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INFLATION AND WELFARE

BY ROBERT E. LUCAS, JR.¹

This paper surveys research on the welfare cost of inflation. New estimates are provided, based on U.S. time series for 1900–94, interpreted in a variety of ways. It is estimated that the gain from reducing the annual inflation rate from 10 percent to zero is equivalent to an increase in real income of slightly less than one percent. Using aggregate evidence only, it may not be possible to estimate reliably the gains from reducing inflation further, to a rate consistent with zero nominal interest.

KEYWORDS: Inflation, welfare, interest rates, monetary policy.

1. INTRODUCTION

IN A MONETARY ECONOMY, it is in everyone's private interest to try to get someone else to hold non-interest-bearing cash and reserves. But someone has to hold it all, so all of these efforts must simply cancel out. All of us spend several hours per year in this effort, and we employ thousands of talented and highly-trained people to help us. These person-hours are simply thrown away, wasted on a task that should not have to be performed at all.

Since the opportunity cost of holding non-interest-bearing money is the nominal rate of interest, we would expect that the time people spend trying to economize on cash holdings should be an increasing function of the interest rate. This observation is consistent with much evidence, and suggests that as long as interest rates are positive people could be made better off if money growth, and hence the average inflation rate and the interest rate, were reduced. The problems of working out the details of this theoretical idea and of applying it to estimate the potential gains in welfare from the adoption of the monetary policies that reduce inflation and interest rates are classic questions of monetary economics, addressed in a long line of research stemming from the contributions of Bailey (1956) and Friedman (1969). The goal of this paper is to provide a substantive summary of where this line of research stands today.

The way the analysis of inflation and its consequences has developed over the years is also interesting from a methodological point of view, as an illustration of the extent to which the quantitative, mathematical vision shared by the founders of the Econometric Society has succeeded in transforming the practice of economics. An applied economist today uses explicit theoretical modelling to organize data from a variety of sources and brings this information to bear on

¹This paper was prepared for the 1997 summer meetings of the Econometric Society in Pasadena, Hong Kong, and Toulouse. Earlier versions, entitled "On the Welfare Cost of Inflation," were given at the 1993 Hitotsubashi International Symposium on Financial Markets in the Changing World, and at conferences in Bergen and the Federal Reserve Bank of San Francisco. I am grateful to many colleagues and discussants, but particularly to Martin Bailey, Lars Hansen, Bennett McCallum, Casey Mulligan, and Nancy Stokey, for helpful discussion and criticism. Michael Beveridge, Vimut Vanitcharearnthum, Tomoyuki Nakajima, and Esteban Rossi-Hansberg provided able research assistance. The National Science Foundation provided research support.

quantitative questions of policy in a way that is almost entirely a development of the last 50 years. As compared to older, more literary methods, the explicit theoretical style of postwar economics can lead to sharper questions and better answers, and at the same time expose the limits of current knowledge in ways that can stimulate improvements in both theory and data. I would like the present paper to exemplify these virtues as well.

In the next section, I will display and discuss evidence on money, prices, production, and interest rates for the 20th century United States. Using this evidence, I replicate essentially Meltzer's (1963a) estimated money demand function, and then use these estimates to replicate Bailey's (1956) welfare cost calculations. The rest of the paper deals with the theoretical interpretation of these calculations.

Section 3 provides one possible general equilibrium rationale for the welfare estimates reported in Section 2, based on a simplified version of Sidrauski's (1967a,b) model. Section 4 then uses the Sidrauski framework to consider the consequences of dropping the assumption, used in Section 3, that the monetary policy that implements any given interest rate can be carried out with lump-sum fiscal transfers. It re-examines the estimation under the alternate assumption that only flat rate income taxes can be used, and that a government sector of given size must be financed either with inflation taxation or with income taxation. This modification introduces theoretical complications but does not, I argue, lead to major quantitative differences from the conclusions of Section 2.

Section 5 provides a second general equilibrium rationale for the welfare estimates of Section 2, using as context a model of a transactions technology proposed by McCallum and Goodfriend (1987). This model provides another theoretical justification of the consumers' surplus formulas used in Section 2, one that turns out to be closely related to Baumol's (1952) inventory-theoretic analysis. Section 6 contains concluding remarks.

2. MONEY DEMAND AND CONSUMERS' SURPLUS

Figure 1 shows plots of annual time series of a short term nominal interest rate, r_t , and of the ratio of M1 to nominal GDP, $m_t = M_t/(P_t y_t)$, for the United States, for the period 1900–1994.² Over this 95 year period, real GDP grew at

²The interest rate is the short term commercial paper rate. For 1900–75, it is from Friedman and Schwartz (1982, Table 4.8, Column 6). For 1976–94, it is from the *Economic Report of the President* (1996, Table B-69).

The money supply is M1 in billions of dollars, December of each year, not seasonally adjusted. For 1900–14, it is from *Historical Statistics of the United States* (1960, Series X-267). From 1915–1947, it is from Friedman and Schwartz (1982, pp. 708–718, Column 7). For 1948–85, it is from the *International Financial Statistics Tape*. From 1986–94, it is from the Federal Reserve Bank of St. Louis *FRED Database*.

Real GDP is in billions of 1987 dollars. From 1900–28, it is from Kendrick (1961, Table A-III). From 1929–58, it is from the *National Income and Product Accounts of the U.S., 1929–1958*, Table 1.2. From 1929–94, it is from *Citibase*, Series GDPQ.

The GDP deflator equals 1.0 in 1987. For 1900–1928, it is from *Historical Statistics of the United States* (1960, Series F-5). For 1929–58, it is from the *National Income and Product Accounts of the U.S., 1929–1958*, Table 7.13. For 1959–94, it is from *Citibase*, Series GDPD.

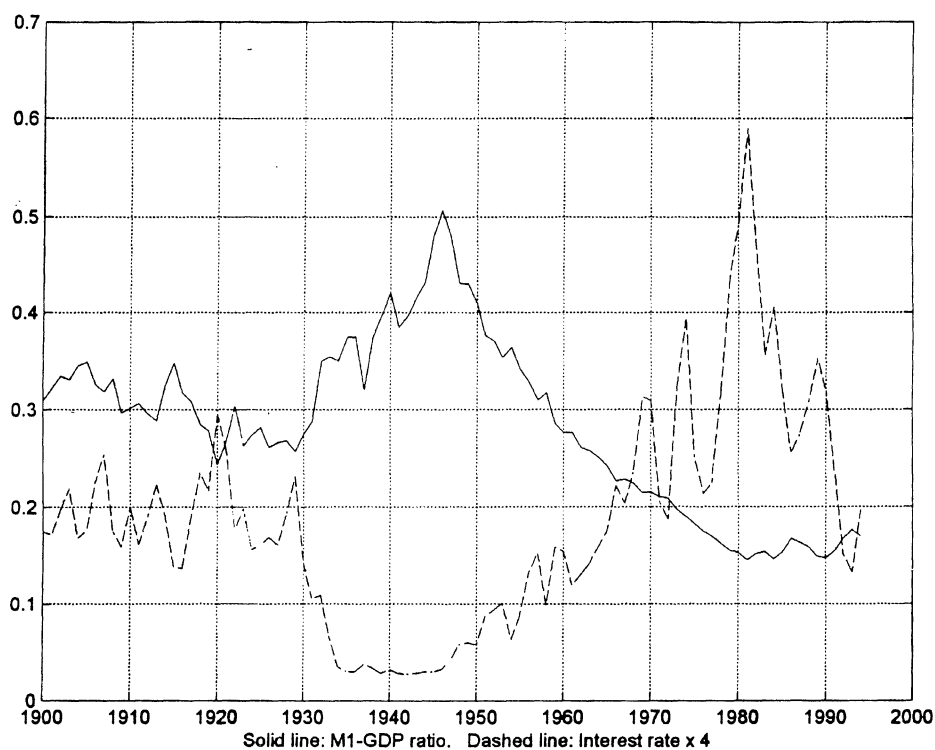


FIGURE 1.—U.S. money, income, and interest rates.

an average annual rate of 3 percent, M1 grew at 5.6 percent, and the GDP deflator grew at 3.2 percent. The money-income ratio is thus essentially trendless over the entire century, although there has been a strong downward trend since World War II. Technical change in the provision of transactions services would, other things equal, produce a downward trend in the money-income ratio m_t . An income elasticity of money demand exceeding one would produce an upward trend. Neither trend appears in the data, though of course both might have been present in an offsetting way.

