Price Theory, III | **Professor Lars Stole**
Economics 33003 | **Spring 2019**

Assignment 1
(Due Friday, April 12, prior to the start of the Review session)

Problem 1 (MWG, Exercise 13.B.3 - variation) Consider a *positive-selection* variation of the model discussed in MWG (13.8) in which $r(\cdot)$ is a continuous, *strictly decreasing* function of θ. Let the density of workers of type θ be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$

(a). Show that the *more capable* workers are the ones choosing to work at any wage for which some workers are employed and others are not.

(b). Show that if $r(\theta) > \theta$ for all θ, then the resulting competitive equilibrium is Pareto efficient.

(c). Suppose that there exists a $\hat{\theta}$ such that $r(\theta) < \theta$ for $\theta > \hat{\theta}$ and $r(\theta) > \theta$ for $\theta < \hat{\theta}$. Show that any competitive equilibrium with strictly positive employment necessarily involves too much employment relative to the Pareto-optimal allocation of workers.

Problem 2 (JR, Exercise 8.7 - variation) Consider the following market for used cars. There are many sellers of used cars. Each seller has exactly one used car to sell and is characterized by the quality of the used car he wishes to sell. Let $\theta \in [0, 1]$ index the quality of a used car and assume that θ is uniformly distributed on $[0, 1]$. If a seller of type θ sells his car (of quality θ) for a price of p, his utility is p. If he does not sell his car, then his utility is $r(\theta)$, which is increasing in θ and $r(0) = 0$. Buyers of used cars receive utility $\theta - p$ if they buy a car of quality θ at price p and receive utility 0 if they do not purchase a car. There is asymmetric information regarding the quality of used cars. Sellers know the quality of the car they are selling, but buyers do not know its quality. Assume that there are not enough cars to supply all potential buyers.

(a). Argue that in a competitive equilibrium under asymmetric information, we must have $E[\theta|p \geq r(\theta)] = p$.

(b). Show that if $r(\theta) = \frac{\theta}{2}$, then every $p \in (0, 1/2]$ is an equilibrium price.

(c). Find the equilibrium price when $r(\theta) = \sqrt{\theta}$. Describe the equilibrium in words. In particular, which cars are traded in equilibrium?

(d). Find an equilibrium price when $r(\theta) = \theta^3$. How many equilibria are there in this case?

(e). Are any of the preceding outcomes Pareto efficient? Describe the inefficiencies (if any) in each setting, relative the the full-information efficient allocations.

Problem 3 Consider a certifiable disclosure game with n players, in which each player i's payoff is simply the market expectation of their type, conditional on the information $m \subseteq \Theta$ revealed:

$$u_i(\theta_i) = E[\theta|\theta \in m].$$

There are two types, $\theta_h > \theta_l > 0$, where $\phi \in (0, 1)$ is the probability of the high type. As a complication, however, suppose that with probability λ, a player cannot use the free certification mechanism and therefore no message is sent (effectively, $m = \Theta$). What is the unique equilibrium? If it involves less than full revelation by those players who can reveal their types, explain why unraveling does not arise. What key condition is violated?

Problem 4 Consider the MWG labor-market signaling game in 13.C. Assume that $c(e, \theta) = e(K - \theta)$ where K is a constant such that $K > \theta_h > \theta_l > 0$. Assume that the probability of the high type is ϕ and the probability of the low type is $(1 - \phi)$.

(a). Characterize a hybrid (mixed-strategy) equilibrium in which the low-type worker randomizes between two levels of education, e_l and e_h, with probabilities λ and $(1 - \lambda)$ respectively, and the high-type plays a pure-strategy always choosing e_h. In particular, compute w_l^*, e_l^*, and give the equation that ties together the low-type's mixing probability, λ^*, with the education level chosen by the high type, e_h^*.

(b). What is special about the kind of separating equilibrium one gets by setting $\lambda = 1$ in the characterization in (a)? What is special about the kind of pooling equilibrium one gets by setting $\lambda = 0$ in the characterization in (a)?

(c). [Optional - not required for assignment.] Does the hybrid equilibrium in (a) pass the Intuitive Criterion?

