Price Theory
Proposed Solution to Problem Set 6, Question 2

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Below is a time series chart of the U.S. prevalence of syphilis, which is a sexually-transmitted disease, in its contagious phases.

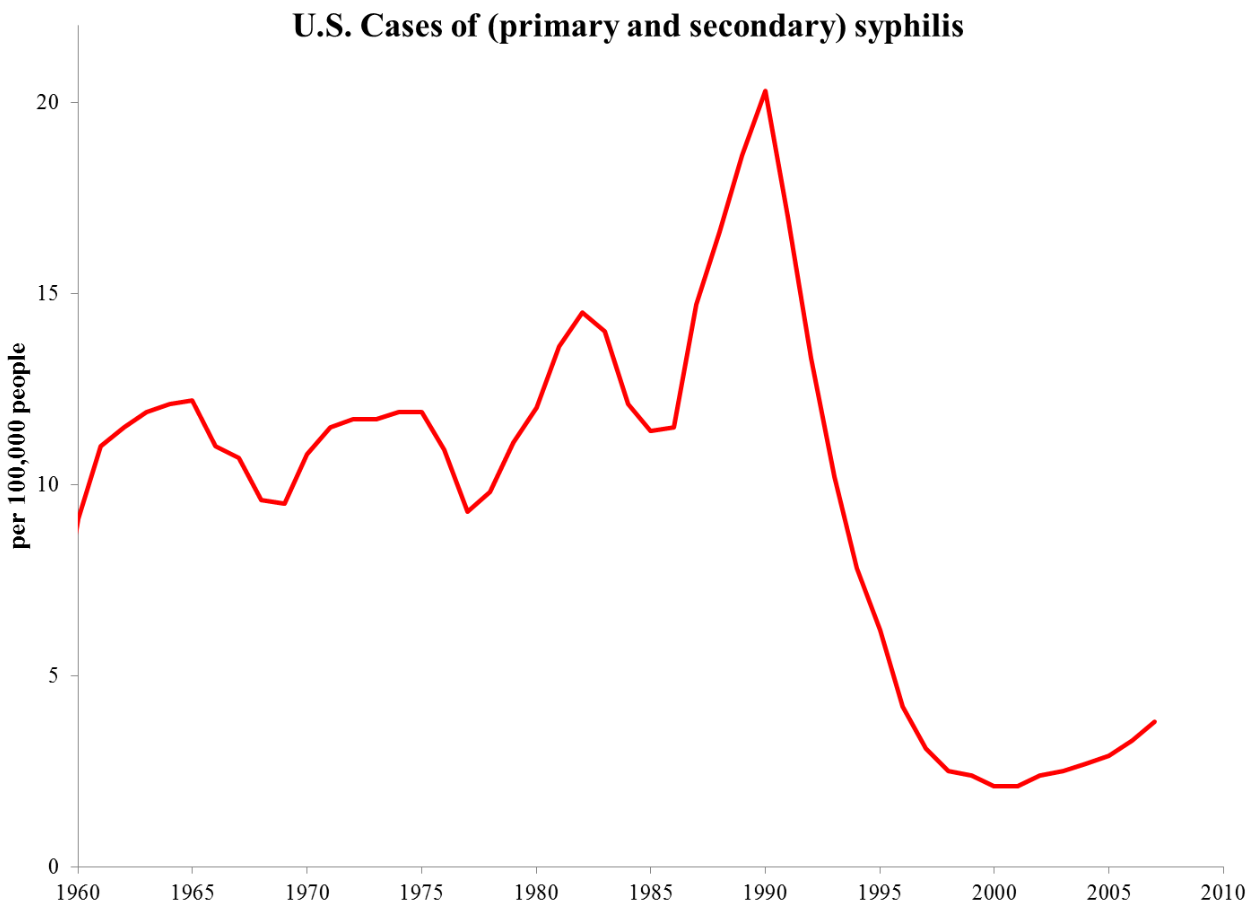

Figure 1: Prevalence of Syphilis, 1960-2010.