In this section, I will interpret these two time series as points on a demand function for real balances of the form $M_t/P_t = L(r_t, y_t)$, where this function L takes the form $L(r, y) = m(r)y$.³ Figure 2 displays a plot of observations (the

³Estimates of the income or wealth elasticity of money (M1 or M2) demand obtained from long U.S. time series tend to be around unity: Meltzer (1963a), Laidler (1977), Lucas (1988), Stock and Watson (1993). Ball (1998), using methods similar to Stock and Watson's but applied to data through 1996, obtains an income elasticity near 0.5. Meltzer (1963b) reports estimates near one for sales elasticities in a cross-section sample of firms. Estimates from post-war quarterly data are generally below one: Goldfeld (1987). Recent estimates by Mulligan and Sala-i-Martin (1992) from panel data on U.S. states are higher, around 1.3.

circles in the figure) on the money-income ratio m_t and the interest rate r_t for the years 1900–1994. The figure also plots the curves $m = Ar^{-\eta}$ for the η -values 0.3, 0.5, and 0.7, where A is selected so the curve passes through the geometric means of the data pairs. Within this parametric family, it is evident that $\eta = 0.5$ gives the best fit. Figure 3 presents the same data, this time along side the curves $m = Be^{-\xi r}$ for the ξ -values 5, 7, and 9. Again, all three curves pass through the geometric means. Within this parametric family, $\xi = 7$ appears to give the best fit. It is also clear, I think, that the semi-log function plotted here provides a description of the data that is much inferior to the log-log curve in Figure 2.⁴

In order to provide some perspective on these estimates, Figure 4 plots actual U.S. real balances (not deflated by income) against the real balances predicted by the log-log demand curve: $Ar_t^{-0.5}y_t$. One sees that the fitted values successfully track the secular increase in the money-income ratio prior to World War II, including the acceleration of this increase in the 1930s and 40s. They also track the decrease in m_t as interest rates rose in the post war period (though they miss the 1990s, when interest rates declined and velocity did not). One also sees, however, that the fitted series exhibits some large, shorter-term, fluctuations that do not appear in the actual series. The interest elasticity needed to fit the long-term trends (and very sharply estimated by these trends) is much too high to permit a good fit on a year-to-year basis. Of course, it is precisely this difficulty that has motivated much of the money demand research of the last 30 years, and has led to distributed lag formulations of money demand that attempt to reconcile the evidence at different frequencies. In my opinion, this reconciliation has not yet been achieved, but in any case, it is clear that the functions plotted in Figures 2 and 3 contribute nothing toward the resolution of this problem.

To translate the evidence on money demand into a welfare cost estimate, we first apply the method of Bailey (1956), defining the welfare cost of inflation as the area under the inverse demand function—the *consumers' surplus*—that could be gained by reducing the interest rate from r to zero. That is, let $m(r)$ be the estimated function, let $\psi(m)$ be the inverse function, and define the welfare cost function $w(r)$ by

$$(2.1) \quad w(r) = \int_{m(r)}^{m(0)} \psi(x) dx = \int_0^r m(x) dx - rm(r).$$

Since the function m has the dimensions of a ratio to income, so does the function w . Its value $w(r)$ has the interpretation, to be made more precise in later sections, as the fraction of income people would require as compensation in order to make them indifferent between living in a steady state with an

⁴Cagan (1956) used the semi-log form in his classic study of the European hyperinflations. It is interesting that the paradox that Cagan noted, of inflation rates during hyperinflations that exceeded the revenue-maximizing levels, is specific to semi-log money demand. With log-log demand, seigniorage is always an increasing function of the money growth rate.

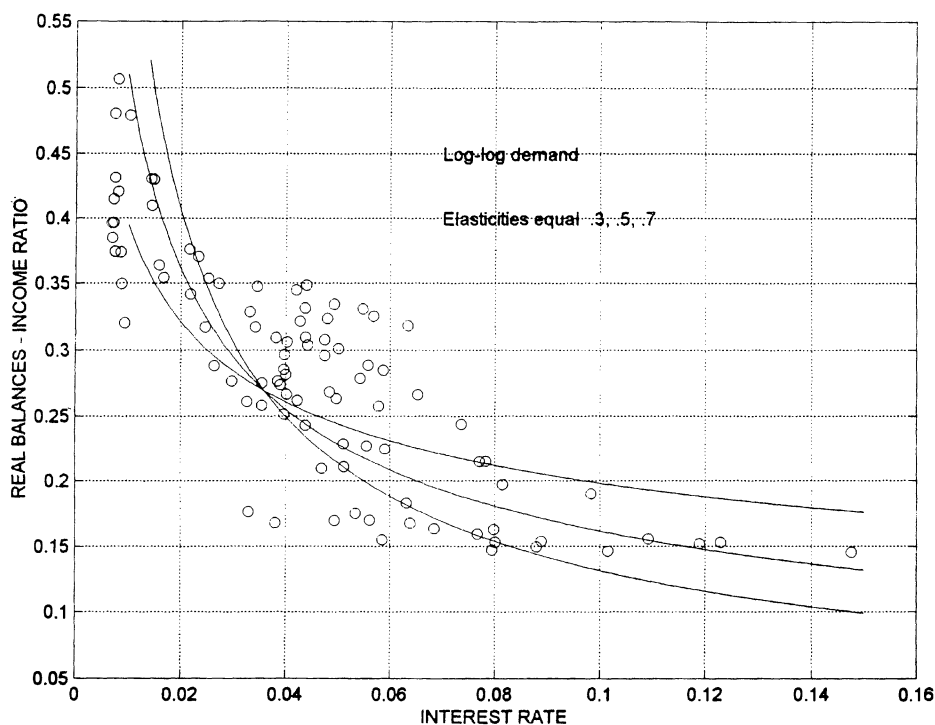


FIGURE 2.—U.S. money demand, 1900–1994.

interest rate constant at r and an otherwise identical steady state with an interest rate of (or near) zero.

For the log-log demand function $m(r) = Ar^{-\eta}$, (2.1) implies

$$w(r) = A \frac{\eta}{1 - \eta} r^{1 - \eta}.$$

For $\eta = 0.5$, this is just a square root function. It is plotted in Figure 5. For the semi-log function $m(r) = Be^{-\xi r}$, (2.1) implies

$$w(r) = \frac{B}{\xi} [1 - (1 + \xi r)e^{-\xi r}].$$

This curve is also plotted, for $\xi = 7$, in Figure 5. This is the parameterization used by Bailey.

Note that the two demand curves imply very different estimates for the welfare cost of moderate inflations. At a six percent interest rate, for example, the log-log curve implies a welfare cost of about one percent of income, while the semi-log curve implies a cost of less than 0.3 percent. But much of this difference is due to the difference in behavior at very low interest rates predicted by these two curves. Figure 6 plots the curves $w(r) - w(0.03)$ for both

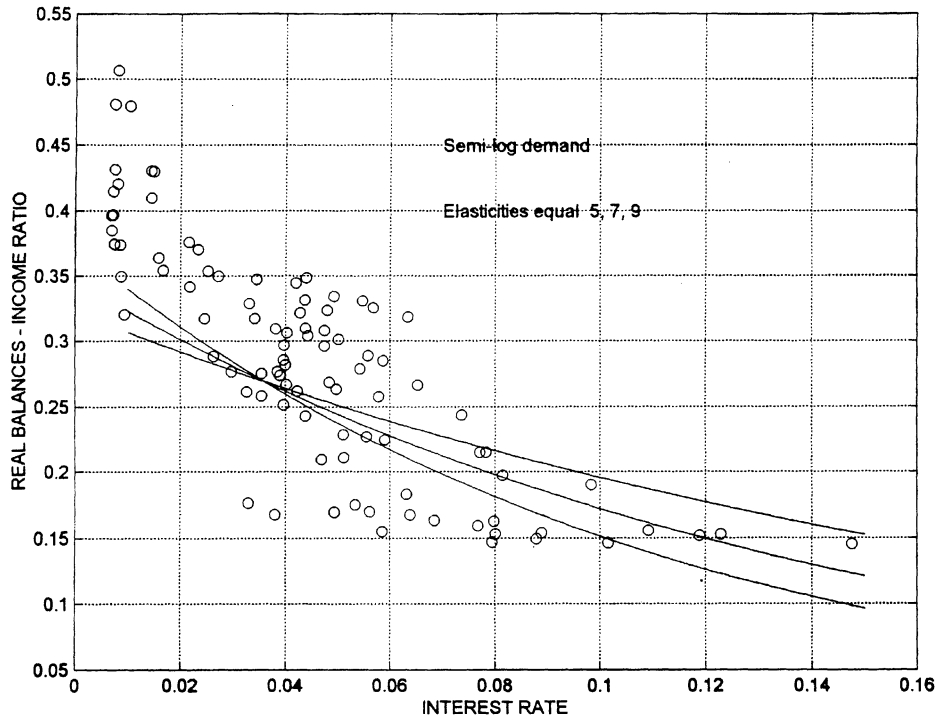


FIGURE 3.—U.S. money demand, 1900–1994.

fitted demand curves, where $r = 0.03$ is chosen as the interest rate that would be associated with an inflation rate of zero. Since the two curves on Figure 5 are nearly parallel between interest rates of 3 and 10 percent, the two curves on Figure 6 imply very similar estimates of the cost of exceeding an inflation rate of zero by moderate amounts. The main difference, then, is that log-log demand implies a substantial gain in moving from zero inflation to the deflation rate needed to reduce nominal interest rates to zero, while under semi-log demand this gain is trivial.

