

PRICE THEORY III  
SPRING 2019  
  
(LARS STOLE)

SOLUTIONS TO  
MIDTERM  
  
UNIVERSITY OF CHICAGO

## Contents

<b>1</b>	<b>Question 1 (20 points)</b>	<b>3</b>
1.1	Part (a) . . . . .	3
1.2	Part (b) . . . . .	4
1.3	Part (c) . . . . .	5
1.4	Part (d) . . . . .	5
<b>2</b>	<b>Question 2 (5 points)</b>	<b>7</b>
<b>3</b>	<b>Question 3 (5 points)</b>	<b>8</b>
<b>4</b>	<b>Question 4 (10 points)</b>	<b>9</b>
<b>5</b>	<b>Question 5 (15 points)</b>	<b>11</b>
<b>6</b>	<b>Question 6 (25 points)</b>	<b>13</b>
6.1	Part (a) . . . . .	13
6.2	Part (b) . . . . .	13
6.3	Part (c) . . . . .	14
6.4	Part (d) . . . . .	15
6.5	Part (e) . . . . .	16

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v1.0 Initial version

## 1 Question 1 (20 points)

Consider the labour-market signalling model from MWG. Recall that: there are two types of workers,  $\theta_H > \theta_L > 0$ , and the probability of the high type is  $\phi \in (0, 1)$ . Type- $\theta$  worker's payoff from wage  $w$  is given by  $u(w, e, \theta) = w - c(e, \theta)$ , where  $c(e, \theta) = e/\theta$  is the cost of obtaining (unproductive) education level  $e \geq 0$ . Worker's reservation utility is zero and there are at least two firms.

### 1.1 Part (a)

What is the range of education levels that can arise in a pooling equilibrium?

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Since firms compete for the workers, they must earn zero profits. Letting  $\mu(e)$  denote the firms' common belief that the worker is of  $\theta_H$  type after observing education level  $e$ , zero profits implies that equilibrium wage function must be given by

$$w^*(e) = \mu(e) \theta_H + (1 - \mu(e)) \theta_L.$$

Bayes rule implies that, in any pooling equilibrium that induces education level  $e^*$ , we must have

$$w^*(e^*) = \phi \theta_H + (1 - \phi) \theta_L.$$

For  $e^*$  to be induced in a pooling equilibrium, it must be the case that, for both types, choosing  $e^*$  is weakly better than choosing some other  $e \neq e^*$  (or taking the outside option of 0). This requires us to firms' off-equilibrium beliefs; i.e.  $\mu(e)$  for  $e \neq e^*$ . Since

$$\theta_H > \theta_L \Rightarrow c(e, \theta_H) < c(e, \theta_L), \forall e,$$

it suffices to check that  $\theta_L$ -type has no incentive to deviate from  $e^*$ . That is,

$$\begin{aligned} u(w^*, e^*, \theta_L) &= \phi \theta_H + (1 - \phi) \theta_L - \frac{e^*}{\theta_L} \geq \mu(e) \theta_H + (1 - \mu(e)) \theta_L - \frac{e}{\theta_L}, \forall e \neq e^* \\ &\Leftrightarrow \phi (\theta_H - \theta_L) - \frac{e^*}{\theta_L} \geq \mu(e) (\theta_H - \theta_L) - \frac{e}{\theta_L}, \forall e \neq e^* \\ &\Leftrightarrow \frac{e^*}{\theta_L} \leq (\phi - \mu(e)) (\theta_H - \theta_L) + \frac{e}{\theta_L}, \forall e \neq e^* \\ &\Leftrightarrow \frac{e^*}{\theta_L} \leq \inf_{e \neq e^*} \left( (\phi - \mu(e)) (\theta_H - \theta_L) + \frac{e}{\theta_L} \right). \end{aligned}$$

We can obtain the highest  $e^*$  by setting  $\phi - \mu(e)$  as large as possible; i.e. setting  $\mu(e) = \mathbf{1}_{\{e=e^*\}}$ . Then, for all  $e^* > 0$ , we need

$$\frac{e^*}{\theta_L} \leq \phi(\theta_H - \theta_L) + \inf_{e \neq e^*} \frac{e}{\theta_L}.$$

