Price Theory, III | **Professor Lars Stole**
Economics 33003 | **Spring 2019**

Assignment 1
(Due Friday, April 12, prior to the start of the Review session)

Problem 1 (MWG, Exercise 13.B.3 - variation) Consider a *positive-selection* variation of the model discussed in MWG (13.8) in which $r(\cdot)$ is a continuous, *strictly decreasing* function of θ. Let the density of workers of type θ be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$

(a). Show that the *more capable* workers are the ones choosing to work at any wage for which some workers are employed and others are not.

(b). Show that if $r(\theta) > \theta$ for all θ, then the resulting competitive equilibrium is Pareto efficient.

(c). Suppose that there exists a $\hat{\theta}$ such that $r(\theta) < \theta$ for $\theta > \hat{\theta}$ and $r(\theta) > \theta$ for $\theta < \hat{\theta}$. Show that any competitive equilibrium with strictly positive employment necessarily involves too much employment relative to the Pareto-optimal allocation of workers.

Problem 2 (JR, Exercise 8.7 - variation) Consider the following market for used cars. There are many sellers of used cars. Each seller has exactly one used car to sell and is characterized by the quality of the used car he wishes to sell. Let $\theta \in [0, 1]$ index the quality of a used car and assume that θ is uniformly distributed on $[0, 1]$. If a seller of type θ sells his car (of quality θ) for a price of p, his utility is p. If he does not sell his car, then his utility is $r(\theta)$, which is increasing in θ and $r(0) = 0$. Buyers of used cars receive utility $\theta - p$ if they buy a car of quality θ at price p and receive utility 0 if they do not purchase a car. There is asymmetric information regarding the quality of used cars. Sellers know the quality of the car they are selling, but buyers do not know its quality. Assume that there are not enough cars to supply all potential buyers.

(a). Argue that in a competitive equilibrium under asymmetric information, we must have $E[\theta | p \geq r(\theta)] = p$.

(b). Show that if $r(\theta) = \frac{\theta}{2}$, then every $p \in (0, 1/2]$ is an equilibrium price.

(c). Find the equilibrium price when $r(\theta) = \sqrt{\theta}$. Describe the equilibrium in words. In particular, which cars are traded in equilibrium?

(d). Find an equilibrium price when $r(\theta) = \theta^3$. How many equilibria are there in this case?

(e). Are any of the preceding outcomes Pareto efficient? Describe Pareto improvements whenever possible.

Problem 3 Consider a certifiable disclosure game with n players, in which each player i's payoff is simply the market expectation of their type, conditional on the information $m \subseteq \Theta$ revealed:

$$u_i(\theta_i) = E[\theta | \theta \in m].$$

There are two types, $\theta_h > \theta_l > 0$, where $\phi \in (0, 1)$ is the probability of the high type. As a complication, however, suppose that with probability λ, a player cannot use the free certification mechanism and therefore no message is sent (effectively, $m = \Theta$). What is the unique equilibrium? If it involves less than full revelation by those players who can reveal their types, explain why unraveling does not arise. What key condition is violated?

Problem 4 Consider the MWG labor-market signaling game in 13.C. Assume that $c(e, \theta) = e(y - \theta)$ where y is a constant such that $y > \theta_h > \theta_l > 0$.

(a). Characterize a hybrid (mixed-strategy) equilibrium in which the low-type worker randomizes between two levels of education, e_l and e_h, with probabilities λ and $(1 - \lambda)$ respectively, and the high-type plays a pure-strategy always choosing e_h. In particular, compute w_l^*, e_l^*, and give the equation that ties together the low-type's mixing probability, λ^*, with the education level chosen by the high type, e_h^*. don't you need the fraction of people?

(b). What is special about the kind of separating equilibrium one gets by setting $\lambda = 1$ in the characterization in (a)? What is special about the kind of pooling equilibrium one gets by setting $\lambda = 0$ in the characterization in (a)?