3. THE SIDRAUSKI FRAMEWORK

In order to decide whether we want to view either of the curves plotted in Figure 5 as describing the consequences of policy changes in the actual U.S. economy, we need to be clear on the nature of the thought experiment the outcome of which is traced out by these curves. For this purpose, we need a model of the entire economy that can let us see what changes in monetary policy might generate the curve $m(r)$ and the associated welfare costs $w(r)$. Simply labelling the points plotted in Figure 2 a “demand function” does not tell us anything about what we are estimating or how accurate these estimates are:

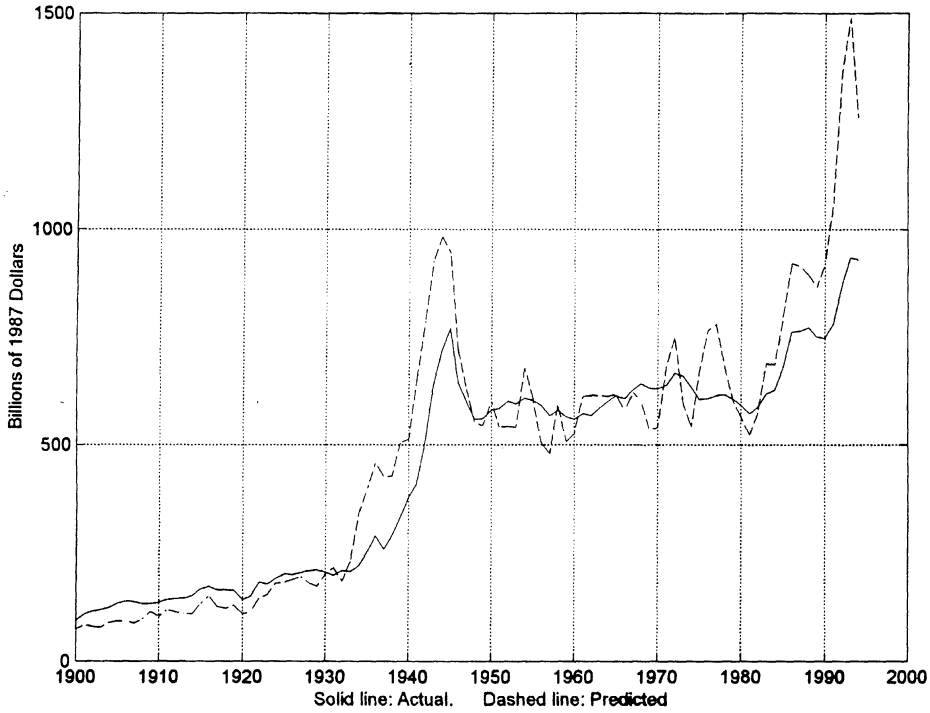


FIGURE 4.—Actual and predicted real balances, 1900–1994.

Giving colorful names to statistical relationships is not a substitute for economic theory.

The following simplified version of the general equilibrium model of Sidrauski (1967a, b) provides one framework that can provide an explicit rationale for the consumers' surplus formula (2.1).⁵ Consider a deterministic, representative agent model, in which households gain utility from the consumption c of a single, nonstorable good, and from their holdings $z = M/P$ of real balances. Household preferences are

$$(3.1) \quad \sum_{t=0}^{\infty} (1 + \rho)^{-t} U(c_t, z_t),$$

where the current period utility function U is given by

$$(3.2) \quad U(c, z) = \frac{1}{1 - \sigma} \left[c \varphi \left(\frac{z}{c} \right) \right]^{1 - \sigma},$$

provided $\sigma \neq 1$. These homothetic preferences are consistent with the absence

⁵Here I follow Brock's (1974) perfect-foresight version of the Sidrauski model.

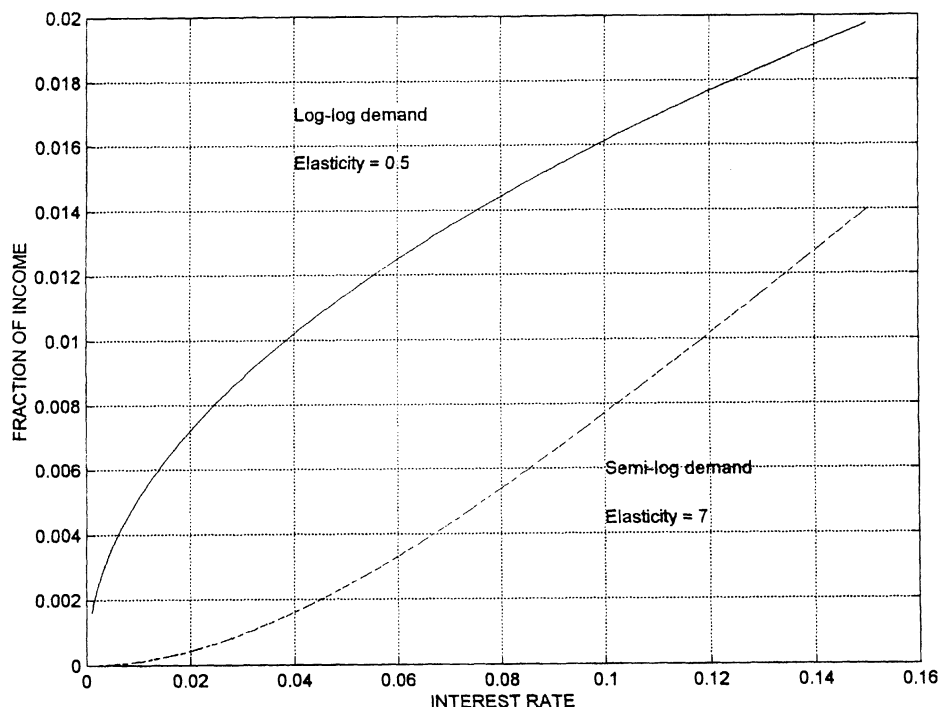


FIGURE 5.—Welfare cost functions.

of trend in the ratio of real balances to income in U.S. data, and the constant relative risk aversion form is consistent with balanced growth.

Each household is endowed with one unit of time, which is inelastically supplied to the market and which produces $y_t = y_0(1 + \gamma)^t$ units of the consumption good in period t .⁶ Hence one equilibrium condition is

$$c_t = y_t = y_0(1 + \gamma)^t.$$

Households begin period t with M_t units of money, out of which they pay a lump sum tax H_t (or, if $H_t < 0$, receive a lump sum transfer). The price level is P_t , so the cash flow constraint for households is

$$M_{t+1} = M_t - H_t + P_t y_t - P_t c_t$$

in nominal terms. In real terms, it is

$$(3.3) \quad (1 + \pi_{t+1})z_{t+1} = z_t - h_t + y_t - c_t,$$

where $h_t = H_t/P_t$ and $1 + \pi_t = P_t/P_{t-1}$.

⁶Throughout this paper I take the real growth rate γ to be independent of monetary policy. The role of inflation when real growth is endogenously determined is examined in De Gregorio (1993), Gomme (1993), Jones and Manuelli (1995), Chari, Jones, and Manuelli (1995), and Dotsey and Ireland (1996).

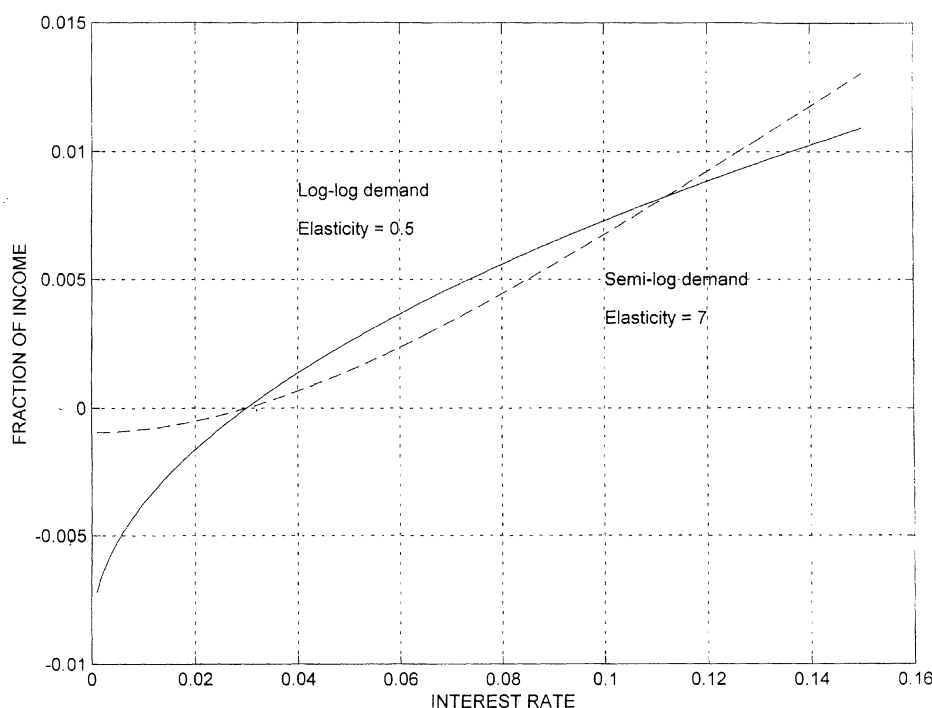


FIGURE 6.—Welfare cost relative to 3% interest.

