Price Theory, III — **Professor Lars Stole**
Economics 33003 — **Spring 2019**

Assignment 3
(Due Friday, April 26, prior to the start of the Review session)

Problem 1 (MWG, Exercise 13.C.2-variation) Reconsider the two-type signaling model with $r(\theta_l) = r(\theta_h) = 0$, assuming that a worker's productivity is $\theta(1 + e)$. As before, $\theta_h > \theta_l > 0$, the probability of θ_h is $\phi \in (0, 1)$, and the worker's cost of education is $c(e, \theta)$, where $c(e, \theta)$ is increasing and convex in e, decreasing in θ, $c(0, \theta) = 0$, and $c_{e\theta}(e, \theta) < 0$ for $e > 0$.

(a). Assume that the worker's productivity is observable and contractible by the labor market. Characterize the competitive equilibrium wages and education levels for each type of worker in this complete information game. Solve the first-order condition

(b). Demonstrate that for some preferences, the outcome in (a) *may* arise in a separating equilibrium in the incomplete information game. Make clever indifference curve positions

(c). Assume that the underlying preferences are such that full-information competitive allocation in (a) does not arise in any separating equilibrium to the incomplete information game. Indicate the range of education levels that can be supported in a PBE separating equilibrium and the range of outputs that can be supported in a PBE pooling equilibrium.

Problem 2 (MWG, Exercise 13.D.1-variation) Extend the screening model of MWG, Chapter 13.D, to the case in which tasks are productive. Assume that a type θ worker produces $\theta(1+t)$ units of output when her task level is t. As before, $\theta_h > \theta_l > 0$, the probability of θ_h is $\phi \in (0, 1)$, and the worker's cost of task is $c(t, \theta)$, where $c(t, \theta)$ is increasing and convex in t, decreasing in θ, $c(0, \theta) = 0$, and $c_{t\theta}(t, \theta) < 0$ for $t > 0$.

Assuming that a subgame perfect Nash equilibrium exists, identify the equilibrium allocations for the low and high-type workers. what's the key difference between signaling and screening?

Problem 3 Consider a principal-agent model with moral hazard in which the principal is risk neutral and the agent is risk averse with $u(w) = \sqrt{100 + w}$. There are two efforts, $e_h > e_l$, with personal cost to the agent of $\psi(e_h) = 1$ and $\psi(e_l) = 0$. The agent's expected utility from wage contract $w(\cdot)$ and effort e is $E[\sqrt{100 + w(x)} \mid e] - \psi(e)$.

There are two outcomes. The high-output outcome is $x_2 = 200$ and the low-outcome output is $x_1 = 100$. When the agent exerts low effort, each output is equally likely. When the agent exerts high effort, the probability of high output is $\frac{3}{4}$ (and the probability of low output is $\frac{1}{4}$). The agent's reservation utility is $\underline{U} = u(0) = 10$.

(a). Does this distribution satisfy MLRP?

(b). Solve for the optimal output-contingent wage contract, $\{w_1^*, w_2^*\}$.

Problem 4 Consider a principal-agent model with moral hazard in which the principal is risk neutral and the agent is also risk neutral. There are two efforts, $e_h > e_l$, with personal cost to the agent of $\psi(e_h) = 10$ and $\psi(e_l) = 0$. The agent's expected utility is $E[w|e] - \psi(e)$.

There are two outcomes. The high-output payoff is $x_2 = 200$, but the low-outcome payoff represents a loss, $x_1 = -100$. When the agent exerts low effort, each output is equally likely. When the agent exerts high effort, the probability of high output is $\frac{3}{4}$ (and the probability of low output is $\frac{1}{4}$). The agent's reservation utility is $\underline{U} = 0$.

(a). Solve for an optimal output-contingent wage contract, $\{w_1^*, w_2^*\}$. Does the worker earn any surplus above his reservation value?

