Further Remarks on Causality and Structural Equations: Excerpt from Econometric Evaluation of Social Programs Part I

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Econ 312, Spring 2019

Policy Evaluation

- Evaluating policy is a central problem in economics.
- Requires economist to construct **counterfactuals**.

- A model of counterfactuals is more widely accepted the more widely accepted are its ingredients:

  1. the rules used to derive a model, including whether or not the rules of logic and mathematics are followed;
  2. its agreement with other theories; and
  3. its agreement with the evidence.

- Models are of **hypothetical worlds** obtained by varying — hypothetically — the factors determining outcomes.
- Frisch: "**Causality is in the mind**."

- Epidemiological and statistical models are incomplete: **they do not specify the sources of randomness generating variability among agents**.
- They do not specify why observationally identical people make different choices and have different outcomes given the same choice.
- They do not distinguish what is in the agent's information set from what is in the observing statistician's information set.
- This distinction is fundamental in justifying the properties of any estimator for solving selection and evaluation problems.

- They are also incomplete because they are recursive.

- They do not allow for **simultaneity** in choices of outcomes of treatment that are at the heart of game theory and models of social interactions.

- The goal of the econometric literature, like the goal of all science, is to model phenomena at a deeper level, to understand the causes producing the effects so that one can use empirical versions of the models to forecast the effects of interventions never previously experienced, to calculate a variety of policy counterfactuals, and to use economic theory to guide the choices of estimators and the interpretation of the evidence.

- These activities require development of a more elaborate theory than is envisioned in the current literature on causal inference in epidemiology and statistics.

- The recent literature sometimes contrasts structural and causal models.
- The contrast is not sharp because the term “structural model” is often not precisely defined.
- There are multiple meanings for this term.

- The essential contrast between causal models and explicit economic models as currently formulated is in the range of questions that they are designed to answer.
- Causal models as formulated in statistics and in the econometric treatment effect literature are typically black-box devices designed to investigate the impact of "treatment" — which are often complex packages of interventions — on some observed set of outcomes in a given environment.

- Explicit economic models go into the black box to explore the mechanism(s) producing the effects: **the causes of effects** .
- Holland (1986): understanding the "**effects of causes**" (the goal of the treatment effect literature).
- Understanding the "causes of effects" (the goal of the literature building explicit economic models).
- By focusing on one narrow black-box question, the treatment effect and natural experiment literatures avoid many of the problems confronted in the econometrics literature that builds explicit economic models.

- At the same time, it produces parameters that are more limited in application.

- By not being explicit about the contents of the black-box (understanding the *causes of effects*), it ties its hands in using information about basic behavioral parameters obtained from other studies as well as economic intuition to supplement available information in the data in hand (limits abduction).

- Lacks the ability to provide explanations for estimated "effects" grounded in economics or to conduct welfare economics.

- When the components of treatments vary across studies, knowledge does not accumulate across treatment effect studies, whereas it does accumulate across studies estimating common behavioral or technological parameters.

Policy Evaluation: Notation and Definition of Individual Level Treatment Effects

- To evaluate is to **value** and to **compare values** among possible outcomes.
- Two distinct tasks.
- We define outcomes corresponding to state (policy, treatment) s for an agent characterized by ω as $Y(s,\omega)$, $\omega \in \Omega$.
- The agent can be a household, a firm, or a country.

- One can think of Ω as a universe of agents each characterized by an element ω.
- The ω encompasses all features of agents that affect Y outcomes.
- $Y(\cdot, \cdot)$ may be generated from a scientific or economic theory.
- It may be vector valued.
- The $Y(s, \omega)$ are outcomes realized after treatments are chosen.
- In advance of treatment, agents will not know the $Y(s, \omega)$ but may make forecasts about them.

- Let $\mathcal{S}$ be the set of possible treatments with elements denoted by s.
- For simplicity, assume that this set is the same for all ω.
- For each ω, we obtain a collection of possible outcomes given by $\{Y(s, \omega)\}_{s \in \mathcal{S}}$.
- The set $\mathcal{S}$ may be finite (e.g., there may be J states), countable, or may be defined on the continuum (e.g., $\mathcal{S} = [0, 1]$), so that there are an uncountable number of states.

