Multivariate Time Series

Time Series, Econ 311: Topic 6

Prof. Harald Uhlig[1]

[1]University of Chicago
Department of Economics
huhlig@uchicago.edu

Winter 2019

Outline

1. Roots, Impulse Responses and Cointegration
   - The characteristic polynomial
   - Impulse Responses
   - Cointegration and error correction

2. Identification of Shocks
   - Cholesky decompositions, structural VARs
   - Sign Restrictions
   - An application: monetary policy shocks

Outline

VAR (n)

- VAR(n) in $y_t \in \mathbb{R}^m$:

$$\begin{aligned} y_t &= \sum_{j=1}^{n} B_j y_{t-j} + A\epsilon_t,\ \epsilon_t \sim \mathcal{N}\left(0, I_m\right), t = 1, \ldots, T \\ (I_m - B(L))y(t) &= A\epsilon_t \end{aligned}$$

- In application: include a constant. Notation more messy.
- We assume that data is available for t = -n+1,..,0,..,T.
- Let

$$u_t = A\epsilon_t,\ \Sigma = AA'$$

- u_t: one-step ahead prediction error.

VAR(1)

This can be rewritten as a VAR(1) for $x_t \in \mathbb{R}^{mn}$: (stacking)

$$x_t = \begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-n+1} \end{bmatrix} = \mathcal{B}x_{t-1} + \mathcal{A}\epsilon_t,$$

where

$$\mathcal{B} = \begin{bmatrix} B_1 & \dots & B_{n-1} & B_n \\ I_m & \dots & 0 & 0 \\ \vdots & \ddots & & \vdots \\ 0 & \dots & I_m & 0 \end{bmatrix} \text{ and } \mathcal{A} = \begin{bmatrix} A \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Wlog, we may assume VAR(1) from here on.

Characteristic polynomial

Definition

The characteristic polynomial is defined as

$$p(\lambda) = det(\lambda I_{mn} - \mathcal{B})$$

Roots

Suppose that $\mathcal{B}$ is diagonalizable. Then,

$$\mathcal{B} = VDV^{-1}$$

where

$$D = \text{diag}(\lambda_1, \ldots, \lambda_{nm})$$

is a diagonal matrix with the zeros of $p(\lambda)$ on the diagonal.

Invertibility

- Suppose, $|\lambda_j| < 1$ for all j. Then

$$\begin{aligned} x_t &= \sum_{j=0}^{\infty} \mathcal{B}^j \mathcal{A} \epsilon_{t-j} \\ &= V \sum_{j=0}^{\infty} \text{diag}(\lambda_1^j, \ldots, \lambda_{nm}^j) V^{-1} \mathcal{A} \epsilon_{t-j} \end{aligned}$$

  with the second line valid, if $\mathcal{B}$ is diagonalizable.
- The first m rows provide an explicit expression for the Wold decomposition for $(I_m - B(L))y_t = u_t$, i.e.

$$y_t = (I_m - B(L))^{-1} u_t$$

- Or: explicitly per recursive substitution. Write $B(L) = \tilde{B}(L)L$. Thus

$$\begin{aligned} y_t &= \tilde{B}(L) y_{t-1} + u_t \\ &= (\tilde{B}(L))^2 y_{t-2} + \tilde{B}(L) u_{t-1} + u_t = \ldots \end{aligned}$$

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Impulse response

Definition

Let a be some m-dimensional vector. The impulse reponse $r_a(k)$ to the vector a is the forecast revision for y_k, given $u_0 = a$, i.e.

$$r_a(k) = E[y_k \mid u_0 = a, y_t, t < 0] - E[y_k \mid y_t, t < 0]$$

for $k \geq 0$.

Remarks

- it is easiest to set $y_t = 0$ for $t < 0$ in order to calculate the impulse responses, since then $E[y_k \mid y_t, t < 0] = 0$.
- More generally (with constants, time trends), set $y_t = E[y_t]$) for $t < 0$ in order to use $E[y_k \mid y_t, t < 0] = E[y_k]$.
- For nonlinear models, the initial conditions matter too.