Problem 5 (MWG, Exercise 13.C.5) Assume a single firm and a single consumer. The firm's product may be either high or low quality and is of high quality with probability $\lambda \in (0, 1)$. The consumer cannot observe quality before purchase and is risk neutral. The consumer's valuation of a high-quality product is v_H; her valuation of a low-quality product is v_L. The costs of production for high (H) and low (L) quality are c_H andc_L, respectively. The consumer desires at most one unit of the product. Finally, the firm's price is regulated and is set at p. Assume that $v_H > p > v_L > c_H > c_L$.

(a). Given the level of p, under what conditions will the consumer buy the product?

(b). Suppose that before the consumer decides whether to buy, the firm (which knows its type) can advertise. Advertising conveys no information directly, but consumers can observe the total amount of money that the firm is spending on advertising, denoted by A. Can there be a separating perfect Bayesian equilibrium, that is, an equilibrium in which the consumer rationally expects firms with different quality levels to pick different levels of advertising?

Problem 6 (MWG, Exercise 13.C.6 - variation) Consider a market for loans to finance investment projects. All investment projects require an outlay of 1 dollar. There are two types of projects: good and bad. A good project has a probability of ϕ_g of yielding profits of $\Pi > 0$ and a probability $(1 - \phi)$ of yielding profits of zero. For a bad project, the relative probabilities are ϕ_b and $(1 - \phi_b)$, respectively, where $\phi_g > \phi_b$. The fraction of projects that are good is $\lambda \in (0, 1)$.

Risk-neutral entrepreneurs go to banks to borrow the cash to make the initial outlay (assume for now that they borrow the entire amount). A loan contract specifies an amount R that is supposed to be repaid to the bank. Entrepreneurs know the type of project they have, but the banks do not. In the event that a project yields profits of zero, the entrepreneur defaults on her loan contract, and the bank receives nothing. Banks are competitive and risk neutral. The risk-free rate of interest (the rate the banks pay to borrow funds) is r. Assume that

$$\phi_g \Pi - (1 + r) > 0 > \phi_b \Pi - (1 + r).$$

To be clear, the timing and strategies of the players are

1. Entrepreneurs privately learn the quality of their investment project
2. Banks either refuse to loan money at any feasible rate, or they offer $R \in [0, \Pi]$ which is repaid if and only if the project succeeds
3. Entrepreneurs accept the best bank offer, or they reject the offer and the project is not funded.

(a). Find the equilibrium level of R and the set of projects financed. How does this depend on ϕ_g, ϕ_b, λ, Π, and r?

(b) Now suppose that the entrepreneur can offer to contribute some fraction x of the 1 dollar initial outlay from her own funds ($x \in [0, 1]$). The entrepreneur is liquidity constrained, however, so that the effective cost of doing so is $(1 + \rho)x$, where $\rho > r$. Also assume that ρ is sufficiently large that

$$1 + \rho > \phi_g \Pi > 1 + r.$$

This implies that even a good project is not profitable if all of the funds must be borrowed at rate ρ rather than r; i.e., some lower-cost bank financing is necessary.

(i). What is an entrepreneur's payoff as a function of her project type, her loan-repayment amount R, and her contribution x?

(ii). Describe the best (from a welfare perspective) separating perfect Bayesian equilibrium of a game in which the entrepreneur first makes an offer that specifies the level of x she is willing to put into a project, banks then respond by making offers specifying the level of R they would require, and finally the entrepreneur accepts a bank's offer or decides not to go ahead with the project. How does the amount contributed by entrepreneurs with good projects change with small changes in ϕ_b, ϕ_g, λ, Π, and r?

(iii). How do the two types of entrepreneurs do in the separating equilibrium of (b)(ii) compared with the equilibrium in (a)?

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Answers to Assignment 1

1 (a). If some of the workers are employed at w and others are not, then there exists a $\hat{\theta} \in (\underline{\theta}, \overline{\theta})$ such that $r(\hat{\theta}) = w$. Because r is strictly decreasing, all workers $\theta \in [\geq \hat{\theta}, \overline{\theta}]$ will choose employment and all workers $\theta \in [\underline{\theta}, \hat{\theta})$ will not accept employment at w.