(b). Now suppose that the agent is still risk neutral, but legally the principal is not allowed to pay a wage that is negative. [Aside: This is sometimes called the *limited-liability* assumption. It is also equivalent to a situation in which an agent is infinitely risk averse at $w < 0$: i.e., $u(w) = w$ for $w \geq 0$ and $u(w) = -\infty$ for $w < 0$.] Solve for an optimal output-contingent wage contract, $\{w_1^*, w_2^*\}$.

What is the expected cost to the principal of the limited-liability constraint?

Problem 5 Consider a simple moral hazard where the principal and the agent are risk neutral (i.e., $v'(\cdot) = u'(\cdot) = 1$), there is no uncertainty over output, but now output depends upon the actions of the agent and the principal. Specifically, the agent chooses $e_1 \in [0, 2]$, the principal chooses $e_2 \in [0, 2]$ and output is deterministic:

$$x = e_1 + e_2 \in \mathcal{X} = [0, 4].$$

Assume that the cost of effort for each player is $\frac{1}{2}e_i^2$, so that first-best production requires $e_i^{fb} = 1$ and $x^{fb} = 2$.

(a). Assume that the principal can offer a contract to the agent which promises $s(x)$ in payment for the outcome x, and the residual profit, $x - s(x)$, is kept by the principal. After the agent accepts the contract, both the principal and agent simultaneously choose their individual efforts, e_i. Once x is revealed, payments are shared as promised with $s(x)$ going to the agent and $x - s(x)$ going to the principal.

If $s(x)$ is required to be continuously differentiable on $\mathcal{X}$, show that the first best cannot be implemented by the principal. [Hint: remember that the principal must also be given incentives. Another way to think of this problem is that there is a team of two players that need to devise a sharing rule $\{s(x), x - s(x)\}$ to give each incentives.]

(b). Suppose that instead of designing a sharing rule, $\{s(x), x - s(x)\}$, the principal can commit to giving some of the output to a third party for some values of x. I.e., the principal can offer $s(x)$ to the agent, and can commit to giving $z(x)$ to a third party, keeping $x - s(x) - z(x)$ for herself. Show that the first best can now be implemented as a Nash equilibrium between the principal and agent and that nothing is given to the third party *along the equilibrium path*. For this part, you may construct the implementing functions using discontinuous s and z functions. [Hint: think about simple contracts which split the output between principal and agent for some x, and give the entire output to a third party for other x.]

(c). Assume we are back to setting (a) without a third party, and thus the share contracts are limited to $\{s(x), x - s(x)\}$. Now, however, assume that the sets of feasible actions are discrete $e_1 \in \{0.5, 1, 1.5\}$ and $e_2 \in \{.67, 1, 1.33\}$. Can the first best $e_1^* = e_2^* = 1$ be implemented?

Problem 6 Consider a risk-averse individual with utility function of money $u(\cdot)$ with initial wealth y who faces the risk of having an accident and losing an amount x of her wealth. She has access to a perfectly competitive market of risk-neutral insurers who can offer coverage schedules $b(x)$ (i.e., in the event of loss x, the insurance company pays out $b(x)$) in exchange for an insurance premium, p. Assume that the distribution of x, which depends on accident-prevention effort e, has an atom at $x = 0$:

$$f(x|e) = \begin{cases} 1 - \phi(e) & \text{if } x = 0 \\ \phi(e)g(x) & \text{if } x > 0, \text{ where } g(\cdot) \text{ is a probability density} \end{cases}$$

Assume $\phi'(e) < 0 < \phi''(e)$. Also assume that the individual's (increasing and convex) cost of effort, separable from her utility of money, is $\psi(e)$.

(a). What is the full information insurance contract when e is contractible? Specifically, what is e and $b(x)$?

(b). If e is not observable, prove that the optimal contract consists of a premium and a deductible; i.e., $b(x) = x - \delta$. You may assume that a solution exists and the first-order approach is valid.