[thm]

Puzzles in Asset Pricing

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Theory of Income II 2019

Plan for the day

Today we focus on:

- Mehra and Prescott, 1985. "The Equity Premium" *Journal of Monetary Economics.*
- Kocherlakota, 1996. "The Equity Premium: It's Still a Puzzle" *Journal of Economic Literature.*

Motivation

- Average real annual yield on S&P 500 index approx. 7%
- Average yield on short-term debt, Treasury Bill, less than 1%.

This two points imply:

1. Excess return on stock is high.
2. Risk free rate is low.

Motivation (cont.)

- **Q:** Can this large difference in yields be accounted for by models that abstract from frictions in the economy? (Mehra and Prescott original question)
- **A:** No, they cannot and this is, primarily, the origin of the puzzles.
- **Q:** Why this constitutes a puzzle?
  - Representative agents models are widely use in macro to explain several facts.
  - Representative agents models of asset returns are integral part of macro and international economics.
  - Failures of this model reflect our ignorance in this topics.

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Mehra and Prescott (1985)

Motivation:

- Average return on equity (6.98%) has far exceeded the average return on short-term default-free debt (0.8%).

Research question:

- *Can this large difference in yields be accounted for by models that abstract from frictions in the economy?*

Methodology

- Quantitative theoretical exercise.
- Calibrate model to match moments of the data.
- Compare implication of the model

Main result

- It cannot!
- Requires impossible high risk aversion!
- *Equity premium puzzle:* the covariances of the return with consumption only explains the observed excess return if the agent is implausibly risk averse.

Model

Variation of Lucas' (1978) pure exchange model.

Preferences

- Representative consumer

$$U = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right], \quad \beta \in (0,1)$$

$$u(c) = \frac{c^{1-\alpha} - 1}{1-\alpha}$$

- α measures the relative risk aversion of the agent.

Model

Endowment

- One Lucas' tree producing the stream of goods and one equity share on this unit that is competitively traded.
- Return on the tree = Market return.
- Tree pays dividends y_t. Growth rate of dividend follows a Markov process

$$y_{t+1} = x_{t+1} y_t$$
$$x_{t+1} \in \{\lambda_1, \ldots, \lambda_n\}$$
$$\Pr\{x_{t+1} = \lambda_j | x_t = \lambda_i\} = \phi_{ij}$$

- In equilibrium consumption equals the dividends $\Longrightarrow$ this is an assumption on consumption growth.

Calibration

- Distinguish preference parameters from technology.
- Technology: two state Markov chain

$$\lambda_1 = 1+\mu+\delta, \quad \lambda_2 = 1+\mu-\delta$$
$$\phi_{11} = \phi_{22} = \phi \qquad \phi_{12} = \phi_{21} = 1-\phi$$

  μ: to mach average growth of consumption,
  δ: to mach its variance,
  ϕ: to match its serial correlation (first-order).
- Preference parameters: search for values that yield the observed equity premium and risk-free rate.
  - Constrained $\alpha \leq 10$ base on the literature
  - $\beta \in (0,1)$

What does the model predicts?