- For example, if $\mathcal{S} = \{0, 1\}$, there are two treatments, one of which may be a no-treatment state—e.g., $Y(0, \omega)$.

- This is the outcome for an agent ω not getting a treatment like a drug, schooling, or access to a new technology, while $Y(1, \omega)$ is the outcome in treatment state 1 for agent ω getting the drug, schooling, or access.

- A prototypical policy evaluation problem.
- Country can adopt a policy (e.g., democracy).
- Choice Indicator:
  - $D = 1$ if it adopts.
  - $D = 0$ if not.

- Two outcomes $(Y_0(\omega), Y_1(\omega)),\ \omega \in \Omega$
  - $Y_0(\omega)$ if country does not adopt
  - $Y_1(\omega)$ if country adopts

- Marshallian *ceteris paribus* causal effect:

$$Y_1(\omega) - Y_0(\omega)$$

Figure 1: Extended Roy economy for policy adoption
Distribution of gains and treatment parameters

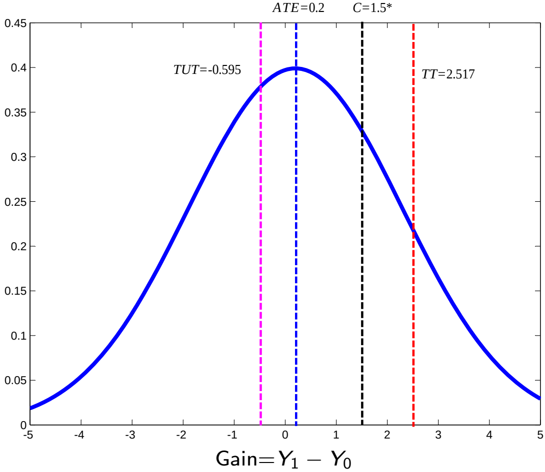

Figure 1 Legend

Suppose that a country has to choose whether to implement a policy. Under the policy, the GDP would be Y_1. Without the policy, the GDP of the country would be Y_0. For the sake of simplicity, suppose that

$$\begin{aligned} Y_1 &= \mu_1 + U_1 \\ Y_0 &= \mu_0 + U_0 \end{aligned}$$

where U_0 and U_1 are unobserved components of the aggregate output. The error terms (U_0, U_1) are dependent in a general way. Let δ denote the additional GDP due to the policy, i.e. $\delta = \mu_1 - \mu_0$. We assume $\delta > 0$. Let C denote the cost of implementing the policy. We assume that the cost is a fixed parameter C.

Figure 1 Legend

The country's decision can be represented as an Extended Roy Model:

$$D = \begin{cases} 1 & \text{if} \quad Y_1 - Y_0 - C > 0 \\ 0 & \text{if} \quad Y_1 - Y_0 - C \leq 0, \end{cases}$$

so the country decides to implement the policy $(D = 1)$ if the net gains coming from it are positive. Therefore, we can define the probability of adopting the policy in terms of the "propensity score" or probability of selection:

$$\Pr(D = 1) = P(Y_1 - Y_0 - C > 0).$$

Assume that $(U_1, U_0) \sim N(\mathbf{0}, \mathbf{\Sigma})$, $\mathbf{\Sigma} = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$, $\mu_0 = 0.67$, $\delta = 0.2$, and $C = 1.5$.

- More generally, define outcomes corresponding to state (policy, treatment) s for an "agent" characterized by ω as $Y(s,\omega)$, $\omega \in \Omega = [0,1]$, $s \in \mathcal{S}$, set of possible treatments.
- The agent can be any economic agent such as a household, a firm, or a country.
- The $Y(s,\omega)$ are *ex post* outcomes realized after treatments are chosen.

- The **individual treatment effect** for agent ω.

$$Y(s,\omega) - Y(s',\omega), \qquad s \neq s', \quad s, s' \in S, \tag{1}$$

Individual level causal effect.

- Comparisons can also be made in terms of utilities $R(Y(s,\omega))$.