For any  $e^* > 0$ ,  $\inf_{e \neq e^*} \frac{e}{\theta_L} = 0$  so that

$$\frac{e^*}{\theta_L} \leq \phi(\theta_H - \theta_L) \Leftrightarrow e^* \leq \theta_L \phi(\theta_H - \theta_L).$$

For  $e^* = 0$ , we need that

$$0 \leq \phi(\theta_H - \theta_L) + \inf_{e \neq 0} \frac{e}{\theta_L}$$

which holds trivially since  $\phi \in (0, 1)$  and  $\theta_H > \theta_L$ . Hence, we conclude that

$$e^* \in [0, \theta_L \phi(\theta_H - \theta_L)].$$

## 1.2 Part (b)

In the set of all separating equilibria, what is the range of education levels that are chosen by the low type,  $\theta_L$ , and what is the range of education levels chosen by the high type,  $\theta_H$ ?

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Since firms must make zero profits in any equilibrium, in any separating equilibrium, equilibrium wage function is given by

$$w^*(e_i^*) = \theta_i, \forall i \in \{H, L\}$$

and  $w^*(e) = \mu(e)\theta_H + (1 - \mu(e))\theta_L$  for all  $e \neq e_H^*, e_L^*$ . The payoff for the  $\theta_L$  type in any separating equilibrium is

$$u(w_L^*, e_L^*, \theta_L) = \theta_L - \frac{e_L^*}{\theta_L}$$

so that  $\theta_L$  type chooses  $e_L^* = 0$ . Hence,  $\mu(0) = 0$ . We must ensure that the low type would not pretend to be high type; i.e

$$\begin{aligned} u(w_L^*, e_L^*, \theta_L) &\geq u(w_H^*, e_H^*, \theta_L) \\ \Leftrightarrow \theta_L &\geq \theta_H - \frac{e_H^*}{\theta_L} \\ \Leftrightarrow e_H^* &\geq \theta_L(\theta_H - \theta_L). \end{aligned}$$

We must also ensure that  $\theta_H$  type wants to choose  $e_H^*$  over any other  $e$ ; i.e.

$$\begin{aligned} u(w_H^*, e_H^*, \theta_H) &= \theta_H - \frac{e_H^*}{\theta_H} \geq \mu(e)\theta_H + (1 - \mu(e))\theta_L - \frac{e}{\theta_H}, \forall e \neq e_H^* \\ \frac{e_H^*}{\theta_H} &\leq \inf_{e \neq e_H^*} \left( (1 - \mu(e))(\theta_H - \theta_L) + \frac{e}{\theta_H} \right). \end{aligned}$$

The right-hand side is largest when we set  $\mu(e) = 0$  for all  $e \neq e_H^*$ , which gives

$$\frac{e_H^*}{\theta_H} \leq (\theta_H - \theta_L) + \inf_{e \neq e_H^*} \frac{e}{\theta_H}.$$

Since we are looking for separating equilibrium, and we already established that  $e_L^* = 0$ , it must be that  $e_H^* > 0$  so that all we need is that

$$\frac{e_H^*}{\theta_H} \leq \theta_H - \theta_L \Leftrightarrow e_H^* \leq \theta_H (\theta_H - \theta_L).$$

Hence, the ranges of separating equilibrium education levels for each type are given by

$$e_L^* = 0, e_H^* \in [\theta_L (\theta_H - \theta_L), \theta_H (\theta_H - \theta_L)].$$

### 1.3 Part (c)

Suppose now that  $c = e$  (independent of  $\theta$ ), but the worker's marginal utility of money depends upon type. Specifically, assume the worker's payoff is

$$u(w, e, \theta) = \theta w - e.$$

How does your answer in (b) change with these new preferences? Explain.

.....