We consider the decision problem of a household in an economy on a balanced growth equilibrium path, on which the money growth rate is constant at μ , maintained by a constant ratio $\nu = h/y$ of transfers to income. In this case, the ratio of money to income will be constant, and the inflation factor $1 + \pi_t$ will be constant at the value $(1 + \mu)/(1 + \gamma)$. Let $\tilde{v}(z, y)$ be the value of the maximized objective function (3.1) for a household in such an equilibrium that has real balances z when the economy-wide income level has reached y . This function \tilde{v} satisfies the Bellman equation:

$$(3.4) \quad \tilde{v}(z, y) = \max_c \left\{ \frac{1}{1 - \sigma} \left[c \varphi \left(\frac{z}{c} \right) \right]^{1 - \sigma} + \frac{1}{1 + \rho} \tilde{v}(z', y(1 + \gamma)) \right\},$$

where next period's real balances z' are

$$z' = \frac{z - h + y - c}{1 + \pi}.$$

Under the homogeneity assumptions I have imposed, the problem (3.4) can be simplified to a single state variable problem as follows. Define the function $v(m)$ by

$$\tilde{v}(z, y) = v(m) y^{1 - \sigma},$$

where $m = z/y$ is the money-income ratio. If we view $\omega = c/y$ as the household's choice variable, we can see that the function $v(m)$ will satisfy:⁷

$$(3.5) \quad v(m) = \max_{\omega} \left\{ \frac{1}{1-\sigma} \left[\omega \varphi \left(\frac{m}{\omega} \right) \right]^{1-\sigma} + \frac{(1+\gamma)^{1-\sigma}}{1+\rho} v(m') \right\}$$

where

$$m' = \frac{z'}{y(1+\gamma)} = \frac{z-h+y-c}{y(1+\gamma)(1+\pi)} = \frac{m-\nu+1-\omega}{1+\mu}.$$

The first-order and envelope conditions for the problem (3.5), evaluated at $\omega = 1$ (which will hold along any equilibrium path) are:

$$[\varphi(m)]^{-\sigma} [\varphi(m) - m\varphi'(m)] = \frac{1}{1+r} v' \left(\frac{m-\nu}{1+\mu} \right),$$

and

$$v'(m) = [\varphi(m)]^{-\sigma} \varphi'(m) + \frac{1}{1+r} v' \left(\frac{m-\nu}{1+\mu} \right),$$

where the nominal interest rate r is defined by

$$(3.6) \quad \frac{1}{1+r} = \frac{(1+\gamma)^{1-\sigma}}{(1+\rho)(1+\mu)}.$$

(Note that this nominal interest r approximately equals $\rho + \sigma\gamma + \mu - \gamma$, the familiar sum of the real rate and the inflation premium.) Along the balanced path, m is constant, and eliminating $v'(m)$ between these two equations and simplifying yields

$$(3.7) \quad \frac{\varphi'(m)}{\varphi(m) - m\varphi'(m)} = r.$$

Let $m(r)$ denote the m value that satisfies (3.7), expressed as a function of the interest rate. Throughout the paper, it is this kind of steady state equilibrium relation $m(r)$ that I call a "money demand function," and that I identify with the curves shown in Figures 2 and 3.

The flow utility enjoyed by the household on the balanced path is $U(y, m(r)y)$, where y is growing at the constant rate γ . Provided $m'(r) < 0$, this utility is maximized over nonnegative nominal interest rates at $r = 0$: the Friedman

⁷If a function v satisfies (3.5), then it is easy to see that the function $\tilde{v}(m, y) = y^{1-\sigma} v(m/y)$ satisfies (3.4). Ruling out other solutions to (3.4) is more difficult. In general, I will not provide a rigorous treatment of the Bellman equations that arise in this paper.

(1969) rule of a deflation equal to the real rate of interest.⁸ In this section, I define the welfare cost $w(r)$ of a nominal rate r to be the percentage income compensation needed to leave the household indifferent between r and 0. That is, $w(r)$ is defined as the solution to

$$U[(1 + w(r))y, m(r)y] = U[y, m(0)y].$$

With the assumed functional form (3.2), this definition is equivalent to

$$(3.8) \quad (1 + w(r))\varphi\left(\frac{m(r)}{1 + w(r)}\right) = \varphi[m(0)].$$

An estimated function $m(r)$ can be used to calculate the function $w(r)$ as follows. Let $m(r)$ be given and let $\psi(m)$ be the inverse function. Then (3.7) implies that the function φ satisfies the differential equation

$$(3.9) \quad \varphi'(m) = \frac{\psi(m)}{1 + m\psi(m)} \varphi(m).$$

Differentiating (3.8) through with respect to r , we obtain

$$(3.10) \quad 0 = w'(r)\varphi\left(\frac{m(r)}{1 + w(r)}\right) + \varphi'\left(\frac{m(r)}{1 + w(r)}\right)\left[m'(r) - \frac{m(r)w'(r)}{1 + w(r)}\right].$$

Now apply (3.9) with $m = m(r)/(1 + w(r))$ to (3.10) and cancel, to obtain the differential equation

$$(3.11) \quad w'(r) = -\psi\left(\frac{m(r)}{1 + w(r)}\right)m'(r)$$

in the welfare cost function w , which has the natural initial condition $w(0) = 0$.

Given any money demand function m (and inverse ψ), (3.11) is readily solved numerically for an exact welfare cost function $w(r)$. But comparing (3.11) and (2.1), one can guess that for small values of r —and hence of $w(r)$ —the solution to (3.11) must be very close to the value implied by the consumers' surplus formula. In fact, on a plot such as Figure 5 the exact and the approximate solutions cannot be distinguished by the eye. (See also Figure 8 in Section 5.)

We can also solve the differential equation (3.9) for the function φ , reconstructing the utility function. For the particular demand function $m(r) = A/\sqrt{r}$, for example, (3.9) has the solution

$$\varphi(m) = \left[1 + \frac{A^2}{m}\right]^{-1}$$

⁸Depending on the way the holding of real balances is motivated, the equilibrium in the limiting economy where $r = 0$ may be ill-defined, or there may be equilibria with $r = 0$ that are not close to equilibria with r positive but arbitrarily small. I will confine attention here to economies with $r > 0$. By referring to 0 as the optimal rate in this context I mean that reducing r is welfare-improving for any $r > 0$.

with the boundary condition $\varphi(0) = 0$. Since the value of A in the U.S. is empirically about 0.05 (see Figure 2), the Sidrauski utility function takes the form

$$U(c, z) = \frac{1}{1 - \sigma} \left(\left[\frac{1}{c} + \frac{0.0025}{z} \right]^{\sigma-1} - 1 \right).$$

The implied elasticity of substitution between goods and real balances is 0.5. The estimated money demand function gives no information on the intertemporal substitution parameter σ .⁹

To interpret the welfare cost functions plotted in Figure 5, then, we think of these curves as tracing out different steady states of deterministic economies subjected to different, constant rates of money growth. The differences in interest rates across these economies are attributed *solely* to differences in inflation premia. This interpretation seems to me to rationalize a focus on low frequency evidence on money demand in the 20th century U.S. time series, and suggests the possibility that accurate estimates of welfare costs, in the sense of across-steady-state comparisons, can be obtained without a good understanding of the behavior of velocity at high frequencies.

Using a general equilibrium framework to interpret the welfare estimates of the last section, even one as simple as my version of Sidrauski's, is helpful—essential, really—in exploring the effects of changes in assumptions on these estimates. Many economists, for example, believe that a deterministic framework like Bailey's or mine misses important costs of inflation that are thought to arise from price or inflation rate *variability*. It would be a straightforward exercise, today, to add stochastic shocks of realistic magnitude and behavior to both real productivity and money supply behavior in this model, and to re-examine the welfare calculations in this new context. Based on the Cooley and Hansen (1989) study of a similar model of the U.S. economy, I am very confident that the effects of such a modification on the welfare costs estimated in Section 2 would be negligible.¹⁰ In the next section, I will illustrate in another way this process of modifying the model in order to examine the importance of its simplifying assumptions.

4. FISCAL CONSIDERATIONS

In the analysis to this point the nominal interest rate r has been treated as a policy variable, and the welfare cost of inflation has been defined by a comparison of resource allocations when $r > 0$ to a benchmark case of $r = 0$. In fact, of course, any particular interest rate policy must be implemented by a specific money supply policy, and this monetary policy must in turn be implemented by a policy of fiscal transfers, open market operations, or both. This fact raises no

⁹The irrelevance of the intertemporal substitution parameter for money demand reflects the fact that, in this model, money is dominated as a store of value by nominal bonds.

¹⁰Burdick (1997) contains an interesting analysis of transition dynamics in a model closely related to Cooley and Hansen's.

difficulties as long as the necessary transfers can be effected through lump-sum payments or assessments, but if this is not possible the optimality of the Friedman rule can cease to obtain. Aspects of this question have been examined by Phelps (1973), Bewley (1983), Kimbrough (1986a,b), Lucas and Stokey (1983a), Woodford (1990), Cooley and Hansen (1991), Eckstein and Leiderman (1992), Miller (1995), and most recently by Guidotti and Vegh (1993), Chari, Christiano, and Kehoe (1993), Correia and Teles (1997), and Mulligan and Sala-i-Martin (1997). This section addresses some of these fiscal questions in the contexts of the Sidrauski model of the last section.