Calculation per recursive substitution

- Example: VAR(1), zero mean,

$$y_t = By_{t-1} + u_t$$

- Substitute recursively,

k	y_k	y_{k-1}
0	a	0
1	Ba	a
2	B^2a	Ba
3	B^3a	B^2a
4	...	...

- Generally, for a VAR(1),

$$r_a(k) = B^k a$$

Numeric example 1

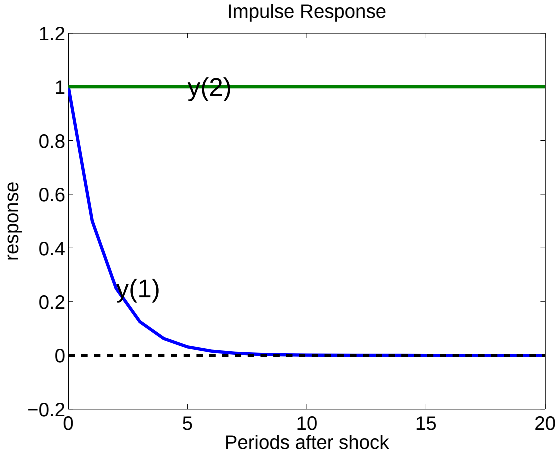
- Suppose that

$$B = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}, a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- Then

$$r_a(k) = B^k a = \begin{bmatrix} 0.5^k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5^k \\ 1 \end{bmatrix}$$

Impulse Response, VAR(1), Numerical Example 1.



Numeric example 2

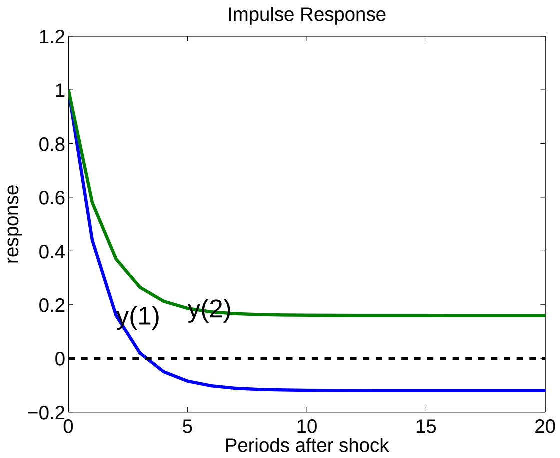
- Suppose that

$$B = \begin{bmatrix} 0.68 & -0.24 \\ -0.24 & 0.82 \end{bmatrix}, a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- Then

$$r_a(k) = B^k a = \begin{bmatrix} 0.68 & -0.24 \\ -0.24 & 0.82 \end{bmatrix}^k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Impulse Response, VAR(1), Numerical Example 2.



Using eigenvectors

- Start with a VAR(1), $y_t = By_{t-1} + u_t$
- To calculate impulse responses, we need to calculate B^k.
- Most quadratic matrices are diagonalizable, i.e. can be written as

$$B = VDV^{-1}$$

  where D is a diagonal matrix. Then,

$$B^k = VD^kV^{-1}$$

- Thus,

$$r_a(k) = VD^kV^{-1}a$$

Numeric example 2

- Note that

$$B = \begin{bmatrix} 0.68 & -0.24 \\ -0.24 & 0.82 \end{bmatrix} = \begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix}$$

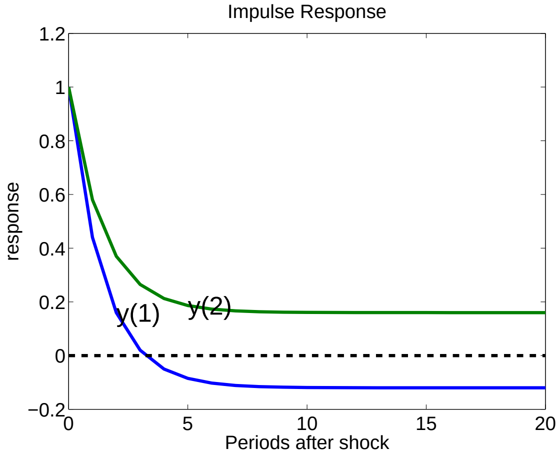
- Then

$$r_a(k) = B^k a = \begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 0.5^k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix} a$$

- Note that

$$\begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.4 \\ 0.2 \end{bmatrix} \text{ and } \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix} * 0.2 = \begin{bmatrix} -0.12 \\ 0.16 \end{bmatrix}$$

Impulse Response, VAR(1), Numerical Example 2.