- $R(Y(s,\omega),\omega) > R(Y(s',\omega),\omega)$ if s is preferred to s'.
- **The difference in subjective outcomes** is $[R(Y(s,\omega),\omega) - R(Y(s',\omega),\omega)]$.
- Another possible definition of a treatment effect.
- Holding ω fixed holds all features of the person fixed except the treatment assigned, s.

- An essential question,

  "What question is the analysis supposed to answer?"

  is often the big unanswered question in the recent policy evaluation literature.

- The question is usually unanswered because it is unasked in much of the modern treatment effect literature which seeks to estimate "an effect" without telling you which effect or why it is interesting to know it.

Adding Uncertainty

- The $Y(s,\omega)$ are outcomes realized after treatments are chosen (ex post).
- In advance of treatment, agents may not know the $Y(s,\omega)$ but may make forecasts about them.
- These forecasts may influence their decisions to participate in the program or may influence the agents who make decisions about whether or not an individual participates in the program.

Treatment May Be Complex

- Each "state" (treatment) may consist of a compound of subcomponent states.
- In this case, one can define s itself as a vector (e.g., $s = (s_1, s_2, \ldots, s_K)$ for K components) corresponding to the different components that comprise treatment.
- Thus a job training program typically consists of a package of treatments.
- We might be interested in the package of one (or more) of its components.

- The outcomes may be time subscripted as well, $Y_t(s, \omega)$ corresponding to outcomes of treatment measured at different times.
- The index set for t may be the integers, corresponding to discrete time, or an interval, corresponding to continuous time.
- The $Y_t(s, \omega)$ are realized or *ex post* (after treatment) outcomes.

What Exactly Is Treatment?

- The term "treatment" is used in multiple ways in this literature and this ambiguity is sometimes a source of confusion.
- In its most common usage:
- **A treatment assignment mechanism is a rule** $\tau : \Omega \rightarrow \mathcal{S}$ **which assigns treatment to each** ω.
- The consequences of the assignment are the outcomes $Y(s, \omega)$, $s \in \mathcal{S}$, $\omega \in \Omega$.
- The collection of possible assignment rules is $\mathcal{T}$ where $\tau \in \mathcal{T}$.
- Two aspects of a policy.
- The policy selects **who gets what**.
- More precisely, it selects individuals ω and specifies the treatment $s \in \mathcal{S}$ received.

- More nuanced definition of treatment assignment that explicitly recognizes the element of choice by agent ω in producing the treatment assignment rule.
- Participation in treatment is usually a choice made by agents.
- Under a more comprehensive definition of treatment, agents are assigned incentives like taxes, subsidies, endowments and eligibility that affect their choices, but the agent chooses the treatment selected.
- Agent preferences, program delivery systems, aggregate production technologies, market structures, and the like might all affect the choice of treatment.
- The treatment choice mechanism may involve multiple actors and multiple decisions that result in an assignment of ω to s.

- For example, s can be schooling while $Y(s, \omega)$ is earnings given schooling for agent ω.
- A policy may be a set of payments that encourage schooling, as in the Progressa program in Mexico, and the treatment in that case is choice of schooling with its consequences for earnings.

- Specify assignment rules $a \in \mathcal{A}$: map individuals ω into constraints (benefits) $b \in \mathcal{B}$ under different mechanisms.
- A constraint assignment mechanism a is a map

$$a : \Omega \to \mathcal{B}$$

  defined over the space of agents.
- The constraints may include endowments, eligibility, taxes, subsidies and the like that affect agent choices of treatment.
- The map a defines the rule used to assign $b \in \mathcal{B}$.
- Formally, the probability system for the model without randomization is $(\Omega, \sigma(\Omega), \mathcal{F})$ where Ω is the probability space, $\sigma(\Omega)$ is the σ-algebra associated with Ω and $\mathcal{F}$ is the measure on the space.

- When we account for randomization we need to extend Ω to $\Omega' = \Omega \times \Psi$, where Ψ is the new probability space induced by the randomization, and we define a system $\left(\Omega', \sigma\left(\Omega'\right), \mathcal{F}'\right)$.

- Some policies may have the same overall effect on the aggregate distribution of b, but may treat given individuals differently.

- Under an anonymity postulate, some would judge such policies as equivalent in terms of the constraints (benefits) offered, even though associated outcomes for individuals and aggregates may be different.