Observe that the new utility function is a monotonic transformation of the original utility functions (divide by  $\theta$ ). It thus follows that all of our previous analysis (in particular, the IC and IR constraints) hold. In other words, the answer to part (b) remains the same in this case.

### 1.4 Part (d)

Suppose again that  $c = e$  (independent of  $\theta$ ) and the worker's marginal utility of money depends upon type, but now the worker's payoff is

$$u(w, e, \theta) = \frac{w}{\theta} - e.$$

How does your answer in (b) change? Explain.

.....

Multiplying both sides by  $\theta$  gives  $w - e\theta$  so that the worker's cost function is, effectively,  $c(e, \theta) = e\theta$ . But this violates the single-crossing assumption that:  $c_{e\theta}(e, \theta) < 0$ . That is, the low-productivity worker finds it cheaper to acquire education. In this case, no separating equilibria exist. If there were and the higher wage was given for more education, then the low-type worker would always prefer to take that wage which is inconsistent with it being the higher wage. If the higher wage has lower education, then both types would pool on the lower education wage.

To show this formally, in any separating equilibria, we need that

$$u(w_L^*, e_L^*, \theta_L) \geq u(w_H^*, e_H^*, \theta_L), \quad u(w_H^*, e_H^*, \theta_H) \geq u(w_L^*, e_L^*, \theta_H).$$

Since  $w_i^* = \theta_i$  for  $i \in \{H, L\}$  in any separating equilibrium, we can rewrite above (using the transformed utility function) as

$$\begin{aligned} \theta_L - e_L^* \theta_L &\geq \theta_H - e_H^* \theta_L \Leftrightarrow (e_H^* - e_L^*) \theta_L \geq \theta_H - \theta_L, \\ \theta_H - e_H^* \theta_H &\geq \theta_L - e_L^* \theta_H \Leftrightarrow \theta_H - \theta_L \geq (e_H^* - e_L^*) \theta_H. \end{aligned}$$

Hence,

$$(e_H^* - e_L^*) \theta_L \geq \theta_H - \theta_L \geq (e_H^* - e_L^*) \theta_H.$$

If  $e_H^* > e_L^*$ , then since  $\theta_H > \theta_L$ , it would contradict the inequality above. If, on the other hand  $e_H^* < e_L^*$ , then we also get a contradiction since

$$0 < (e_H^* - e_L^*) \theta_L \geq \theta_H - \theta_L > 0$$

and so we would have to have  $e_H^* = e_L^*$ .

## 2 Question 2 (5 points)

Consider a moral-hazard contracting problem in which a risk-averse agent can choose  $e \in \{e_L, e_H\}$  at personal cost  $\psi(e)$  and the risk-neutral principal desires to induce the agent to choose the high effort. The influence of effort on output is stochastic and captured by the conditional distribution  $f(x|e)$  on  $[\underline{x}, \bar{x}]$ . The agent's payoff given a wage schedule,  $w(x)$ , output  $x$  and effort  $e$  is

$$u = u(w(x)) - \psi(e),$$

while the principal's payoff is  $x - w(x)$ . Assume that the agent's outside option is 0. Assume also that an optimal contract exists, and that  $w(x) \in (\underline{w}, \bar{w})$  for all  $x$  (i.e., there are no binding constraints on the range of wages). Explain why MLRP implies that the cost minimising wage schedule is increasing in output.

[You may reproduce supporting equations and results from the lectures without proof, but you must state and use them correctly.]

.....

Recall from class that the optimal wage is defined by

$$\frac{1}{u'(w(x))} = \lambda + \mu \left( \frac{f_H(x) - f_L(x)}{f_H(x)} \right),$$

where  $\lambda$  and  $\mu$  are the Lagrange multipliers on the IR and IC constraints, respectively. Since  $u$  is strictly concave,  $w$  is increasing in  $x$  if the right-hand side of the equation is increasing in  $x$ . We argued in class that  $\mu > 0$  so that wages are increasing in  $x$  if and only if the ratio on the right-hand side is increasing in output. This is exactly the MLRP condition.

