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THE PERMANENT INCOME HYPOTHESIS AND CONSUMPTION DURABILITY: ANALYSIS BASED ON JAPANESE PANEL DATA*

FUMIO HAYASHI

The permanent income hypothesis with durability of commodities is tested on a panel of about 2,000 Japanese households for several commodity groups. Under static expectations about real interest rates and for some class of utility functions, consumption, which is a distributed lag function of current and past expenditure, follows a martingale. Main empirical results are (i) the durability of commodities usually classified as services is substantial, (ii) the hypothesis applies to about 85 percent of the population consisting of wage earners, and (iii) income changes explain only a small fraction of the movements in expenditure.

I. INTRODUCTION AND SUMMARY

The empirical validity of the permanent income hypothesis[1] is a long-standing issue that has been debated for nearly three decades. At the heart of the debate is the question of whether consumption is "too sensitive" to income fluctuations. Its operational meaning was not given until Hall [1978], who has shown

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1. In this paper the permanent income hypothesis is taken to mean that households optimize their intertemporal utility function subject to the lifetime budget constraint.

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that the marginal utility of consumption is a martingale under the permanent income hypothesis with constant real interest rates. Since then, quite a few papers have studied the issue of the excess sensitivity of consumption.[2] Many of them have also tried to estimate the fraction of "liquidity-constrained" (rule-of-thumb) households whose consumption simply tracks disposable income.

As impressive as it is, the literature has failed to pay enough attention to the distinction between consumption and expenditure.[3] As the permanent income hypothesis is a theory about the service flow of consumption, the literature has either looked at perishables (nondurables or services or both) alone or singled out durables for special treatment. But it is not entirely clear that most commodities labeled as nondurables or services are perishable so that consumption and expenditure can be equated. A good example is dental services. People go to a dentist not because they enjoy the treatment but because it is hoped that their teeth will be in good shape for some time to come. So dental services are physically durable. Another example is a pleasure trip. It is physically perishable, but it may have a lasting psychological effect on preferences as people derive utility from the memory of a trip. If so, recreational expenditure should be treated as if it is durable.

The present study attempts to address this durability issue as well as to obtain a sharper estimate of the fraction of households in the population for which total expenditure tracks disposable income. It uses a four-quarter panel of Japanese households for several commodity groups. The unique feature of this data set is its inclusion of respondents' expectations about expenditure and income. A surprising fact revealed by the data set is that expenditure changes are negatively correlated over time, a fact that appears to be inconsistent with consumption smoothing. This can, however, be reconciled with the permanent income hypothesis by allowing for "transitory consumption" (measurement error and preference shocks) and the durability of commodities.

The theoretical model in this study takes durability into account by taking consumption to be a distributed lag function of expenditure. If the commodity is perfectly perishable, only current expenditure shows up in the distributed lag. The model also allows for preference shocks by making the utility function depend on a

2. See King [1983] for a survey of recent contributions.

3. Most recently, Eichenbaum and Hansen [1984] and Dunn and Singleton [1984] have studied the durability of commodities using U. S. aggregate time-series data.

preference shift parameter. The model is standard in other respects: a household's objective function is a time-separable function of consumption, and households can freely borrow and lend at the nominal risk-free interest rate. Under the assumption of static expectations about the real interest rates and for some specific preferences, it is shown that *consumption* follows a martingale (with an intercept term that depends on the real rate), so that a change in *expenditure* on the commodity in question is a univariate autoregression. Thus, the model provides a unified treatment of commodities with differing degrees of durability.

The estimation of the model is carried out with no restrictions on the serial correlation structure of income, preference shocks, and measurement error but with the restriction of geometric decay on the distributed lag function for consumption. The crucial identifying assumption is that neither preference shocks nor measurement error in expenditure is correlated with income and that there is no measurement error in income. Since expectations are directly measured, there is no need to make a specific assumption about expectations of future income and expenditure. The main findings are as follows. Clothes and recreation and education are highly durable. The fraction of the population of wage earners for which total expenditure tracks income is sharply estimated to be around 15 percent. If these "liquidity-constrained" households are allowed for, the model is successful in mimicking the sample covariances between (expected and unexpected) expenditure and income changes. However, only a small fraction of expenditure changes is explained by income.

The plan of the paper is as follows. Section II describes the nature of the data set. Section III examines some summary statistics of the (actual and expected) expenditure and income variables. The theoretical model is presented, and the martingale property of consumption is derived in Section IV. The estimation procedure is discussed in Section V. The parameter estimates are presented in Section VI.

II. The Data and the Variables Used

The data set for the present study is obtained from the 1982 Survey of Family Consumption compiled by the Economic Planning Agency of the Japanese government. This is an interview panel survey in which families reported to visiting interviewers every three months over a four-quarter period (1981:Q2–1982:Q1).

More specifically, the respondents were asked at the end of each quarter to provide the following information: (i) expenditures on eleven different and mutually exclusive commodity groups for the quarter,[4] (ii) "normal" income consisting of regular wages and salaries, net of income, social security, and national health insurance taxes, (iii) "temporary" income consisting of (after-tax) bonuses, income from interest, dividends and estates, insurance payments, severance pay, and tax returns at the end of a calendar year, (iv) the respondent's expectations (at the end of each quarter) of all the variables in the above three items for the following quarter and (v) family characteristics (occupation, family size, age of the head, and housing tenure). Since the survey's distinction between "normal" and "temporary" income seems arbitrary, this study uses disposable income (the sum of "normal" and "temporary" income) as the income variable. The survey does not cover one-person households. Although this is not a diary survey, interviewers actually visited the households every quarter and the respondents filled out the questionnaire in the presence of the interviewer. There is practically no attrition: for any quarter at least 99.5 percent of the surveyed 5,837 households responded. Information about food, for example, is elicited by the question: "How much did your family spend on food for the last three months?" and "How much do you expect your family will spend on food for the next three months?"

Our analysis will focus on "workers' households," i.e., households whose head is on a payroll, which are about 58 percent of the original sample. This is because identification of the fraction of the population for which expenditure tracks income rests on the assumption of no measurement error in income. (The results for all households will be mentioned when necessary.) It became necessary to delete the sample from the entire Tokyo prefecture and some other parts of the country, because a four-quarter panel could not be formed due to some coding problem. At this stage the sample size became 2,707. From this, households with missing values[5] (497 cases), households whose head's age either increases by more than a year or decreases (150 cases), and then households which changed their housing tenure (from a nonhomeowner to a

4. The expenditure data refer to the full cost of purchases even though full payment may not have been made at the date of purchase.

5. Of the 2,707 cases, 331 cases did not report actual values, and 399 cases did not report expected values for all the relevant variables. Thus, we are not deliberately dropping households that cannot form expectations.

homeowner or from a homeowner to a nonhomeowner)[6] (46 cases) are deleted. This left a sample of 2,014 cases. For this sample the empirical distribution was examined and a decision was made to remove seven cases that reported extreme values.[7] The final sample size for workers' households became 2,007. (The final sample size for all households under a similar sample selection rule became 3,520.)[8]

Although expenditures are classified into eleven groups in the original survey, this study uses a broader classification of seven commodity groups because some of the original commodity groups (e.g., recreation, education, and "cultural" expenses) seem to be close substitutes. The variables used in this study are as follows:

C_1 = food (including liquor and beverages and excluding meals away from home);

C_2 = rents, fuel, and utilities;

C_3 = clothing and household textiles;

C_4 = consumer durables (including furniture, electric appliances, musical instruments, cameras, automobiles, bikes, bicycles, sports equipment, and stainless sinks);

C_5 = recreation and education (including recreational expenditure such as vacation expenses, movies, admission fees, and meals away from home; plus educational expenses such as tuition, books, supplies and equipment for kindergarten, elementary and high schools, colleges and universities; plus "cultural" expenses such as reading materials, tuition for such cultural activities as flower arrangement, cooking, tea ceremony, music and dance; plus "social" expenses such as gifts and contributions);

6. The reason for doing this is to treat homeowners and nonhomeowners symmetrically. Rents should include imputed rents for homeowners, which will disappear when expenditure changes are taken.

7. One case reported about 3.8 million yen for "other expenses" in period 1. This was a clear outlier. (The next highest value was about 1.2 million yen.) Four cases reported temporary income for period 1 in excess of 10 million yen. If the four cases are included, the sample standard deviation of temporary income in period 1 nearly quadruples. Another case reported temporary income in excess of 10 million yen in period 2. The remaining seventh case reported expected normal income for period 5 of 19.89 million yen. Since its actual normal income in periods 1,2,3, and 4 is 1.989 million yen, we concluded that the number was wrong by one decimal.

