

Summary of Market Insurance, Self-Insurance, and Self-Protection

Albert Shyn Kwan Tan

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1 Market Insurance

- An individual is faced with two states (0,1) with probability(w/p) p and $1-p$ respectively
- His real world income endowment in each state is given at certainty by I_0^e and I_1^e respectively where $I_0^e - I_1^e$ is prospective loss if state 0 occurs. If income in state 1 can be exchange for income in state 0 at the fixed rate

$$-\frac{dI_1}{dI_0} = \pi \quad (1)$$

- $\pi =$ 'Price of insurance' measured in terms of income in state 1
- The amount of insurance purchased in state 0 can be defined as difference between the actual and endowed incomes:

$$s = I_0 - I_0^e \quad (2)$$

- The expenditure on insurance measured in terms of state 1's income is

$$b = I_1^e - I_1 = s\pi \quad (3)$$

- Substituting (2) into (3) gives the opportunity boundary or the budget line AB in Figure 1

$$I_1^e - I_1 = \pi(I_0 - I_0^e) \quad (4)$$

- Individual choose optimal income in states 1 and 0 by maximizing the expected utility of the income prospect,

$$U^* = (1-p)U(I_1) + pU(I_0) \quad (5)$$

subject to the constraint given by the opportunity constraint (4)

- The first order optimality condition is

$$\frac{U_{I_0}^*}{U_{I_1}^*} = \frac{pU(I_0)'}{(1-p)U(I_1)'} = \pi \quad (6)$$

where $\frac{pU(I_0)'}{(1-p)U(I_1)'}$ is the slope of the indifference curve, and π is the slope of the budget line. In equilibrium, they must be equal (See point P)

- One can separate taste from environmental factors by dividing $p/(1-p)$ through in (6) to obtain

$$\bar{\pi} = \frac{1-p}{p}\pi = \frac{U(I_0)'}{U(I_1)'} \quad (7)$$

- $\bar{\pi}$ = Price of insurance deflated by the actuarially fair price $p/(1-p)$, is a measure of the 'real' price of insurance because a fair price is costless to individual. Equation (7) implies that the real price of insurance equals to the ratio of the marginal utility of I_0 to that of I_1 .
- Imposing marginal utility of income is strictly declining makes the indifference curve convex from the origin

$$D = -pU_0'' - \pi^2(1-p)U_1'' > 0 \quad (8)$$

- If the slope of indifference curve exceeded the price of insurance at endowment point E, some insurance would be demanded-some I_1 would be traded for I_0 :

$$\bar{\pi} < \frac{U(I_0)'}{U(I_1)'} \quad (9)$$

If the opposite was true, "gambling would occurred" (Figure 2) i.e. Individual would give up I_0 for I_1 .

- Note that inferences about attitudes toward risk cannot be made independently of existing market opportunities; a person may appear to be a 'risk avoider' under one combination of prices and potential loss and a 'risk taker' on another. (Compare Figure 1 and Figure 2)
- If the insurance were actuarially fair, equation 7 would reduce to $1 = U(I_0)'/U(I_1)'$; incomes would equalized in both states of the world.

2 Substitution Effects

Find the effect of an exogenous increase in price of insurance. Differentiate the FOC's and use Cramer's rule, we have:

$$\frac{dI_0}{d\pi} = \frac{1}{D} [-(1-p)U_1' + (I_0 - I_0^e)\pi(1-p)U_1''] \quad (10)$$

The denominator D is positive. From (10), an increase in the relative cost of insurance would necessarily decrease the demand for I_0 in state 0 if we assume $I_0 - I_0^e > 0$. Moreover, it also reduces the amount of insurance purchased, since I_0^e remains the same: $ds/d\pi = dI_0 - dI_0^e/d\pi = dI_0/d\pi$

Similarly, the effect of increase in π on I_1 and thus the amount spent on insurance is

$$\frac{dI_1}{d\pi} = \frac{1}{D}[(1-p)U_1'\pi + (I_0 - I_0^e)pU_1''] \quad (11)$$

Here, the result is ambiguous since U_1' is positive whereas $(I_0 - I_0^e)pU_1'' = spU_0''$ is negative if $U_0'' < 0$ and $s > 0$.

- The result is ambiguous because, although an increase in π reduces the amount of insurance purchased, each unit becomes more expensive. Consequently, the amount spent on insurance would decline only if the price elasticity of demand for insurance exceeded unity. If both I_1 and I_0 are superior goods (income elasticity greater than 1), the income and substitution effects reduce the demand for I_1 whereas they have opposite effects on demand for I_0 . Note that even if I_0 is an inferior good, however, an expenditure compensated increase in π must always reduce the demand for I_0 and increase I_1 .