Let $m(r)$ be steady state real balances. Define the parameter δ by $1 + \delta = (1 + \rho)/(1 + \gamma)^{1-\sigma}$, so that $\delta \equiv \rho + \sigma\gamma - \gamma$ is the amount by which the real interest rate exceeds the growth rate of output. Recall that $r = \delta + \mu$ and $\nu = -\mu m(r)$. Then the consumer budget constraint and the resource constraint together imply that to implement a nominal interest rate r , the fraction

$$(4.1) \quad \nu = -\mu m(r) = (\delta - r)m(r)$$

of income y_t must be transferred from the private sector to the government in a steady state, in the form of real balances withdrawn from circulation. (If $\delta < r$, the negative of this magnitude is seigniorage revenue, relative to income.)

For the function $m(r) = A/\sqrt{r}$ that fits U.S. data, $m(r) \rightarrow \infty$ as $r \rightarrow 0$, so if the flow (4.1) must be withdrawn using a fractional tax on income, the policy $r = 0$ is not feasible. The need to resort to income taxation thus places a positive lower bound on r . But with $\delta = 0.02$ and $A = 0.05$, an income tax rate of 0.03 would implement an interest rate of 0.001 (that is, one-tenth of one percent). The Friedman rule requires qualification in this case, but the qualification is of no quantitative interest.

The cases considered by most of the authors cited above, however, have the additional complications that labor is elastically supplied, so an income tax distorts resource allocation, and there is a positive amount of government consumption, necessitating a resort to distorting taxation of some kind. In these circumstances, it is not impossible that a positive inflation tax might have a useful role to play in the overall tax structure. In this section, these two features will be added to the model of Section 3, and the welfare cost calculations described there will be re-done. The results of these calculations are given in Figure 7.

We modify the current period preferences (3.1) to include the consumption of leisure, x :

$$U(c, m, x) = \frac{1}{1-\sigma} \left[c \varphi\left(\frac{z}{c}\right) \phi(x) \right]^{1-\sigma}.$$

Modify the resource constraint to include government consumption, g_t :

$$c_t + g_t = (1 - x_t)y_t = (1 - x_t)y_0(1 + \gamma)^t.$$

Modify consumers' budget constraints to reflect income taxation at a flat rate τ :

$$(1 + \mu)m_{t+1} = m_t + (1 - \tau)(1 - x_t) - \omega_t,$$

where $m_t = z_t/y_t$ is the ratio of money to full income, and $\omega_t = c_t/y_t$.

If government consumption is a constant ratio g to full income y_t , this model has an equilibrium path with constant ratios of consumption and real balances to income and with leisure constant as well. Using the same normalization employed in Section 3, an individual household's Bellman equation on such a path is

$$v(m) = \max_{\omega, x} \left\{ \frac{1}{1 - \sigma} \left[\omega \varphi\left(\frac{m}{\omega}\right) \phi(x) \right]^{1 - \sigma} + \frac{(1 + \gamma)^{1 - \sigma}}{1 + \rho} v(m') \right\}$$

where

$$(1 + \mu)m' = m + (1 - \tau)(1 - x) - \omega.$$

The first order and envelope conditions for this problem are

$$\left[\omega \varphi\left(\frac{m}{\omega}\right) \phi(x) \right]^{-\sigma} \left[\varphi\left(\frac{m}{\omega}\right) - \frac{m}{\omega} \varphi'\left(\frac{m}{\omega}\right) \right] \phi(x) = \frac{1}{1 + r} v'(m'),$$

$$\left[\omega \varphi\left(\frac{m}{\omega}\right) \phi(x) \right]^{-\sigma} \omega \varphi\left(\frac{m}{\omega}\right) \phi'(x) = \frac{1}{1 + r} v'(m')(1 - \tau),$$

and

$$v'(m) = \left[\omega \varphi\left(\frac{m}{\omega}\right) \phi(x) \right]^{-\sigma} \varphi'\left(\frac{m}{\omega}\right) \phi(x) + \frac{1}{1 + r} v'(m'),$$

where again the nominal interest rate r is defined by (3.6). Along the balanced path, m is constant, and eliminating $v'(m)$ from these equations and simplifying yields

$$(4.2) \quad r \left[\varphi\left(\frac{m}{\omega}\right) - \frac{m}{\omega} \varphi'\left(\frac{m}{\omega}\right) \right] = \varphi'\left(\frac{m}{\omega}\right),$$

$$(4.3) \quad \omega \varphi\left(\frac{m}{\omega}\right) \phi'(x) = \left[\varphi\left(\frac{m}{\omega}\right) - \frac{m}{\omega} \varphi'\left(\frac{m}{\omega}\right) \right] \phi(x)(1 - \tau),$$

Additional steady state equilibrium conditions are

$$(4.4) \quad \omega + g + x = 1,$$

$$(4.5) \quad \mu m = (1 - \tau)(1 - x) - \omega.$$

Condition (4.2) just repeats (3.6). Condition (4.3) equates the marginal rate of substitution between goods and leisure to the after tax real wage, $1 - \tau$. Conditions (4.4) and (4.5) are the resource and consumer budget constraints; together, they imply the government budget constraint. For any given nominal interest rate r and government consumption rate g , (4.2)–(4.5) are four equa-

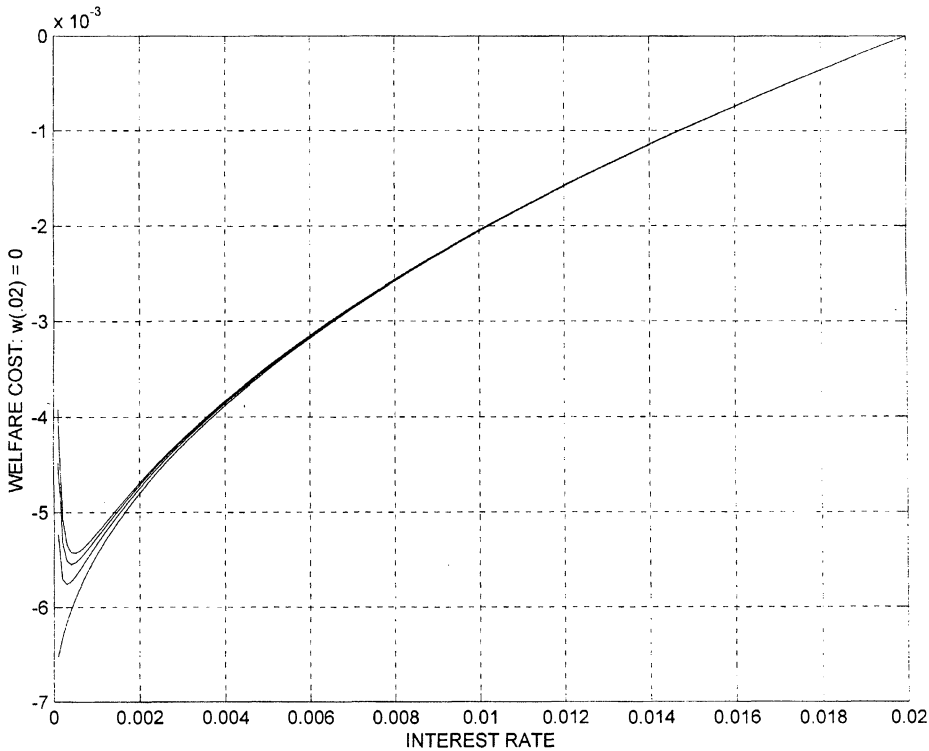


FIGURE 7.—Welfare cost with income taxation.

tions that can be solved for the steady state allocation (ω, x, m) and the income tax rate τ . Any monetary policy dictates a tax policy, depending on the extent to which seigniorage revenues help to finance g , or the extent to which the need to withdraw cash from the public adds to the burden on the tax system.

Figure 7 tabulates a welfare cost function $w(r)$, defined as

$$(4.6) \quad U[(1 + w(r))c(r), m(r), x(r)] = U[c(\delta), m(\delta), x(\delta)].$$

I use $r = \delta$ as a benchmark rather than $r = 0$ because, depending on the assumed functions φ and ϕ , the system (4.2)–(4.5) may not have a solution at $r = 0$.

The figure is based on the following parameterization. For the function φ , I used $\varphi(m) = (1 + 1/(km))^{-1}$, which follows from the money demand function $m(r) = A\sqrt{1/r}$; A was set equal to 0.05, to fit the U.S. data. For the function ϕ , I used $\phi(x) = x^\beta$. With these assumptions, the definition (4.6) of the function $w(r)$ implies

$$(1 + w(r)) \frac{m(r)}{1 + k(m(r)/\omega(r))} x(r)^\beta = \frac{m(\delta)}{1 + k(m(\delta)/\omega(\delta))} x(\delta)^\beta.$$

I let the elasticity β range over the values 0.0001, 0.3, 0.6, and 0.9. Reading from bottom to top, these are the four curves plotted in Figure 7. I set $1 - g = 0.35$, so that if $x = 0$, $\omega = 1$. Finally, I set $\delta = 0.02$.