8. If (i) actual or expected expenditure exceeds 3 million yen in any of the commodity groups excluding durables, or (ii) actual or expected temporary income exceeds 10 million yen, or (iii) expenditure or income variables change by one decimal, then the case is deleted.

C_6 = medical expenses not paid by the national health insurance, glasses and medical appliances for personal use;
C_7 = other (including housewares, repairs, personal care services, transportation and communication, telephone charges, private insurance premiums, shoes, umbrellas, and vehicle operations; plus money given to family members other than the head);
YD = disposable income, namely the sum of "normal" income and "temporary" income;
AGE = age of the household head;
FSZ = family size (i.e., the number of people in the family).

We use the second subscript to denote the quarter. For example, C_{11} is food expenditure in the first quarter of the panel (1981:Q1), and C_{14} is food expenditure in the fourth quarter (1982:Q1). Expenditure and income variables are all deflated by the relevant components of the implicit price deflator for personal consumption expenditures in the National Income Accounts. The overall deflator is used to deflate C_7 and YD. Since prices were very stable during the period covered by the panel (the inflation rate during the period was 2.9 percent), the choice of the deflator is immaterial.

The present data set also contains information on expectations held by households. Since the period of the survey is from 1981:Q2 to 1982:Q1, reported expectations refer to the period of 1981:Q3 to 1982:Q2. We shall put superscript "e" to denote expectations. For example, C_{1t}^e denotes the household's expectation, formed at the end of period $t - 1$, of C_1 in period t ($t = 2,3,4,5$, or 1981:Q3 through 1982:Q2). As the interview was conducted at the end of each quarter, expectations about variables dated t are based on information available at the end of period $t - 1$ that includes actual values of variables dated $t - 1$. We assume that relevant components of the implicit price deflator are correctly foreseen by households one quarter in advance, so that actual values of the relevant deflators are used to convert expected values into real terms.

III. Some Summary Statistics

The sample means and standard deviations of expenditure and income variables in real terms are shown in Table I. Both expenditure and income exhibit seasonality; they all rise in the

TABLE I

MEANS AND STANDARD DEVIATIONS OF LEVELS[a]

Variable	1981:Q2	1981:Q3	1981:Q4	1982:Q1
C_1 (food)	213.9 (84.1) [0.0]	218.0 (84.3) [0.0]	241.6 (96.8) [0.0]	216.3 (86.7) [0.0]
C_2 (rents and utilities)	66.7 (44.8) [0.002]	65.2 (43.4) [0.001]	74.9 (47.9) [0.001]	74.7 (44.3) [0.001]
C_3 (clothes)	52.8 (74.5) [0.023]	50.4 (78.1) [0.027]	72.9 (75.6) [0.013]	53.4 (61.5) [0.036]
C_4 (durables)	59.4 (167.4) [0.418]	61.0 (183.2) [0.410]	66.0 (144.0) [0.314]	42.3 (121.7) [0.448]
C_5 (recreation & education)	178.4 (187.4) [0.0]	186.9 (157.2) [0.0]	198.8 (165.3) [0.0]	182.9 (180.7) [0.0]
C_6 (medical)	20.8 (38.3) [0.093]	21.5 (31.3) [0.084]	23.8 (39.3) [0.079]	21.8 (38.6) [0.084]
C_7 (other)	154.6 (109.5) [0.000]	161.0 (113.3) [0.000]	180.6 (123.4) [0.0]	147.7 (95.6) [0.001]
CON (total)	743.5 (399.1) [0.0]	760.7 (377.8) [0.0]	857.6 (390.4) [0.0]	733.2 (356.0) [0.0]
YD (disposable income)	887.6 (493.7) [0.0]	904.9 (422.6) [0.000]	1,151.2 (559.4) [0.0]	745.8 (326.5) [0.000]

a. In thousands of 1980 yen. Sample standard deviations are in parentheses. The numbers in brackets are the fraction of the sample (of 2,007 households) which reported zeros for the variable in question. If no households reported zeros, "0.0" is entered in brackets.

fourth quarter of the year. The lumpiness of durables is reflected in the large standard deviation and the high fraction of households that report zero expenditure on durables.

The theory to be presented in the next section is stated in terms of expenditure changes. Table II shows the sample means and standard deviations of (actual and unexpected) expenditure and income changes. As expected, the most volatile commodity group is durables. Both the level and the change of durables expenditure vary a lot across households. The standard deviation is larger for actual changes than for unexpected changes because part of actual changes is foreseen.

TABLE II

MEANS AND STANDARD DEVIATIONS OF ACTUAL AND UNEXPECTED EXPENDITURE AND INCOME CHANGES[a]

Variable (X)	$X_2 - X_1$	$X_2 - X_2^e$	$X_3 - X_2$	$X_3 - X_3^e$	$X_4 - X_3$	$X_4 - X_4^e$
C_1	4.1	−0.7	23.6	13.6	−25.2	−0.6
	(47.7)	(48.1)	(49.2)	(49.1)	(53.5)	(46.7)
C_2	−1.5	−0.6	9.7	2.3	−0.1	3.3
	(32.5)	(28.7)	(33.1)	(32.2)	(32.3)	(27.0)
C_3	−2.4	6.8	22.4	16.1	−19.5	11.0
	(78.2)	(53.0)	(76.7)	(59.6)	(71.6)	(50.0)
C_4	1.5	27.5	5.0	19.8	−23.7	16.7
	(228.5)	(172.7)	(214.0)	(150.3)	(179.3)	(111.1)
C_5	8.5	16.5	11.9	21.4	16.5	20.8
	(165.2)	(104.4)	(147.5)	(101.7)	(168.9)	(116.7)
C_6	0.7	4.3	2.3	2.8	−2.0	2.1
	(41.9)	(29.6)	(41.8)	(36.8)	(48.1)	(33.2)
C_7	6.4	12.6	19.6	15.7	−32.9	3.6
	(89.7)	(79.2)	(91.1)	(83.9)	(84.0)	(56.0)
CON	17.2	64.9	96.8	90.9	−124.3	55.3
	(322.5)	(233.5)	(290.2)	(226.1)	(292.4)	(196.2)
YD	17.3	29.6	246.4	67.1	−405.5	23.0
	(443.0)	(198.6)	(367.2)	(177.5)	(359.2)	(123.4)

a. Sample standard deviations are in parentheses. In thousands of 1980 yen.

Table III reports the sample autocorrelation of changes.[9] The first-order autocorrelation coefficients are uniformly negative and large in absolute value. This is surprising, because Hall's [1978] permanent income hypothesis implies that changes in consumption are serially uncorrelated as the level of consumption is changed only when the consumer receives new information. That implication appears to be inconsistent with the data. It is usually suspected that one of the most likely reasons for the failure of the permanent income hypothesis is that it takes time for consumers to adjust to new information [Hall, 1978]. But this cannot be the reason for the strong negative autocorrelation, because the lagged responses should induce *positive* autocorrelation. Another explanation for the negative autocorrelation is the seasonality in expenditure. In particular, the negative correlation between changes from 1981:Q3 to Q4 and from 1981:Q4 to 1982:Q1 must be at least

9. The summation in the calculation of autocorrelations is over households, not over time. This becomes relevant when we examine the property of unexpected changes. See the last paragraph of this section.

TABLE III

SAMPLE AUTOCORRELATION OF CHANGES

	Seasonally unadjusted		Seasonally adjusted	
	Autocorrelation		Autocorrelation	
Variable	First-order[a]	Second-order	First-order[a]	Second-order
C_1	−0.269***	−0.102***	−0.304***	−0.085***
	−0.496***		−0.421***	
C_2	−0.303***	−0.034	−0.338***	−0.034
	−0.540***		−0.503***	
C_3	−0.408***	−0.128***	−0.504***	−0.102***
	−0.439***		−0.263***	
C_4	−0.555***	0.000	−0.588***	0.020
	−0.469***		−0.355***	
C_5	−0.418***	0.007	−0.409***	0.000
	−0.483***		−0.452***	
C_6	−0.353***	−0.031	−0.349***	−0.028
	−0.500***		−0.448***	
C_7	−0.459***	−0.068**	−0.493***	−0.061**
	−0.541***		−0.460***	
CON	−0.450***	−0.049*	−0.467***	−0.050*
	−0.454***		−0.382***	
YD	−0.563***	0.071**	−0.639***	0.023
	−0.683***		−0.460***	

a. Two first-order autocorrelation coefficients are calculated. The numbers that first appear in the column for first-order autocorrelation are the correlation coefficient between $X_3 - X_2$ and $X_2 - X_1$, and the numbers that appear below them are the correlation coefficient of $X_4 - X_3$ and $X_3 - X_2$ ($X = C_1, C_2, \ldots, C_7$, *CON*, *YD*).

