Micro and Macro Labor Elasticity

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Theory of Income II 2019

Before I share my experience...

To be able to pass the course you should focus on two broad aspects

- Material cover in class:
  - Tools on Bellman equations
  - Know how to apply the techniques to models cover in class(** papers on Syllabus).
  - Train with previous exams and problem sets, ask TAs during OH
- Go through the papers on the syllabus.
  - Why? Midterm and Finals have questions about the papers.
  - What do you need? Broad knowledge of the paper. Be able two write two paragraph (what is about, the main result, assumption on which it depends, modeling technique, etc.)

    Example: Review Questions Winter 2018

    Briefly describe the main idea of Rogerson and Wallenius (JET 2009) paper "Micro and macro elasticities in a life cycle model with taxes"
  - I'll cover some.

... and a query

- There will be **two** computational problem sets at some points.
- Requires solving a model in the computer
- You can use your prefer language for coding. I recommend,
  - Matlab
  - Julia
- How comfortable do you feel coding (in any language)? Do you think it would be useful to have an Introduction to Matlab?

Plan for the day

- Cover some of the require readings (one * papers)
  Today we focus on:
  - Rogerson and Wallenius, 2009. "Micro and Macro elasticities in a lify cycle model with taxes" *Journal of Economic Theory.*
  - **(I'll add some comments)** Keane and Rogerson, 2012. "Reconciling micro and macro labor supply elasticities: a reassessment of conventional wisdom" *Journal of Economic literature.*
- Discussed *Story Problem* from Professor Stokey class
  - Home Production Economy

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Micro vs. Macro Labor elasticity

- Crucial parameter in macro models.
  - Determines the response of hours work to changes in taxes.
  - Determines the degree of distortions introduce by taxes.
  - Determines how employment (hence output) responds to fluctuations in productivity.
- Important but its magnitude is controversial
  - Studies that employ micro data found relatively small values ($\hat{b}_1 \in [0, 0.4]$)
  $$\log h_{it} = b_0 + b_1 \log w_t + \varepsilon_t$$
  - Representative agent models parametrization imply large values ($\hat{b}_1 \in [2, 4]$)
  $$\log H_t = b_0 + b_1 \log w_t + \varepsilon_t$$
- Which literature is right? Is it possible to reconcile the two?

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Micro vs. Macro Labor elasticity

- First paper to propose an explanation for the discrepancies on the micro and macro estimates.
- **[Spoiler Alert]** Low values at the micro level can be consistent with high values at macro level. The key is that there exists extra margins of adjustment.
    - h_{it} captures the intensive margin of adjustment (choice of hours given employment)
    - H_t captures the intensive and extensive margin of adjustment (work/not work)
- Extensive margin is important: 2/3 of total fluctuations in aggregate hours are fluctuations in employment.

Rogerson and Wallenius (2009)

Motivation:

- Time devoted to market work in continental Europe is (was?) 70% of the US. Prescott (2002), among others, argues that differences in tax and transfer can account for a large share of this differences. (See Prescott (2002) in the Syllabus).
- Prescott (2002) $\rightarrow$ representative household model (stand-in household).
  - No distinction between employment and hours per employee.
  - Calibration yield a high labor supply elasticity. Not consistent with micro estimates.
- Main critique: macro elasticity far from micro estimates. Calibration using micro elasticity $\rightarrow$ taxes do not account for the differences in hours worked.

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Rogerson and Wallenius (2009) (cont.)

Research question:

"Are low micro labor elasticity consistent with high macro estimates?"

Methodology:

- Model of life cycle labor supply that allows micro and macro estimates of labor supply.
- Calibrate the model to replicate features of life cycle labor supply.
- Study how tax and transfer policies affects hours work in the steady state.

Key elements:

- Non-linearity in the mapping between time at work and labor services provided.
- Heterogeneity across cohorts. (minor effect).

Main results:

- Macro elasticities are unrelated to micro elasticities.
- Micro elasticities in the range $[0.5, 1.25]$ can coexist with macro elasticities in the range $[2.25, 3]$

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Model

Preference

- Continuous time overlapping generation framework.
- Unit mass of identical, finitely lived individuals born at each instant t. Length of lifetime equals 1.
- Utility over consumption $(c(a))$ and hours worked $(h(a))$, $a \equiv$ age of individual,

$$\int_0^1 \log(c(a)) - \alpha \frac{h(a)^{1+\gamma}}{1+\gamma} da$$

  $\gamma \approx$ Frisch elasticity of labor supply

- No discounting

Model (cont.)

Technology

- Aggregate prod. function only depends on aggregate labor services,

$$Y(t) = L(t)$$

- Hours worked $\rightarrow$ Effective labor services depends on the productivity of the agent $(e(a))$ and the effective hours worked $(g(h))$,

$$l = e(a)\, g(h)$$

  - g(h): endogenous, mapping between hours worked and labor services is not one-to-one.

$$g(h) = \max\left\{0, h - \bar{h}\right\}, \quad \bar{h} > 0$$

  - $e(a)$: exogenous, it drives variation in hours worked when employed. We assumed single peaked

Government:

- Taxes all labor earning at rate τ
- Revenue redistributed with uniform lump-sum transfers (balanced budget rule)

Two important propositions

Proposition

The optimal solution $h^*(a)$ has a reservation property. In particular, there exists a value e^* such that $h^*(a) > 0$ if $e(a) > e^*$ and $h^*(a) = 0$ if $e(a) < e^*$.