Because the disease is acquired x years before it becomes contagious we model the dynamics of prevalence y with a difference equation:

$$y_{t+1} = NewInfections_t + (1-r)y_t$$

where x is the length of the time period, and r is the recovery rate. For simplicity, we ignore births and deaths and assume that the recovered people go back into the susceptible population.

Setting

Syphilis is a sexually transmitted disease, so that one can only acquire syphilis by having sex. This question will explore how the costs and benefits of sex determine syphilis prevalence.

Decisions

Our attempt to understand the observed pattern of syphilis prevalence will recognize two margins over which people make sex decisions. The extensive margin is a discreet decision to engage in casual sex. The intensive margin is the decision of how much casual sex to have in the event of engaging in casual sex.

In principle, casual sex is is not measured unidimensionally, since it involves choices like how many partners to have, how much sex to have with each partner or the sex intensity at each sex encounter. We will work with a boiled-down, unidimensional measure of sex, and call it *sex*.[2]

[2] Among other things, a reason not to use the number of sex partners as our sex measure is that market clearing is a complicated concept. See the Erdos and Gallai theorem (1961).

Timing

Timing is crucial to clarify and simplify our analysis. In line with the question, our setup has discreet time, with period length equal to x years. At the beginning of each period, the extensive and intensive margins of sex are decided and executed, and new infections occur. Finally, a fraction r of those who started the period with syphilis is cured. This timing yields the difference equation for prevalence that is stated in the question:

$$y_{t+1} = NewInfections_t + (1-r)y_t.$$

Note that, because of our timing, anyone who got infected in time t will be contagious at the start of time $t+1$ (i.e. date t plus x years). Therefore, y_t is also equal to the contagious syphilis prevalence and we do not have to incorporate the decisions of those with syphilis that are not contagious into our analysis.

a.

What can you say about the relationship between prevalence and new infections? Separate your answer into a mechanical component, related to random pairings of partners, and a behavioral component.

Intuitively, a healthy person in the casual sex pool will get syphilis depending on how much sex she has (sex_i), how much sex other healthy people in the pool have (sex^{casual}), how much sex infected people in the pool have ($sex^{infected}$) and the share of infected people in the pool (y^{casual}).

Under the assumption of random pairings, this induced conditional probability can be simplified:

$$\Pr(\text{Get syphilis} \mid sex_i, sex^{healthy}, sex^{infected}, y^{casual})$$
$$= p\left(sex_i;\ \frac{y^{casual} sex^{infected}}{y^{casual} sex^{infected} + (1 - y^{casual}) sex^{healthy}}\right),$$

where p is a primitive function (taking values in $[0, 1]$) whose form is determined by epidemiology.

Notice that when everyone makes the same *sex* decisions in the pool – an assumption we will maintain for parts a., b. and c. – this probability is simply $p(sex_i;\ y^{casual})$.

For a given casual sex pool with sex level *sex* and syphilis prevalence y, new infections are simply:

$$NewInfections = (1 - y)p(sex;\ y)$$

The form for p is the mechanical component in the relation between new infections and syphilis prevalence. The behavioral component is the sex level *sex*, which is a choice of our agents.

b.

Based on your answer to (a), how many prevalence steady states are there?

We use the equation derived in part a. and the law of motion for syphilis prevalence to find that a steady state must satisfy:

$$ry = (1 - y)p(sex;\ y) \iff r\frac{y}{1 - y} = p(sex;\ y).$$

Although we do not know the exact form for p, it should satisfy some basic properties. For any given amount of *sex* one has,

No prevalence, no danger $p(sex;\ 0) = 0$.

More prevalence, more danger $p(sex;\ \cdot)$ is strictly increasing.

Marginal danger is decreasing in prevalence $p(sex;\ \cdot)$ is concave.