Outline

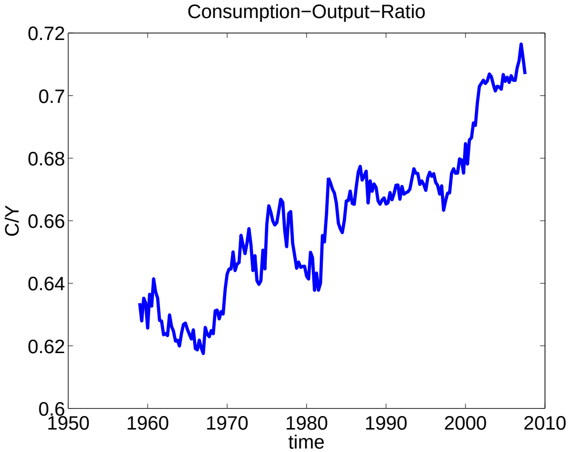
Cointegration

Definition

The vector time series y_t is said to be cointegrated of rank r, if each of the series taken individually is I(1), while some linear combination of the series $\beta' y_t$ is stationary for some cointegrating matrix $m \times r$ matrix β of rank r, $r \geq 1$.

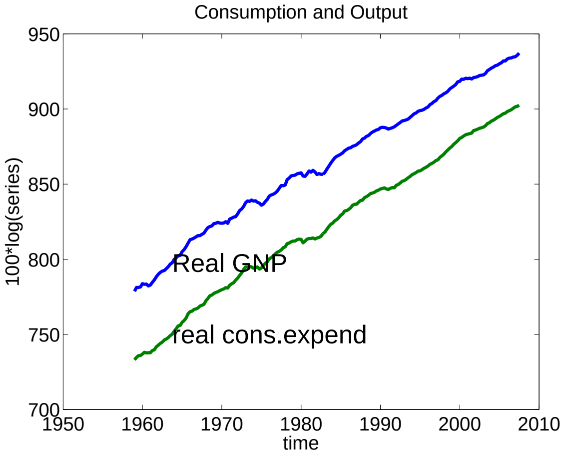
More generally, y_t may be nonstationary, e.g. contain a time trend or a time plus a unit root or be integrated of higher order, while some linear combination is stationary.

Consumption-Output-Ratio

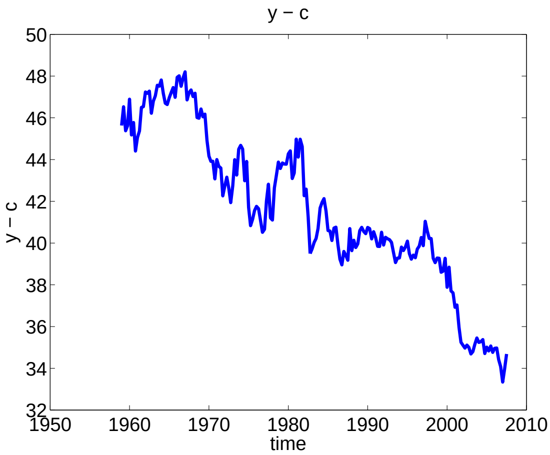


Output: GNPC96, Consumption: PCEC96, both from the Fed St. Louis Data Base. Units: Chained 2000 Dollars.

Log Output vs Log Consumption



Log Output minus Log Consumption



See also: Lettau-Ludvigson “cay” (consumption-assets-output)

A cointegrating matrix

Proposition

For a VAR(1), $Y_t = BY_{t-1} + u_t$, let $\mid z_1 \mid < 1, \ldots, \mid z_r \mid < 1$ be the stationary roots and let $z_{r+1} = \ldots = z_m = 1$ be the unit roots. Suppose, B is diagonalizable, $B = VDV^{-1}$. A cointegrating matrix β is given by the transpose of the first r rows of V^{-1}. I.e., with

$$V^{-1} = \begin{bmatrix} -- & w_1 & -- \\ -- & w_2 & -- \\ \vdots & \vdots & \vdots \\ -- & w_m & -- \end{bmatrix}, \text{ write } \beta = \begin{bmatrix} | & | & \vdots & | \\ w_1' & w_2' & \ldots & w_r' \\ | & | & \vdots & | \end{bmatrix}$$