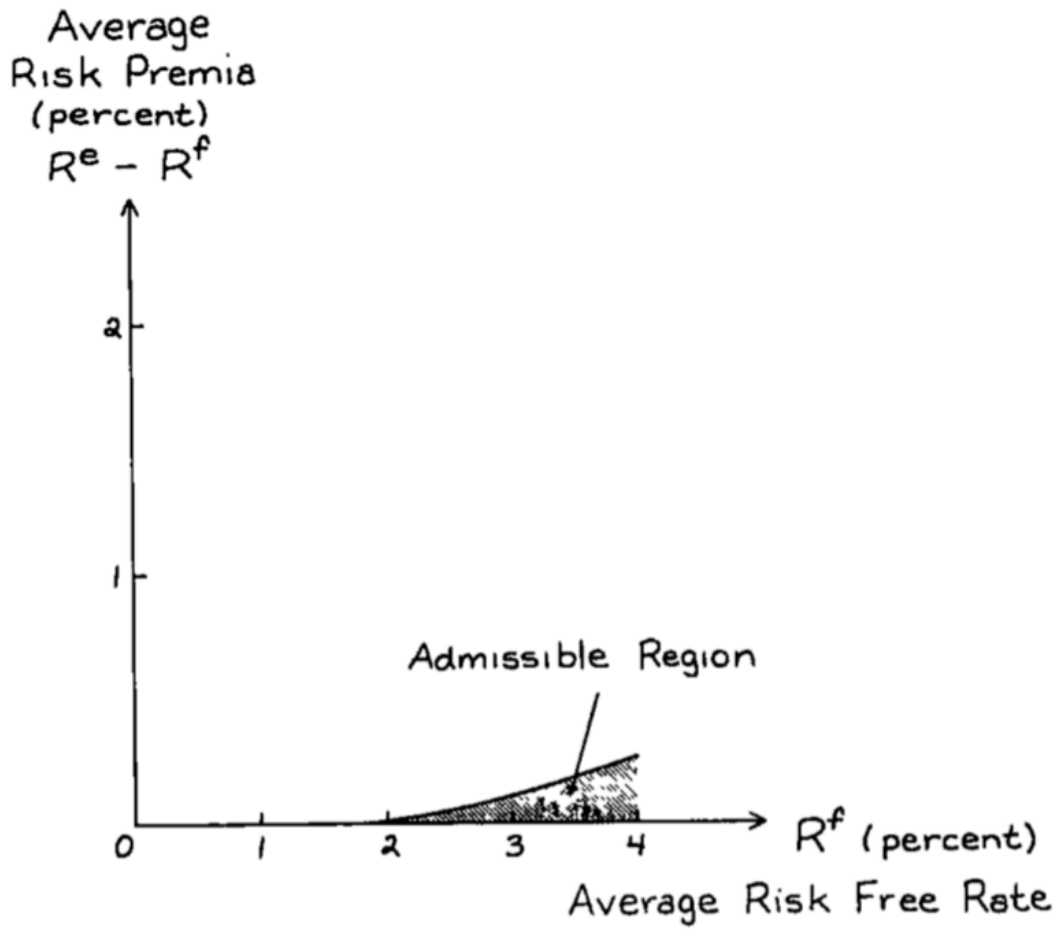

Fig. 4. Set of admissible average equity risk premia and real returns.

Largest equity premium obtainable with the model is 0.35%. The result is robust to other specification of technology and its calibrated values.

We knew this already - Hansen & Singleton (1982)

- Econometric test that start with Euler Equation of Consumption:

$$\mathbb{E}_t \left[e^{-\rho} \left(\frac{c_{t+1}}{c_t} \right)^{-\alpha} R_i^{t+1} \right] = 1$$

  Assumed period utility $u(c) = \frac{c^{1-\alpha}-1}{1-\alpha}$.
- Estimate (ρ, α) through GMM using instruments z_t. Large value of α.
- Restrict α to "realistic" values and test if the system of Euler equations on the set of assets hold with equality.

Kocherlakota (1996)

- Focus on two puzzle (arguably the most important)
  - **Equity premium puzzle:** we need impossible high risk aversion to match the equity premium.
  - **Risk free rate puzzle:** the yield on short-term default-free debt is too low.

