# Price Theory I Fall 2018

*Problem Set 1, Question 1 Solutions*

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## (a) Production Allocation with Mobile Factors

If labor and capital are mobile across regions, then arbitrage implies that the real wage and real capital rental price must be the same in all regions. With a CRS production function, price equals unit costs. Thus we have:

$$P = C(W, R, Q)$$

Where $W$ is the (nominal) wage rate, $R$ is the (nominal) rental rate of capital, and $Q$ is the (nominal) rental price of land. By homogeneity, we can divide both sides of this equation by the price, yielding:

$$1 = \frac{1}{P} \cdot C(W, R, Q) = C\left(\frac{W}{P}, \frac{R}{P}, \frac{Q}{P}\right)$$

In each region. But remember that $W/P$ and $R/P$ are constants across regions, since capital and labor are freely mobile. But then $Q/P$ must also be constant across regions (since the LHS is a constant).

With equal factor prices, factor *ratios* must also be equal across regions (with land measured in efficiency units). Since land receives a constant price per efficiency unit, land prices per acre will be proportional to land quality. Factor quantities per acre will also be proportional to land quality.

## (b) Fixed Land Consumption

Now, people will get equal *utility* across regions, rather than equal *wages*. Since land prices vary across regions, wages will need to be higher in regions with higher land prices per acre. This is necessary in order to give people the same level of utility. Because utility is equal across regions but wages vary, people will work more in regions with higher wages. In other words, wage differences across regions are compensated in the Hicksian sense: regions in which people have to pay more for houses would yield lower utility *ceteris paribus*, so they must pay higher wages in order to compensate people for the higher price of housing. We still have:

$$1 = C\left(\frac{W}{P}, \frac{R}{P}, \frac{Q}{P}\right)$$

in each region. Since capital is mobile, $R/P$ will once again be constant across regions. But as noted, $W/P$ will be higher in regions with higher land quality (alternatively, higher land prices). This means that $Q/P$ will be *lower* in regions with quality land.

To understand what happens to factor ratios, we need to use our expressions for the output-constant factor demands. Denote the quantity of land in efficiency units by $E = QX$. Now note that we are holding unit cost constant (because higher land prices are fully compensated by higher wages), and so $S_L d\log W + S_E d\log q = 0$. Why is this equation true? Any change in total spending on wages must be equal to the change in total spending on land. This is a direct implication of the Hicksian compensation. This then implies that $S_L d\log W = -S_E d\log q$.

We can write:

$$\begin{aligned}
d\log L &= S_L[\sigma_{LL} - \sigma_{LE}]d\log W \\
d\log E &= S_L[\sigma_{LE} - \sigma_{EE}]d\log W \\
d\log K &= S_L[\sigma_{LK} - \sigma_{EK}]d\log W
\end{aligned}$$

What do these equations say? Note that we can write $S_L\sigma_{LL} = \frac{LW}{PY}\frac{C_{ww}C}{C_LC_L} = LW \times \frac{C_{ww}}{L^2} = \frac{W}{L}\frac{\partial L}{\partial W}$ (the first equality uses the definitions of $\sigma_{LL}$ and $S_L$, the second equality uses $C = PY$ and $C_L = L$, and the third equality uses $C_{WW} = \partial L/\partial W$). Thus, we have $S_L\sigma_{LL} = \epsilon_{LW}$ - the elasticity of labor with respect the wage. Similarly, we have $\sigma_{LE} = \epsilon_{EW}$. Thus, the first equation says that the change in labor is equal to the change in the wage multiplied by the difference between the elasticities of labor and land w.r.t. wages. In other words, if wages change, there is some substitution between land and labor, and some change in the scale of labor used. These 'substition' and 'scale' effects are similar to the substitution and income effects in demand. The logic is the same for the other two equations.

*Note: You will see detailed derivations of these equations and the logic behind them when Prof. Murphy covers CRS production.*

From these equations, we can see that the usage of each factor per unit output could go up or down, depending on the relative elasticities of substitution. The only thing we can say for sure is that $L/E$ must fall, since

$$d \log L - d \log E = S_L[\sigma_{LL} - 2\sigma_{LE} + \sigma_{EE}] d \log W < 0$$

With the inequality because $(\sigma_{LL} - 2\sigma_{LE} + \sigma_{EE}) < 0$ by concavity of the cost function. Hence, we could have $L$ rise or $E$ fall, but not both. What happens to $K$ depends on the relative complementarity of $K$ with $L$ and $E$.

## (c) Productivity

In part **(b)**, $q$ falls with the quality of land in equilibrium. Why? Because higher land quality implies workers must pay more to live in a location, which in turn means firms must pay workers more to live there. The intuition is that *land quality is equivalent to productivity in output, but not to productivity in a full production sense.* This is because land of higher quality is not more productive in *housing.*

Because of this, the relationship between land rents and land quality will be flatter in **(b)** than in **(a)**. Higher-quality land gets a lower return per unit quality, since wages must be higher in order to compensate workers who live in regions with higer-quality land.