Price Theory, III | **Professor Lars Stole**
Economics 33003 | **Spring 2019**

Assignment 6
(Due Friday, May 24, prior to the start of the Review session)

Problem 1 Consider a natural (but regulated) monopolist with a constant marginal cost of production equal to c which is private information at the time of contracting and uniformly distributed on $[1, 2]$. The monopolist's profit for producing output q at marginal cost c with transfer t from the regulator is

$$u(q, c, t) \equiv t - cq.$$

Note that the firm does not get any revenue from selling q; all payments are through t. The firm's participation constraint is $\underline{U} = 0$.

The regulator's objective function is to maximize

$$\int_{\underline{c}}^{\overline{c}} [vq(c) - \frac{1}{2}q(c)^2 - t(c)]f(c)dc,$$

over the class of all DRM's which satisfy IC and IR constraints. Assume $v \geq 3$. Among other things, this implies the first-best, full-information output would be $q^{fb}(c) = v - c > 0$.

(a). What are the IC and IR constraints for this problem? State and prove a characterization theorem for the set of all IC mechanisms $\{q(\cdot), t(\cdot)\}$ in terms of a monotonicity condition and an integral condition.

(b). Solve for the regulator's optimal revelation mechanism, $\{q(\cdot), t(\cdot)\}$, and compare $q(c)$ to $q^{fb}(c)$. Explain the inequality that you find.

(c). Set $v = 3$. Compute an indirect tariff, $T(q)$, with the property that the regulator can offer the firm this schedule, $T(q)$, letting the firm freely choose its output q in exchange for $T(q)$, and the resulting choice and payments are equivalent to those from (b).

Problem 2 Suppose that an airline it selling tickets to business customers (b) and tourists (t). Each category of customer draws its value from a different distribution: $F_b(\theta)$ and $F_t(\theta)$. For now, we don't make any assumptions about how these relate. At the time of purchase, the customer knows whether they are a business or tourist flyer, but the airline does not; the customer does not know his ex post type, θ. Business customers represent the proportion $\phi \in (0, 1)$ of potential consumers.

The airline has decided to restrict attention to a simple pricing scheme where. A ticket costs p at the time of purchase, but also has a refund provision that allows the consumer to return the ticket for r after the consumer learns θ. Hence, a ticket is defined by its price and the amount that is refundable, (p, r). The airline wants to design a menu of tickets, $\{(p_b, r_b), (p_t, r_t)\}$, to maximize its profits. It has constant unit cost of serving either customer of c.

(a). Write down the airline's program, including the IC and IR constraints for both types.

(b). Argue that the IR constraint of the business customer can be ignored if either F_b first-order stochastically dominates F_t, or F_b is a mean-preserving spread of F_t.

(c). Consider the relaxed program in which the IC constraint for the t consumer is ignored. Show in the relaxed program that IR for t must bind and IC for b must bind.

(d). State and solve the relaxed program in which IC_t and IC_b bind (and IC_t is ignored). Describe the optimal refund policy for the two classes of tickets for the case when $F_b \geq_{FOSD} F_t$. [You may assume that IC_t is slack, though this can be proved.] Given your solution, why this may not be such a great model of the airline pricing we observe in the real world.

(e). Suppose instead that F_b is a mean-preserving spread of F_t. In particular, let's suppose that $F_b(\theta) = \theta$ on $[0, 1]$, but the tourists have a triangular distribution,

$$F_t(\theta) = \begin{cases} 2\theta^2 & \text{if } \theta \leq \frac{1}{2} \\ 4\theta - 2\theta^2 - 1 & \text{if } \theta > \frac{1}{2}. \end{cases}$$

Let $c = \frac{3}{8}$ and $\phi = \frac{1}{2}$. Describe the optimal refund policy for the two classes of tickets. [Hint: Look for the solution to the FOC for r_t where $r_t \leq \frac{1}{2}$.] Describe your solution. Does this do a better job at fitting actual airline pricing?

Problem 3 (From B. Szentes.) Consider a trade of a divisible good between a seller (principal) and a buyer (agent). The buyer's payoff is $vq - p$ where v is her valuation, q is the quantity traded, and t the payment to the seller. The seller's payoff is $t - q^2/2$, where $q^2/2$ is the cost of producing q units of the good.

Suppose that the buyer's valuation is

$$v = \lambda\theta + (1 - \lambda)\varepsilon,$$

where θ is the buyer's private information and ε is a publicly observable and contractible shock. Assume that both θ and ε are independently and uniformly distributed on $[0, 1]$. The seller can offer a contract to the buyer prior to the realization of ε, but θ is privately known by the buyer at the time of the contract. The buyer's outside option is zero.

(a). What is the set of contracts to which it is without loss of generality to restrict attention?

(b). Consider an incentive compatible contract and let $U(\theta)$ denote the equilibrium payoff of the buyer with type θ. What is $U(\theta) - U(0)$?

(c). Use your result in part (b) to express the buyer's ex-ante expected payoff, $E_\theta[U(\theta)]$.

(d). Express the seller's expected payoff as the difference between social surplus and the buyer's payoff.

(e). Use your result is part (d) to derive the optimal quantity produced as a function of θ and ε.

(f). Suppose that ε is not observable by the seller, but instead remains private information to the buyer. Otherwise, the timing is as before: the contract is offered after the buyer has observed θ but before the buyer observes ε. Using a result from class, argue that the same allocation in (e) will be implemented by the seller.

Problem 4 Consider an IPV auction environment with two bidders, one "strong" and one "weak". The strong bidder's type θ_s, is uniformly distributed on $[2, 3]$ and the weak bidder's type, θ_w, is uniformly distributed on $[0, 1]$.

(a). Compute the equilibrium bidding functions in the second-price auction. Compute the expected revenue to the seller.

(b). Compute the equilibrium bidding functions in the first-price auction for the equilibrium in which the weak player bids $\bar{b}_w(\theta_w) = \theta_w$. Compute the expected revenue to the seller.

(c). Compare the expected revenues. Explain why they are the same (i.e., explain how the revenue equivalence theorem applies to this setting), or explain why they are different (i.e., why the revenue equivalence theorem does not apply to this situation).

Problem 5 (JR, Exercise 9.8) In a first-price, all-pay auction, the bidders simultaneously submit sealed bids. The highest bid wins the object and every bidder pays the seller the amount of his bid. Consider the independent private values model with symmetric bidders whose values θ_i are each distributed according to the distribution function F, with density f.

(a). Find the unique symmetric equilibrium bidding function.

(b). Do bidders bid higher or lower than in a first-price auction?

(c). Find an expression for the seller's expected revenue.

(d). Both with and without using the revenue equivalence theorem, show that the seller's expected revenue is the same as in a first-price auction.

Problem 6 (JR, Exercise 9.9) Suppose there are just two bidders. In a second-price, all-pay auction, the two bidders simultaneously submit sealed bids. The highest bid wins the object and both bidders pay the second-highest bid.

(a). Find the unique symmetric equilibrium bidding function. Interpret

(b). Do bidders bid higher or lower than in a first-price, all-pay auction?

(c). Find an expression for the seller's expected revenue.

(d). Both with and without using the revenue equivalence theorem, show that the seller's expected revenue is the same as in a first-price auction.