1 Short Questions

1. (5 points) Define what it means for a preference relation $\succsim$ on $\mathbb{R}^n_+$ to be continuous.

2. (5 points) Prove that if two utility functions u and v represent the same preference relation on $\mathbb{R}^n_+$, then there is a strictly increasing function $f : \mathbb{R}_+ \to \mathbb{R}_+$ such that $u(x) = f(v(x))$ for every $x \in \mathbb{R}$. You should assume that the range of both $u(\cdot)$ and $v(\cdot)$ is all of $\mathbb{R}_+$.

3. (5 points) Provide an Edgeworth box showing a Walrasian equilibrium allocation that is in the interior of the box but that is not Pareto efficient even though preferences are continuous.

4. (5 points) State the first welfare theorem for the exchange economy.

2 Exchange Economy

Consider an economy with four consumers:

$$u^1(x_1, x_2) = u^2(x_1, x_2) = x_1 x_2, \quad \mathbf{e}^1 = \mathbf{e}^2 = (18, 2)$$
$$u^3(x_1, x_2) = u^4(x_1, x_2) = x_1 x_2, \quad \mathbf{e}^3 = \mathbf{e}^4 = (2, 18)$$

1. (5 points) Show that the allocation $\mathbf{x}^1 = \mathbf{x}^3 = (9, 9)$ and $\mathbf{x}^2 = \mathbf{x}^4 = (11, 11)$ is Pareto efficient.

2. (10 points) Show that the allocation in part (a) is not in the core. (Hint: Find a blocking pair)

3. (15 points) Show that the allocation $\mathbf{x}^1 = \mathbf{x}^2 = (9, 9)$ and $\mathbf{x}^3 = \mathbf{x}^4 = (11, 11)$ is in the core but that it is not a Walrasian equilibrium allocation. (If needed, you may use the fact that $29\sqrt{19/11} > 38$)

3 Production Economy

1. (5 points) Define the profit function, $\Pi(\mathbf{p})$, for a price-taking firm with production set $Y \subset \mathbb{R}^n$.

2. (5 points) Suppose that $\mathbf{y}^0 \in Y$ is a profit-maximizing production plan at the price vector $\mathbf{p}^0 \gg 0$. Suppose that the price of good k increases and that all other prices remain fixed, leading to the new price vector $\mathbf{p}^1$. Prove that if $\mathbf{y}^1 \in Y$ is profit-maximizing at the price vector $\mathbf{p}^1$, then $y^1_k \geq y^0_k$ i.e. the supply of good k does not fall when its price goes up. Do not use envelope theorem or Hotelling's lemma.

3. (5 points) Let Y be the *aggregate* production set of an economy with $J > 1$ firms. That is,

$$Y = \left\{ y \in \mathbb{R}^n : y \in \sum_{j=1}^{J} y^j, y^j \in Y^j \right\}$$

Fix some $\mathbf{p}$ and suppose that $\hat{\mathbf{y}} \in Y$ solves $\max \mathbf{p} \cdot \mathbf{y}$ subject to $\mathbf{y} \in Y$. Prove that there exists $\hat{\mathbf{y}}^1 \in Y^1, \ldots, \hat{\mathbf{y}}^J \in Y^J$ such that for each $j \in \{1, \ldots, J\}$, $\hat{\mathbf{y}}^j$ solves $\max \mathbf{p} \cdot \mathbf{y}^j$ subject to $\mathbf{y}^j \in Y^j$.

4 Social Welfare

Consider a society with three individuals $(1, 2, 3)$ and three social alternative (x, y, z). Let f be a social welfare function that yields:

Individual 1	Individual 2	Individual 3	Society f
x	y	z	x
y	z	x	z
z	x	y	y

1. (5 points) Provide precise statements of Arrow's four axioms using notations from class.

2. (5 Points) State Arrow's theorem. Without changing any individuals' rankings in the table above, use Arrow's theorem to conclude that f must violate at least one of U, WP or IIA.

3. (5 Points) Suppose that f satisfies U. By changing the ranking(s) of one or more individuals in the table above, show that f cannot also simultaneously satisfy both WP and IIA. You must answer this question without appealing to Arrow's theorem.