Bubbles

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Theory of Income III 2019

Road Map

- Bubbles and Long-Lived Assets
- Heterogeneous belief (Midterm 2017)
    - Bubbles due to expeculative behavior of agents.

Bubbles and Long-Lived Assets

- Two-periods OLG moodel.
- Young pref. : $\log c_{1,t} + \log c_{2,t+1}$,
  - Endowment $(e_1, e_2) = (e, 0)$.
- Initial Old pref.: $\log c_{2,0}$.
  - 1 unit of money $\rightarrow$ price $p_{m,t}$ in each t
  - 1 unit of real asset $\rightarrow$ price $p_{a,t}$ yields dividends d.

Competitive Eq.

A CE is a sequence of prices $\{p_{a,t}, p_{m,t}\}$ and allocations $\{c_{1,t}, c_{2,t} a_t, m_t\}$ such that:

- Optimality for each Young given prices:

$$\begin{aligned}
&\max \log c_{1,t} + \log c_{2,t+1} \\
&s.t.\ c_1 + p_{a,t} a_t + p_{m,t} m = e_1 \\
&\qquad c_2 = (p_{a,t+1} + d) a_t + p_{m,t+1} m \\
&\qquad a_t \geq 0, \qquad m \geq 0.
\end{aligned}$$

- Optimality for each Initial-Old: $c_{2,0} = (p_{a,0} + d) + p_{m,0}$.
- Market clears:

$$\begin{aligned}
c_{1,t} + c_{2,t} &= e_1 + d \\
a_t &= 1 \\
m_t &= 1.
\end{aligned}$$

No-Bubble equilibria

- The existence of a real asset with money generates different equilibria.
- Money becomes obsolete when there is a real asset available $\Longrightarrow$ No Bubble!

$$p_{m,t} = 0, \quad \text{for all } t.$$

- Result holds even if dividends are really small!
- **Intuition:**
  - No endowment in the second period.
  - Both assets only mechanism to generate consumption in second period.
  - Non-arbitrage condition if both asset in positive demand requires explosive sequence of prices.

No-Bubble equilibria

- Impose $p_{m,t} = 0$ for all $t \Longrightarrow$ look for $\{p_{a,t}, c_{1,t}, c_{2,t}, a_t, m_t = 1\}$.
- Young in t solves:

$$\max_{a_t \geq 0} \log\left(e_1 - p_{a,t} a_t\right) + \log\left[\left(p_{a,t+1} + d\right) a_t\right].$$

- Constraint not binding$\rightarrow$ consumption equal to zero not a solution.
- Optimality implies:

$$\frac{1}{c_{1,t}} = \frac{1}{c_{2,t+1}} \frac{\left(p_{a,t+1} + d\right)}{p_{a,t}}, \qquad \forall t \geq 0.$$

No-Bubble equilibria

- Market clearing requires:

$$m_t = 1, \quad a_t = 1.$$

- Mkt Clearing + Period budget constraint:

$$c_{1,t} = e_1 - p_{a,t},$$
$$c_{2,t+1} = p_{a,t+1} + d.$$

- Plugg-into Euler Equation to obtain a difference equation for prices:

$$\frac{1}{e_1 - p_{a,t}} = \frac{1}{p_{a,t+1} + d} \frac{(p_{a,t+1} + d)}{p_{a,t}}, \qquad \forall t \geq 0.$$

No-Bubble equilibria

- Hence,

$$p_{a,t} = \frac{e_1}{2}, \qquad \forall t \geq 0.$$

  Independent of the size of the dividend!!

- Equilibrium with no Bubbles:

$$c_{1,t} = \frac{e_1}{2}, \quad c_{2,t} = \frac{e_1}{2} + d$$
$$a_t = 1, \quad m_t = 1.$$

- The real asset precludes existence of bubble!
- Robustness: check PS8 from 2018. What happens there and why?