Defining more matrices

- Let $D_r = \text{diag}(z_1, \ldots, z_r)$ the $r \times r$ diagonal matrix, containing only the stationary roots.
- Let ν be the $m \times r$ matrix of the first r columns of V,

$$\nu = \begin{bmatrix} | & | & \vdots & | \\ v_1 & v_2 & \ldots & v_r \\ | & | & \vdots & | \end{bmatrix}$$

- Let β_* be the $m \times (m - r)$ matrix of the transpose of the last $m - r$ rows of V^{-1}.
- Let ν_* be the $m \times (m - r)$ matrix of the last $m - r$ columns of V.

The stationary and nonstationary dynamics

- In total, we have now split our matrices V and V^{-1}:

$$V = \left[\begin{array}{cc} \nu & \nu_* \end{array} \right], V^{-1} = \left[\begin{array}{c} \beta' \\ (\beta_*)' \end{array} \right]$$

- The dynamics is therefore given by

$$y_t = \left[\begin{array}{cc} \nu & \nu_* \end{array} \right] \left[\begin{array}{cc} D_r & 0_{r,m-r} \\ 0_{m-r,r} & I_{m-r} \end{array} \right] \left[\begin{array}{c} \beta' \\ (\beta_*)' \end{array} \right] y_{t-1} + u_t$$

Splitting the impulse responses

- For a vector a, write

$$a_r = \beta' a \in \mathbb{R}^r \text{ and } a_* = (\beta_*)' a \in \mathbb{R}^{m-r}$$

- The impulse response is now

$$r_a(k) = \nu D_r^k a_r + \nu_* a_*$$

- The first part is the transitory part, the second part is the permanent part.
- In particular, the long-run impulse response is

$$r_a(\infty) = \lim_{k \to \infty} r_a(k) = \nu_* a_*$$

Error Correction

- Define

$$\alpha = \nu(I_r - D_r)$$

- The VAR(1) can alternatively be rewritten as

$$\Delta y_t = -\alpha\beta' y_{t-1} + u_t$$

- Note: $\alpha\beta' = I_m - B$.
- This representation is called the error-correction representation
- Idea: the change of y_t from one period to the next is driven
  - by new shocks, u_t,
  - by the convergence of the stationary component $\beta' y_{t-1}$ back to zero.

Error Correction

Proposition

A VAR(k), which is cointegrated of rank r can be given an error correction representation,

$$A^*(L)\Delta y_t = -\alpha\beta' y_{t-1} + u_t$$

where $A^*(L)$ only has stable roots.

Source: Robert F. Engle and C.W.J. Granger, "Co-Integration and Error Correction: Represenation, Estimation and Testing," Econometrica, vol. 55, no. 2, March 1987, pp. 251-276.

Numeric example 2

- Suppose

$$\begin{aligned} B &= \begin{bmatrix} 0.68 & -0.24 \\ -0.24 & 0.82 \end{bmatrix} \\ &= \begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix} \end{aligned}$$

- With that,

$$\nu = \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix}, \nu_* = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}, \beta = \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix}, \beta_* = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}$$

and

$$\alpha = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}, \alpha\beta' = \begin{bmatrix} 0.32 & 0.24 \\ 0.24 & 0.18 \end{bmatrix} = I_2 - B$$

Unit Root Tests, Cointegration Tests

- A number of tests have been developed during the last 20 years to test for unit roots and/or cointegration, to estimate the cointegrating rank and the cointegrating vectors.
- Johansen procedure.
- Standard econometric packages implement these.
- Often, the cointegrating vector is known.
- Conditionality. “Size = Power”.
- Bayesian vs classical perspective.
- Pretesting, superconsistency?

Pretesting? Superconsistency?