One can see from Figure 7 that above about half a percent, estimated welfare costs are the same as in the inelastic labor supply, lump sum tax case studied in earlier sections. The effects of distorting taxation appear only at extremely low interest rates. Thus for a leisure elasticity of $\beta = 0.3$, the optimal interest rate is about 0.03 percent, while at $\beta = 0.9$, it is about 0.04 percent. For any $\beta > 0$, the optimal r is strictly positive, but the deviations from $r = 0$ are minute. The differences in welfare are small too. The minimized welfare costs are in all cases less than -0.0045 , while the intercept of the benchmark curve, $-w(\delta)$, is -0.006 , a difference of 0.0015 times income.

These second-best tax problems have so many logical possibilities that I thought it would be useful to work one case through, quantitatively, to see what kind of magnitudes are at stake. But the case I selected for study is, in some respects, arbitrary, and the literature cited above is helpful in isolating crucial assumptions. The model underlying Figure 7 is a special case of the model analyzed in Section 2 of Chari, Christiano, and Kehoe (1993), where it is shown that the Friedman $r = 0$ policy is optimal in the sense of Ramsey, provided that the private sector begins with a net nominal position (money plus nominal debt) of zero. If, on the other hand, the net nominal position of the private sector is positive, a monetary-fiscal policy that is efficient in Ramsey's sense entails an initial hyperinflation to exploit the capital levy possibilities. In my analysis, there is no government debt and the public holds a positive initial nominal position (its cash), but I have constrained the money growth rate and the tax rate to be constant, precluding a capital levy. Under these assumptions, Woodford (1990) shows that $r = 0$ is not optimal, a fact that Figure 7 reflects.

In short, the optimality of the Friedman rule can be studied in a very wide variety of second-best frameworks, with a wide variety of different qualitative conclusions. In the specific context I have used, the Friedman rule needs qualification, but the magnitude of the needed amendment is trivially small. The fact is that real balances are a very minor "good" in the U.S. economy, so the fiscal consequences of even sizeable changes in the rate at which this good is taxed—the inflation rate—are just not likely to be large.¹¹

5. THE MCCALLUM-GOODFRIEND FRAMEWORK

The Sidrauski theory takes us behind the estimated money demand function to possible underlying preferences and technology, and by so doing certainly clarifies the welfare interpretation of Figure 5. It is also a convenient framework

¹¹In the U.S. tax structure, inflation also has an indirect effect on the effective tax rates on income from capital (due to its effects on allowable deductions for depreciation, for example). These effects, if not offset by indexing or legislative changes, can be sizeable. See Feldstein (1996) and Bullard and Russell (1997).

for exploring the consequences of different assumptions that may affect welfare cost estimates, such as the fiscal considerations examined in the last section. It is less helpful in thinking about cash management behavior at very low interest rates. The same criticism can be raised about Friedman's (1969) argument: What does it mean, exactly, to satiate an economy with cash? To make progress on this question, one needs to think more concretely about what people *do* with their money holdings.

The cash-in-advance formulation used in Lucas and Stokey (1983b) provides a specific image of a cash-using society that could be useful for this purpose. In this section, though, I will use a version of McCallum and Goodfriend's (1987) proposed variation on the Sidrauski model. In their model, the use of cash is motivated by an assumed transactions technology, rather than by an assumption that real balances yield utility directly. One can also see useful connections between this assumed technology and earlier inventory-theoretic studies of money demand.

In the McCallum-Goodfriend model, household preferences depend on goods consumption only:

$$(5.1) \quad \sum_{t=0}^{\infty} (1 + \rho)^{-t} \frac{1}{1 - \sigma} c_t^{1-\sigma}, \quad \sigma \neq 1.$$

Each household is endowed with one unit of time, which can be used either to produce goods or to carry out transactions. Call s the fraction devoted to transacting. The goods production technology is assumed to be

$$(5.2) \quad c_t = (1 - s_t)y_t = (1 - s_t)y_0(1 + \gamma)^t.$$

The cash flow constraint in real terms is

$$(1 + \pi_{t+1})z_{t+1} = z_t - h_t + (1 - s_t)y_t - c_t,$$

where $z_t = M_t/P_t$. In terms of the money-income ratio m_t , this constraint reads

$$(1 + \mu_{t+1})m_{t+1} = m_t - \nu_t + 1 - s_t - \omega_t,$$

where $\nu_t = h_t/y_t$ and $\omega_t = c_t/y_t$.

The new element in the model is a transactions constraint, relating household holdings of real balances and the amount of household time devoted to transacting to the spending flow the household carries out. I assume that in real terms this constraint takes the form

$$(5.3) \quad c_t = z_t f(s_t),$$

which will be consistent with a unit income elasticity of money demand.¹²

As in the last section, I consider the decision problem of a household in an economy on a balanced growth equilibrium in which the money growth rate is

¹² Brock (1974) proposes a similar formulation, and shows that it is equivalent to a utility-based formulation in which utility depends on leisure as well as goods and real balances.

constant at μ , maintained by a constant ratio $\nu = h/y$ of transfers to income, the ratio of money to income is a constant m , and the inflation factor $1 + \pi_t$ is constant at the value $(1 + \mu)/(1 + \gamma)$. Think of the household's choice variables as the time allocation s and the consumption-income ratio ω . Let $y^{1-\sigma}v(m)$ be the value of the maximized objective function (5.1) for a household in this balanced path equilibrium that has a ratio of money balances to income of $m = M_t/(P_t y)$ when the economy-wide income level has reached y . Then the function v satisfies the Bellman equation

$$v(m) = \max_{\omega, s} \left\{ \frac{1}{1-\sigma} \omega^{1-\sigma} + \frac{(1+\gamma)^{1-\sigma}}{1+\rho} v(m') \right\}$$

subject to

$$\omega = mf(s),$$

where

$$(5.4) \quad m' = \frac{m - \nu + 1 - s - \omega}{1 + \mu}.$$

We use the transactions constraint to eliminate ω as a decision variable:

$$(5.5) \quad v(m) = \max_s \left\{ \frac{1}{1-\sigma} [mf(s)]^{1-\sigma} + \frac{(1+\gamma)^{1-\sigma}}{1+\rho} v\left(\frac{m - \nu + 1 - s - mf(s)}{1 + \mu}\right) \right\}.$$

The value function that satisfies (5.5) need not be concave, so one cannot use standard arguments to show that a time allocation that satisfies the first-order condition for (5.5) is in fact optimal. Even so, I will begin, as in Sections 3 and 4, by using the first-order and envelope conditions to characterize a balanced path equilibrium. Then I will carry out a numerical analysis of (5.5) to determine the conditions under which consumer utility is maximal along this balanced path.

The first-order and envelope conditions for (5.5) are

$$[mf(s)]^{-\sigma} mf'(s) = \frac{1}{1+r} v'(m') [1 + mf'(s)]$$

and

$$v'(m) = [mf(s)]^{-\sigma} f(s) + \frac{1}{1+r} v'(m') [1 - f(s)],$$

where as in Section 3, the nominal interest rate r is given by (3.6). Along the balanced path, $m = m'$, and eliminating $v'(m)$ and simplifying yields

$$(5.6) \quad f(s) = r mf'(s).$$

A second equilibrium condition follows from the transactions constraint and the fact that $\omega = c/y = 1 - s$ on a balanced path:

$$(5.7) \quad 1 - s = mf(s).$$

Given f , we can solve (5.6) and (5.7) for s and m as functions of r .

In this model, the time spent economizing on cash use, $s(r)$, has the dimensions of a percentage reduction in production and consumption, and hence is itself a direct measure of the welfare cost of inflation, interpreted as wasted time. To estimate this function $s(r)$, we work backward from the function $m(r)$ as estimated in Section 2 to the transactions technology function f . As in Section 3, we do this by finding a first order differential equation in the welfare cost $s(r)$.

Given f , let $m(r)$ and $s(r)$ satisfy (5.6) and (5.7). Then differentiating (5.7) through with respect to r and using (5.6) and (5.7) to eliminate $f(s)$ and $f'(s)$ yields

$$(5.8) \quad s'(r) = - \frac{rm'(r)(1 - s(r))}{1 - s(r) + rm(r)}.$$

Comparing (5.8) and (2.1), one can see that for small r —and hence small $s(r)$ —solutions to (5.8) and the area under the inverse money demand function will be very close. Figure 8 plots the solution $s(r)$ with initial condition $s(0) = 0$ for the log-log and semi-log demand cases, for interest rates ranging from 0 to 2 (200%). Also plotted are the areas under the two demand curves, as in Figure 5. For the semi-log case, the exact and approximate welfare cost estimates cannot be distinguished. For the log-log case, the two curves are also virtually identical at interest rates below 20%. Thus the McCallum-Goodfriend model yields simply a new interpretation of estimates already obtained.

