE $\Rightarrow$ $y_1 + y_2 = 2$

$2P + \frac{1}{2}(P+1) = 2$

$4P + P + 1 = 4$

$\cancel{3P = 3}$ $5P = 3$

$P = \frac{3}{5}$

This is not correct!

$\because$ ① will consume negative $X_1$.

Check ①'s budget constraint:

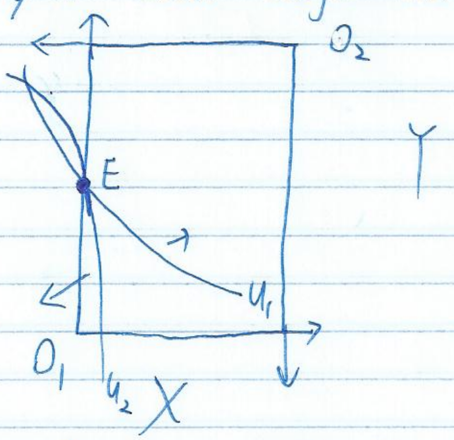
$$PX_1 + y_1 = 1$$

$$PX_1 = 1 - y_1$$

$$X_1 = \frac{1-y_1}{P} = \frac{1-2P}{P} = \frac{1-2\cdot\frac{3}{5}}{\frac{3}{5}} = \frac{\frac{-1}{5}}{\frac{3}{5}} = \frac{-1}{3} < 0 !$$

$\therefore$ If you strart from corner solution, you may get a wrong answer.

If you draw the Edgeworth box:




$\Rightarrow$ Tangent point is outside the box!

$\therefore$ E must be WEA.

for ①, E is a corner solution.

for ②, E is a inner solution.

$\therefore$ $P = MRS_2(1, 1)$

$\Rightarrow P = 1$

Under $P = 1$,

① 's choice is a corner solution:

$$MU_1(0,1) = 1$$

$$MU_2(0,1) = \frac{2}{y_1} = 2$$

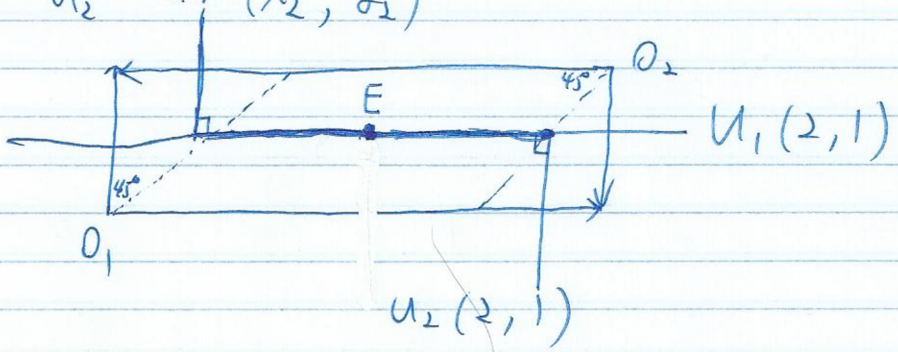
$$\Rightarrow \frac{MU_1(0,1)}{MU_2(0,1)} = \frac{1}{2} < \frac{P_1}{P_2} = 1$$

$\therefore$ ① is willing to have $(0,1)$ under $\frac{P_1}{P_2} = 1$.

2. Check whether $P_1 = 0$ or $P_2 = 0$ is possible.
(You may have this under Leontit utility function.)

Example: (one price = 0.)

$$U_1 = \min(X_1, Y_1) \qquad e_1 = e_2 = (2, 1)$$
$$U_2 = \min(X_2, Y_2)$$




In this case, $(P_1, P_2) = (0, P)$ $P > 0$ is WE.

(Thus, if you set $(P_1, P_2) = (1, P)$, $P > 0$, you won't get the solution!)

3. Check if MRS is unique for every point.
(If not, you may find multiple equilibria.)

Example:

$U_1 = \min(X_1, y_1)$ $\quad e_1 = e_2 = (1, 1)$

$U_2 = \min(X_2, y_2)$

Then, any $(P_1, P_2) \geq (0, 0)$ and $(P_1, P_2) \neq 0$ is WE.

(If you set $(P_1, P_2) = (P, 1)$, $P > 0$

$\Rightarrow X_1 = y_1 \Rightarrow PX_1 + y_1 = P + 1$

$X_1 = 1$,

$y_1 = 1$

⋮

It doesn't tell you anything about P.

In addition, I it is possible $P_1 = 0$ or $P_2 = 0$!)

4. If (1) Not corner solution
(2) $P_1 \neq 0$, $P_2 \neq 0$
(3) MRS is unique at each point.

$\Rightarrow$ you may use $MRS_1 = MRS_2 = \frac{P_1}{P_2}$ to solve WE.

5. If you're not sure 4. can work

⇒ Draw Edgeworth Box.

For example, two cases with Leontif utility function can be solved quickly if you draw the box!

6. Check all possible $(P_1, P_2)$

(This is useful to show W.E. doesn't exist!)

Example: 5.21 in PS 1.

Cases that 2nd Welfare theorem can't work.

Example: Q2 from Final (Winter 2013)

* If you foresee $\frac{P_1}{P_2} = 1$ is WE,

you may not want to solve it directly.

(Thus, the following way to solve it is tricky!)

(1) Check $P_1 \neq 0$ and $P_2 \neq 0$

(Either $P_1 = 0$ or $P_2 = 0$ can be possible for WE, so we need to check it.)

(2) let $\frac{P_1}{P_2} = P$.

If $P < 1$ ⇒ consumption outcome:

$$X_{11} > X_{12}, \quad X_{21} > X_{22}$$

$$X_{11} + X_{21} > X_{12} + X_{22}$$

⇒ market can't clear ∵ aggregate endowment is the same for 2 goods.

If $P > 1 \Rightarrow$ Similarly,
market can't clear.

$\therefore$ We can just try $P = 1$
$\Rightarrow$ check conditions of WE
$\Rightarrow$ It satisfies all conditions
$\therefore$ WE is $\frac{P_1}{P_2} = 1$ !

(It is tricky because you know what WE is.
$\therefore$ you can use a proper way to show
$P > 1$, $P < 1$ is not possible.
Then you get that the last hope is $P = 1$ ! )

Last Note:
All of above shouldn't be a formal way to find WE.
I just want to show that use

$$|MRS_1| = |MRS_2| = \frac{P_1}{P_2}$$

sometimes it may not work or you will miss some
possible eqm.