(b). If $r(\theta) > \theta$, then the full-information, Pareto efficient outcome is for no worker to work. We will show that this is also a competitive equilibrium.

Notice that $\min_\theta r(\theta) = r(\overline{\theta}) > \overline{\theta}$, thus no worker will be demanded at any wage w that is acceptable to a worker and to a firm. Indeed, any wage $w^* \in [\overline{\theta}, r(\bar{})$) is a competitive equilibrium which induces zero labor supply and zero labor demand.

(c). Define

$$g(w) \equiv E[\theta | r(\theta) \leq w]$$

which is a decreasing function (because $r(\cdot)$ is a decreasing function).

Consider a wage at $w = r(\hat{\theta})$. By assumption,

$$E[\theta | r(\theta) \leq r(\hat{\theta})] = E[\theta | \theta \geq \hat{\theta})] > \hat{\theta} = r(\hat{\theta}).$$

Thus, $g(w) - w > 0$ at $w = r(\hat{\theta})$ and there is excess labor demand. Indeed, $g(w) - w > 0$ holds for any wage lower than $r(\hat{\theta})$ as well. To see this, choose $\tilde{w} = r(\tilde{\theta})$ where $\tilde{w} < r(\hat{\theta})$, $\tilde{\theta} > \hat{\theta}$ and therefore $r(\tilde{\theta}) < \tilde{\theta}$. At $\tilde{w}$, we have

$$E[\theta | r(\theta) \leq \tilde{w}] = E[\theta | \theta \geq \tilde{\theta}] > \tilde{\theta} > r(\tilde{\theta}) = \tilde{w}.$$

Hence, all equilibria must arise at wages above $r(\hat{\theta})$ and therefore the marginal worker type will be lower than $\hat{\theta}$ and the set of employed workers will be larger than $[\hat{\theta}, \overline{\theta}]$.

Next consider a wage at $w = r(\underline{\theta})$ that induces full employment. Here we have $g(w) - w = E[\theta] - r(\underline{\theta}) < 0$ (by assumption). Given that g is continuous, the equilibrium wage must lie between these extremes: $w^* \in (r(\hat{\theta}), r(\underline{\theta}))$. Hence $w^* > r(\hat{\theta})$ and therefore the set of employed workers will be larger than $[\hat{\theta}, \overline{\theta}]$.

Aside: Note that we really didn't need the assumption in the question about $f > 0$ over $[\underline{\theta}, \overline{\theta}]$ because the question did not require that we show a competitive equilibrium exists. The continuity of F allows us to show, however, that

$$g(w) \equiv E[\theta | r(\theta) \leq w] = \int_{r^{-1}(w)}^{\overline{\theta}} \frac{\theta f(\theta)}{1 - F(r^{-1}(w))} d\theta,$$

where $\theta = r^{-1}(w)$ is the inverse of $r(\theta) = w$ ($R^{-1}(\cdot)$ is well defined because r is strictly decreasing), is a *continuous* function. Moreover, because $g(w)$ is decreasing, g can have at most one fixed point. Once we establish that $g(w) - w > 0$ at $w = r(\hat{\theta})$, we need to only check $g(w) - w$ at $w = r(\underline{\theta})$, the wage that induces full employment. If $E[\theta] - r(\underline{\theta}) < 0$, then there is a unique competitive wage between $r(\hat{\theta})$ and $r(\underline{\theta})$. If $E[\theta] - r(\underline{\theta}) \geq 0$, then the equilibrium wage increases to $w^*[\theta]$ and there is full employment. In either case there is a unique competitive equilibrium with over employment.

MWG gives us the assumption that $f > 0$ so that we can conclude g is continuous and a competitive equilibrium exists. (Note that Tarski's fixed-point theorem only works for increasing functions.