*Significant at the 5 percent level.

**Significant at the 1 percent level.

***Significant at the 0.1 percent level.

partly due to the general rise in expenditure in 1981:Q4. The right half of Table III shows the same autocorrelations for seasonally adjusted data.[10] They are not much different from the numbers in the left half of the table.

The negative autocorrelation in expenditure changes can be made consistent with the permanent income hypothesis in several ways. First, survey data on expenditure are subject to measurement error. When expenditure levels are measured with error,

10. *The Annual Report on the Survey of Family Consumption* (the Economic Planning Agency) has quarterly time-series data from 1977:Q2 on average expenditure. We estimate multiplicative seasonality factors using this time-series data and use them to perform seasonal adjustment on the present data set.

changes in measured expenditure will have a moving average term that can induce negative autocorrelation even if true expenditure changes are not serially correlated. Second, preference shocks that shift the marginal utility will introduce another moving average term. Third and most important, commodities may be durable, so that expenditure and consumption are not the same thing. It is consumption, not expenditure, that should be serially uncorrelated under the permanent income hypothesis. A higher level of expenditure means a larger stock of consumption, which will depress expenditure in the next period if households behave in a way to smooth out consumption (rather than expenditure) over time. The theoretical model to be presented in the next section will incorporate preference shocks and the durability of commodities, and the estimation of the model will allow for measurement error.

Another useful way to look at the data is to fit a vector autoregression (VAR) consisting of the eight variables (seven commodity groups and income) and examine their dynamic structure. Since the panel is four quarters long and only three successive changes can be calculated, the lag length is two.[11] As is clear from Table IV, there is virtually no feedback (particularly from income to expenditures), so that the VAR looks very much like a collection of univariate autoregressions. Own lags are negative and significant, which of course is a reflection of the strong negative autocorrelation of expenditure changes.

The results presented so far are not inconsistent with the permanent income hypothesis with measurement error and preference shocks. But there is also evidence that is not favorable to the hypothesis: the correlation between current expenditure changes and lagged income changes is generally negative and significant. For example, the sample autocorrelation between the change in food expenditure from 1981:Q4 to 1982:Q1 and the lagged change in income is -0.080 and highly significant. If preference shocks and measurement error are uncorrelated with income, then the correlation should be zero, because expenditure changes are forecast errors according to the hypothesis.[12] Using an annual panel data set, Hall and Mishkin [1982] found the

11. Four additional variables, AGE, its square, $FSZ_4 - FSZ_3$, and its square, are also included in the autoregressions. Many of them are insignificant.

12. This argument implicitly assumes that the correlation (across households) between forecast errors and lagged information is zero, which is not necessarily true even under rational expectations. See the next paragraph.

TABLE IV

EIGHT-VARIABLE VECTOR AUTOREGRESSION

Equation[a]	Own lags: First / Second	Significant Feedback from:[b]	SE[c]	R^2
C_1 (food)	−0.60*** −0.28***	C_6*	44.3	0.31
C_2 (rents and utilities)	−0.60*** −0.21***	C_6,* YD*	25.9	0.36
C_3 (clothes)	−0.54*** −0.34***	none	58.9	0.32
C_4 (durables)	−0.57*** −0.30***	none	147.5	0.33
C_5 (recreation & education)	−0.67*** −0.23***	none	142.1	0.29
C_6 (medical)	−0.68*** −0.28**	none	39.9	0.31
C_7 (other)	−0.66*** −0.38***	C_2,* C_3*	63.1	0.43
YD (disposable income)	−0.92*** −0.35**	none	219.5	0.63

a. The dependent variable in the C_1 equation, for example, is the change in food expenditure from 1981:Q4 to 1982:Q1, namely $C_{14} - C_{13}$.
b. The presence of feedback is determined by the significance of two lags as a whole. Heteroskedasticity-robust standard errors are used.
c. The sample standard deviation of the residuals.
*Significant at the 5 percent level.
**Significant at the 1 percent level.
***Significant at the 0.1 percent level.

same correlation to be −0.055. Their explanation is that some households are "liquidity constrained" in the sense that their consumption tracks income. Consumption by these households will introduce negative autocorrelation because income changes are negatively autocorrelated. There is, however, another explanation, which draws on the durability of commodities. If at least a part of income changes is unexpected, then a rise in income causes expenditure (and hence the stock of consumption to be carried over to the next period) to rise, which tends to depress expenditure next period. Thus, what appears to be the excess sensitivity of consumption to income may be attributable to the durability of commodities and not to liquidity constraints. The estimation procedure in this paper will distinguish between the

TABLE V

CORRELATION COEFFICIENTS OF UNEXPECTED CHANGES WITH LAGGED UNEXPECTED AND ACTUAL CHANGES

	Correlation coefficient of $X_4 - X_4^e$ with:			
Variable (X)	$X_3 - X_3^e$	$X_2 - X_2^e$	$X_3 - X_2$	$X_2 - X_1$
C_1	−0.137***	−0.033	−0.145***	−0.067***
C_2	−0.181***	−0.019	−0.134***	−0.033
C_3	−0.010	0.019	−0.035	−0.007
C_4	0.005	0.093***	−0.099***	0.043
C_5	0.022	0.034	−0.054*	0.048*
C_6	−0.004	−0.097***	−0.008	−0.111***
C_7	−0.007	−0.043	−0.035	−0.074**
YD	0.010	0.017	0.102***	−0.093***

*Significant at the 5 percent level.
**Significant at the 1 percent level.
***Significant at the 0.1 percent level.

two competing explanations of the observed correlation of expenditure with income changes.

Before turning to the theoretical model, we briefly examine the reported expectations data. The sample means and standard deviations of unexpected changes in expenditure and income are already reported in Table II. Table V shows the sample correlations of changes from 1981:Q4 to 1982:Q1 with lagged unexpected and actual changes. Although they are mostly smaller than 0.1 in absolute value, some of them are statistically significant. This, however, is not a rejection of rational expectations, because the rational expectations hypothesis implies that the correlation between unexpected changes and lagged changes is zero, if the average is taken *over time,* not *across households.*[13] In particular, the mean (across households) of unexpected changes can differ from zero as everyone can be wrong in the same direction at any given point in time. So it is generally incorrect in panel or cross-section contexts to impose the usual rational expectations or-

13. This is pointed out by Chamberlain [1984]. Suppose, to provide our own example, that there is a totally unexpected income tax reform in period t that slashes the tax rates for the rich. The forecast error $YD_t - YD_t^e$ will be positive for the rich and negative for the poor. So the covariance (across households) between the forecast error and YD_{t-1}, which equals

$$\operatorname*{plim}_{N\to\infty} N^{-1} \sum_{i=1}^{N} (YD_{it} - YD_{it}^e)\, YD_{i,t-1}$$

(where N is the sample size and i is the household index), is positive.

thogonality condition that forecast errors are uncorrelated with lagged information.[14] (Our estimation procedure, to be presented in Section V, will *not* use this type of orthogonality condition.) Some of the results in Table II are favorable to rational expectations, and some are not. Except for food and rents and utilities (C_1 and C_2), the sample standard deviation of the unexpected change from 1981:Q4 to 1982:Q1 in Table II is smaller than that of the VAR residuals reported in Table IV. On the other hand, the sample mean of unexpected changes is generally positive, which suggests consistent underprediction over time.