- Example:

$$y_t = By_{t-1} + u_t, \text{ where } B = \begin{bmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{bmatrix}$$

- Suppose, truth is $\rho_1 = 1, \rho_2 = 0.9$
- $\hat{\rho}_1 \to 1$: superconsistent, non-normal asymptotics.
- $\hat{\rho}_1 + \hat{\rho}_2 \to 1.9$: root-T convergence, normal asymptotics.
- Note: the unit roots are those aspects of the data, we ought to be particularly sure about. It is the "stationary" aspects that generate larger estimation errors.
- Often, one may be interested in the "stationary" issues in the data: usual root-n convergence. Then, why pretest for unit roots?
- In practice, both types of uncertainty are often equally large. Another reason for Normal-Wishart-Bayesian procedures.

Differencing or not?

- Given that there might be unit roots in the data, should one run VARs on the differenced data $\Delta y_t = y_t - y_{t-1}$ instead?
- Overdifferencing: Suppose y_t is cointegrated. A VAR in Δy_t will then "miss" the information coming from $\beta' y_t$:

$$(I_m - \tilde{B}(L))\Delta y_t = u_t \quad \text{vs} \quad \Delta y_t = -\alpha\beta' y_{t-1} + u_t$$

- Not as bad as it seems. With increasing lag length on the lhs, convergence to true dynamics (Marcet).
- Or: could first determine the cointegrating rank and vectors and proceed with the appropriate error correction representation, as if one were sure that one got the right cointegrating vectors.
- Or estimate in levels. Unit roots will produce superconsistency "on occasions".
- Religion? It all "more or less" works in practice, if used sensibly.

Outline

Impulse and Propagation

- The raw estimation results for a VAR are rarely interesting. Alternatively, one can represent a VAR as responses to impulses:
- Wold decomposition:

$$
\begin{aligned}
y_t &= (I_m - B(L))^{-1} u_t \\
&= \sum_{s=0}^{\infty} C_s u_{t-s}
\end{aligned}
$$

  The impulse response to the j-th one-step ahead prediction error is given by the j-th column of the C_s.
- ... however, responses to one-step ahead prediction errors are rarely economically interesting.

Identifying Shocks

- To interpret the VAR in an economically meaningful way, one needs to disentangle u_t into "structural" shocks ϵ_t, like e.g. monetary policy shocks, productivity shocks, etc.,

$$u_t = A\epsilon_t = \begin{bmatrix} | & | & \vdots & | \\ a_1 & a_2 & \ldots & a_m \\ | & | & \vdots & | \end{bmatrix} \begin{bmatrix} \epsilon_{t,1} \\ \vdots \\ \epsilon_{t,m} \end{bmatrix} = a_1\epsilon_{t,1} + \ldots + a_m\epsilon_{t,m}$$

- Let a_j be the j-th column of A. The impulse response of all variables at horizon s to the j-th structural shock $\epsilon_t' = e_j$ is then given by

$$r_s = C_s a_j$$

Identification

- Assuming $E[\epsilon_t \epsilon_t'] = I_m$, the only restriction on A is

$$\Sigma = E[u_t u_t'] = AE[\epsilon_t \epsilon_t']A' = AA'$$

- So, one needs (at least) $m(m-1)/2$ additional identifying assumptions to identify ϵ_t.
- Furthermore, restrictions / conventions for the sign of the columns of A need to be imposed.
- Surveys: Christiano-Eichenbaum-Evans, Leeper-Sims-Zha, Favero, Canovas book, others.

Identification via Cholesky decomposition

$$u_t = A\epsilon_t = a_1\epsilon_{t,1} + \ldots + a_m\epsilon_{t,m}, \; \Sigma = AA'$$

- Assume A is lower triangular (Cholesky decomposition)

$$A = \begin{bmatrix} * & 0 & 0 & \cdots & 0 \\ * & * & 0 & \cdots & 0 \\ * & * & * & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \cdots & * \end{bmatrix}$$

- Sign convention: diagonal elements are nonnegative.
- Identifying assumption: Shock j has no contemporaneous impact on variables $i < j$.
- Recursive identification, Wold causal chain, causal ordering.
- Granger causality: “helping to predict”.

Block Recursion

- Start from a Cholesky decomposition. Focus on some shock j “of interest”.
- Consider an alternative decomposition

$$\Sigma = \tilde{A}\tilde{A}'$$

  per linear recombining shocks within $i < j$ and within $i > j$.
- Identifying assumption for shock j remains unchanged.

