~~Q4(a)~~ 9

Q5 = 21

Name Elena Istomina Part Score_______/55 Q4 - 26

Economics 331, Winter 2018 Professor Stokey

FINAL EXAM: PART II

4. Two markets (30 points)

Consider an individual who moves back and forth between two markets. During each period she spends in market A, she meets exactly one potential employer, who offers her work for a wage. The wage offer w is drawn from a fixed distribution $F(w)$ with density $f(w)$. The wage is paid in dollars. Her only decision is whether to accept or reject the offer.

During each period she spends in market B, she meets exactly one seller, who offers her goods for sale in exchange for dollars. The price p, which must be paid in dollars, is a random draw from a fixed distribution $G(p)$ with density $g(p)$.

She can buy any quantity x of goods she wants at the price p, and add them to any stock she already has on hand.

She also chooses her consumption $c \geq 0$, which cannot exceed her current stock, including goods purchased in the current period.

She can costlessly hold any amount of money in either market and carry money back and forth between markets. While she remains in market B, she can also costlessly hold any amount of goods, consuming over several periods. But she cannot carry goods from market B to market A.

She can stay as long as she wants in either market, moving back and forth between the two markets when she wants. There is a fixed cost ϕ_f of traveling between the two markets, paid in dollars.

Consuming c in any period provides utility $u(c)$, and the utility cost of working in any period is -1. Her discount factor is $\beta \in (0, 1)$ per period.

She cannot borrow: her stocks of cash and goods must always be nonnegative.

In either market, the timing is: At the beginning of the period she meets a trading partner and makes a decision about transacting. At the end of each period she makes her decision about moving. Let V^k and Z^k, $k = A, B$, denote her value functions in markets A, B, at the beginning of the period (V^k) and the end (Z^k). Discounting occurs after the end of one period, before the beginning of the next period.

a. For each market:

what decisions does she make?

what are constraints, if any, on those decision?

what are the state variables?

what are constraints, if any, on the state variables?

9/10

Market A:
- beginning of the period
  - choose if accept an offer or not +1
  - choice is either "yes" or "no"
  - the wage offer w is a state variable, as well as stock of cash +1
  - stock of cash is non-negative
- end of the period
  - move to B or stay at A
  - state variable is the stock of cash +1
  - stock of cash is non-negative.

Market B:
- beginning of the period +1
  - choose how much to buy and how much to consume +1
  - expenditures on goods no more than the stock of cash +1
  - consumption is no more than the stock of goods +1
  - state vars: price p, stock of cash, stock of goods +1
  - stocks are nonnegative +1

$M \geq 0$

- end of period
  - choose if move to A or stay at B
  - state are stocks of cash and goods
  - both stocks must be non-negative

b. Write her Bellman equations. [Turn the page for space.]

$$V^A(m,w) = \max\{-1 + Z^A(m+w) \; ; \; Z^A(m)\}$$ ✓ +17/20

$$Z^A(m) = \max\{-\phi_f + \beta \mathbb{E}_p[V^B(m;0;p)] \; ; \; \beta \mathbb{E}_w V^A(m;w)\}$$ +9/10

$$V^B(m;g;p) = \max_c \{u(c) - \cancel{px} + Z^B(m - px; g + x - c)\}$$

$$\text{s.t. } c \le x + g, \quad px \le m$$

+8/10

$$\& \; Z^B(m;g) = \max\{-\phi_f + \beta \mathbb{E}_w V^A(m;w) \; ; \; \beta \mathbb{E}_p V^B(m,g,p)$$

where

$$\mathbb{E}_p V^B(m,g,p) = \int V^B(m,g,p)\, g(p)\, dp$$

$$\mathbb{E}_w V^A(m;w) = \int V^A(m,w)\, f(w)\, dp$$

5. Cash-in-advance: pricing a console (25 points)

21/25

Consider an economy in which consumers face a cash-in-advance (CIA) constraint. The only asset traded is a nominal console, a claim to one unit of money each period, forever. The exogenous state variable $s = (y, \omega)$ follows a first-order Markov process, where y is the current endowment and $\omega = \overline{M}/\overline{M}_{-1}$ is the (gross) rate of money growth.