3 Income Effects

Equation (4) can be written as

$$I_1^e + \pi I_0^e = W = I_1 + \pi I_0 \quad (12)$$

where W is a measure of the total opportunities available. Differentiate FOC to get

$$\begin{aligned} \frac{dI_0}{dW} = \frac{dI_0}{dI_0^e} = \frac{dI_0}{dI_1^e} \frac{dI_0^e}{dW} &= -\frac{D_{31}}{D} \\ \frac{dI_1}{dW} = \frac{dI_1}{dI_1^e} = \frac{dI_1}{dI_0^e} \frac{dI_0^e}{dW} &= -\frac{D_{32}}{D} \end{aligned} \quad (13)$$

where $D_{31} = \pi(1-p)U_0''$ and $D_{32} = pU_1''$. The income demanded in each state necessarily increases with opportunities if the marginal utility of income is falling. Hence, an increase in each state's endowment increases the demand for income in other state as well.

However, effects on the demand for insurance differ depending on how different endowments change. If I_1^e alone increased,

$$\frac{ds}{dI_1^e} = \frac{dI_0}{dI_1^e} > 0 \quad (14)$$

and demand for insurance would increase. Similarly, if I_0^e alone increased,

$$\frac{ds}{dI_0^e} = \frac{dI_0}{dI_0^e} - 1 < 0 \quad (15)$$

and the demand for insurance would decreased.

- Equations (14) and (15) imply that if the difference in endowed income—the endowed loss from hazard—increased either because I_0^e decreased or I_1^e increased, the demand of insurance would increase i.e. a person is likely to insure large rather than small loss.

If both endowments are change by the same percentage,

$$(\epsilon_s W - 1) = I_0(\eta_0 - 1) \quad (16)$$

where $\epsilon_{sw} = ds/dW * W/s$ and $\eta_0 = dI_0/dW * W/I_0$ are the opportunity elasticities for s and I_0 , respectively. From (16), we have two cases to consider :

- If the slopes of the indifference curves are constant along a given ray from origin—there is constant risk aversion: $\eta_0 = \eta_1 = 1$. An increase endowments would then increase the demand of insurance by the same proportion
- If there is increasing relative risk aversion, η_0 and thus ϵ_{sw} would exceed unity. Increasing relative risk aversion implies that elasticities of substitution between I_1 and I_0 tends to decline as opportunities increase. However, relative risk aversion remains constant about the certainty line.

4 Rare losses

Changes in p , the probability of loss do not affect the incentive to insure as long as the real price of insurance is independent of p . Note that if insurance is actuarially fair, the real price would always equal to unity, and hence independent of p . Now, introduce loading

$$\pi = (1 + \lambda)p/(1 - p) \quad (17)$$

where λ is the “loading factor”. If λ were independent of p , so would also be the real price of insurance and p would no effect on the incentive to insure.

- However, since rare losses are frequently insured, λ is presumably positively related to p , perhaps because processing and investigating cost increases as p increases. Even if incentive to insure is independent of p , the amount insured would decline and the expenditure on insurance would increase as p increase.

5 Self-Insurance and Self-Protection

Introduction: It is somewhat artificial to distinguish behaviour that reduces the probability of a loss from behaviour that reduces the size of a loss, since many actions do both. Nevertheless, Becker do that;.

5.1 Self-Insurance

Assume that market insurance is unavailable. Loss to a person is $L = L(L^e, c)$, where $L^e = I_1^e - I_0^e$ is the endowed loss, c is the expenditure on self-insurance, and $\frac{dL}{dc} = L'(c) \leq 0$. The expected utility is

$$U^* = (1-p)U(I_1^e - c) + pU(I_1^e - L(L^e, c) - c) \quad (18)$$

The value c that maximize equation (18), c^0 , satisfies the following FOC

$$\frac{-1}{L'(c^0) + 1} = \frac{pU'_0}{(1-p)U'_1} \quad (19)$$

This maximizes expected utility if the marginal utility of income and marginal productivity of self-insurance are decreasing-the indifference curves are convex and the production transformation curve between income in states 1 and 0 is concave to the origin. A necessary condition for a positive self-insurance is $-L'(c^0) > 0$ or that there be a net addition to income in state 0. A sufficient condition is that

$$\frac{-1}{L'(L^e, 0) + 1} < \frac{pU'_0}{(1-p)U'_1} \quad (20)$$

- An increase in unit cost of self-insurance, measured by the marginal productivity of self-insurance, would reduce the demand of self-insurance, measured by c^0 :

$$\frac{dc^0}{d\alpha} < 0 \quad (21)$$

where α is a parameter that reduces the absolute value of L' given c . Similarly, a reduction in I_0^e would increase the demand for self-insurance:

$$-\frac{dc^0}{dI_0^e} > 0 \quad (22)$$

- From equation (20), the incentive to self-insure is smaller for rare losses compare to market for insurance. The reason is that the loading factor of self-insurance is larger for rare losses because its price can be presumed to be independent of the probability of loss.