On the other hand, $r\frac{y}{1-y}$ is a convex function that equals zero for $y = 0$ and tends to infinity as y tends to one. Thus, associated to each *sex* level, there are two steady states: one without prevalence, and one with positive prevalence. In particular, since $p(sex;\ y)$ should also be increasing in *sex* for any given y, we see that the steady state with positive prevalence associated to a given *sex* level is increasing in *sex* – this is no surprise.

c.

Focusing on the period 1960-85, can incentives explain why prevalence follows a cycle? Take the case where people are ex ante identical (that is, conditional on disease status and their knowledge of market conditions, make the same decisions). Do you need to assume anything about the magnitude of x?

We now provide more details on our setting. We will model a continuum of people with mass of $100,000$, to make it consistent with the quantities in the question.

In this world, earn M dollars every time period, and they care about how much *sex* they have and about consumption of a composite good x that we take as our numeraire. Given consumption levels, the period utility of a healthy person is then $U(sex, x)$.

Having syphilis is a bad thing, and we will model this as simply as possible: people with syphilis suffer a utility loss of L. Their utility is then $U(sex, x) - L$.

In the event of not joining the casual sex pool, individuals will have a utility level of U_0, given by

$$U_0 = \begin{cases} U(sex_0, M) & \text{if healthy} \\ U(sex_0, M) - L & \text{if infected,} \end{cases}$$

where sex_0 is a given amount of sex people can have outside of the casual sex pool, which we assumed can be consumed for free.

Inside the casual sex pool, consuming *sex* requires one to forego resources. In particular, it is cheaper to consume *sex* the more *sex* others choose to consume! This cost, measured in dollars, is given by $c(sex;\ sex^{others}$, and it is then decreasing in sex^{others}.

Problem for infected people

Suppose an infected person joins the casual sex pool. Does the sex amount they choose affect future payoffs? No, so that their problem is of a static nature.

Under quasilinear preferences, the optimal sex^I given sex^{others} is a transparent version of marginal benefit equals marginal cost:

$$U'(sex^I) = c'(sex^I;\ sex^{others}).$$

On the extensive margin then, for some sex^{others}, infected people then choose between U_0 and $U(sex^I, M - c(sex^I))$.

In what follows, we will be interested in equilibria in which sex^{others} is sufficiently large to make the jump to the casual sex pool worthwhile for infected people.[3]

[3] Equilibria in which infected people don't join the casual sex pool are such that healthy people won't join it either, since they face additional costs, as we will see. They are therefore not empirically relevant.

Problem for healthy people

In contrast with infected people, their problem is dynamic: current sex decisions affect future disease status and, hence, future welfare.

However, notice that we can simplify this problem by thinking ahead about the kinds of equilibrium that can possibly occur: could it be the case that we have equilibria for which (i) no healthy person joins the casual sex pool, (ii) some healthy people join it, or (iii) everybody joins it?

(i) is certainly possible: in the event of no other healthy person joining the casual sex pool, it does not make sense for any one of them to join it![4] (ii) is knife-edge. This requires healthy people to be indifferent between joining and not joining the casual sex pool. Note however that the value of joining the casual sex pool has to be increasing in the amount of healthy people joining it. Thus, one additional person joining or leaving the pool would drive every healthy person in or out, respectively. By this reasoning, (iii) could certainly be a stable equilibrium outcome.

[4] Yes, implicitly L and $p(sex;\ 1)$ are assumed to be big enough.

Answering c.

We view the syphilis prevalence trend prior to 1990 as an equilibrium outcome among the set of equilibria in which every healthy person joins the casual sex pool.