For the log-log case with interest elasticity of 0.5, the implied transactions time function is simply a straight line through the origin, $f(s) = ks$, for some constant k . This case is of particular interest, since a multiplicative transactions technology kms corresponds to the celebrated inventory-theoretic model introduced by Baumol (1952), and developed by Tobin (1956), Miller and Orr (1966), Dvoretzky and Patinkin (1965), Frenkel and Jovanovic (1980), and Chang (1992).¹³ If one can sustain a given pattern of transactions with average balances m and s units of time in trips to the bank, then the same pattern can be sustained by halving average cash and doubling the number of trips. In this special case, the

¹³ Karni (1973), Kimbrough (1986a,b), Den Haan (1990), Cole and Stockman (1992), and Gillman (1993) have also used monetary models featuring a time-using technology for transactions. Karni is explicit about the links with the inventory-theoretic literature that I am here using to motivate a specific form for this technology. The construction of an explicit general equilibrium model in which agents solve Baumol-like cash management problems has not been carried out in any of these papers, nor is it in this one. See Fusselman and Grossman (1989) or Grossman (1987) for interesting results along this line. A useful recent contribution is Rodriguez (1996).

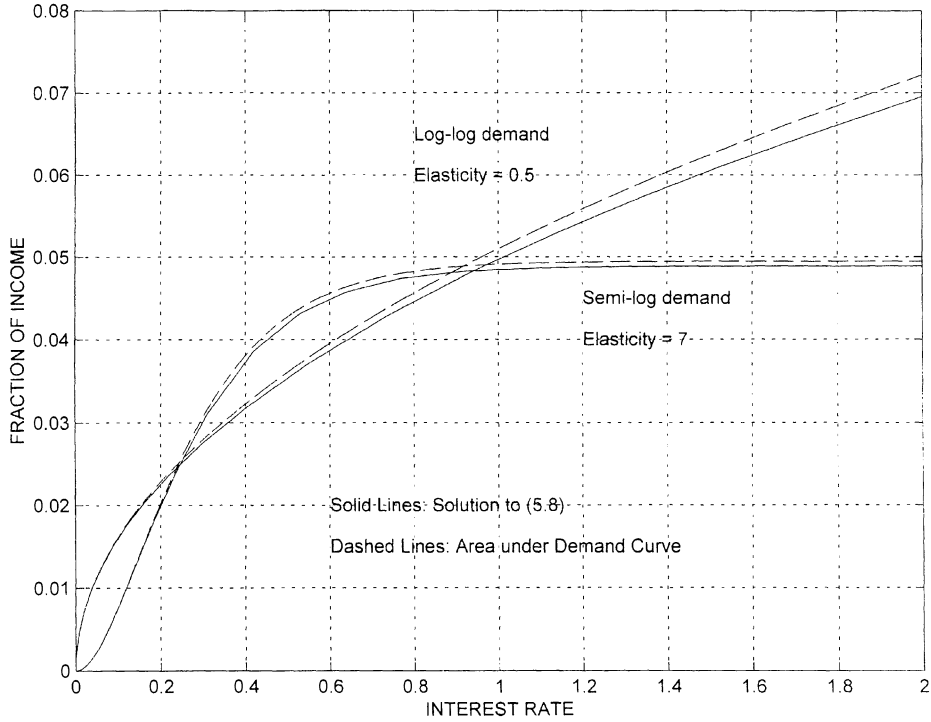


FIGURE 8.—Approximate and exact welfare cost functions.

two steady state equations (5.6) and (5.7) become

$$s = rm$$

and

$$1 - s = kms,$$

and eliminating the money-income ratio m between the two yields a quadratic in the steady state value of s :

$$(5.9) \quad \frac{k}{r}s^2 = 1 - s.$$

For large values of the ratio k/r , the unique positive solution to (5.9) is very well approximated by the square root rule¹⁴

$$s(r) = \sqrt{\frac{r}{k}},$$

¹⁴Jovanovic (1982) contains another derivation of the square root formula from an aggregative general equilibrium model.

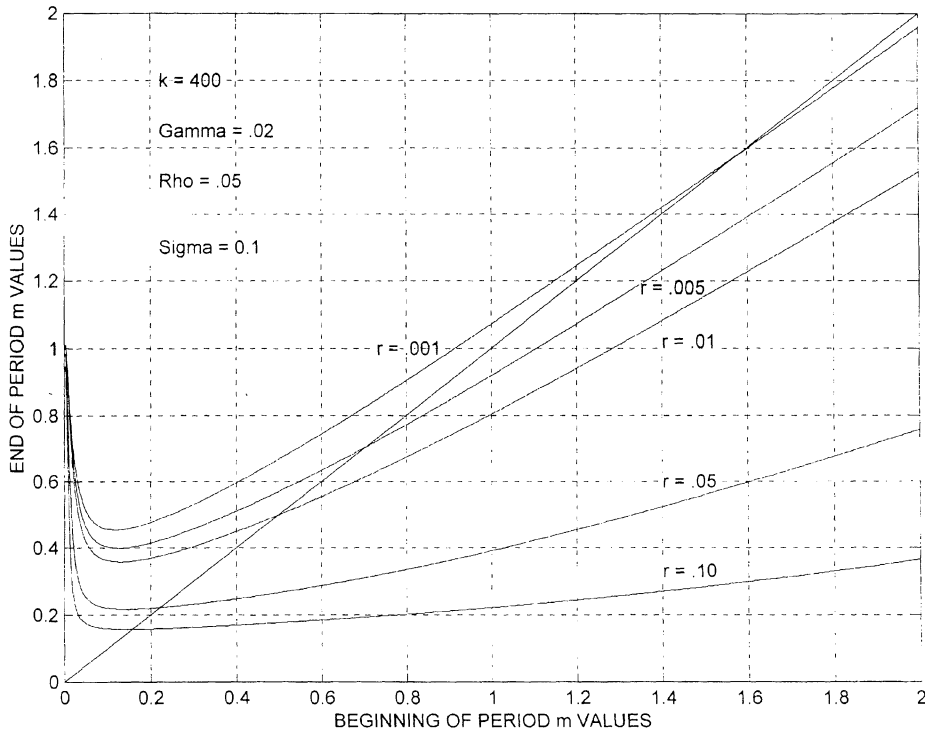


FIGURE 9.—Policy functions for different interest rates.

and the money-income ratio by

$$(5.10) \quad m(r) = \sqrt{\frac{1}{rk}}.$$

The parameter k can be calibrated from the intercept $A = 0.05$ of the money demand function: $k = (0.05)^{-2} = 400$.

Could it be simply coincidence that the interest elasticity predicted by Baumol's theory—one-half—is the value that best fits U.S. time series evidence? This is a possibility, certainly, but attributing striking results to coincidence is not the way science tends to move forward!¹⁵

Figures 9 and 10 report results of numerical calculations designed to check whether consumer utility is in fact maximized along the balanced path that I

¹⁵ Depending on the way one interprets the Baumol theory, one may take it as also predicting that the *income* elasticity of money demand is one-half. If this is right, the theory fails badly on U.S. time series evidence. The issue is whether we interpret the growth in the economy's aggregate production as growth in the size of the cash flows to be managed, or in the number of flows, or somewhere in between. The constant returns, unit income elasticity that I have built into the aggregate theory requires the assumption that it is the *number* of cash flows to be managed that doubles whenever real GDP doubles, not their average size.

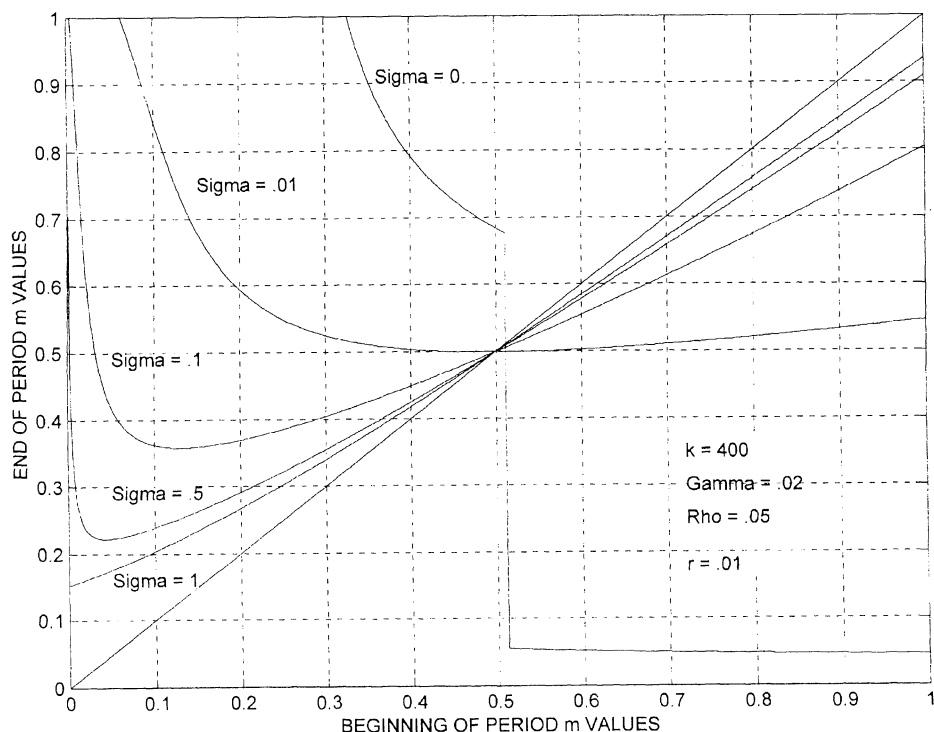


FIGURE 10.—Policy functions for different sigma values.

have constructed from the first-order conditions for the dynamic program (5.5). In all calculations, the technology $f(s) = ks$ is assumed, with $k = 400$. I assumed the real growth rate $\gamma = 0.02$ and a subjective discount rate of $\rho = 0.05$. The coefficient of risk aversion σ and the nominal interest rate r were varied over several values, as indicated. For each (σ, r) pair, I used (3.6) to calculate the rate of money growth μ that is implied by given values of γ , ρ , σ , and r . Then I used the condition $\mu m = -\nu$, with m at the balanced path value given in (5.10), to calculate the implied fiscal policy. These parameter values completely specify the consumer's problem, (5.5).