### 3 Question 3 (5 points)

Consider the competitive screening labour-market model in MWG. There are two workers, with  $\theta_H > \theta_L$ , and the firms screen workers using tasks,  $t$ . Suppose that, in the pure-strategy separating equilibrium,  $\theta_H$  workers choose  $(w_H, t_H)$  and  $\theta_L$  workers choose  $(w_L, t_L)$ . Assume the standard single-crossing property: the cost of task  $t$  is  $c(t, \theta)$  and  $c_{t\theta}(t, \theta) < 0$  for  $t > 0$ . Thus, a type- $\theta$  worker earning wage  $w$  while undertaking  $t$  tasks, has payoff

$$w - c(t, \theta).$$

Prove that  $t_H \geq t_L$ .

.....

Let write down the two incentive compatibility constraints:

$$\begin{aligned} w_H - c(t_H, \theta_H) &\geq w_L - c(t_L, \theta_H), \\ w_L - c(t_L, \theta_L) &\geq w_H - c(t_H, \theta_L). \end{aligned}$$

Summing them together, we get

$$\begin{aligned} w_H + w_L - c(t_H, \theta_H) - c(t_L, \theta_L) &\geq w_H + w_L - c(t_L, \theta_H) - c(t_H, \theta_L) \\ \Leftrightarrow c(t_H, \theta_L) - c(t_H, \theta_H) &\geq c(t_L, \theta_L) - c(t_L, \theta_H). \end{aligned} \quad (3.1)$$

Since  $\theta_H > \theta_L$  the single crossing property of  $c$  then implies that  $t_H \geq t_L$ .

More formally, by way of contradiction, suppose that  $t_H < t_L$ . Then,

$$\begin{aligned} &[c(t_H, \theta_L) - c(t_H, \theta_H)] - [c(t_L, \theta_L) - c(t_L, \theta_H)] \\ &= \left[ - \int_{\theta_L}^{\theta_H} c_{\theta}(t_H, \theta) d\theta \right] - \left[ - \int_{\theta_L}^{\theta_H} c_{\theta}(t_L, \theta) d\theta \right] \\ &= \int_{\theta_L}^{\theta_H} [c_{\theta}(t_L, \theta) - c_{\theta}(t_H, \theta)] d\theta \\ &= \int_{\theta_L}^{\theta_H} \int_{t_H}^{t_L} \underbrace{c_{\theta t}(t, \theta)}_{<0} dt d\theta < 0 \end{aligned}$$

but this contradicts (3.1).<sup>1</sup> Hence, it must be that  $t_H \geq t_L$ .

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<sup>1</sup>I'm using that  $c_{t\theta} \equiv c_{\theta t}$ .



## 4 Question 4 (10 points)

Consider the linear-contract setting of Holmström and Milgrom. Suppose that the principal receives the price  $p$  per unit of output,  $x = e + \varepsilon$ , (i.e. the principal's return is  $px$ ), and  $\varepsilon$  is normally distributed with mean 0 and variance  $\sigma^2$ . The agent's monetary cost of effort is  $C(e) = ke^2/2$ . The principal is risk neutral and the agent has CARA utility with parameter  $r > 0$  and outside option 0 (in units of money).

Derive the value of  $\alpha$  for the principal's optimal linear contract,  $w(x) = \alpha x + \gamma$ .

Hint: If  $z$  is normally distributed with mean  $\mu$  and variance  $\sigma^2$ , and if the agent has CARA utility with risk-aversion parameter  $r > 0$ , then

$$CE = \mu - \frac{1}{2}r\sigma^2.$$

.....