Heterogeneous Expectations (Midterm 2017)

- Time is discrete and infinite, $t = 0, 1, \ldots$.
- Unit measure of risk-neutral, infinitely-lived individuals.
  - Discount factor β.
  - "Deep pockets" $\implies$ large endowments in each period $\implies$ non-negativity of consumption not binding.
  - Short-selling is not permitted.
- Unit measure of an asset $\rightarrow$ produce dividends $d \in \mathscr{D} = \{d_1, \ldots, d_n\}$
- **Belief:**
  - Everyone agrees that the dividend is independently and identically distributed over time.
  - Disagree about probability distribution over the set $\mathscr{D}$.
  - Fraction α_i, $i \in 1, \ldots, m$ of the population believes that $d = d_j$, $j \in \{1, \ldots, m\}$ with probability $\pi_j^i > 0$, where $\sum_{j=1}^{n} \pi_j^i = 1$.
- Expected dividend under type $i's$ belief to be $\delta_i \equiv \sum_j d_j \pi_j^i$.

What could cause a bubble?

- An agent that holds an assets could have high belief that the dividends it pays are low.
- Why holds the asset?
- He knows that other agent might place a high value to the asset if they think the asset is good.
- I can earn a profit by selling the asset to those guys.
- **Important:** in the argument what matters are belief on dividend's distribution not fundamentals.

A more general set-up

- The price of the asset is pin down by the individual with the highest valuation of the next period's dividends based on their belief.
- The post dividend price of the asset is given by:

$$p(d) = \max_i \beta \sum_{d'} \pi^i (d'|d) (d' + p(d')) .$$

- Each individual i's buy-and-hold price for the asset is:

$$q^i(d) = \beta \sum_{d'} \pi^i (d'|d) (d' + q^i(d')) .$$

- Note that the highest valuation of the asset can differ with $p(d)$. Define:

$$q(d) = \max_i q^i(d) .$$

Definition of a Bubble

Definition

We say that there is a speculative behavior (bubble) if and only if

$$p(d) > q(d).$$

- Bubble $\rightarrow$ there is speculative behavior.
  - Each individual believes that it is not worth holding the asset at that price,
  - Someone is willing to purchase at that price.
  - The one willing to purchase do so because he thinks he can sell it for higher price.

Buy-and-hold price

- First we compute the buy-and-hold price for each type of household:

$$q_i\left(d_j\right) = \beta \sum_{\ell=1}^{n} \pi^i_{\ell,j}\left(d_\ell + q_i\left(d_\ell\right)\right).$$

Since dividends are i.i.d for all belief:

$$\begin{aligned} q_i\left(d_j\right) &= \beta \sum_{\ell=1}^{n} \pi^i_{\ell}\left(d_\ell + q_i\left(d_\ell\right)\right) \\ &= \beta \sum_{\ell=1}^{n} \pi^i_{\ell} d_\ell + \beta \sum_{\ell=1}^{n} \pi^i_{\ell} q_i\left(d_\ell\right) \\ & \beta \delta_i + \beta \sum_{\ell=1}^{n} \pi^i_{\ell} q_i\left(d_\ell\right). \end{aligned}$$

- Notice $q_i\left(d_j\right)$ is independent of d_j:

$$q_i = \beta \delta_i + q_i \beta \sum_{\ell=1}^{n} \pi^i_{\ell} \implies q_i = \frac{\beta}{1-\beta}\delta_i$$

Equilibrium price

- The equilibrium price satisfies:

$$p(d_j) = \max_i \left\{ \beta \sum_{\ell=1}^{n} \pi^i_{\ell,j} (d_\ell + p(d_\ell)) \right\}.$$

- Independence of dividends implies:

$$p(d_j) = \max_i \left\{ \beta \delta_i + \beta \sum_{\ell=1}^{n} \pi^i_\ell p(d_\ell) \right\}.$$

- So the realization of dividends today is not a relevant state to compute today's asset price. Hence:

$$\begin{aligned} p &= \max_i \{\beta \delta_i + \beta p\} \\ &= \beta \max_i \{\delta_i\} + \beta p. \end{aligned}$$

- The equilibrium price is:

$$p(d) = \frac{\beta}{1-\beta} \max_i \delta_i.$$

Do we have a Bubble?

- Use the definition developed before. We compare $p(d)$ with $q(d) = \max_i q_i(d)$.
- In this case $p(d) = q(d)$ for all d. No bubble in this example.
- Why?
    - Everyone belief that the dividend is independent in time.