Proposition

Let $h^*(a)$be the optimal solution for hours of work over the life cycle. Let a_1 and a_2 be distinct ages for which $h^*(a) > 0$. Then $e(a_1) > e(a_2)$ implies $h(a_1) \geq h(a_2)$.

- Individual should work when productivity is high. Also conditional on working, hours work should be increasing in $e(a)$.
  - Intensive margin: conditional on working adjust worked hours.
  - Extensive margin: enter and exit of individual to the labor force given by e^*.

Two important propositions

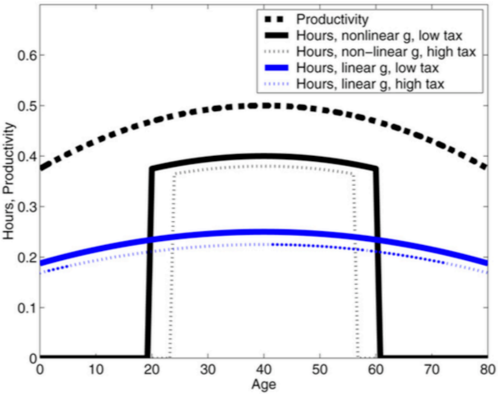

Fig. 1. Hours and productivity over the life cycle.

A change in tax and transfer policies

Table 2

Relative outcomes for $\tau = .5$.

γ	H	λ	$h^{\max}$
.50	.777	.857	.856
1.00	.784	.825	.918
2.00	.788	.808	.956
10.00	.790	.794	.991

- Change in aggregate hours is large.
- Frisch elasticities has no impact on aggregate hours worked. Important for the break between length of working life (λ) vs. hours worked while employed (h^{max}).
- Consistent with data $\rightarrow$ change in H come from changes in extensive and intensive margin.

Conclusions

- General equilibrium life cycle model of labor supply that incorporates the intensive and extensive margins of labor supply.
    - life cycle variation in hours and wages for workers $\rightarrow$ micro labor elasticity
    - variation in aggregate hours worked across economies $\rightarrow$ macro labor elasticities.
- Main findings: (summary from the paper)
    - Macro and micro elasticities are unrelated. (Table 1 and 2)
    - Macro elasticity is high is high even when micro elasticity is low. (Table 1 and 2)
    - Higher taxes lead to less work on both margin.
    - Employment differences generated by differences in tax and transfer programs are concentrated among young and old workers.
    - Non-linearity of labor services in work hours imply a bias in the mapping from Frisch elasticity to parameter in utility function.
- Key in the result is the non-linearity of the function $g(h)$.

Story Problem: Home Production Economy

Technology

- Two types of goods: Home (h) and Market (m)
  - $h \rightarrow$consumption: $c_h \leq A_h [(1-\gamma) k]^{\eta} \nu^{1-\eta}$
  - $m \rightarrow$ consumption or investment: $c_m + [k' - k(1-\delta)] \leq A_m [\gamma k]^{\eta} n^{1-\eta}$
- γ share of capital use in market goods (endogenous)
- One unit of time: work home(ν), work market (n) and leisure $(1-\nu-n)$

Preferences

- Utility over consumption (c_m, c_h) and leisure $(1-\nu-n)$
- $\beta \in (0,1)$ discount factor

Story Problem: Home Production Economy (cont.)

- State variable $\rightarrow$ capital stock k.
- Choice variables $\rightarrow$ (ν, n, γ, k').
- Feasibility set $\Gamma(k)$ $\rightarrow$ defined by the constraints of the problem.
  - Capital cannot be negative: $k' \geq 0$.
  - Capital is produce with market goods

$$k' = A_m [\gamma k]^\eta n^{1-\eta} + (1-\delta) k - c_m$$

  - Consumption is non-negative

$$k' \leq A_m [\gamma k]^\eta n^{1-\eta} + (1-\delta) k$$

  - $\gamma, \nu, n \in (0,1)$
  - $\Gamma(k) = \left\{(\gamma, \nu, n, k') : \gamma, \nu, n \in [0,1], k' \in \left[0, A_m [\gamma k]^\eta n^{1-\eta} + (1-\delta) k\right]\right\}$
- Bellman Equation,

$$v(k) = \max_{(\gamma,\nu,n,k') \in \Gamma(k)} u(c_h, c_m, 1-\nu-n) + \beta v(k)$$

$$c_h = A_h [(1-\gamma) k]^\eta \nu^{1-\eta}$$

$$c_m = A_m [\gamma k]^\eta n^{1-\eta} + k(1-\delta) - k'$$