To calculate the optimal value and policy functions for (5.5), the values of m and m' were restricted to a grid of 1000 values ranging from 0 to 2 in Figure 9 and 0 to 1 in Figure 10. Maximization was carried out by comparing values at all points of the grid: No first-order conditions were used. Each figure plots a different family of policy functions (the optimal m' as a function of m) for (5.5).

In Figure 9, σ is set at the low value of 0.1, and the nominal interest rate is varied from 0.001 (one-tenth of one percent) up to 0.10. In all cases, the cash holdings of a single consumer with arbitrary initial balances converges to the steady state value given by (5.10). As the interest rate increases above 0.10, the policy function continues to flatten above balanced path values, reflecting the

fact that at high interest rates, consumers very quickly reduce cash holdings to long run levels. Similar results are obtained at higher values of σ .

In Figure 10, the interest rate is held fixed at 0.01 and the parameter σ is varied from the linear case $\sigma = 0$ through the log utility case $\sigma = 1$. For $\sigma > 0$, all of these policy functions have a fixed point at $m = 0.5 = 1/\sqrt{rk} = 1/\sqrt{(400)(0.01)}$, consistent with the analysis based on first-order conditions that leads to (5.10). For linear utility, however, the policy function has a discontinuity at $m = 0.5$: The optimal policy in this case is to set $s = 0$ for a while, consuming nothing, earning maximum income, and accumulating cash, and then to enjoy a consumption orgy in which all cash is spent at once. The consumer then returns to the cash-accumulation phase, and the cycle is repeated. Similar behavior emerges at positive but very small (smaller than 0.01) values of σ .

In summary, then, it is possible that in this nonconvex problem the first-order conditions can fail to hold under optimal behavior. In such cases, the McCallum-Goodfriend theory cannot be used to rationalize the money demand function (5.10). But these difficulties arise only under near-linear utility, with values of σ far below any available estimates. For realistic values of the risk aversion parameter, and in particular even for very low interest rates, (5.10) is an implication of the theory.

6. CONCLUSIONS AND FURTHER DIRECTIONS

There are several research developments that hold promise for sharpening our knowledge on the cost of inflation that I have not yet mentioned. I will discuss these briefly, and then offer some conclusions.

I have emphasized that money holding behavior at very low interest rates is central for estimating welfare costs. In this paper, I have pursued the idea that models parameterized to fit time series behavior under interest rates as low as two percent could be used to predict behavior at interest rates in the zero to two percent range. Recent work by Mulligan and Sala-i-Martin (1996) provides reason to believe that this extrapolation will not be reliable, and proposes a quite different empirical approach to the problem. They begin from the hypothesis that there is a fixed cost (renewable annually, say) of holding positive amounts of interest-bearing securities, and that households who hold only cash do not incur this cost. In this case, if a monetary policy driving interest rates to zero were implemented, more and more households would decide not to incur this fixed cost, which is to say that fewer and fewer households would be using resources to economize on cash holdings. The presence of such a cost might be undetectable in aggregate time series, yet important enough to completely negate any welfare gain from reducing interest rates from, say, 1.5 percent to zero.

Mulligan and Sala-i-Martin then observe that in deciding whether to incur the fixed cost, a household will compare it to something like the *product* rA of the

interest rate r and asset holdings A . If so, then the portfolio behavior of people with low asset holdings should resemble behavior at low interest rates, and we should be able to see the effects of the fixed cost by looking at people with low financial wealth in a cross section. According to the Survey of Consumer Finances, as described in Avery et al. (1984), about 59% of American households in 1989 hold no financial assets beside cash and their checking account. Mulligan and Sala-i-Martin interpret this fact as evidence that the fixed cost described in the last paragraph are sizeable. I think this interpretation is right, and conclude that the construction of models that can utilize cross-section and time series evidence together has real promise for learning about behavior under very low interest rates. If so, then there is good reason to doubt that accurate estimates of cash holding at very low interest rates can be obtained from aggregate U.S. time series evidence alone.

Another set of questions about the time series estimates concerns the fact that M1—the measure of money that I have used—is a sum of currency holdings that do not pay interest and demand deposits that (in some circumstances) do. Moreover, other interest bearing assets beside these may serve as means of payment. One response to these observations is to formulate a model of the banking system in which currency, reserves, and deposits play distinct roles. Such a model seems essential if one wants to consider policies like reserve requirements, interest on deposits, and other measures that affect different components of the money stock differently. See Yoshino (1993) for a promising start in this direction.¹⁶

A second response to the arbitrariness of M1, more fully developed so far than the first, is to replace M1 with an aggregate in which different monetary assets are given different weights. The basic idea, as proposed in Barnett (1978, 1980), and Poterba and Rotemberg (1987), is that if a treasury bill yielding 6 percent is assumed to yield no monetary services, then a bank deposit yielding 3 percent can be thought of as yielding half the monetary services of a zero-interest currency holding of equal dollar value. Implementing this idea avoids the awkward necessity of classifying financial assets as either entirely money or not monetary at all, and lets the data do most of the work in deciding how monetary aggregates should be revised over time as interest rates change and new instruments are introduced. The Divisia monetary aggregates constructed by Barnett and others can behave quite differently from “simple sum” aggregates like M1 or M2.¹⁷ For most of the U.S. time series data used in this paper, though, demand deposits were required by law not to pay interest. I doubt that this issue is of much importance for Meltzer’s (1963a) estimates, nor do I think it is of much importance for my extension of Meltzer’s estimation to later years. But one can see from Figure 4 that my estimated money demand functions do very badly in the 1990s. I share the widely held opinion that M1 is

¹⁶Other recent work that treats components of M1 separately includes Dotsey (1988) and Marty (1993).

¹⁷See, for example, Belongia (1996).

too narrow an aggregate for this period, and I think that the Divisia approach offers much the best prospects for resolving the difficulty.

As in any active research area, then, there are many interesting avenues left to pursue. But I began this paper with the substantive question of estimation of the welfare gains available to a society that reduces the long run growth rates of money and prices, and I owe the reader a summary of what is known, now, on this question.

In all of the models I have reviewed, the estimated gains of reducing inflation and interest rates are positive, starting from any interest rate above, say, one tenth of one percent. Even when fiscal considerations make a strictly positive interest rate optimal, the necessary qualification to the Friedman (1969) rule is quantitatively trivial. According to Figure 5 (or 6) reducing interest rates from 14 percent to 3 percent would yield a benefit equivalent to an increase in real income of about 0.008, eight tenths of one percent. This estimate is about the same whether one uses the fitted log-log demand curve for money or the semi-log version. It is based on observations that contain a great deal of information on behavior over this entire range of interest rates. I have argued that this estimate is not at all sensitive to assumptions about the fiscal policy used to effect the interest rate reduction, and that adding realistic productivity or money supply shocks to the model of Section 3 or to that of Section 5 will not alter the estimated welfare cost by much. I regard all of these conclusions as solidly, though of course not conclusively, established.

A 3 percent interest rate is about the rate that would arise in the U.S. economy under a policy of zero inflation. The optimal monetary policy, within the class of theories discussed in this paper, entails a *deflation* consistent with interest rates at or near *zero*. Based on the theory and evidence I have reviewed, the estimated welfare gain of a reduction in interest rates to near zero levels can vary considerably, depending on the specific model one uses. According to the estimates based on a log-log demand curve, as reported in Figure 5, the welfare gain from a monetary policy that reduces interest rates from 3 percent to zero, measured as a fraction of real GDP, is about 0.009, which is to say slightly larger than the gain from reducing rates from 14 to 3 percent! Using the semi-log estimates, however, the estimated gain from reducing interest rates from 3 percent to zero is less than 0.001. Insofar as the fixed costs postulated by Mulligan and Sala-i-Martin are important, even this figure may be an overstatement.

Successful applied science is done at many levels, sometimes close to its foundations, sometimes far away from them or without them altogether. As Simon (1969) observes, "This is lucky, else the safety of bridges and airplanes might depend on the correctness of the 'Eightfold Way' of looking at elementary particles." The analysis of sustained inflation illustrates this observation, I think: Though monetary theory notoriously lacks a generally accepted "microeconomic foundation," the quantity theory of money has attained considerable empirical success as a positive theory of inflation. Beyond this, I have argued in this survey that we also have a normative theory that is quantitatively reliable over a wide

range of interest rates. There are indications, however, that theory at the level of the models I have reviewed in this paper is not adequate to let us see how people would manage their cash holdings at very low interest rates. Perhaps for this purpose theories that take us farther on the search for foundations, such as the matching models introduced by Kiyotaki and Wright (1989), are needed.

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