Since  $x \sim N(e, \sigma^2)$ ,

$$w(x) | e \sim N(\alpha e + \gamma, \alpha^2 \sigma^2).$$

The agent's chooses effort to maximise his utility, which is equivalent to the effort that maximises his certainty equivalent; i.e.

$$\arg \max_e \alpha e + \gamma - \frac{k}{2}e^2 - \frac{1}{2}r\alpha^2\sigma^2.$$

The objective function is globally concave in  $e$  so that the first-order condition is both necessary and sufficient:

$$\alpha - ke^* = 0 \Leftrightarrow e^* = \frac{\alpha}{k}.$$

Given this, the problem for the principal is

$$\begin{aligned} \max_{\alpha, \gamma} \quad & \mathbb{E}[px - \alpha x - \gamma | e^*] \\ \text{s.t.} \quad & \alpha e^* + \gamma - \frac{k}{2}(e^*)^2 - \frac{1}{2}r\alpha^2\sigma^2 \geq 0. \end{aligned}$$

Observe that the principal's objective is decreasing in  $\gamma$ , and that  $e^*$  does not depend on  $\gamma$ . Thus, the optimal  $\gamma$  is such that the IR constraint binds; i.e.

$$\begin{aligned} \alpha e^* + \gamma^* - \frac{k}{2}(e^*)^2 - \frac{1}{2}r\alpha^2\sigma^2 &= 0 \\ \Leftrightarrow \gamma^* &= \frac{\alpha^2}{2} \left( r\sigma^2 - \frac{1}{k} \right). \end{aligned}$$

Since  $\mathbb{E}[x | e^*] = \alpha/k$ , the principal's problem then becomes

$$\max_{\alpha} (p - \alpha) \frac{\alpha}{k} - \frac{\alpha^2}{2} \left( r\sigma^2 - \frac{1}{k} \right).$$

Observe that the objective is globally concave so that the first-order condition is both necessary and sufficient:

$$\begin{aligned}\frac{p - \alpha^*}{k} - \frac{\alpha^*}{k} - \alpha^* \left( r\sigma^2 - \frac{1}{k} \right) &= 0 \\ \Leftrightarrow \alpha^* &= \frac{p}{1 + kr\sigma^2}.\end{aligned}$$

## 5 Question 5 (15 points)

Consider a competitive market with adverse selection as in the labour market model of MWG. The seller's product quality,  $\theta$ , is known only to the seller.  $\theta$  is distributed uniformly on  $[\underline{\theta}, \bar{\theta}]$  and the distribution is common knowledge. A buyer who purchases a good of quality  $\theta$  for price  $p$  earns  $\theta - p$ . The seller's reservation value depends upon  $\theta$  and is  $r(\theta) = \theta - \delta$ , where  $\delta > 0$ . A seller who sells a good at price  $p$  earns  $p - r(\theta)$  (on net) given the lost opportunity  $r(\theta)$ . Determine the competitive equilibrium (equilibria), indicating the equilibrium price,  $p^*$ , and the set of seller's who participate in the market,  $\Theta^*$ . Note that the size of  $\delta$  should be considered in your answer, but you may assume that  $p^* \geq \underline{\theta}$ .

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Since the market is competitive, buyers must earn zero profits in equilibrium, and so  $w^*$  must equal the expected value of  $\theta$  among those that participate in the market. Since sellers sell only if the price of the good is above  $r(\theta)$ , the two conditions for the competitive equilibria can be written as

$$\begin{aligned}\Theta^* &= \{\theta \in \Theta : p^* \geq r(\theta)\} = \{\theta \in [\underline{\theta}, \bar{\theta}] : \theta \leq p^* + \delta\}. \\ p^* &= \mathbb{E}[\theta | \theta \in \Theta^*].\end{aligned}$$

There are three cases to consider:

- ▷  $p^* + \delta < \underline{\theta} < \bar{\theta}$ . In this case, no one sells the good so that  $\Theta^* = \emptyset$ . Then,  $p^*$  can be set arbitrarily. But this case is ruled out by the question since  $p^* \geq \underline{\theta}$  and  $\delta > 0$ .
- ▷  $p^* + \delta = \underline{\theta} < \bar{\theta}$ . Then,  $p^* = \underline{\theta}$  and  $\Theta^* = \underline{\theta}$ . But this contradicts the fact that  $\delta > 0$ . So this cannot be an equilibrium. In any case, the question rules out this case since  $p^* \geq \underline{\theta}$  and  $\delta > 0$ .
- ▷  $\underline{\theta} < p^* + \delta \leq \bar{\theta}$ . Then,  $\Theta^* = [\underline{\theta}, p^* + \delta]$  and so

$$\begin{aligned}p^* &= \mathbb{E}[\theta | \theta \in [\underline{\theta}, p^* + \delta]] = \underline{\theta} + \frac{p^* + \delta - \underline{\theta}}{2} \\ \Leftrightarrow p^* &= \underline{\theta} + \delta.\end{aligned}$$