2 (a). If $p > E[\theta | p \geq r(\theta)]$, then no buyers would purchase car but some sellers would offer the car for sale (because $p > 0$ and $r(0) = 0$), so the market would not be in equilibrium. Therefore $p \leq E[\theta | p \geq r(\theta)]$. If $p < E[\theta | p \geq r(\theta)]$, however, all buyers would want to buy a car, but because there are not enough cars to supply all buyers by assumption, this cannot be an equilibrium. Hence

$$p = E[\theta | p \geq r(\theta)].$$

(b). For a given p, the set of sellers willing to sell are $\{\theta | \theta \leq \min\{2p, 1\}\}$. If $p > \frac{1}{2}$, then all sellers wish to sell, but $E[\theta | \theta \leq 1] = \frac{1}{2}$, but then no buyers would purchase. Instead, consider a price $p \in \left(0, \frac{1}{2}\right]$. Now we have

$$E[\theta | \theta \leq 2p] = \int_0^{2p} \frac{\theta}{2p} d\theta = \frac{\theta^2}{4p} |_0^{2p} = p.$$

This is true for all $p \in (0, \frac{1}{2}]$. Note: if $p = 0$, supply has measure zero, so no measure of trade takes place and even the measure-zero traders are indifferent to the transaction. It is still an equilibrium, but not an interesting one.

(c). For any price p, the set of sellers willing to trade is $\{\theta | \theta \leq p^2\}$. If $p \leq 1$, we have

$$E[\theta | \theta \leq p^2] = \int_0^{p^2} \frac{\theta}{p^2} d\theta = \frac{\theta^2}{2p^2} |_0^{p^2} = \frac{p^4}{2p^2} = 2p^2.$$

Solving $p = 2p^2$, we obtain $p = 2$ (which is not in $[0, 1]$ and thus not an equilibrium) and $p = 0$. Thus, $p^* = 0$ and no meaningful trade takes place because the market has unravelled.

(d). For any price p, the set of sellers willing to trade is $\{\theta | \theta \leq \sqrt[3]{p}\}$. If $p \leq 1$, we have

$$E[\theta | \theta \leq \sqrt[3]{p}] = \int_0^{\sqrt[3]{p}} \frac{\theta}{\sqrt[3]{p}} d\theta = \frac{\theta^2}{2\sqrt[3]{p}} |_0^{\sqrt[3]{p}} = \frac{1}{2}\sqrt[3]{p}.$$

Solving $p = \frac{1}{2}p^{\frac{1}{3}}$, we get two solutions: $p^* = 0$ (no meaningful trade) and $p^* = \frac{1}{2\sqrt{2}} \approx 0.354$.

(e).

- $r(\theta) = \frac{\theta}{2}$. In this case, $\theta > r(\theta)$ for all $\theta \in (0, 1]$, so complete trade is efficient. The range of equilibrium prices runs from $p^* = 0$ (no trade) to $p^* = \frac{1}{2}$ (complete trade). The equilibrium with $p^* = \frac{1}{2}$ is Pareto efficient.

- $r(\theta) = \sqrt{\theta}$. In this case, $\theta < r(\theta)$ for all $\theta \in (0, 1)$, so complete trade is efficient. The unique equilibrium price of $p^* = 0$ is therefore Pareto optimal.

- $r(\theta) = \theta^3$. In this case, $\theta > r(\theta)$ for all $\theta \in (0, 1)$, so complete trade is efficient. At a price of p^*, no trade takes place; at a price of $p^* = \frac{1}{2\sqrt{2}}$, some trade takes place which is Pareto superior to the $p^* = 0$ equilibrium, but gains from trade still remain.