- Note that the effect on self-insurance from a change in income has to be seperated from the effect of the associated change in marginal productivity.

If market and self-insurance were both available , then

$$U^* = (1-p)U(I_1^e - c - s\pi) + pU(I_1^e - L(L^e, c) - c + s) \quad (23)$$

If the price of insurance were independent of the amount of self-insurance, FOC is

$$\begin{aligned} -(1-p)U_1' - U_0'[L'(c) + 1] &= 0 \\ U_1'\pi + pU_0' &= 0 \end{aligned} \quad (24)$$

By combining (24), we have

$$\pi = -\frac{1}{L'(c) + 1} \quad (25)$$

In equilibrium, “shadow price” of self-insurance would be equal to market price of insurance.

- Can infer that market insurance and self-insurance are substitutes in the sense that an increase in π , the probability of loss being the same, would decrease the demand for market insurance and increase the demand for self-insurance. Similarly, the purchase of market insurance would reduce the demand for self-insurance.

When market insurance is available at fair price, (25) become

$$-L'(c) = \frac{1}{p} \quad (26)$$

precisely the condition that maximizes expected income. The following are the important points from these equations:

- Even if the price of insurance were not fair, the optimal value of self-insurance would maximize the market value of income regardless of the shape of indifference curve or the probability distribution of states
- Effect of specific parameters on the demand for market and self-insurance when market or self insurance when both are available are different when they are alone available. For instance, an increase in endowed losses increases the demand for self- or market insurance when either is alone. However, when both are available, the indirects effect and offset the direct effect. Similarly, because a decrease in probability of loss reduces the demand for self insurance, it increases the demand of market insurance. Therefore, people more likely to use the market to insure rare losses because of a substitution between market and self-insurance.

5.2 Self-Protection, Subjective Probabilities and Moral Hazard

Self-insurance and market insurance both redistribute income toward hazardous states, whereas self-protection reduces the probabilities of these states. Unlike insurance, self-protection does not redistribute income, because the amount spent reducing the probability of a loss decreases income in all states, leaving unchanged the absolute size of the loss.

Although an appropriate definition of states would produce state probabilities that are independent of human actions, it would not produce a distribution of outcomes-relevant probability distribution- that is independent of these actions i.e. the human can't change the probability of a lightning storm but can change the probability of loss occurring due to a lightning storm.

Look at Figure 6.

- Let's say we are endowed with outcomes given by AB. Self-insurance(CD) lowers the probability of low and high outcomes, thereby unambiguously reducing the dispersion of outcomes. Self-protection, shift the distribution to the left, say EF, reduces the probability of low outcomes and raises the probability of high one and does not have an unambiguous effect on the dispersion.

5.2.1 Model for self-protection without market insurance

$$U^* = [1 - p(p^e, r)]U(I_1^e - r) + p(p^e, r)U(I_0^e - r) \quad (27)$$

$$-p'(r^0)(U_1 - U_0) = (1 - p)U'_1 + pU'_0 \text{ (Optimal Condition)} \quad (28)$$

Let us assume the probability of a hazardous state can be reduced by appropriate expenditure: $p = p(p^e, r)$, where p^e is the endowment probability of hazard and r is the expenditure on self-protection, and $dp/dr = p'(r) \leq 0$. In equation (28), LHS is marginal gain from reduction in p ; RHS is decline in utility due to decline in both income, is the marginal cost. The second order condition is

$$U_{*rr} = -p''(r^0)(U_1 - U_0) + 2p'(r^0)(U'_1 - U'_0) + (1 - p)U''_1 + pU''_0 < 0 \quad (29)$$

Decreasing marginal utility of income is not a sufficient nor necessary condition. If $p''(r^0) > 0$, equation (29) is always satisfied if the MU is constant but not if MU is decreasing or increasing. Models implications

- Incentive to self-protect is not so dependent on attitudes toward risk, and could be as strong for risk preferences as for risk avoiders.
- A decline in I_0^e alone might not increase the demand for self-protection, even if the marginal utility of income were falling because a decline in I_0^e would increase the marginal cost of protection

- A decline in marginal productivity of protection-an increase in the shadow price of protection- always decreases the demand for self-protection regardless of attitudes towards risks. Therefore, if the endowed probabilities and income are the same, more efficient providers of self-protection would have lower equilibrium probabilities of hazard.