IV. THE THEORETICAL MODEL

This section presents a model of households with preference shocks and the durability of commodities. Consider a household whose intertemporal decision problem is to maximize

$$E_t\left[\sum_{s=0}^{T} \beta^s U(\overline{C}_{t+s};\ \eta_{t+s})\right], \tag{1}$$

where E_t is the expectations operator associated with the subjective probability distribution (assumed by the household) of future variables that are uncertain to the household, β is a discount factor, T is the length of the remaining life, $\overline{C}$ is a vector of *consumption* of n commodities, $U(.)$ is the instantaneous utility function, and η is a vector of variables (preference shocks) that shift the instantaneous utility function.[15] Preference shocks may exhibit seasonal variations. We shall make no assumptions about the correlation structure of preference shocks (across components and over time). The relationship between the vector of consumption $\overline{C}$ and the associated n-dimensional vector of *expenditure* C is given by

$$\overline{C}_{jt} = \sum_{k=0}^{M} (\rho_{jk}\, C_{j,t-k}) \equiv \rho_j(L)C_{jt}, \qquad j = 1,2, \ldots, n, \tag{2}$$

where $\rho_j(L) = \Sigma\rho_{jk}L^k$ is a polynomial in the lag operator L. That is, current consumption is a distributed lag function of current

14. Examples that use this type of orthogonality condition are Hall and Mishkin [1982]; Hotz, Kydland, and Sedlacek [1982]; and Bernanke [1984].

15. We omit leisure choices by assuming that utility is separable across leisure and consumption goods. Anyway, there is no information on labor supply in the present data set. This point is further discussed in Section V.

and past expenditure. This is a generalization of the usual formula for durables, where $\overline{C}_{jt}$ is service flow from the stock of durables and the distributed lag coefficients ρ_{jk} $(k = 0,1,2, \ldots)$ are of the Koyck type.

We assume that the household has access to asset markets including access to a nominal risk-free security whose nominal interest rate is R_t. The maximization of (1) subject to (2) gives the first-order necessary condition for the nominal risk-free security:[16]

$$(3) \qquad E_t\left\{\sum_{k=0}^{M'} [\beta^k MU_j(t+k)\,\rho_{jk}]\right\} = E_t\left\{(1+r_{j,t+1})\,\beta\sum_{k=0}^{M''}[\beta^k MU_j(t+k+1)\rho_{jk}]\right\}, \qquad j = 1,2,\ldots,n.$$

Here, M' is min (M,T), M'' is min $(M,T-1)$, $MU_j(t) = \partial U(\overline{C}_t; \eta_t)/\partial\overline{C}_{jt}$ is the marginal utility of commodity j, and $r_{j,t+1}$ is the real rate on commodity j; i.e., $1 + r_{j,t+1} = (1 + R_t)\, p_{jt}p_{j,t+1}$, where p_{jt} is the price of commodity j in period t. The left-hand side of this equation is the marginal cost of forgoing one unit of expenditure on the jth commodity. This involves a summation from 0 to M' because a change in current expenditure influences current and future consumption. The right-hand side is the marginal benefit of increasing $1 + r_{j,t+1}$ units of expenditure on commodity j in the next period. This also involves a summation from 0 to M'' for the same reason. In Appendix A it is shown that under (3),

$$(4) \qquad E_t[(1+r_{j,t+1})\beta\, MU_j(t+1)/MU_j(t)] = 1, \qquad j = 1,2,\ldots,n$$

holds approximately if M (the length of the distributed lag) is small relative to T (the length of remaining life) and the household has static and point expectations about future real rates, and holds exactly if (as is usually the case for durables) ρ_{jk} is geometrically declining in k and $r_{j,t+1}$ is known in t and $r_{j,t+1} = r_{j,t+2}$. Equation (4) is the usual first-order condition without durability. So, essentially, the cost of having the familiar expression like (4) is the assumption of static and point expectations about future real rates.[17]

16. A first-order condition similar to (3) has been derived by Dunn and Singleton [1984].

17. Of course, this does not imply that the real rates actually do not change over calendar time.

Equation (4) can be made tractable under two alternative assumptions. First, assume that the instantaneous utility function $U(.)$ takes the following separable form:

$$(5) \qquad U(\overline{C}_t; \eta_t) = \sum_{j=1}^{n} \left\{ -\mu_j \exp\left[\frac{\eta_j - \overline{C}_{jt}}{\mu_j}\right]\right\}, \qquad \mu_j > 0.$$

Rewrite (4) as

$$(6) \quad (1 + r_{j,t+1})\beta\, MU_j(t+1)/MU_j(t) = 1 - e'_{j,t+1}, \qquad j = 1,2,\ldots,n,$$

where $e'_{j,t+1}$ is the difference between the left-hand side of (4) and the left-hand side of (6); that is, $e'_{j,t+1}$ is the forecast error of the left-hand side of (6) and satisfies $E_t(e'_{j,t+1}) = 0$. It therefore consists of new information the household receives in period $t + 1$ about labor income, real rates, and preference shocks. Take the log of both sides of (6) and use the approximation $\ln(1 + x) \simeq x$ to obtain[18]

$$(7) \quad \ln(1 + r_{j,t+1}) + \ln(\beta) + \ln[MU_j(t+1)] - \ln[MU_j(t)] = -\,e'_{j,t+1}.$$

Under the assumed utility function (5) this becomes

$$(8) \qquad \overline{C}_{j,t+1} - \overline{C}_{jt} = d_{j,t+1} + e_{j,t+1}, \qquad j = 1,2,\ldots,n,$$

where

$$(9) \qquad d_{j,t+1} = \mu_j[\ln(1 + r_{j,t+1}) + \ln(\beta)] + \eta_{j,t+1} - \eta_{jt},$$

and $e_{j,t+1} = \mu_j e'_{j,t+1}$ is the forecast error that is uncorrelated (in the household's judgment) with any information that is known to the household at the end of period t.

Another way to make (4) tractable is to assume that the instantaneous utility function $U(.)$ is quadratic:

$$(10) \qquad U(\overline{C}_t; \eta_t) = a'\overline{C}_t - (\tfrac{1}{2})\,(\overline{C}_t - \eta_t)'B(\overline{C}_t - \eta_t);$$

and that the real rates are constant and the same across commodities:

18. Equation (7) can also be obtained by imposing conditional normality assumption on expenditure levels. See Hansen and Singleton [1983].

$$(11)\qquad (1 + r_{j,t+1})\beta = 1, \qquad j = 1,2,\ldots,n.$$

It then is easy to show that (4) reduces to (8) with $d_{j,t+1} = \eta_{j,t+1} - \eta_{jt}$. This is the multi-commodity version of Hall's [1978] martingale hypothesis.

Under either assumption about the instantaneous utility function, substitution of (2) into (8) gives equations stated in terms of expenditure changes:

$$(12)\qquad \rho_j(L)(C_{j,t+1} - C_{jt}) = d_{j,t+1} + e_{j,t+1}, \qquad j = 1,2,\ldots,n,$$

which shows that, conditional on the intercept $d_{j,t+1}$, expenditure changes are a collection of univariate autoregressions. (Appendix B shows that approximately the same equation can be derived from a continuous-time model where C's in (12) are unit averages over periods of an arbitrary length.) If we can ignore the potential (across households) correlation between the forecast error $e_{j,t+1}$ and lagged expenditure changes mentioned in the last paragraph of Section IV, which may be quantitatively unimportant in view of the result in Table V, lagged expenditure changes in other commodities will help explain the current change in expenditure on the commodity in question only when η_t has a contemporaneous correlation across commodities. The result in Table IV that there is no significant feedback among commodities, then, indicates that our maintained assumption about the instantaneous utility function is consistent with the data, since a significant feedback means either that η_t has a contemporaneous correlation (which our model allows for), or that the true instantaneous utility function differs from (5) or (10).

The autoregressive form (12) is not convenient for our purposes because our data set contains only three successive expenditure changes. Our empirical implementation will assume geometric decay for $\rho_j(L)$:

$$(13)\qquad \rho_j(L) = 1 + \rho_j L + (\rho_j)^2 L^2 + \ldots = (1 - \rho_j L)^{-1},$$

so that the autoregressive representation (12) can be inverted to obtain a moving average representation:[19]

19. As long as the household's economic age is greater than one quarter of 1981:Q2, the initial condition problem does not arise here. Equation (14) is a slight generalization of the equation derived by Mankiw [1982].

$$C_{j,t+1} - C_{jt} = (d_{j,t+1} - \rho_j d_{jt}) + (e_{j,t+1} - \rho_j e_{jt}), \quad j = 1,2,\ldots,n. \tag{14}$$

This will be utilized in our later estimation.

The *intra*temporal first-order condition for the case of geometric decay can be derived from equation (A.4) in Appendix A as

$$\frac{MU_j(t)}{MU_l(t)} = \frac{(1 + r_{j,t+1} - \rho_j)p_{j,t+1}}{(1 + r_{l,t+1} - \rho_l)p_{l,t+1}}, \qquad j,l = 1,2,\ldots,n,$$

where $p_{j,t+1}$ and $p_{\ell,t+1}$ are assumed to be known in period t. We will not exploit this marginal condition for our four-quarter panel data because there is no way (unless $\rho_j = \rho_\ell$) to transform it into an equation like (14) that involves only a small number of successive expenditure changes.