Remark

When the focus is on a particular shock, it is enough for A to be block-diagonal. I.e., take a stand on which variables come “before j” and which variables come “after j”. More is not needed.

- Can do this also with a subset of shocks.
- Block recursion.

Structural VARs

$$u_t = A\epsilon_t, \; \Sigma = AA'$$

- "Structural" VAR: Impose equations in $u_{t,i}$ and $\epsilon_{t,j}$. Often: impose zero restrictions on entries in A or A^{-1}.
- Exact identification vs overidentification.
- See Bernanke, Blanchard-Watson, Sims.

Long-run identification

- Blanchard-Quah, Gali, etc: effect of technology shocks.
- Decompose into permanent ("supply"/"technology") shocks and transitory ("demand") shocks.
- Cointegrated VAR:

$$y_t \quad = \quad \begin{bmatrix} \nu & \nu_* \end{bmatrix} \begin{bmatrix} D_r & 0_{r,m-r} \\ 0_{m-r,r} & I_{m-r} \end{bmatrix} \begin{bmatrix} \beta' \\ (\beta_*)' \end{bmatrix} y_{t-1} + A\epsilon_t$$

- $u_t = a_1 \epsilon_{t,1} + \ldots + a_m \epsilon_{t,m}$
- Long-run impulse response to shock j is

$$r_{a_j}(\infty) = \nu_* (\beta_*)' a_j$$

- Identifying assumption 1: transitory shocks. $r_{a_j}(\infty) = 0$.
- Identifying assumption 2: permanent shocks. Orthogonal to transitory shocks

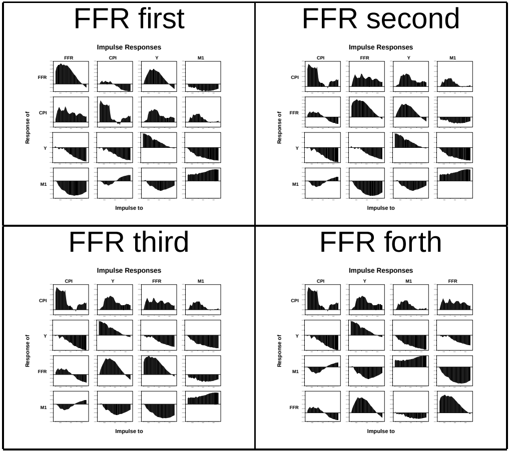
Mix-and-match

- Long-run identifying assumptions typically only identify subspaces, ...
- ... plus one may have some view on a partial contemporaneous ordering of variables or a partial set of structural equations.
- In principle, several of these restrictions can be combined.
- ... and/or combined with sign restrictions (below).
- Exact identification, over-identification.

Example: What is a monetary policy shock?

- A shock to the Federal Funds Rate,
  - ... that is ordered last in a Cholesky decomposition
  - ... that has no permanent effect on output
  - ... that solves a set of structural equations, sorting out demand from supply shocks.
  - ...
- Often, the problem is: the reactions of the other variables do not look "as they should".

Example: FFR with CPI infl, real GNP, M1, 1986-2001



Typical puzzles

- The liquidity puzzle: when identifying monetary policy shocks as surprise increases in the stock of money, interest rates tend to go down, not up.
- The price puzzle: after a contractionary monetary policy shock, even with interest rates going up and money supply going down, inflation goes up rather than down.

What to do?

- Accept the results as they come out or
- continue the specification search, until one set of identifying restrictions delivers the ”right result” or
- impose the ”right results” as part of the identifying restrictions
- Continuation of Sims agenda: avoid unreasonable identication restrictions.

Outline

Sign restrictions: some literature

- Mark Dwyer (1997), "Dynamic Response Priors for Discriminating Structural Vector Autoregressions," draft, UCLA.
- Jon Faust (1998), "The Robustness of Identified VAR Conclusions about Money," Carnegie-Rochester, 49(0), Dec., 207-44. Uhlig (1998): comment.
- Canova, Fabio and Gianni de Nicolo (2002), "Monetary Disturbances matter for business fluctuations in the G-7," Journal of Monetary Economics, September, 49(6), 1131-59.
- Uhlig, Harald (2005), "What are the effects of monetary policy shocks on output?", Journal of Monetary Economics, 381-419.
- Blanchard, Olivier Jean, “A Traditional Interpretation of Macroeconomic Fluctuations,” American Economic Review, vol. 79, no. 5 (Dec. 1989), 1146-1164.
- Leamer, ...