- Another definition of equivalent policies is in terms of the distribution of aggregate outcomes associated with the treatments.

- This lecture characterizes policies at the individual level recognizing that sets of $\mathcal{A}$ that are characterized by some aggregate distribution over elements of $b \in \mathcal{B}$ may be what others mean by a policy.

- Given $b \in \mathcal{B}$ allocated by constraint assignment mechanism $a \in \mathcal{A}$, agents pick treatments.
- Treatment assignment mechanism: $\tau : \Omega \times \mathcal{A} \times \mathcal{B} \to \mathcal{S}$.
- Map taking agent $\omega \in \Omega$ facing constraints $b \in \mathcal{B}$ assigned by mechanism $a \in \mathcal{A}$ into a treatment $s \in \mathcal{S}$.
- Note: including $\mathcal{B}$ in the domain of definition of τ is redundant since the map $a : \Omega \to B$ selects an element $b \in \mathcal{B}$.
- I make b explicit to remind the reader that agents are making choices under constraints.
- In settings with choice, τ is the choice rule used by agents where $\tau \in \mathcal{T}$, a set of possible choice rules.
- It is conventional to assume a unique $\tau \in \mathcal{T}$ is selected by the relevant decision makers.
- Not required in our definition.

- A policy regime $p \in \mathcal{P}$ is a pair $(a, \tau) \in \mathcal{A} \times \mathcal{T}$ that maps agents denoted by ω into elements of s.
- In this notation, $\mathcal{P} = \mathcal{A} \times \mathcal{T}$.

- Incorporating choice into the analysis of treatment effects is an essential and distinctive ingredient of the econometric approach to the evaluation of social programs.
- The traditional treatment-control analysis in statistics equates mechanisms a and τ (e.g., "Rubin model").
- An assignment in that literature is an assignment to treatment, not an assignment of incentives and eligibility for treatment with the agent making treatment choices.
- This approach has only one assignment mechanism and treats noncompliance with it as a problem rather than as a source of information on agent preferences, as in the econometric approach.

- Under **“full compliance”**, $a : \Omega \to \mathcal{S}$ and $a = \tau$, where $\mathcal{B} = \mathcal{S}$.

- Policy invariance is a key assumption for the study of policy evaluation.
- It allows analysts to characterize outcomes without specifying how those outcomes are obtained.
- **Policy invariance has two aspects.**
- The first aspect is that, for a given $b \in \mathcal{B}$ (incentive schedule), the mechanism $a \in \mathcal{A}$ by which ω is assigned a b (e.g. random assignment, coercion at the point of a gun, etc.) and the incentive $b \in \mathcal{B}$ are assumed to be irrelevant for the values of realized outcomes for each s that is selected.
- Second: for a given s for agent ω, the mechanism τ by which assigns ω to s under assignment mechanism $a \in \mathcal{A}$ is irrelevant for the values assumed by realized outcomes.

- **Both** assumptions define what we mean by policy invariance.

- Policy invariance allows us to describe outcomes by $Y(s, \omega)$ and ignore features of the policy and choice environment in defining outcomes.
- If have to account for the effects of incentives and assignment mechanisms on outcomes, must work with $Y(s, \omega, a, b, \tau)$ instead of $Y(s, \omega)$.
- The following policy invariance assumptions justify collapsing these arguments of $Y(\cdot)$ down: $Y(s, \omega)$.

- Policy invariance for objective outcomes:

PI-1 1

For any two constraint assignment mechanisms $a, a' \in \mathcal{A}$ *and incentives* $b, b' \in \mathcal{B}$*, with* $a(\omega) = b$ *and* $a'(\omega) = b'$*, and for all* $\omega \in \Omega$*,* $Y(s, \omega, a, b, \tau) = Y(s, \omega, a', b', \tau)$*, for all* $s \in \mathcal{S}_{\tau(a,b)}(\omega) \cap \mathcal{S}_{\tau(a',b')}(\omega)$ *for assignment rule* τ *where* $\mathcal{S}_{\tau(a,b)}(\omega)$ *is the image set for* $\tau\,(a, b)$*.*

- For simplicity assume $\mathcal{S}_{\tau(a,b)}(\omega) = \mathcal{S}_{\tau(a,b)}$ for all $\omega \in \Omega$.