3 There are effectively 4 types,

$$\theta_{he}, \theta_{hn}, \theta_{le}, \theta_{ln},$$

which correspond to high types with evidence, high types with no evidence, etc. The respective probabilities are

$$\{\phi(1-\lambda), \phi\lambda, (1-\phi)(1-\lambda), (1-\phi)\lambda\}.$$

In any equilibrium, players without evidence cannot reveal anything. We need only determine the equilibrium strategies of the two types with evidence. If the high type with evidence reveals it, the market's assessment will be $\mu(\theta_h|\text{evidence}) = 1$, which is strictly higher than not revealing the information and being pooled with a set of types that include the low type, θ_{ln}, and possibly θle (if the low-type with evidence chooses not to reveal it.). If the low type with evidence reveals it, the market assessment will be $\mu(\theta_h|\text{evidence}) = 0$, which is strictly lower than not revealing the information and being pooled with a set that include θ_{hn}. In any equilibrium, the low type with evidence chooses not to disclose. Upon seeing no evidence, the market infers (using Bayes rule),

$$E[\theta|\text{no evidence}] = \frac{\phi\lambda}{(1-\phi)+\phi\lambda}\theta_h + \frac{(1-\phi)}{(1-\phi)+\phi\lambda}\theta_l \quad \in \quad (\theta_l, \theta_h).$$

Relating this back to the general unraveling result in Okuno-Fujiwara, Postlewaite and Suzumura (OFPS) , their key condition requires that each player has the ability to disclose evidence that their type is at least as good as their true type. In this problem, players without access to evidence do not have the ability to disclose anything about their type.

4 (a). We want to find the conditions required for a hybrid equilibrium. Suppose that the low type chooses e_l with probability λ and e_h with probability $(1-\lambda)$. First, note that $e_l^* = 0$ for exactly the same reasons as it is true in a fully separating equilibrium. With that in mind, the following conditions are necessary and sufficient for a BNE:

$$w_l = \theta_l,$$

$$w_h = \frac{\phi}{\phi+(1-\phi)(1-\lambda)}\theta_h + \frac{(1-\phi)(1-\lambda)}{\phi+(1-\phi)(1-\lambda)}\theta_l,$$

$$\theta_l = w_h - c(e_h, \theta_l),$$

$$w_h - c(e_h, \theta_h) \geq \theta_l.$$

The first two equations are the market wages are equal to expected productivity in equilibrium; the third equation is the indifference constraint required for the θ_l type to be willing to mix between 0 and e_h; the fourth constraint is the requirement that the high-type worker doesn't want to pretend to be the low type worker. The single-crossing property, however, implies that this constraint is slack (given the third constraint). Substituting for w_h in the third equation, using the second equation, we have a single equation for determining λ and e_h:

$$\theta_l + c(e_h, \theta_l) = \frac{\phi}{\phi+(1-\phi)(1-\lambda)}\theta_h + \frac{(1-\phi)(1-\lambda)}{\phi+(1-\phi)(1-\lambda)}\theta_l.$$

Using our assumption that $c(e, \theta) = e(y - \theta)$, we have

$$\theta_l + e_h(y - \theta_l) = \frac{\phi}{\phi+(1-\phi)(1-\lambda)}\theta_h + \frac{(1-\phi)(1-\lambda)}{\phi+(1-\phi)(1-\lambda)}\theta_l.$$

Thus, for any given e_h, the value of λ is

$$\lambda = \frac{e_h(y-\theta_l) - \phi(\theta_h - \theta_l)}{e_h(y-\theta_l)(1-\phi)}.$$

There is one degree of freedom and hence a continuum of hybrid equilibria.

(b). When $\lambda = 1$, we recover the least-cost full-separating equilibrium:

$$e_h = \tilde{e} = \frac{(\theta_h - \theta_l)}{(y-\theta_l)}.$$

For $\lambda < 1$, however, the corresponding e_h is lower than the least-cost separating equilibrium, but note also that we are no longer fully separating.

For $\lambda = 0$, we recover the highest-cost pooling equilibrium:

$$e_h = \frac{\phi(\theta_h - \theta_l)}{(y-\theta_l)} = \frac{E[\theta] - \theta_l}{(y-\theta_l)}.$$

(c). No. We will show for any $\lambda < 1$, the intuitive criterion fails for this equilibrium. To do that, let's first compute the equilibrium payoffs

$$u_l^* = \theta_l,$$

$$u_h^* = w_h - c(e_h, \theta_h) = (\theta_l + c(e_h, \theta_l)) - c(e_h, \theta_h).$$

The first equation is from the indifference condition of the low-type agent. The second equation was further simplified by using the same indifference condition.