The additional condition we require on  $\delta$  is

$$\begin{aligned}p^* + \delta &= \underline{\theta} + 2\delta \leq \bar{\theta} \\ \Leftrightarrow \delta &\leq \frac{\bar{\theta} - \underline{\theta}}{2}.\end{aligned}$$

Thus, for all  $\delta \in (0, (\bar{\theta} - \underline{\theta})/2]$ ,

$$p^* = \underline{\theta} + \delta, \Theta^* = [\underline{\theta}, \underline{\theta} + 2\delta].$$

▷  $\underline{\theta} < \bar{\theta} < p^* + \delta$ . Then,

$$\Theta^* = \Theta, \quad p^* = \mathbb{E} [\theta | \theta \in [\underline{\theta}, \bar{\theta}]] = \underline{\theta} + \frac{\bar{\theta} - \underline{\theta}}{2} = \frac{\bar{\theta} + \underline{\theta}}{2}$$

For this to hold, it must be that

$$\bar{\theta} < \frac{\bar{\theta} + \underline{\theta}}{2} + \delta \Leftrightarrow \delta > \frac{\bar{\theta} - \underline{\theta}}{2}.$$

## 6 Question 6 (25 points)

Two life insurance companies offer policies to consumers with unknown health types,  $\theta_H > \theta_L > 0$ , where the healthy (high) type occurs in proportion  $\phi \in (0,1)$  in the population. Buyers of insurance know which type they are.

The expected cost of a life insurance policy to a type  $\theta$  customer is  $C - \theta$ , a life insurance company earns a profit of  $p - (C - \theta)$  for each policy it sells to a  $\theta$ -type consumer. The buyers gain value  $v > C$  from purchasing a life insurance policy and have net surplus of  $v - p$  if they purchase a policy at price  $p$ .

Both insurance companies have decided to start using fitbits in their pricing strategies. Specifically, customers will be given free fitbits monitors that will track and certify how many steps they walk each day. The insurance companies can price their policies conditional on the customer reaching a certain number of daily steps.

Assume (unrealistically), that walking has no benefit on prolonging someone's life, but walking is costly for people who have low types, and not as costly for people with high-health types. Specifically, suppose that the cost of reaching  $s$  daily steps to a type- $\theta$  consumer is

$$c(s, \theta) := \frac{\sqrt{s}}{\theta}.$$

The insurance companies simultaneously announce a menu of prices and minimum daily steps for their life insurance policies. The utility of a consumer of type  $\theta$  who purchases a policy  $(p, s)$ , at price of  $p$  and a requirement of  $s$  steps per day, is

$$u(v, p, s, \theta) := v - p - \frac{\sqrt{s}}{\theta}.$$

Lastly, assume that  $\theta_H = 20$  and  $\theta_L = 10$ .

### 6.1 Part (a)

Show that the expected profit of each company is zero in any pure-strategy equilibrium.

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Here, we mean equilibrium in a game theoretic sense (SPNE, to be precise). By way of contradiction, suppose that in an equilibrium, a company makes strictly positive profits via contract  $(p_i, s_i)_{i \in \{L, H\}}$ . Then, the other firm could offer contract  $(p_i - \epsilon, s_i)_{i \in \{L, H\}}$  for some  $\epsilon > 0$  small to capture all of the industry profit, contradicting that this  $(p_i, s_i)_{i \in \{L, H\}}$  was an equilibrium.

### 6.2 Part (b)

Show that there does not exist a pure-strategy pooling equilibrium.