Example

CRRA utility with RRA equal to γ, discount factor $e^{-\rho}$. Define $x_{i,t} \equiv \log R_{it}$ and $\hat{x}_{it} \equiv x_{it} - x_{it-1}$ and let $\begin{pmatrix} \hat{c}_t \\ r_{it} \end{pmatrix} \sim N(\mu, \Sigma)$. It can be shown that the FOC implies,

$$r_{f,t} = \rho + \gamma \mathbb{E}[\hat{c}_t] - \frac{\gamma^2}{2} \mathbb{V}[\hat{c}_t]$$

$$\mathbb{E}[r_{it}] + \frac{1}{2} \mathbb{V}[r_{it}] - r_{ft} = \gamma cov[\hat{c}_t, r_{it}]$$

Data: $\bar{R}_{M,t} = 1.07$; $\bar{R}_{f,t} = 1.01$, $\bar{\hat{c}}_t = 0.017$, $Var[\hat{c}] = 0.036^2$ and $cov[\hat{c}, r_{m,t}] = 0.003$

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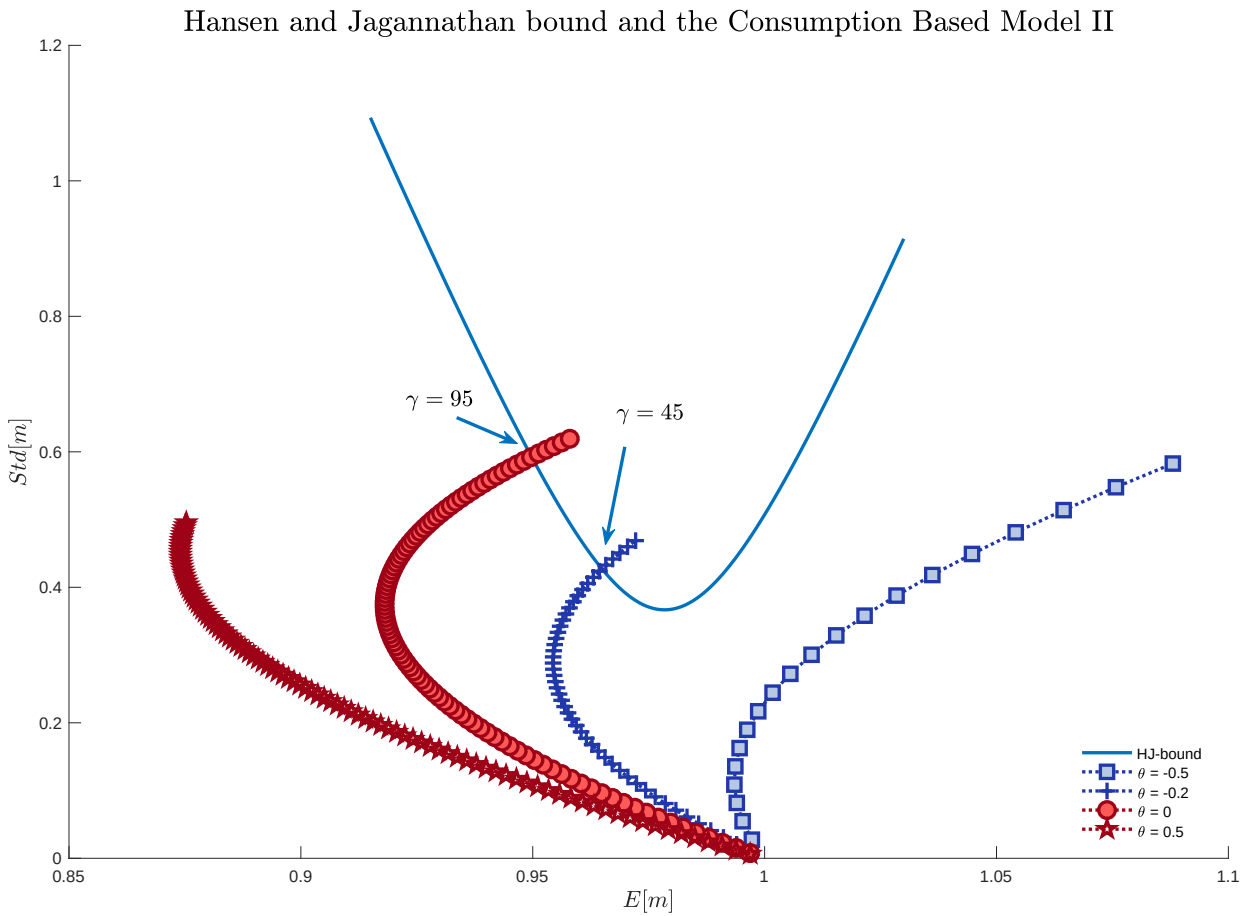
Hansen - Jaganathan bound for