Define $\tilde{e}$ to be the least-cost separating education level in the full-separation equilibrium:

$$\theta_l = \theta_h - c(\tilde{e}, \theta_l).$$

Consider a deviation of $e = \tilde{e} + \varepsilon$, where $\varepsilon > 0$ is small. Such a deviation would generate less than θ_l in payoff to the low-type agent, even at the best beliefs. Thus, $\theta_l \in \hat{\Theta}(\tilde{e} + \varepsilon)$. The high type agent who makes the same deviation and is thought to be the high type, however, would do better than his equilibrium payoff if the following is the case:

$$u_h^* = \theta_l + c(e_h, \theta_l) - c(e_h, \theta_h) < \theta_h - c(\tilde{e} + \varepsilon, \theta_h).$$

Rewriting this we have

$$c(e_h, \theta_l) - c(e_h, \theta_h) < (\theta_h - \theta_l) - c(\tilde{e} + \varepsilon, \theta_h).$$

By definition of the least-cost separating $\tilde{e}$, we can substitute for $(\theta_h - \theta_l)$ and obtain

$$c(e_h, \theta_l) - c(e_h, \theta_h) < c(\tilde{e}, \theta_l) - c(\tilde{e} + \varepsilon, \theta_h).$$

Note that because $\tilde{e} > e_h$, the single-crossing property implies

$$c(e_h, \theta_l) - c(e_h, \theta_h) < c(\tilde{e}, \theta_l) - c(\tilde{e}, \theta_h).$$

Hence, as ε becomes small, the inequality is satisfied. Thus, we conclude $\hat{\Theta}(\tilde{e}+\varepsilon) = \{\theta_l\}$ for ε sufficiently small.

The second part of the intuitive criterion requires that we evaluate θ_h payoff when the worst possible belief is assigned to him from the set $\Theta \setminus \hat{\Theta}(\tilde{e}+\varepsilon)$ which requires $\mu(\theta_h|\tilde{e}+\varepsilon) = 1$. In this case, as we showed above, the high type would want to deviate for sufficiently small ε. We conclude that for any $e_h^* < \tilde{e}$ (i.e., for any $\lambda < 1$), the equilibrium is not "intuitive".

5 (a). The consumer will buy the product if the expected value of the product is higher than the price, i.e., if

$$\lambda v_h + (1-\lambda)v_l \geq p.$$

(b). The answer is no, there cannot be a separating equilibrium. Suppose to the contrary that there exists a separating equilibrium in which the high-quality producers spends A on advertising and only the high quality product will be bought (in a separating equilibrium consumers know the quality of a product, so low quality products will not be bought since $p > v_l$). This implies that the low-quality producer makes no profit and the high quality producer makes a non-negative profit, $p - c_H - A \geq 0$. However, a low quality producer can make a positive profit by spending A on advertising, since the consumer will then mistake him for a high-quality producer and buy the good from him. The low quality producer's profit will equal $\pi_l = p - c_l - A > p - c_h - A \geq 0$. Therefore, no separating equilibrium can exist.

6 (a). Suppose that both types of projects are financed at R. In order for the risk-neutral banks to earn nonnegative a competitive return (of zero), it must be that

$$\lambda\phi_g R + (1-\lambda)\phi_b R = 1 + r,$$

or

$$R = \frac{(1+r)}{(\lambda\phi_g + (1-\lambda)\phi_b)}.$$

At this rate, and entrepreneur of type $i = g, b$ will obtain funds and pursue the project if

$$\phi_i(\Pi - R) + (1-\phi_i)0 \geq 0,$$

or equivalently

$$\Pi \geq R.$$

Given our value of R, the entrepreneur will obtain financing of their projects if

$$\Pi \geq \frac{(1+r)}{(\lambda\phi_g + (1-\lambda)\phi_b)}.$$

Because $\Pi > \frac{(1+r)}{\phi_g}$, if λ is sufficiently high, both types of entrepreneurs will secure financing and pursue their projects. The required level of funding λ is

$$\hat{\lambda} = \frac{1 + r - \phi_b \Pi}{\Pi(\phi_g - \phi_b)}.$$

Note that it is not possible that an equilibrium can arise with only the good projects being funded because the banks' requirement for funding is the same for both: invest if $\Pi \geq R$. Hence, either both projects are financed, or neither project is financed. We conclude that for $\lambda < \hat{\lambda}$, the equilibrium outcome must exhibit no financing (e.g., banks set rates at $R > \Pi$ and Entrepreneurs do not secure funding).