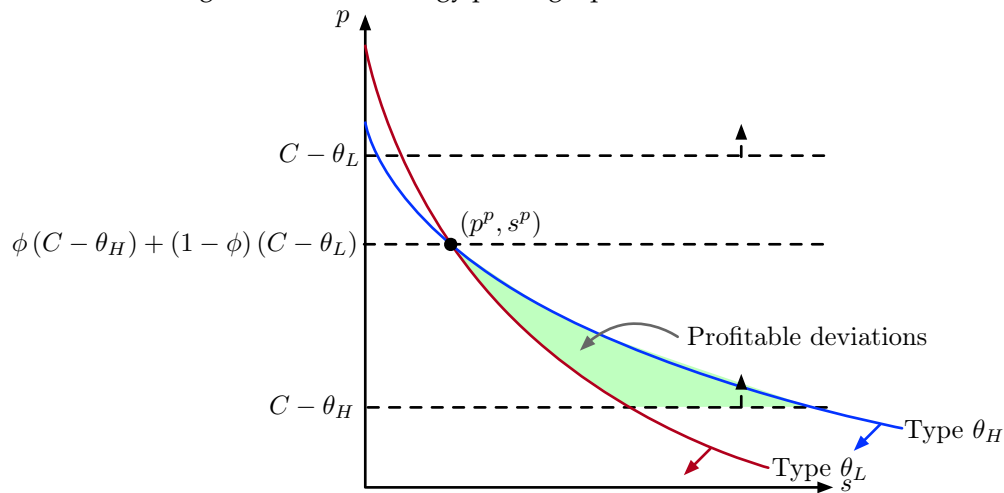
.....

By definition, in any pooling equilibrium, both types purchase the same contracts, say  $(p^p, s^p)$ . Since firms must make zero profits in any equilibrium (part (a)), it must be that

$$p^p = \phi (C - \theta_H) + (1 - \phi) (C - \theta_L).$$

But then, single-crossing implies that there exists a profitable deviation contract with higher number of steps but a lower price that is preferred by type  $\theta_H$  but not  $\theta_L$  (shaded region in figure below).

Figure 6.1: Pure-strategy pooling equilibria do not exist.



### 6.3 Part (c)

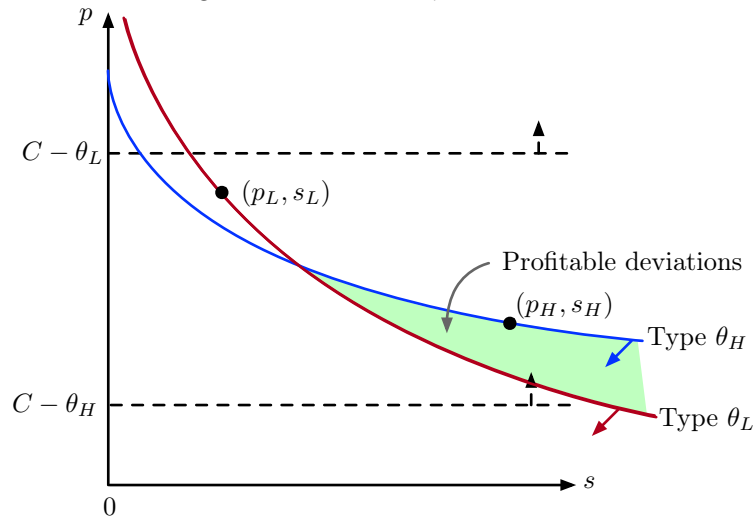
Prove that, in any pure-strategy separating equilibrium, the price paid by *type-θ* consumer is  $p = C - \theta$ .

.....

Let  $(p_i, s_i)_{i \in \{L, H\}}$  be a separating equilibrium. We wish to show that  $p_i = C - \theta_i$  for each  $i \in \{L, H\}$ .

By way of contradiction, suppose first that  $p_L > C - \theta_L$ . Then, either firm could deviate and offer  $(p_L - \varepsilon, s_L)$  for  $\varepsilon > 0$  sufficiently small to attract the  $\theta_L$  workers and still make a strictly positive profit (even more so if  $\theta_H$  is also attracted). Hence,  $p_L \leq C - \theta_L$ .

