Price Theory, III **Professor Lars Stole**
Economics 33003 **Spring 2019**

Assignment 5
(Due Friday, May 17, prior to the start of the Review session)

Problem 1 Consider the managerial contracting problem in MWG 14.C, but with continuously distributed efforts. Specifically, suppose that a firm hires a manager of type θ, distributed with CDF $F(\theta)$ on $[\underline{\theta}, \overline{\theta}]$, where $[1 - F(\theta)]/f(\theta)$ is nondecreasing. The manager chooses effort at personal cost $g(e, \theta)$ which is increasing and strictly convex in $e \in [0, \infty)$ and decreasing in θ. Effort increases profit in a deterministic way, $\pi(e)$, where $\pi(\cdot)$ is strictly increasing and concave. Assume g and π three times continuously differentiable, $g_{e\theta}(e, \theta) < 0$ for all $e > 0$, $\pi(0) = g(0, \theta) = 0$ for all θ, $\pi'(0) - g_e(0, \theta) = \infty$ and $\lim_{e\to\infty} \pi(e) - g(e, \theta) = -\infty$ for all θ.

The type-θ manager's payoff from a wage w and a choice of effort, e, is

$$w - g(e, \theta).$$

We assume that the manager is risk neutral for positive payoffs (i.e., for all $w - g(e, \theta) \geq 0$), but is infinitely risk averse of negative payoffs: $u = -\infty$ if $w - g(e, \theta) < 0$. As such, any contract will need to promise the manager a nonnegative return in exchange for participation. The firm's profits are simply $\pi(e) - w$, and the firm maximizes expected profits. Profits are observable and contractible.

The timing is standard: nature chooses the manager's type; the firm offers the manager a contract; the manager accepts (or rejects) and chooses effort to maximize his payoff.

(a). Characterize the first-best level of effort when θ is observable and contractible using the first-order condition.

From now on, assume θ is private information to the manager. Note that because $\pi(e)$ is strictly increasing, it is invertible and the firm can contract on e as well as π.

(b). State the revelation principle for this problem using deterministic direct mechanisms. Specifically, consider direct-revelation mechanisms of the form $\{e(\cdot), w(\cdot)\}$ where $e(\hat{\theta})$ is the required effort (i.e., $\pi(\hat{\theta}) = \pi(e(\hat{\theta}))$ is the required profit level) for a worker who reports $\hat{\theta}$ in exchange for wage $w(\hat{\theta})$.

(c). Define the effort-utility allocation as $\{e(\cdot), U(\cdot)\}$. State and prove the two conditions for such a profile to be implementable with wage payment $w(\theta) = U(\theta) + g(e(\theta), \theta)$.

(d). Solve for the optimal effort allocation, $e(\cdot)$. You may make any additional assumptions on π and g, but be explicit about them. How does this compare to the first-best allocation in (a).

Problem 2 (MWG, Exercise 14.C.6) Reconsider the labor-market screening model in MWG Exercise 13.D.1 (Problem set 3, Problem 2) in which tasks are productive and a type-θ worker produces $\theta(1 + t)$ in output for the firm. Now, however, suppose that there is a single employer. Characterize the solution to this firm's screening problem (assume that both types of workers have a reservation utility level of 0). Compare the task levels in this solution with those in the equilibrium of the competitive screening model (assuming an equilibrium exists) that you derived in Exercise 13.D.l.

Problem 3 (MWG, Exercise 14.C.8 - variation) Air Shangri-la is the only airline allowed to fly between the islands of Shangri-la and Nirvana. There are two types of passengers, tourist and business. Business travelers are willing to pay more than tourists. The airline, however, cannot tell directly whether a ticket purchaser is a tourist or a business traveler. The two types do differ, though, in how much they are willing to pay to avoid having to purchase their tickets in advance. (Passengers do not like to commit themselves in advance to traveling at a particular time.) More specifically, the utility levels of each of the two types net of the price of the ticket, p, for any given amount of time w prior to the flight that the ticket is purchased are given by

Business: $v - \theta_b p - w$,

Tourist: $v - \theta_t p - w$.

where $\theta_t > \theta_b > 0$. (Note that for any given level of w, the business traveler is willing to pay more for his ticket. Also, the business traveler is willing to pay more for any given reduction in w.) The proportion of travelers who are business customers is $\phi \in (0, 1)$ and the proportion who are tourists is $(1 - \phi)$. Assume that the cost of transporting a passenger is c where $c \leq \frac{v}{\theta_t}$ so that it is optimal to serve both types in a full-information world.