5.2.2 Model for self-protection with market insurance

The function

$$U^* = [1 - p(p^e, r)]U(I_1^e - r) + p(p^e, r)U(I_0^e - r) \quad (30)$$

would be maximized w.r.t. r and s . FOC are

$$-(1 - p)'_1 \pi + pU'_0 = 0 \quad (31)$$

$$-p'(r^*) * (U_1 - U_0) - (1 - p)U'_1[1 + s * \pi'(r^*)] - pU'_0 = 0 \quad (32)$$

The term $\pi'(r)$ measures the effect of change in self-protection on the price of market insurance through its effect on p and loading fraction λ

$$\frac{d\pi}{dr} = \pi'(p)p'(r) + \pi'(\lambda)\lambda'(r) \quad (33)$$

The effect of market insurance on the demand for self-protection is called “Moral Hazard”. In particular, moral hazard refers to an alleged deterrent effect of market insurance on self-protection that increases actual probabilities of hazardous events. Market insurance has two opposite effects on self-protection:

- Self protection is discourage because marginal gain is reduced by the reduction of the difference between the incomes, and thus the utilities in different states(Effect 2)
- It is encourage if the price of market insurance is negatively related to the amount spent on protection through the effect of these expenditures on probabilities(Effect 1)

Effect 1

Consider equation (31). $\pi = p/(1 - p)$ implies that

$$\pi'(r) = p'(r)/(1 - p)^2 \quad (34)$$

Substiting $U_1 = U_0$ and $U'_1 = U'_0$ into equation (32) and using (33a), we have

$$p'(r^*) = -(1 - p)/s = \frac{-1}{I_1^e - I_0^e} \quad (35)$$

By (33a), there is incentive to spend on protection since π is negatively related to r . By (34), fair price market insurance encourages an expenditure on self-protection that maximizes expected income. Hence, no moral hazard. The optimal self-protection, r^* , can be larger than the amount spent in the absence of market insurance, r^0 . By (34) and (38), and the condition $p'(r) < 0$, r^* would be larger than r^0 if

$$\frac{U(I_1^e - r^0) - U(I_0^e - r^0)}{I_1^e - I_0^e} = \bar{U}' < (1 - p)U'(I_1^e - r^0) + pU'(I_0^e - r^0) \quad (36)$$

which is likely provided p is not very small and U is concave. Implies that market insurance and self-protection are complements in the sense that:

- Availability of the former could increase demand for the latter
- Increase in productivity of self-protection or a decrease in the real cost of market insurance would increase demand for both

Effect 2

- The price of market insurance was independent of expenditures on self-protection - the loading factor increased sufficiently to offset exactly the reduction in the probability of loss. Self-protection would then usually be discouraged by market insurance-moral hazard would exist-because the main effect of introducing market insurance would be to narrow the differences between income of different states. Moreover, since the demand for market insurance is negatively related to the loadings, it would be negatively related to expenditures on self-protection. Implications: For those market insurance whose prices that are largely independent of expenditures on self-protection, one should either observed a large demand for insurance and a small demand for self-protection, or the converse.

6 Misc

- Since the price of self-insurance is independent of probability of hazard, and thus expenditures on self-protection, our analysis of market insurance implies that self-insurance is likely to create moral hazard. That is, availability of self-insurance would discourage self-protection and vice versa.
- Analysis of moral hazard can be used not only on the relation between self-protection and insurance, but also to the relation between protection and insurance for all uncertain events that can be influenced by human actions.

7 T/F/U Questions

1. Unemployment compensation relief encouraged someone becomes unemployed. **ANS:** Uncertain. If cost in time and embarrassment of applying for compensation relief is sufficiently positively correlated with its frequency, the availability of insurance might encourage insured to make find a job. On the other hand, unemployment benefit would encourage someone to search longer for jobs, thus increasing the number of unemployed.
2. Unemployment compensation should be extended. **ANS:** Uncertain. Extension on unemployment compensation poses a risk on creating the culture of unemployment. Once the period of unemployment becomes long enough, people begin to get the habits from being unemployed for a long time: they sleep late, develop various leisure interests, and at the same time their work skills depreciate from not using them for an extended period. On the other hand, extending unemployment compensation while setting a “deductible”(a person need to be unemployed for at least some time to qualify for unemployment benefit) may increase welfare since those experiencing long-term unemployment experience most economic hardships. This is only true if people who are long-term unemployed are unemployed not because of their own fault, but due to general economic recession.
3. Use of seat belts increases would increase with the introduction of market insurance. **ANS:** Uncertain. It depends on the degree to which seat belts is of self-insurance or self-protection.
 - Seat belts are ways to self-insure in the sense that it reduces the severity of an accident. Since market insurance is a substitute for self-insurance, use of seat belts would decrease whenever market insurance is purchased.
 - Seat belts is a self-protection in the sense that it reduces the probability of an accident being fatal. Reducing the probability of a fatal accident would reduce the probability of a fatal accident, and hence the price of insurance on fatal accidents if the insurance company can access the seat belts usage of drivers. Hence there is an incentive of drivers to use seat belts more often. However, it is unlikely that insurance is able to do so.