In the empirical implementation, we wish to test the permanent income hypothesis with durability just presented against the alternative model of "liquidity constraints." A most natural way to incorporate liquidity constraints would be to let the interest rate be endogenous and depend on the household's income and asset position. The present data set has no information on households' assets and liabilities, so that we cannot tell whether the household is borrowing or lending at the margin. Thus, it is impossible to implement the idea of endogenous interest rates in a satisfactory fashion.[20] Our version of liquidity constraints, which is also the one employed in Hall [1978], Hall and Mishkin [1982], and Hayashi [1982], is that the marginal propensity to spend out of current disposable income is unity. If the marginal budget share α_j of commodity j is independent of income, the behavior of liquidity constrained households is described by

20. The difficulty is that the relevant interest rate, which may depend on disposable income when the household is borrowing, is exogenous when the household is lending at the margin. The after-tax real rate $r_{j,t+1}$ in $d_{j,t+1}$ also depends on the marginal tax rate on interest income. Since interest income is virtually tax-free in Japan, it is reasonable to assume that the real rates are the same across households. (Interest income for individuals from a principal of up to 3 million yen is tax-free. One can, however, avoid taxes on interest completely by maintaining accounts at several different financial institutions.) Shapiro [1984] exploits the cross-section variation in the after-tax real rates caused by variation in the marginal tax rates among households in the United States. At any rate, on our panel data the model in which the real rate is a time-invariant function of disposable income fails to fit the sample covariances of expected and unexpected expenditure changes with actual disposable income changes.

$$C_{j,t+1} - C_{jt} = \alpha_j(YD_{t+1} - YD_t), \quad j = 1,2,\ldots,n, \quad \sum_{i=1}^{n} \alpha_j = 1, \tag{15}$$

where YD is real disposable income.[21]

We assume that a constant fraction λ of the population is liquidity constrained in the sense just defined, while the remaining households in the population follow the permanent income hypothesis with durability. The model to be estimated is a weighted average of (14) and (15). This model will be referred to as the *augmented model.*

V. Econometric Issues

In this section we derive a set of overidentifying restrictions on the covariances of relevant variables implied by the augmented model. These restrictions will serve as a basis for estimation and inference that will be carried out in Section VI. Nontechnical readers can skip to the last paragraph of this section without losing continuity.

The discussion can be made clearer if we temporarily drop the commodity subscript j and introduce the household subscript i. To exploit the information on expectations included in our data set, we derive two equations that involve expectations from each of the two equations, (14) and (15). Taking expectations of both sides of (14) and nothing that $E_{it}(e_{i,t+1}) = 0$, we obtain

$$C^e_{i,t+1} - C_{it} = -\rho e_{it} + (d^e_{i,t+1} - \rho d_{it}), \quad i = 1,2,\ldots,N, \tag{16a}$$

where $d^e_{i,t+1} = E_{it}(d_{i,t+1})$ is the expectation of $d_{i,t+1}$ held by household i at the end of period t, and N is the number of households in the sample. Equations (14) and (16a) imply that

$$C_{it} - C^e_{it} = e_{it} + (d_{it} - d^e_{it}), \quad i = 1,2,\ldots,N. \tag{16b}$$

21. (i) If the distinction between consumption and expenditure is ignored, (ii) if the household is myopic in the sense that it maximizes the instantaneous utility function subject to the budget constraint that total expenditure equals disposable income, and (iii) if the utility function is given by (5), then (15) with a time-dependent intercept term can be derived with $\alpha_j = \mu_j$ and YD_t = nominal disposable income deflated by $\Sigma\, \mu_j p_{jt}$. See Pollak [1971]. The intercept term is ignored in (15) because it does not affect the subsequent discussion.

We note here that d_{it} and d^e_{it} differ across households because they depend on preference shocks η_{it} and on the preference parameters (β,μ) that may differ across households (see (9)). Similarly for (15) we obtain

$$(17a) \quad C^e_{i,t+1} - C_{it} = \alpha(YD^e_{i,t+1} - YD_{it}),$$

$$(17b) \quad C_{it} - C^e_{it} = \alpha(YD_{it} - YD^e_{it}), \qquad i = 1,2, \ldots, N.$$

We now allow for measurement error in (actual and expected) expenditure, so that (16) and (17) are rewritten as

$$(16a') \qquad C^e_{i,t+1} - C_{it} = -\rho e_{it} + u_{1it},$$

$$(16b') \qquad C_{it} - C^e_{it} = e_{it} + u_{2it},$$

$$(17a') \qquad C^e_{i,t+1} - C_{it} = \alpha(YD^e_{i,t+1} - YD_{it}) + v_{1it},$$

$$(17b') \qquad C_{it} - C^e_{it} = \alpha(YD_{it} - YD^e_{it}) + v_{2it},$$

$$i = 1,2, \ldots, N,$$

where C_{it} and C^e_{it} now stand for measured (actual and expected) expenditure. The "error terms" $(u_{1it}, u_{2it}, v_{1it}, v_{2it})$ are composed of preference shocks, individual (household-specific) differences in (β,μ), and measurement for error in expenditure.

Let λ be the fraction of "liquidity-constrained" households in the population (from which our random sample was drawn). For the ith draw from the cross-section joint distribution of $(e_{it}, YD^e_{i,t+1} - YD_{it}, YD_{it} - YD^e_{it}, u_{1it}, u_{2it}, v_{1it}, v_{2it}; \quad t = 2,3,4)$, expenditure changes $C^e_{i,t+1} - C_{it}$ and $C_{it} - C^e_{it}$ are generated by (16a',b') with probability $1 - \lambda$ and by (17a',b') with probability λ. So, for example, the population mean of $C^e_{i,t+1} - C_{it}$ is written as

$$E(C^e_{i,t+1} - C_{it}) = (1 - \lambda)[-\rho E(e_{it}) + E(u_{1it})] + \lambda[\alpha E(YD^e_{i,t+1} - YD_{it}) + E(v_{1it})].$$

Here, the expectations operator "E," which denotes the population mean associated with the cross-section joint distribution, should be clearly distinguished from the expectations operation "E_{it}," which is associated with the *subjective* distribution assumed by household i of future variables that are uncertain to the household. In particular, we have that $E_{it}(e_{it}) = 0$ but $E(e_{it})$ is not necessarily zero as explained in the last paragraph of Section III.

Our basic assumption that is required for identification is that preference shocks, measurement error in expenditure, and individual differences in the preference parameter $(\beta,\mu_1,\mu_2,\ldots,\mu_n)$ are all uncorrelated (across households) with disposable income. So u_{1it}, u_{2it}, v_{1it} and v_{2it} are uncorrelated with $YD^e_{i,t+1} - YD_{it}$ and $YD_{it} - YD^e_{it}$. Is this assumption plausible? We have assumed implicitly about the utility function (5) that leisure is separable from commodities. If leisure is nonseparable from commodities, this could be a source of preference shocks η_t, resulting in their correlation with disposable income changes. The same results if leisure is separable (or the utility function is quadratic) but the shock to leisure is correlated with η_t. Either way, it seems that a contemporaneous correlation in the components of η_t is a sign of the importance of the correlation of η_t with disposable income, although it is possible that each component of η_t is correlated with disposable income without a contemporaneous correlation. But, as discussed in the paragraph containing equation (12), the result in Table IV is consistent with the assumption of no strong contemporaneous correlation in η_t. At any rate, the assumption of no correlation between η_t and disposable income is absolutely essential because without it any theory that allows for preference shocks is consistent with any observed correlation between expenditure and income changes.