Intuition for each puzzle

- **Equity premium puzzle**
  We explained the excess of return of an asset by its co-movements with typical investor consumption (not the variance of the asset). Risk aversion implies that the typical investor wants to keep consumption stable across states. So if the asset positively covariates with consumption (the return is high when consumption is high) the agent demands an excess return to keep the asset. RRA in standard value cannot explained the observed excess return (it predicts low value) $\Longrightarrow$ the agent needs to be more risk averse.
  Why individuals are so averse to highly procyclical risk associated with returns?
- **Risk free rate puzzle**
  The intertemporal elasticity of substitution implies smoothness of consumption across time as well. Large equity premium implies that investors are highly risk averse $\Longrightarrow$ Agent dislike growth.
  However, despite the low yield of Treasure bills individual save, consumption grows at a high rate (2%).
  Why people save when returns are low?

Kocherlakota (1996)

Puzzles robust to different specifications. They relay in three assumptions:

1. **Individual preference:** representative consumer, expected discounted value of a stream of utilities generated by power utility.
2. **Asset market structure:** markets are complete.
3. **Asset trading is costless:** the only impediment to keep the assets is its price. Asset markets are frictionless

Any model that 'explains' these puzzles must break one of these assumptions. Are those models reasonable?

1. Low risk-free rate can be explained by: changing the preferences (Epstein-Zin), borrowing constraint, among others.
2. Equity risk premium: agents highly averse to consumption risk, or trading cost costly relative to bonds (little evidence on those)

$\Longrightarrow$ Equity premium puzzle still unresolved

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Preference Modifications

Two possible modifications:

1. **Generalized Expected Utility:** Epstein-Zin preferences recursive way to describe preferences.

$$U_t = \left\{ c_t^{1-\rho} + \beta \left[\mathbb{E}_t U_{t+1}^{1-\alpha} \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}$$

Separates risk aversion α from IES ρ^{-1}.

$$\mathbb{E}[r_{w,t}] + \frac{1}{2}\mathbb{V}[r_{w,t}] - r_{ft} = \rho \frac{1-\alpha}{1-\rho} cov[\hat{c}_t, r_{w,t}] + \frac{\alpha-\rho}{1-\rho}\mathbb{V}[r_{w,t}]$$

2. **Habit formation:** consumption in $t-1$ affects MU_t (aggregate cons. *catching up with the Joneses*, or individual cons. *internal habit).*

$$U = \mathbb{E}\left[\sum_{s=0}^{\infty} \beta^s \frac{(c_{t+s} - \lambda c_{t+s-1})^{1-\alpha}}{1-\alpha} \right], \quad \lambda > 0$$

"If I buy a BMW rather than a [Fitito] today, then I am certainly better off today; however, the having the BMW "spoils" me and reduces my utility from all future car purchases"