(b)-(i). The entrepreneur's expected payoff from a project of type $i = g, b$ is

$$\phi_i(\Pi - (1-x)R) + (1-\phi_i)(0) - (1+\rho)x = \phi_g(\Pi - R) - x((1+\rho) - \phi_g R).$$

(b)-(ii). The amount of skin in the game that the entrepreneur chooses, x, will serve as a signal of her project quality. Because $\rho > r$, it is a costly signal (the banks have a lower cost of funds than the entrepreneur). Thus, we are looking for the least-cost separating equilibrium (the one with the lowest x_g for this solution).

Suppose that the equilibrium is for the good entrepreneur to choose x_g and the bad one to choose x_b, where $x_g \neq x_b$. In equilibrium, the banks will know the project type and will offer an entrepreneur with a good project a rate of $R_g = (1+r)/\phi_g < \Pi$ and will refuse a loan to the entrepreneur with a bad project because $R_b = (1+r)/\phi_b > \Pi$ and bad entrepreneur cannot fully repay the loan in the positive state.

We want the minimum level of x that achieves separation (i.e., does not induce the bad entrepreneur from mimicking the good one). The bad entrepreneur will not pursue funding at R_g if it requires a personal stake, x, such that

$$\phi_b(\Pi - (1-x)R_g) - x(1+\rho), .$$

Substituting for R_g, rearranging and setting this equal to zero,

$$\phi_b\left(\Pi - \frac{(1+r)(1-x)}{\phi_g}\right) - x(1-\rho) = 0.$$

We now solve for the value $x = x_g$ which makes this true:

$$x_g = \frac{\phi_b(\phi_g\Pi - (1+r))}{\phi_g(1+\rho) - \phi_b(1+r)}.$$

Obviously, $x_g > 0$. Rewriting x_g and using the assumption that a good project is not profitable at interest rate ρ, we have

$$x_g = \frac{\Pi - \frac{1+r}{\phi_g}}{\frac{1+\rho}{\phi_b} - \frac{1+r}{\phi_g}} < \frac{\frac{1+\rho}{\phi_g} - \frac{1+r}{\phi_g}}{\frac{1+\rho}{\phi_b} - \frac{1+r}{\phi_g}} < 1.$$

Because the bad entrepreneur is indifferent between x_g and abandoning his project, the good entrepreneur strictly prefers to obtain financing at x_g (by the single-crossing proeprty).

Given our formula for $x_g < 1$, it follows that x_g increases in Π, ϕ_g, and ϕ_b, but decreases in r. x_g is independent of λ.

(b)-(iii). The entrepreneurs with the bad project will not be better off, and may be worse off in the separating equilibrium of part (ii). For large enough λ, all projects will be financed in the equilibrium of part (i) and the entrepreneurs with the bad project will make a strictly positive expected profit. In the separating equilibrium of part (ii), however, entrepreneurs with bad projects will always obtain a payoff of zero. For small λ, entrepreneurs with bad projects will obtain a payoff of zero in both equilibria.

For small λ, entrepreneurs with good projects will be better off in the separating equilibrium, since they obtain a positive payoff in the separating equilibrium and a zero payoff in the equilibrium of part (i) (since the projects will not be financed).

As λ becomes larger, projects will also be funded in the equilibrium of part (i). Now, the entrepreneur will have to pay a higher interest rate R on the bank loan in the equilibrium of part (i). In the separating equilibrium, however, the entrepreneur has to contribute his own funds, which is costly to him since he is liquidity constraint. Thus as λ becomes large enough, the entrepreneur will be better off in the equilibrium of part (i).