Under this assumption we can obtain the following expressions for the covariances of expected and unexpected expenditure and income changes (hereafter we drop the household subscript i and reintroduce the commodity subscript j):

$$\text{(18a)}\quad \operatorname{cov}(C^e_{j,t+1} - C_{jt}, YD^e_{t+1} - YD_t) = -(1-\lambda)\rho_j \operatorname{cov}(e_{jt}, YD^e_{t+1} - YD_t) + \lambda\alpha_j\gamma_{1t},$$

$$\text{(18b)}\quad \operatorname{cov}(C^e_{j,t+1} - C_{jt}, YD_t - YD^e_t) = -(1-\lambda)\rho_j \operatorname{cov}(e_{jt}, YD_t - YD^e_t) + \lambda\alpha_j\gamma_{2t},$$

$$\text{(18c)}\quad \operatorname{cov}(C_{jt} - C^e_{jt}, YD^e_{t+1} - YD_t) = (1-\lambda)\operatorname{cov}(e_{jt}, YD^e_{i,t+1} - YD_t) + \lambda\alpha_j\gamma_{2t},$$

$$\text{(18d)}\quad \operatorname{cov}(C_{jt} - C^e_{jt}, YD_t - YD^e_t) = (1-\lambda)\operatorname{cov}(e_{jt}, YD_t - YD^e_t) + \lambda\alpha_j\gamma_{3t},$$

$$j = 1,2,\ldots,n;\ t = 2,3,4,$$

and

$$\text{(19a)}\quad \operatorname{var}(YD^e_{t+1} - YD_t) = \gamma_{1t},$$

(19b) $$\operatorname{cov}(YD^e_{t+1} - YD_t, YD_t - YD^e_t) = \gamma_{2t},$$

(19c) $$\operatorname{var}(YD_t - YD^e_t) = \gamma_{3t} \qquad t = 2,3,4.$$

There are $3 \times 4 = 12$ covariances for each commodity j in (18) and $3 \times 3 = 9$ covariances for income in (19). So in total there are $12n + 9$ covariances. They involve the following $8n + 9$ parameters:

(20) $$\begin{aligned}&\lambda;\ \rho_1, \ldots, \rho_n;\ \alpha_1, \ldots, \alpha_{n-1} \text{ (note that } \Sigma\alpha_j = 1);\\ &\operatorname{cov}(e_{jt}, YD^e_{t+1} - YD_t),\ \operatorname{cov}(e_{jt}, YD_t - YD^e_t),\\ &j = 1, \ldots, n \text{ and } t = 2,3,4;\ \gamma_{1t}, \gamma_{2t}, \gamma_{3t},\ t = 2,3,4.\end{aligned}$$

That the $12n + 9$ covariances are functions of the $8n + 9$ parameters is the set of overidentifying restrictions implied by the augmented model.

Let θ be a vector of the $8n + 9$ parameters in (20) and g be a vector of the $12n + 9$ covariances in (18) and (19). The set of overidentifying restrictions can be written compactly as $g = g(\theta)$. It is easy to see that $g(\theta)$ is twice continuously differentiable and that $\partial g(\theta)/\partial\theta$ is of full rank unless $\lambda = 0$ or 1. Let θ^* be the true value of θ. It can be shown that $g(\theta) = g(\theta^*)$ implies that $\theta = \theta^*$; namely the model is identifiable. Since our data are a random sample, the $12n + 9$ dimensional vector of sample covariances $\bar{g}$ converges almost surely to $g(\theta^*)$ and the limiting distribution of $N^{1/2}[\bar{g} - g(\theta^*)]$ is normal with mean zero and some variance matrix Δ. If expenditure and income changes have fourth moments, then the sample variance $\bar{V}$ of cross products of expenditure and income changes converges almost surely to Δ.[22] We can then apply Propositions 1 and 2 in Chamberlain [1984] to show that the minimum distance estimator $\hat{\theta}$ of θ, which is obtained from

(21) $$\text{minimize } S(\theta) = N[\bar{g} - g(\theta)]'\bar{V}^{-1}[\bar{g} - g(\theta)],$$

converges almost surely to the true value θ^*, is asymptotically normal, and the asymptotic variance is consistently estimated by $\{[\partial g(\hat{\theta})/\partial\theta]'\bar{V}^{-1}[\partial g(\hat{\theta})/\partial\theta]\}^{-1}$. Furthermore, the minimized distance $S(\hat{\theta})$ is asymptotically distributed as chi-squared with the degrees

22. See (4.5) in Chamberlain [1982] for the formula for $\bar{V}$.

of freedom equal to the number of overidentifying restrictions. We can use this statistic to test the overidentifying restrictions.

We now make several remarks on the estimation strategy just presented. (i) A natural question arises as to why we do not look at the covariances between current expenditure changes and lagged income changes like $\mathrm{cov}(C^e_{j,t+1} - C_{jt}, YD^e_t - YD_{t-1})$. As explained in the last paragraph of Section III, even if expectations are rational, such covariances are not necessarily zero and have to be estimated. Adding those to the list of covariances merely increases the number of parameters without increasing the number of restrictions. (ii) We have made no assumptions about the serial correlation of income, preference shocks, and measurement error in expenditure. This is why the set of covariances in (18) does not include autocovariances in expenditure and income changes. (iii) Since expected changes from 1981:Q2 to Q3 are available, (18a) and (19a) could be used not just for $t = 2,3,4$ but also for $t = 1$. Doing so would only introduce an unidentifiable parameter $\mathrm{cov}(e_{j1}, YD^e_2 - YD_1)$ if $\rho_1 = 0$. (iv) Without the expectations data, it is impossible to identify the parameters $(\lambda, \alpha_1, \ldots, \alpha_n, \rho_1, \ldots, \rho_{n-1})$, unless we impose the orthogonality condition that the correlation between forecast errors and lagged variables across households is zero. Unfortunately, as shown in Table V, the correlation is statistically significant, although its quantitative importance is small. This study is probably the first *not* to impose this type of orthogonality condition. (v) Our minimum distance estimator does not require expenditure and income changes to be normally distributed. As discussed in Chamberlain [1982], the quasi-maximum likelihood estimator—which maximizes the normal likelihood function even though the distribution is not necessarily normal—is consistent, but the true asymptotic variance is not given by the standard information matrix formula.[23] In addition, our minimum distance estimator is more efficient than the quasi-maximum likelihood estimator. (vi) It is necessary to assume that (at least one) income change is measured without error. If income is measured with error, the γ's in (18) and the γ's in (19) are no longer the same, which renders γ unidentifiable, unless we place an arbitrary restriction on one of the γ's in (18). But this model with income measurement error but with one arbitrary restriction on the γ's in (18) did not improve the fit

23. Hall and Mishkin [1982] and Bernanke [1984] use the quasi-maximum likelihood procedure.

significantly.[24] We therefore assume for the rest of this paper that there is no income measurement error.

To summarize: under the assumption that preference shocks and expenditure measurement error are uncorrelated with income and that there is no income measurement error, the augmented model (14) and (15) imposes a set of overidentifying restrictions that $12n + 9$ covariances in (18) and (19) are functions of $8n + 9$ parameters listed in (20). Autocovariances in expenditure changes are not included in (18) because we make no assumptions about the serial correlation of preference shocks and measurement error. Our estimation strategy is the minimum distance procedure of choosing parameter values that minimize a suitably defined "distance" between the sample covariances and the corresponding covariances predicted by the model.

VI. Results

To obtain good starting parameter estimates, the joint minimum distance procedure (of estimating all the parameters simultaneously) is first broken down into seven commodity-wise estimation procedures. In other words, for each commodity j the twelve covariances in (18) are paired with the nine covariances in (19), and a minimum distance procedure is applied to those twenty-one covariances.[25] This gives consistent estimates of ρ_j and $\lambda\alpha_j$ for each commodity j, and an estimate of λ of 0.16 is obtained by summing the commodity-wise estimates of $\lambda\alpha_j$ over j. However, for C_1 (food), the durability parameter ρ_1 fluctuates around zero and does not converge in the minimization iterations, and for C_2 (rents, fuel, and utilities), the estimate of ρ_2 is unrea-

24. The model with income measurement error and with one arbitrary restriction has eight additional parameters. The reduction in the distance (21) due to the increased number of parameters is 7.8, which is not surprisingly large for a variable from $\chi^2(8)$ distribution. In this minimum distance estimation with income measurement error, data on C_2 are not used and $\rho_1 = 0$ is imposed for reasons explained in the next section. The estimates of ρ and α are similar to those in the left half of Table VI. If the sample includes all households, the reduction in the distance (21) is about 19, which is significant at close to the 1 percent level. The estimate of λ if the sample includes all households is about 0.22 with a standard error of 0.036 when the identifying restriction with income measurement error is that the γ_{33} in (18) equals the γ_{33} in (19c).

25. The minimization algorithm that we use first transforms the distance into the sum of squares and then applies a locally available subroutine for minimizing a sum of squares by a modified Levenberg-Marquardt-Morrison method. The convergence criterion is that iterations continue until no parameter changes by more than 0.01 percent. In the commodity-wise estimation, two starting values, 0.01 and 0.99, are tried for ρ_j.