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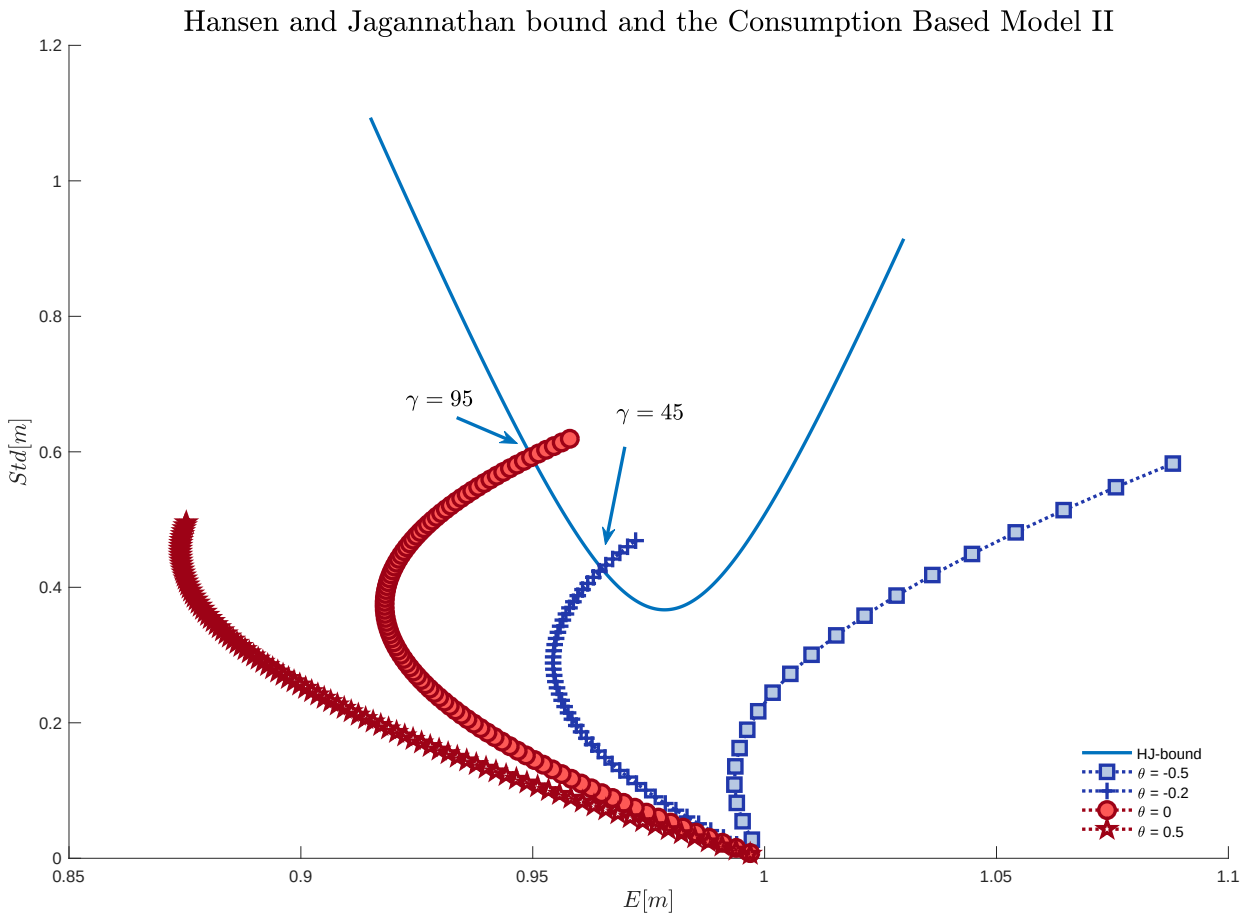

Asset market

This literature tries to explain the existence of the puzzles through the presence of incompleteness and trading cots.

1. **Incomplete markets:** breaks existence of representative consumer, hence individual consumption cannot be understood from per-capita consumption. Individual consumption presents idiosyncratic un-insurable risk $\Longrightarrow$ individual consumption might covariate more with stock explaining the risk premia.
   Dynamic models such as Aiyagari (1994) individuals can insure against idiosyncratic risk by trading with other individuals.
2. **Trading costs:** (i) borrowing constraint, (ii) difference in cost to trade stocks relative to bonds.
   To generate sizeable equity premium is to make trading in stock market more costly than in the bond market. The source of this cost is unclear.
   People held bond because their cost is low equivalently people held money because they derive utility from it (a bit depressing as a possible answer).

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