Assume in (a)-(c) below that Air Shangri-la wants to carry both types of passengers so the per-unit cost of transporting a passenger is immaterial.

(a). Draw the indifference curves of the two types in (p, w)-space. Draw the airline's isoprofit curves. Now formulate the optimal (profit-maximizing) price discrimination problem mathematically that Air Shangri-la would want to solve. [Hint: Impose nonnegativity of prices as a constraint since, if it charged a negative price, it would sell an infinite number of tickets at this price.]

(b). Show that in the optimal solution, tourists are indifferent between buying a ticket and not going at all.

(c). Show that in the optimal solution, business travelers never buy their ticket prior to the flight and are just indifferent between doing this and buying when tourists buy.

(d). Describe fully the optimal price discrimination scheme under the assumption that the airline sells to both types.

(e). Describe fully the optimal price discrimination scheme under the assumption that the airline sells to only the high type. Give a condition as a function of the underlying parameters for when selling to both is more profitable than selling only to the high types.

Problem 4 (MWG, Exercise 14.C.9) Consider a risk-averse individual who is an expected utility maximizer with a Bernoulli utility function over wealth $u(\cdot)$. The individual has initial wealth y and faces a probability π of suffering a loss of size L, where $y > L > 0$. An insurance contract may be described by a pair (c_1, c_2), where c_1 is the amount of wealth the individual has in the event of no loss and c_2 is the amount the individual has if a loss is suffered. That is, in the event no loss occurs the individual pays the insurance company an amount $w - c_1$, whereas if a loss occurs the individual receives a payment $c_2 - y + L$ from the company.

(a). Suppose that the individual's only source of insurance is a risk-neutral monopolist (i.e., the monopolist seeks to maximize its expected profits). Characterize the contract the monopolist will offer the individual in the case in which the individual's probability of loss, π, is observable.

(b). Suppose, instead, that π is not observable by the insurance company (only the individual knows π). The parameter π can take one of two values $\pi_H > \pi_L > 0$ and Prob$[\pi = \pi_L] = \phi \in (0,1)$. Characterize the optimal contract offers of the monopolist. Can one speak of one type of insured individual being "rationed" in his purchases of insurance (i.e., he would want to purchase more insurance if allowed to at fair odds)? Intuitively, why does this rationing occur? [Hint: It might be helpful to draw a picture in (c_1, c_2)-space. To do so, start by locating the individual's endowment point, that is, what he gets if he does not purchase any insurance.]

(c). Compare your solution in (b) with your what we know about the competitive screening outcome in the insurance-market model of JR, chapter 8.

Problem 5 Consider the price discrimination model considered in the lecture, except that there are now three types, where $\theta_3 > \theta_2 > \theta_1 > 0$. The utility of agent θ_i is

$$u(q, \theta_i) = \theta_i q - t.$$

Denote the allocations to agent θ_i by (q_i, t_i). Now there are three (IR) constraints, one for each type. There are also six (IC) constraints, since we must ensure type θ_1 does not want to copy type θ_2 or θ_3, and similarly for the other agents. For example, (IC12) says that θ_1 must not want to copy θ_2, i.e.

$$\theta_1 q_1 - t_1 \geq \theta_1 q_2 - t_2.$$

The firm's profit is

$$\sum_{i=1}^{3} \phi_(t_i - C(q_i)),$$

where ϕ_i is the proportion of type θ_i agents and $C(q)$ is increasing and convex.

(a). Show that (IR2) and (IR3) can be ignored.

(b). Show that $q_3 \geq q_2 \geq q_1$.

(c). Assume that $q_3 \geq q_2 \geq q_1$ in (b) holds. Using (IC12) and (IC23) show that we can ignore (IC13). Using (IC32) and (IC21) show that we can ignore (IC31).

(d). Show that (IR1) will bind.

(e). Show that (IC21) will bind.

(f). Show that (IC32) will bind.

(g). Assume that $q_3 \geq q_2 \geq q_1$. Show that (IC12) and (IC23) can be ignored.

(h). Suppose that $C(q) = \frac{1}{2}q^2$, $\theta_1 = 4$, $\theta_2 = 5$, $\theta_3 = 6$, $\phi_1 = \phi_2 = \phi_3 = \frac{1}{3}$. State the firm's optimization program given your conclusions in (a)-(g) and find the optimal qualities to sell, (q_3, q_2, q_1).