Price Theory, III
Economics 33003

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Review Problems for Review Session 1
This is NOT an assignment

Problem 1 Part 1: Consider a Cournot duopoly with linear demand, $p = a - bQ$, and constant unit costs, $C_i(q) = c_i q$. Both firms initially have unit costs equal to c, but with probability $\phi \in (0, 1)$, firm 1 obtains a cost reduction equal to δ before outputs are chosen. Thus, firm 1's unit cost is $c_{1h} = c$ with probability $1 - \phi$) and $c_{1l} = c - \delta$ with probability ϕ; firm two's unit cost is $c_2 = c$.

(a). Complete information: Assume that any cost reduction to firm 1 is publicly observable to both firms before they choose their outputs. Solve the Nash equilibrium outputs and firm 1's equilibrium profit following the two possible cost realizations. (Assume throughout that a is sufficiently high that each firm produces positive output in equilibrium.)

(b). Incomplete information: Suppose instead that any cost reduction is private information to firm 1. Solve for the Bayes-Nash equilibrium outputs, $(q^*_{1h}, q^*_{1l}, q^*_2)$, and compute firm one's profit following for each possible cost.

(c). Incomplete information with the possibility of evidence. Sticking with the private information assumption in (b), assume that if a cost reduction is achieved, firm 1 can choose to reveal evidence proving the reduction to firm 2 before outputs are chosen. To be precise, at date 0 nature chooses firm 1's cost type; at date 1, if firm 1's cost is low, it can reveal evidence proving this to firm 2; at date 2 outputs are simultaneously chosen by the firms. What is the equilibrium to this disclosure-output game (i.e., with what probability does the low-cost firm reveal evidence at stage 1; what are the corresponding outputs at stage 2)? [Hint: use your profit computations from (a) and (b).]

Part 2: Consider a differentiated Bertrand duopoly game with linear demand,

$$q_1 = \alpha - \beta p_1 + \gamma p_2$$

$$q_2 = \alpha - \beta p_2 + \gamma p_1,$$

and constant unit costs as in Part 1: $c_{1h} = c$ (with probability $(1 - \phi)$, $c_{1l} = c - \delta$ (with probability ϕ), and $c_2 = c$.

(d). In the incomplete information game with cost-reduction evidence, will the low-cost firm reveal evidence to firm 2? Note that evidence only exists regarding cost reductions; if a cost reduction didn't happen, firm 1 cannot prove that it didn't happen. (I.e., evidence is not symmetric.) Explain. You do not need to explicitly prove your statement using profit computations as you did in Part 1, but you do need to give the intuition. [Hint: drawing a set of reaction functions can be illuminating.]

Problem 2 Consider a public goods game with two players, $i = 1, 2$, and private information. Each player can choose to either contribute to the public good ($s_i = 1$) or not, ($s_i = 0$), $S_i = \{0, 1\}$. If one or both players contribute, then the public good is produced and each player receives a benefit of 1. Player i's cost of contributing is $c_i \in [\underline{c}, \overline{c}]$, where $\underline{c} < 1 < \overline{c}$; c_i is independently distributed for both players according to the continuous distribution function $F(c_i)$. If player i contributes, her payoff is

$1 - c_i$; if player i does not contribute, her payoff is 1 if player $j \neq i$ contributed, and 0 otherwise. Formally,

$$u_i(s_i, s_j, c_i) = \max\{s_1, s_2\} - c_i s_i.$$

A pure-strategy equilibrium in this game is a profile of contribution functions, $\{s_1^*(\cdot), s_2^*(\cdot)\}$ where $s_i^* : [\underline{c}, \overline{c}] \to \{0, 1\}$.

(a). Characterize the symmetric pure-strategy Bayesian-Nash equilibrium. Prove that it is unique and always exists.

(b). Characterize a pair of asymmetric pure-strategy equilibria in which one player never contributes. Give the condition on the distribution of costs that guarantees the existence of such equilibria.

Problem 3 Consider a first-price auction (without a reserve price) between two buyers. Valuations are $\theta_h > \theta_l > 0$ and the probability that $\theta = \theta_h$ is $\phi \in (0, 1)$.

There is no pure-strategy equilibrium to the first-price auction with discrete types. Find the symmetric Bayesian-Nash equilibrium in which type θ_l bids $b_l = \theta_l$, but type θ_h randomizes her bid according to some equilibrium distribution $G(b_h)$ on $[\underline{b}, \overline{b}]$.

[Hint: you *may* need to solve a differential equation. If you are unfamiliar with solving *ODE*'s, try using software like *Mathematica*. Alternatively, here is an ODE formula that should work for you:

$$\alpha + f(x) = (\beta - x) f'(x) \implies f(x) = \frac{\alpha x + C}{\beta - x},$$

where C is an arbitrary constant. You can also solve this problem without needing to solve ODE.]