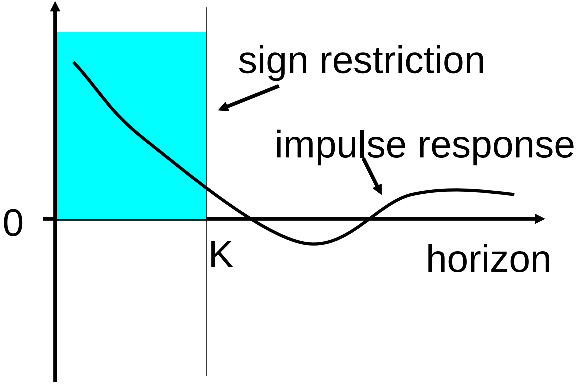
Example: monetary policy shocks

The conventional wisdom says: after a contractionary monetary policy shock,

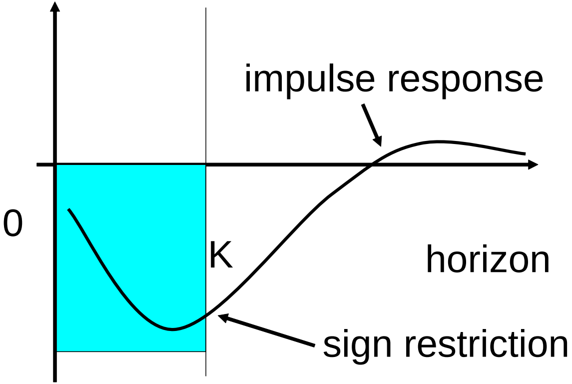
- Interest rates go up
- Inflation goes down
- Nonborrowed reserves go down
- real output goes down

Let's impose that (or a subset) in order to identify the monetary policy shock!

Sign restrictions on interest rates



Sign restrictions on prices



Impulse vectors

- Recall:

$$u_t = A\epsilon_t,\ AA' = \Sigma,\ y_t = \sum_{s=0}^{\infty} C_s u_{t-s}$$

- Recall: let a be the j-th column of A. The impulse response of all variables at horizon s to the j-th structural shock $\epsilon_t' = e_j$ is then given by

$$r_s = C_s a$$

Definition

The vector $a \in \mathbb{R}^m$ is called an impulse vector, iff there is some matrix A, so that $AA' = \Sigma$ and so that a is a column of A.

- Let $\tilde{A}\tilde{A}' = \Sigma$ be some decomposition. Then, $a \in \mathbb{R}^m$ is an impulse vector, if and only if for some vector α of unit length, $a = A\alpha$.

Identification

- Task: find e.g. the monetary policy shock impulse vector.
- Identifying restrictions: the responses of a subset of the variables. This can mean
  - a particular shape of the response (Dwyer, 1997),
  - the instantaneous response, i.e. the sign of the entries in the impulse vector (Faust, 1998),
  - sign restrictions on the correlations of impulse responses, i.e. sign restrictions on the product of the impulse response of e.g. variable 1 at horizon j_1 and variable 2 at horizon j_2 (Canova-de Nicolo, 2002).
  - sign restrictions on the impulse response for horizons $k = 0, \ldots, K$ after the shock (Uhlig, 2005),
- Additional restrictions (long-run neutrality, some partial ordering) can be imposed without problem.

Imposing Sign Restrictions: A Bayesian Approach

- Form a prior for the reduced-form VAR. Using the data, form the posterior.
- Take a draw (B, Σ) from the posterior. Calculate the Cholesky decomposition $\Sigma = AA'$.
- Take a draw α from the unit sphere (dimensions = number of variables). Calculate $a = A\alpha$.
- Calculate the impulse responses to a. If they satisfy the sign restrictions, keep the joint draw (B, Σ, a), otherwise discard.
- Given the draws kept, calculate statistics of interest.