Let (a, x) denote the consumer's portfolio of money and consoles when the asset market meets, which is **after** the console has paid its current-period return. The consumer chooses how much money to hold m, and how many consoles x' to carry forward.

Her Bellman equation is

$$v\left(a, x; s\right) = \max_{c,m,x'} \left[u(c) + \frac{1}{1+\rho} \int v\left(a', x'/\omega'; s'\right) dF\left(s'; s\right)\right]$$

$$\begin{aligned} \text{s.t.} \quad p\left(s\right) c - m &\leq 0, & \mu \\ m + q_x(s)\left(x' - x\right) &\leq a, & \lambda \end{aligned}$$

$$a' = \frac{1}{\omega'}\left[m + x' + p\left(s\right)\left(y - c\right) + \left(\omega' - 1\right)\right],$$

where the console price $q_x(s)$ and goods price $p(s)$ depend on the current state, the second line is the CIA constraint, the third is the budget constraint, and the last describes her money holdings in the asset market next period. Note that the quantity of consoles in her value function next period is renormalized: it appears as x'/ω', since the claims are nominal.

The market clearing conditions are

$$c\left(s\right) = y\left(s\right), \qquad m(s) = 1, \qquad x(s) = 0, \qquad \text{all } s.$$

a. Using μ and λ for the multipliers on the CIA and budget constraints, write the first order and envelope conditions. Notice that since there are two state variables, there are two envelope conditions. [Turn the page for more space.]

5/5

$$\frac{\partial}{\partial c}: \quad u'(c) - \mu\, p(s) - \cancel{p(s)} \frac{1}{1+\rho} \int \frac{v_a'\left(a', \frac{x'}{w'}; s'\right)}{w'} dF(s',s) \checkmark \quad \begin{cases} \leq 0, & c = 0 \\ = 0 & c \in (0, \dots \\ \geq 0 & c = m \dots \end{cases}$$

$$\frac{\partial}{\partial m}: \quad \mu - \lambda + \cancel{\ } \frac{1}{1+\rho} \int \frac{v_a'\left(a', \frac{x'}{w'}; s'\right)}{w'} dF(s',s) \checkmark \quad \begin{cases} \leq 0 & m = 0 \\ = 0 & \text{else} \\ \geq 0 & m = a + \dots \end{cases}$$

$$\frac{\partial}{\partial x'}: \quad -q_x(s) \cdot \lambda + \frac{1}{1+\rho} \int \frac{v_a'\left(a', \frac{x'}{w'}; s'\right)}{\omega'} + \frac{v_x'\left(a'; \frac{x'}{w'}; s'\right)}{w'} dF(\dots \checkmark \quad \begin{cases} \leq 0 & x' = 0 \\ = 0 & \text{else} \\ \geq 0 & x' = \frac{a - \dots}{q_x \dots} \end{cases}$$

EV:

$$v_a'(a,x,s) = \lambda \checkmark$$

$$v_x'(a,x,s) = \lambda q_x(s) \checkmark$$

b. Assume that the CIA constraint binds. Using the market clearing, derive a system of equations in the functions $p(s), \lambda(s), \mu(s), q_x(s)$.