To understand the costs and benefits of *sex* faced by healthy people, we now pose the problem they solve. At the beginning of a period t, any one healthy person will observe current prevalence y_t, hypothesize a sex level chosen by everybody else now and in the future – $\{sex_t\}_t$ – and have value

$$
\begin{aligned}
V(y_t, \{sex_t\}_t) = \max_{sex,x} \quad & U(sex, x) + \beta \left\{ p(sex;\ y_t) V^I(y_{t+1}, \{sex_{t+1}\}_t) + [1 - p(sex;\ y_t)] V(y_{t+1}, \{sex_{t+1}\}_t) \right\} \\
s.t. \quad & x + c(sex, sex_t) = M \\
& y_{t+1} = (1 - y_t) p(sex_t;\ y_t) + (1 - r) y_t.
\end{aligned}
$$

where V^I is the value of having syphilis:[5]

[5] Recall the optimal sex level for them depends on sex_t.

$$
\begin{aligned}
V^I(y_t, \{sex_t\}_t) = \quad & U(sex^I, M - sex^I) + \beta \left\{ r V(y_{t+1}, \{sex_{t+1}\}_t) + (1 - r) V^I(y_{t+1}, \{sex_{t+1}\}_t) \right\} \\
s.t. \quad & y_{t+1} = (1 - y_t) p(sex_t;\ y_t) + (1 - r) y_t.
\end{aligned}
$$

Again, with quasilinear preferences, the first order condition is:

$$
U'(sex^*) = c'(sex^*;\ sex_t) + \beta p'(sex^*;\ y_t) \left[V(y_{t+1}, \{sex_{t+1}\}_t) - V^I(y_{t+1}, \{sex_{t+1}\}_t) \right].
$$

The cyclical pattern comes to life upon a closer inspection of this relation of marginal costs and benefits of sex under different y_t. This in turn depends on how $p'(sex^*;\ y_t)$ changes for different y_t. Note first that, in the data, the number of people with syphilis per 100,000 is actually pretty low. Focusing on prevalences in this range $(9-20)$, the added likelihood of getting syphilis given by an additional unit of sex should increase in the prevalence level! In other words, we are far from a situation in which prevalence is so high that an increase in prevalence reduces the probability increase of getting syphilis given by an additional unit of sex.

Thus, healthy people face an additional cost of sex that has to do with the risk of having syphilis in the future, and this cost component is increasing in the prevalence level. Therefore, large prevalences are coupled with low sex levels for healthy people. The outcome observed therefore corresponds to an oscillating equilibrium.[6]

[6] We actually did not solve for this equilibrium explicitly. In reality it is not possible to do so if we assume that healthy and infected people make the same sex decisions.

The magnitude of x matters, as it should coincide with half the period observed, or about 3 years.

d.

How is your answer different if you recognize that people differ in terms of the number of partners that they have?

Infected and healthy people would naturally choose a different amount of partners, and more generally a different amount of *sex*, since infected people bear only a fraction of the cost faced by healthy people.

People will now have to additionally keep track of the sex level chosen by each subgroup when forming the probability that they get syphilis given their chosen sex level.

However, the analysis of the first order condition for healthy people remains unchanged.

e.

Prostitution was illegal throughout the period shown in the chart. Using your answers from above, what do you predict would have been different about the prevalence series if prostitution had been legal?

When a market is legalized, such as the one for prostitution here, transactions can be enforced by the state. We would then expect this market to become much more sophisticated: contracts for safe sex with prostitutes would be much easier to enforce, and people signing this contract would have access to safe, casual sex. In this sense, (i) remaining outside the "casual sex" group would now become more appealing, possibly driving healthy people out of the casual sex pool, and (ii) since safe sex with prostitutes is a substitute for sex in the casual sex pool, the amount of casual sex on the intensive margin would decrease.

Overall, this line of reasoning implies that legalizing prostitution would have a negative effect on syphilis prevalence.

f.

In about 1990, another sexually transmitted disease was rapidly spreading: AIDS. Does the emergence of AIDS help explain why syphilis prevalence dropped to new lows?

Yes: this is a shift from an equilibrium in which all healthy people engage in casual sex to one where only infected people do, driven by a rise in L, reinterpreted as the combined utility loss of having syphilis and AIDS.

Crucially, this trend is driven by the sex decisions taken on the extensive margin.