Outline

The effects of monetary policy on output

- Source: Uhlig, Harald (2005), ”What are the effects of monetary policy shocks on output?”, Journal of Monetary Economics, forthcoming.
- The conventional wisdom says: after a contractionary monetary policy shock,
  - interest rates go up
  - Inflation goes down
  - Nonborrowed reserves go down
  - real output goes down
- Do we really know?

Searching for an answer

- The research question: what is the effect of monetary policy shocks on output?
- Traditional identification may be tainted by the specification search (Leamer, 1978): it can be hard to tell what is assumed and what is concluded.
- This is avoided with the sign restriction approach.
- Method: impose all the other sign restrictions, but not the restriction on output (”agnostic identification”).

The data

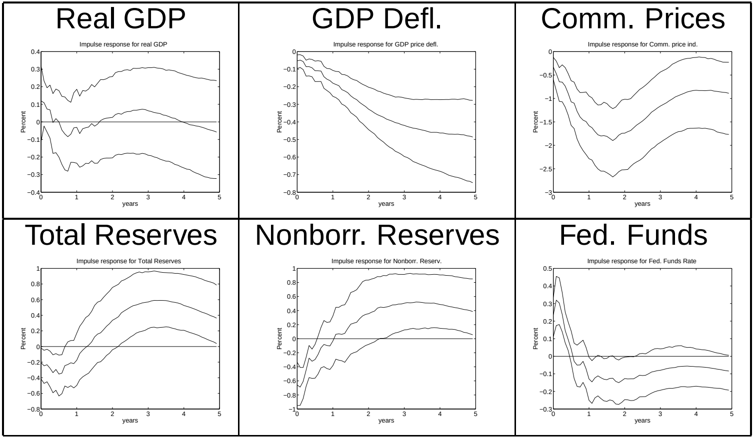
Monthly data, January 1965 to December 1996.

Series	Response restricted?
Real output.	no
GDP deflator.	Negative or zero.
Commodity price index.	Negative or zero.
Total Reserves.	no
Nonborrowed Reserves.	Negative or zero.
Federal Funds Rate.	Positive or zero.

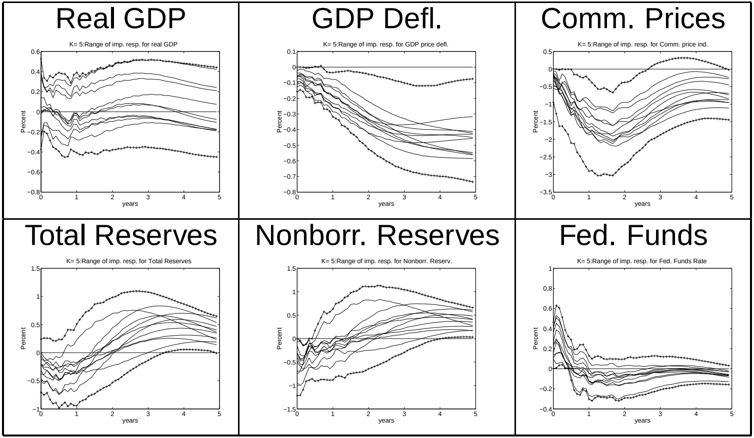
The restrictions are for horizons $k, k = 0, \ldots, K$.

Results

Restriction Horizon: $K = 5$.

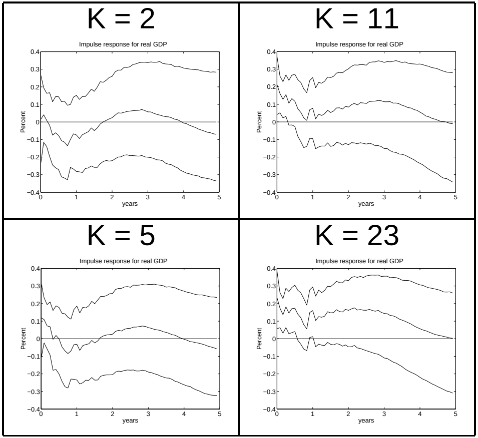


Sample impulse responses



Results for GDP, varying K

Varying the Restriction Horizon $K = 2, 5, 11, 23$.



Summary of the results

Remark

While it may be that contractionary monetary policy shocks lead to a reduction in real GDP, the evidence in the data is inconclusive on this point.