5/5

Assuming utility is well-behaved and we have no corners.

$$u'(y(s)) = p(s) \left[\mu(s) + \frac{1}{1+\rho} \int \frac{\lambda(s')}{w'} d F(s', s) \right] = 0 \qquad (1)$$

$$\mu(s) - \lambda(s) + \frac{1}{1+\rho} \int \frac{\lambda(s')}{w'} d F(s', s) = 0 \qquad (2)$$

$$q_x(s) \cdot \lambda(s) = \frac{1}{1+\rho} \int \frac{\lambda(s')}{w'} + \frac{q_x(s') \cdot \lambda(s')}{w'} d F(s', s) \qquad (3)$$

$$p(s) \cdot y(s) = m \quad \Rightarrow \quad p(s) = \frac{1}{y(s)}$$

c. Assume that the shock is i.i.d. and specialize to the case of log utility, $u(c) = \ln c$. 4/5

Solve for the multipliers and the asset price q_x as explicitly as you can.

$$\frac{1}{y(s)} = p(s)\cdot\lambda(s) \Rightarrow \frac{1}{y(s)} = \frac{1}{y(s)}\cdot\lambda(s) \Rightarrow \boxed{\lambda(s) = 1}$$

$$\mu(s) - 1 + \frac{1}{1+\rho}\int\frac{1}{w'}\,dF(s') = 0$$

$$\Rightarrow \boxed{\mu(s) = 1 - \frac{1}{1+\rho}\int\frac{1}{w'}\,dF(s')}$$ ✓

$$q_x(s) = \frac{1}{1+\rho}\int\frac{1}{w'} + \frac{q_x(s')}{w'}\,dF(s')$$ ✓

* RHS isn't multiplied by $\frac{1}{\omega}$ / Instead, RHS is constant → q constant → $\int q\,dF$

$$\int\frac{q_x(s)}{w}\,dF(s) = \frac{1}{1+\rho}\int\frac{q_x(s')}{w'}\,dF(s) + \frac{1}{1+\rho}\int\frac{1}{w'}\,dF(s')$$

$$\int\frac{q_x(s)}{w}\,dF(s) = \frac{\frac{1}{1+\rho}\int\frac{1}{w'}\,dF(s')}{1-\frac{1}{1+\rho}} = \frac{\int\frac{1}{\omega'}\,dF(s')}{\rho}$$

$$\Rightarrow \boxed{q_x(s) = \int\frac{1}{w'}\,dF(s')}$$

d. What is the economic interpretation of the expression for the console price? 3/5

Does it vary with the current state?

When the shocks are iid the price of a console does not vary with a price. ✓

The price of a console is ~~the~~ ~~the~~ expected ~~...~~ "real value" of a console next period. + dividend

e. What is the price of the console when the monetary authority follows a policy constant money growth, with $\overline{\omega} > 1$? Is the price increasing or decreasing in $\overline{\omega}$. Briefly explain the economic reasoning. 3/5

Then, $\int \frac{1}{\overline{\omega}} dF(s') = \frac{1}{\omega'} \Rightarrow q_x(s) = \frac{1}{\overline{\omega}}$

the price is decreasing in $\overline{\omega}$.

This is because the console is nominal, and the quantity theory holds.

This leads to the price (value of ~~a~~ console) to fall with $\overline{\omega}$ as their purchasing power is ~~the~~ lower now.

f. Continue to assume log utility. With i.i.d. shocks for money growth and real income, what is the real interest rate r? Does it vary with the current state? + 1/3

$$1+r(s) = \frac{\frac{p(s)}{q_x(s)} \int \frac{\cancel{q_x(s)}\, u'(y(s'))}{\omega' p(s')} dF(s')}{\int u'(y(s')) dF(s')}$$

$q_x(s)$ is independent of s, as are both integrals but $p(s)$ is.

So the real interest rate does~~n't~~ depend on current state.

$$1+r(s) = \frac{1}{y(s)} \left(\int \frac{1}{\omega'} dF(s') \right) \Big/ \left(\int \left(\frac{1}{y(s')}\right) dF(s') \right)$$

$$1+r(s) = \frac{p(s)}{q_x(s)} \left(\int \frac{q_x(s')}{\omega' p(s')} u'(y(s'))\, dF(s') \right) \Big/ \left(\int u'(y(s'))\, dF \right)$$