(c). [Optional - not required for assignment.] Does the hybrid equilibrium in (a) pass the Intuitive Criterion?

Problem 5 (MWG, Exercise 13.C.5) Assume a single firm and a single consumer. The firm's product may be either high or low quality and is of high quality with probability $\lambda \in (0, 1)$. The consumer cannot observe quality before purchase and is risk neutral. The consumer's valuation of a high-quality product is v_H; her valuation of a low-quality product is v_L. The costs of production for high (H) and low (L) quality are c_H andc_L, respectively. The consumer desires at most one unit of the product. Finally, the firm's price is regulated and is set at p. Assume that $v_H > p > v_L > c_H > c_L$.

(a). Given the level of p, under what conditions will the consumer buy the product?

(b). Suppose that before the consumer decides whether to buy, the firm (which knows its type) can advertise. Advertising conveys no information directly, but consumers can observe the total amount of money that the firm is spending on advertising, denoted by A. Can there be a separating perfect Bayesian equilibrium, that is, an equilibrium in which the consumer rationally expects firms with different quality levels to pick different levels of advertising?

Problem 6 (MWG, Exercise 13.C.6 - variation) Consider a market for loans to finance investment projects. All investment projects require an outlay of 1 dollar. There are two types of projects: good and bad. A good project has a probability of ϕ_g of yielding profits of $\Pi > 0$ and a probability $(1 - \phi)$ of yielding profits of zero. For a bad project, the relative probabilities are ϕ_b and $(1 - \phi_b)$, respectively, where $\phi_g > \phi_b$. The fraction of projects that are good is $\lambda \in (0, 1)$.

Risk-neutral entrepreneurs go to banks to borrow the cash to make the initial outlay (assume for now that they borrow the entire amount). A loan contract specifies an amount R that is supposed to be repaid to the bank. Entrepreneurs know the type of project they have, but the banks do not. In the event that a project yields profits of zero, the entrepreneur defaults on her loan contract, and the bank receives nothing. Banks are competitive and risk neutral. The risk-free rate of interest (the rate the banks pay to borrow funds) is r. Assume that

$$\phi_g \Pi - (1 + r) > 0 > \phi_b \Pi - (1 + r).$$

To be clear, the timing and strategies of the players are

1. Entrepreneurs privately learn the quality of their investment project
2. Banks either refuse to loan money at any feasible rate, or they offer $R \in [0, \Pi]$ which is repaid if and only if the project succeeds
3. Entrepreneurs accept the best bank offer, or they reject the offer and the project is not funded.

(a). Find the equilibrium level of R and the set of projects financed. How does this depend on ϕ_g, ϕ_b, λ, Π, and r?

(b) Now suppose that the entrepreneur can offer to contribute some fraction x of the 1 dollar initial outlay from her own funds ($x \in [0, 1]$). The entrepreneur is liquidity constrained, however, so that the effective cost of doing so is $(1 + \rho)x$, where $\rho > r$. Also assume that ρ is sufficiently large that

$$1 + \rho > \phi_g \Pi > 1 + r.$$

This implies that even a good project is not profitable if all of the funds must be borrowed at rate ρ rather than r; i.e., some lower-cost bank financing is necessary.

(i). What is an entrepreneur's payoff as a function of her project type, her loan-repayment amount R, and her contribution x?

(ii). Describe the best (from a welfare perspective) separating perfect Bayesian equilibrium of a game in which the entrepreneur first makes an offer that specifies the level of x she is willing to put into a project, banks then respond by making offers specifying the level of R they would require, and finally the entrepreneur accepts a bank's offer or decides not to go ahead with the project. How does the amount contributed by entrepreneurs with good projects change with small changes in ϕ_b, ϕ_g, λ, Π, and r?

(iii). How do the two types of entrepreneurs do in the separating equilibrium of (b)(ii) compared with the equilibrium in (a)?