TABLE VI

PARAMETER ESTIMATES[a]

Commodity group	Seasonally unadjusted		Seasonally adjusted	
	ρ	α	ρ	α
C_1 (food)	0.0[b]	0.10 (0.013)	−0.10 (0.23)	0.12
C_2 (rent & utilities)	—	—	—	—
C_3 (clothes)	1.36 (0.30)	0.17 (0.025)	1.07 (0.21)	0.20
C_4 (durables)	0.58 (0.18)	0.22 (0.036)	0.38 (0.24)	0.16
C_5 (recreation & education)	1.23 (0.20)	0.21 (0.043)	1.17 (0.19)	0.26
C_6 (medical)	0.29 (0.26)	0.02 (0.010)	0.48 (0.21)	0.03
C_7 (other)	0.12 (0.19)	0.27 (0.030)	0.23 (0.13)	0.24
λ	0.158 (0.020)		0.126	

a. Standard errors in parentheses.
b. Constrained to be 0.0 in the minimum distance estimation.

sonably large (about 6.5 and significant). This is not surprising because most expenditures in C_2 cannot actually be changed on a quarterly basis.

In the joint estimation, we therefore drop C_2 and set $\rho_1 = 0$. Fortunately, the commodity-wise estimate of the marginal expenditure share α_2 for C_2 is less than 5 percent, so dropping C_2 will not change our estimate of λ significantly. There are $12 \times 6 + 9 = 81$ covariances to match and $8 \times 6 + 9 - 1 = 56$ parameters to be estimated. The number of overidentifying restrictions (which include $\rho_1 = 0$) is thus 25. The results from the joint estimation are reported in the left half of Table VI.[26] The durability parameter ρ exceeds unity for C_3 (clothes) and C_5 (rec-

26. As the reader may have noticed, our estimation ignores the role of family size and age. If we use as our basic income and expenditure changes the residuals from regressions of raw changes on AGE, AGE^2, change in FSZ, and its square, it makes little difference to the results. In fact, we have $\lambda = 0.165$ (0.021), $\rho_3 = 1.47$ (0.37), $\rho_4 = 0.72$ (0.19), $\rho_5 = 1.26$ (0.20), $\rho_6 = 0.32$ (0.26), and $\rho_7 = 0.13$ (0.19).

reation and education), but it is not significantly different from, say, 0.9. On the other hand, C_4 (durables) does not come out to be highly "durable." This may be explained by the lumpiness of durables, an element that is not incorporated in our theoretical model but is present in Table I. The fraction λ of liquidity-constrained households whose total expenditure tracks income is sharply estimated to be 0.16. With C_2 dropped, the augmented model is successful in mimicking the sample covariances: the "distance" between the sample covariances and the fitted covariances implied by our parameter estimates is 34, which is not a surprisingly large value from a $\chi^2(25)$ distribution. Table VII shows the two sets of covariances for food.

Seasonality in our model is represented by preference shocks η, which show up additively in the expenditure equation (14) and hence do not change the covariances in (18) and (19). To see whether this additive specification of seasonality is appropriate, seasonally adjusted sample covariances are calculated using the same multiplicative seasonality factors that we used in the right half of Table III. Since equation (15) for liquidity-constrained households should be in seasonally unadjusted variables, disposable income as well as expenditure on commodity j are multiplied by the *same* seasonality factor specific to commodity j. This means that seasonally adjusted income changes are different across com-

TABLE VII

SAMPLE AND FITTED COVARIANCES: FOOD[a]

	Sample	Fitted
$\text{cov}(C^e_{13} - C_{12}, YD^e_3 - YD_2)$	1,282	1,478
$\text{cov}(C^e_{13} - C_{12}, YD_2 - YD^e_2)$	33	−174
$\text{cov}(C_{12} - C^e_{12}, YD^e_3 - YD_2)$	−864	−1,080
$\text{cov}(C_{12} - C^e_{12}, YD_2 - YD^e_2)$	611	775
$\text{cov}(C^e_{14} - C_{13}, YD^e_4 - YD_3)$	2,463	1,806
$\text{cov}(C^e_{14} - C_{13}, YD_3 - YD^e_3)$	−738	−364
$\text{cov}(C_{13} - C^e_{13}, YD^e_4 - YD_3)$	−1,361	−697
$\text{cov}(C_{13} - C^e_{13}, YD_3 - YD^e_3$	1,159	817
$\text{cov}(C^e_{15} - C_{14}, YD^e_5 - YD_4)$	959	763
$\text{cov}(C^e_{15} - C_{14}, YD_4 - YD^e_4)$	−95	−68
$\text{cov}(C_{14} - C^e_{14}, YD^e_5 - YD_4)$	−88	−136
$\text{cov}(C_{14} - C^e_{14}, YD_4 - YD^e_4)$	743	621

a. Income and expenditure are stated in thousands of 1980 yen.

TABLE VIII

REGRESSION OF CHANGE IN FOOD EXPENDITURE ON EXPECTED AND UNEXPECTED DISPOSABLE INCOME CHANGES*

Change in food expenditure $C_{1,t+1} - C_{1t}$	Coefficient of income change: Expected $YD^e_{t+1} - YD_t$	Coefficient of income change: Unexpected $YD_{t+1} - YD^e_{t+1}$	R^2
1981:Q2 to 1981:Q3	0.014	0.022	0.021
	(0.004)	(0.009)	
1981:Q3 to 1981:Q4	0.015	0.035	0.036
	(0.005)	(0.011)	
1981:Q4 to 1982:Q1	0.025	0.035	0.036
	(0.006)	(0.014)	

* Other variables included in the regression are *AGE* and its square, the change in *FSZ*, and its square. They contribute only marginally to the R^2. Heteroskedasticity-robust standard errors are in parentheses.

modities, so the commodity-wise minimum distance estimation is used. The resulting parameter estimates are shown in the right half of Table VI. They are not grossly different from the parameter estimates on seasonally unadjusted data. We note that ρ_1 is now estimated and is insignificant.

If $\rho_1 = 0$, less formal but probably more intuitively appealing results can be obtained from a regression of food expenditure changes on expected and unexpected income changes. Apart from the potential bias arising from the correlation between the forecast error $e_{1,t+1}$ and the expected income change $YD^e_{t+1} - YD_t$, the augmented model predicts that the coefficient of the expected income change is $\lambda\alpha_1$. Its OLS estimate reported in Table VIII is consistent with that prediction. We also note in Table VIII that income variables explain only a small fraction of expenditure changes. The dominant source of fluctuations of expenditure at the individual level is preference shocks and measurement error.[27]

On the whole, then, the empirical evidence points to a high degree of durability for most of the commodities usually labeled as services or semidurables. Is this consistent with time-series evidence? This is an important question to ask because on the U.S. aggregate time-series data quarterly changes in expenditure on nondurables and services as a whole are very much like white

27. In a study that uses diary panel data [Hayashi, 1985], it is also found that income explains only a small fraction of expenditure changes. Thus, it is preference shocks that are the dominant source of fluctuations in expenditure.

TABLE IX

AGGREGATE TIME-SERIES ESTIMATES[a]

Commodity group in the National Income Accounts	a_{j0}	a_{j1}	a_{j2}	a_{j3}	a_{j4}
1. Food, beverages, and tobacco	0.10	−0.10	−0.32	−0.09	0.025
	(0.11)	(0.18)	(0.14)	(0.14)	(0.016)
2. Rents, fuel, and utilities[b]	0.10	−0.17	−0.38	0.04	−0.025
	(0.08)	(0.21)	(0.19)	(0.21)	(0.014)
3. Clothes and footware	0.00	−0.31	−0.54	−0.39	−0.005
	(0.10)	(0.16)	(0.13)	(0.15)	(0.003)
4. Durables	0.18	0.04	−0.16	0.07	−0.009
	(0.06)	(0.15)	(0.14)	(0.15)	(0.008)
5. Recreation and education	0.27	−0.07	−0.49	−0.05	0.002
	(0.10)	(0.17)	(0.15)	(0.18)	(0.009)
6. Medical care	0.03	−0.41	−0.24	0.14	−0.008
	(0.40)	(0.23)	(0.20)	(0.34)	(0.006)
7. Other[c]	0.16	−0.25	0.06	0.21	−0.006
	(0.11)	(0.15)	(0.15)	(0.16)	(0.011)

a. Standard errors are in parentheses. The estimated equation is (23) in the text. The data on expenditure are in real and per capita terms. The interest rate on one-year time deposits is used for the nominal risk-free interest rate. The sample period is 1971:Q1–1983:Q1.