- This assumption says that for the same treatment s and agent ω, different constraint assignment mechanisms a and a' and associated constraint assignments b and b' produce the same outcome.
- **“Many ways to skin a cat.”**

- A second invariance assumption invoked in the literature:
- For a fixed a and b, the outcomes are the same independent of the treatment assignment mechanism:

PI-2 2

For each constraint assignment $a \in \mathcal{A}$, $b \in \mathcal{B}$ and all $\omega \in \Omega$, $Y(s, \omega, a, b, \tau) = Y(s, \omega, a, b, \tau')$ for all τ and $\tau' \in \mathcal{T}$ with $s \in \mathcal{S}_{\tau(a,b)} \cap \mathcal{S}_{\tau'(a,b)}$, where $\mathcal{S}_{\tau(a,b)}$ is the image set of τ for a given pair (a, b).

- Again, exclude the possibility of ω-specific image sets $\mathcal{S}_{\tau(a,b)}$ and $\mathcal{S}_{\tau'(a,b)}$.

- If treatment effects based on subjective evaluations are also considered, we need to broaden invariance assumptions 1 and 2 to produce invariance in rewards for certain policies and assignment mechanisms.
- It would be unreasonable to claim that utilities $R(\cdot)$ do not respond to incentives.
- Suppose, instead, that we examine subsets of constraint assignment mechanisms $a \in \mathcal{A}$ that give the same incentives (elements $b \in \mathcal{B}$) to agents, but are conferred by different delivery systems, a.

- For each $\omega \in \Omega$, define the set of mechanisms delivering the same incentive or constraint b as $\mathcal{A}_b(\omega)$:

$$\mathcal{A}_b(\omega) = \{a \mid a \in \mathcal{A}, a(\omega) = b\}, \qquad \omega \in \Omega.$$

  The set of delivery mechanisms that deliver b may vary among the ω.

- Let $R(s, \omega, a, b, \tau)$ represent the reward to agent ω from a treatment s with incentive b allocated by mechanism a with an assignment to treatment mechanism τ.

PI-3 3

For any two constraint assignment mechanisms $a, a' \in \mathcal{A}$ *and incentives* $b, b' \in \mathcal{B}$ *with* $a(\omega) = b$ *and* $a'(\omega) = b'$*, and for all* $\omega \in \Omega$*,* $Y(s, \omega, a, b, \tau) = Y(s, \omega, a', b', \tau)$ *for all* $s \in \mathcal{S}_{\tau(a,b)}(\omega) \cap \mathcal{S}_{\tau(a',b')}(\omega)$ *for assignment rule* τ*, where* $\mathcal{S}_{\tau(a,b)}(\omega)$ *is the image set of* $\tau(a, b)$ *and for simplicity we assume that* $\mathcal{S}_{\tau(a,b)}(\omega) = \mathcal{S}_{\tau(a,b)}$ *for all* $\omega \in \Omega$*. In addition, for any mechanisms* $a, a' \in \mathcal{A}_b(\omega)$*, producing the same* $b \in \mathcal{B}$ *under the same conditions postulated in the preceding sentence, and for all* ω*,* $R(s, \omega, a, b, \tau) = R(s, \omega, a', b, \tau)$.

- This assumption says, for example, that utilities are not affected by randomization or the mechanism of assignment of constraints.
- Corresponding to 2 we have a policy invariance assumption for the utilities with respect to the mechanism of assignment:

PI-4 4

For each pair (a, b) *and all* $\omega \in \Omega$*,*

$$\begin{aligned} Y(s, \omega, a, b, \tau) &= Y(s, \omega, a, b, \tau') \\ R(s, \omega, a, b, \tau) &= R(s, \omega, a, b, \tau') \end{aligned}$$

for all $\tau, \tau' \in \mathcal{T}$ *and* $s \in \mathcal{S}_{\tau(a,b)} \cap \mathcal{S}_{\tau'(a,b)}$.

- This assumption rules out general equilibrium effects, social externalities in consumption, etc. in both subjective and objective outcomes.