Now suppose that  $p_H > C - \theta_H$ . Since firms must make zero profits in expectation, it follows that  $p_L < C - \theta_L$ . Consequently, there exists a contract (in the shaded region in the figure) such that only  $\theta_H$  would choose if offered and that leads to a strictly positive profit for the offering firm. Hence,  $p_H \leq C - \theta_H$ .

Figure 6.2: Cannot be  $p_H > C - \theta_H$ .

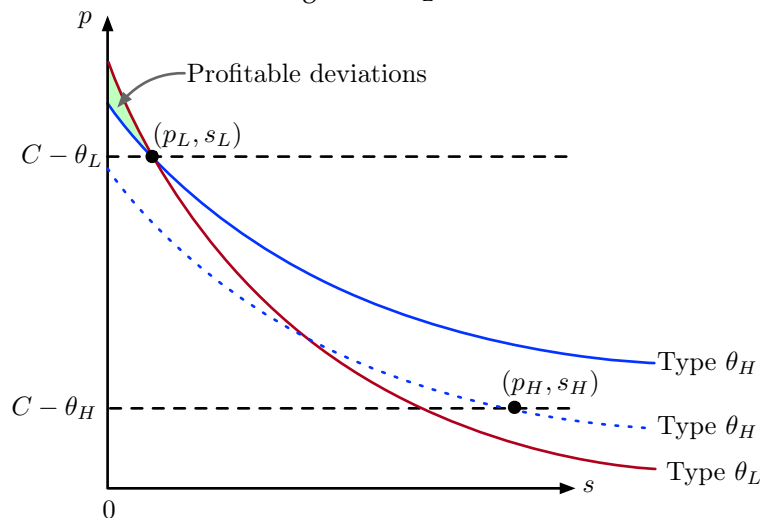
We have now shown that  $p_i \leq C - \theta_i$  for each  $i \in \{L, H\}$ . Since firms must make zero profits in expectation, it must be that  $p_i = C - \theta_i$  for each  $i \in \{L, H\}$ .

#### 6.4 Part (d)

Prove that, in any pure-strategy separating equilibrium, the  $\theta_L$  consumer never walks anywhere (i.e.,  $s_L = 0$ ).

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By way of contradiction, let  $(p_i, s_i)_{i \in \{L, H\}}$  be some separating equilibrium such that  $s_L > 0$ . But then a firm could offer a contract with fewer steps that would be strictly preferred by the low type at a higher price (that would not attract type- $\theta_H$  consumers). So this cannot be an equilibrium. That is,  $s_L = 0$  in any equilibrium (recall we already ruled out pooling equilibria).

Figure 6.3:  $s_L = 0$ .

## 6.5 Part (e)

Characterise the pure-strategy equilibrium of the game assuming that one exits. How many steps per day does a type- $\theta_H$  walk to get a discounted policy?

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So far, we established that

$$p_H = C - \theta_H,$$

$$p_L = C - \theta_L,$$

$$s_L = 0$$

in any (separating) equilibrium. So it remains to specify the equilibrium  $s_H$  (since we get to assume that an equilibrium exists). In any separating equilibrium, the type- $\theta_H$  consumers are offered a step requirement that just makes type- $\theta_L$  consumers indifferent between the two policies (there is only “envy” case in this set up):

$$\begin{aligned} v - (C - \theta_L) &= v - (C - \theta_H) - \frac{\sqrt{s_H}}{\theta_L} \\ \Leftrightarrow \frac{\sqrt{s_H}}{\theta_L} &= \theta_H - \theta_L \\ \Leftrightarrow s_H &= \theta_L^2 (\theta_H - \theta_L)^2 \\ &= 10,000. \end{aligned}$$

If this were not the case, then a firm could profitably deviate and offer a contract to the  $\theta_H$  type with fewer steps and a lower price that attracts only type- $\theta_H$  consumers.

Figure 6.4: Pure-strategy separating equilibrium.