b. Rents here include imputed rents.

c. Communication and transportation are not included.

noise [Hall, 1978].[28] The Japanese National Income Accounts have two different classifications (by type of product and type of expenditure) of personal consumption expenditures. Durables in the first classification seem to correspond to C_4 in our data set. The second classification includes six types of expenditure which are listed in Table IX along with durables. They correspond roughly to C_1, C_2, C_3, C_5, C_6 and C_7 in our data set. For those commodity groups, we estimate the following equation using the National Income Accounts data:

$$\begin{aligned} C_{j,t+1} - C_{jt} &= \text{const.} + \text{seasonal dummies} + a_{j0}\ln(1 + r_{j,t+1}) \\ &\quad + a_{j1}(C_{jt} - C_{j,t-1}) + a_{j2}(C_{j,t-1} - C_{j,t-2}) \\ &\quad + a_{j3}(C_{j,t-2} - C_{j,t-3}) + a_{j4}(YD_{t+1} - YD_t) + \text{error}, \\ &\qquad j = 1,2,\ldots,7. \end{aligned} \tag{23}$$

28. However, using updated data for the United States, Christiano [1984] finds that expenditure on nondurables and services has a positive serial correlation.

This is a weighted average of (12)—the autoregressive equation for the permanent income hypothesis—and (15). So $a_{j0} = (1 - \lambda)\mu_j$, $a_{jk} = -(1 - \lambda)\rho_{jk}$ $(k = 1,2,3)$, and $a_{j4} = \lambda\alpha_j$ and the error term includes forecast errors. In (23), $\ln(1 + r_{j,t+1})$ and $YD_{t+1} - YD_t$ are instrumented by $\ln(1 + r_{jt})$ and $(YD_t - YD_{t-1})$ as they can be correlated with forecast errors. The resulting parameter estimates are given in Table IX. The coefficients of lagged expenditure changes are mostly negative, which is consistent with the permanent income hypothesis with durability. As in Table VI, C_3 (clothes and footware) and C_5 (recreation and education) are most durable, but their estimated durability is not so large.

APPENDIX A: PROOF OF (4)

This appendix proves (4) under two alternative assumptions.

CASE 1. M is small relative to T and $r_{j,s} = r_j$ *for all* $s > t$ and j.

Let ${}_ty_{t+k} = E_t[(1 + r_j)\beta MU_j(t + k + 1) - MU_j(t + k)]$. Then (3) becomes

$$\sum_{k=0}^{M} ({}_ty_{t+k}\,\beta^k\,\rho_{jk}) = 0.$$

This must be true at any future point in the remaining lifetime, so

$$\sum_{k=0}^{K} ({}_sy_{s+k}\,\beta^k\,\rho_{jk}) = 0, \qquad s = t, t + 1, \ldots, t+T - 1,$$

where $K = \min(t + T - s, M)$ and ${}_sy_{t+T} = -E_s[MU_j\,(t + T)]$. Apply the expectations operator E_t on both sides of this equation to obtain

$$\text{(A.1)} \qquad \sum_{k=0}^{K} (x_{s+k-t}\,\beta^k\,\rho_{jk}) = 0, \qquad s = t, t + 1, \ldots, t + T - 1,$$

where $x_\tau = {}_ty_{t+\tau}$. This is an mth-order difference equation in x_τ. If the terminal value x_T is given, (A.1) can determine the remaining value $x_0, x_1, \ldots, x_{T-1}$. (The lifetime budget constraint is necessary to determine the value of x_T.) Since $\beta^k\rho_{jk}$ is declining in k and positive, the difference equation is unstable, so that the initial value x_0 must be small relative to the terminal value. In fact, if T is infinite, then $x_0 = 0$.

CASE 2. ρ_{jk} is geometrically declining in k, $p_{j,t+1}$ is known in t, and $r_{j,t+1} = r_{j,t+2}$.

With $\rho_{jk} = (\rho_j)^k$, (2) implies that $\overline{C}_{jt} = C_{jt} + \rho_j C_{j,t-1}$. Consider the following small deviation from the optimal decision rule:

reduce current expenditure on commodity j by one unit and increase the next period's expenditure by ρ_j units. Since this deviation means an additional saving of p_{jt} in period t, the additional income in period $t + 1$ is $(1 + R_t)p_{jt} - \rho_j p_{j,t+1}$. Note that this deviation leaves $C_{j,t+1}$ unchanged from its level implied by the optimal decision rule. This small change should neither decrease nor increase the objective function, so that

$$MU_j(t) = \beta E_t\{v_{t+1}[(1 + R_t)p_{jt} - \rho_j p_{j,t+1}]\}, \tag{A.2}$$

where v_{t+1} is the marginal utility of income in $t + 1$, which follows

$$v_t = (1 + R_t)\beta E_t(v_{t+1}). \tag{A.3}$$

If $p_{j,t+1}$ is known at t, (A.2) can be rewritten as

$$MU_j(t) = (1 + r_{j,t+1} - \rho_j)p_{j,t+1}\beta E_t(v_{t+1}). \tag{A.4}$$

The right-hand side of (A.4) has the interpretation of the "user cost of capital" adjusted for the marginal utility of income. Now

$$\begin{aligned} E_t[(1 + r_{j,t+1})\beta MU_j(t + 1)], &= E_t[(1 + r_{j,t+1})\beta(1 + r_{j,t+2} - \rho_j), \\ &\quad p_{j,t+2}\beta E_{t+1}(v_{t+2})] \qquad \text{(by (A.4))} \\ &= E_t[(1 + r_{j,t+1})\beta(1 + r_{j,t+2} - \rho_j)\,(1 + r_{j,t+2})^{-1}p_{j,t+1}v_{t+1}] \\ &\qquad \text{(by (A.3))}. \end{aligned}$$

If $r_{j,t+1} = r_{j,t+2}$, this is equal to $MU_j(t)$ by (A.4).

APPENDIX B: TIME AGGREGATION

There is no reason that the length of the unit period for the household's optimization is exactly one quarter. The purpose of this appendix is to show that the quarterly model—equation (12) of the text—can be derived as an approximation to the continuous-time model.

The continuous-time version of (8) is

$$\overline{C}(\tau') - \overline{C}(\tau) = e(\tau,\tau'), \qquad E_\tau e(\tau,\tau') = 0 \text{ for } \tau' > \tau, \tag{B.1}$$

where the commodity subscript j is dropped and the intercept term $d_{j,t+1}$ is ignored for simplicity. Let $t = 0,1,2, \ldots$ be points in continuous calendar time that mark the end of each quarter. Set $\tau = t$ and $\tau' = t + 1$ to obtain

$$\overline{C}(t + 1) - \overline{C}(t) = e(t,t + 1), \qquad E_t e(t,t + 1) = 0. \tag{B.2}$$

In the continuous-time model, $\overline{C}$ is related to C by

$$\overline{C}(\tau) = \int_0^\infty \rho(v)C(\tau - v)dv. \tag{B.3}$$

Combine (B.2) and (B.3) to get

$$\text{(B.4)} \quad e(t,t+1) = \int_0^\infty \rho(v)C(t+1-v)dv - \int_0^\infty \rho(v)C(t-v)dv.$$

Now, consider the following step function as an approximation to $\rho(v)$:

$$\text{(B.5)} \quad \bar{\rho}(v) = \rho(0) \text{ for } 0 \leqslant v < 1,\ \bar{\rho}(v) = \rho(1) \text{ for } 1 \leqslant v < 2, \text{ etc.}$$

Then we obtain

$$\text{(B.6)} \quad \begin{aligned} e(t,t+1) &\simeq \int_0^\infty \bar{\rho}(v)C(t+1-v)dv - \int_0^\infty \bar{\rho}(v)C(t-v)dv \\ &= \rho(0)(C_{t+1} - C_t) + \rho(1)(C_t - C_{t-1}) + \ldots, \end{aligned}$$

where C_t is a unit average; i.e.,

$$\text{(B.7)} \quad C_t = \int_{t-1}^{t} C(\tau)d\tau.$$

Note that $e(t,t+1)$ is orthogonal to information available at the *end* of period t, since $E_t e(t,t+1) = 0$. In (12) e_{t+1} is used for $e(t,t+1)$.

OSAKA UNIVERSITY, JAPAN, AND
NATIONAL BUREAU OF ECONOMIC RESEARCH, U.S.A.

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