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Economics 331, Winter 2019

Professor Stokey

FINAL EXAM: PART I

Answer each question in the space provided. Points for each are as indicated.

1. Short answers (10 points) 8

a. Briefly describe the main idea in Atkeson and Ohanian's 2001 *QR* paper, "Are Phillips curves useful for forecasting inflation?" What is their main conclusion, and what evidence do they base it on?

They speculate that using of NAIRU models for forecasting inflation based on unemployment (or other indeces of economic activity) is hardly better than naive forecasting (that future inflation will be the same as the current one).

They do a couple of simulations to see what a forecast would look like it ~~if~~ it was based on various specifications of NAIRU models and compare the resulting RMSE with that of naive forecast.

For a level of inflation naive model is better than NAIRU. For a rate of inflation NAIRU models are slightly better but the difference in RMSE is trivial.

The authors hypothesize that the failure of Philips curves ~~lies in the a~~ is due to the attempt to capture a redeced form dependence between unemployment and inflation — this dependence itself should depend on the economic situation as a whole which is too changeable.

(5)

b. Briefly describe the main idea Mehra and Prescott's 1985 *JME* paper "The equity premium: a puzzle." [Turn the page for space.]

The puzzle of equiti premium lies in the fact that the difference between a risk-free bond and risk-bearing assets is just too big. Moreover, if high risk aversion is assumed, ~~the~~ under the classic CRRA utility function, this would also lead the consumer to be ~~....~~ willing to smooth across ~~states~~ time the same way they do across the states, which would also ~~the~~ drive the risk-free rate up.

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2. Asset pricing (10 points)

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Recall from class the asset pricing formula

$$u'\left(\Sigma_n y_{nj}\right) p^e_{nj} = \frac{1}{1+\rho} \sum_{k=1}^{J} q_{jk} u'\left(\Sigma_m y_{mk}\right)\left(y_{nk} + p^e_{nk}\right), \qquad \text{all } j, n,$$

where $u(c)$ is the utility function, $\rho > 0$ is the discount rate, $j, k = 1, \ldots, J$ index states; $Q = [q_{jk}]$ is a $J \times J$ transition matrix for the state; $m, n = 1, \ldots, N$ index assets; $Y = [y_{nj}]$ is an $N \times J$ matrix of dividends; and $P^e = [p^e_{nj}]$ is an $N \times J$ matrix of equilibrium asset prices.

Suppose the equilibrium asset prices P^e have been calculated. Consider a new asset, with dividend vector $y_{N+1} = (y_{N+1,1}, \ldots, y_{N+1,J})$. Suppose the quantity of the asset is very small relative to the aggregate payoffs of the old assets.

a. Describe as precisely as you can the condition(s) under which it is possible to characterize its price vector $p^e_{N+1} = \left(p^e_{N+1,1}, \ldots, p^e_{N+1,J}\right)$ in terms of P^e, the prices of existing assets.

If there exists a $\Lambda_{J\times N}$, such that $\Lambda Y = y_{N+1}$, is a column

then $P^e_{N+1} = \Lambda_{J\times N} P^e$.

So the condition is ~~is a li~~ the dividend vector of a new asset can be represented as a linear combination of the existing assets.

(where in $\Lambda_{J\times N}$ the rows are replicate of one another)

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b. Suppose the condition(s) in (a) are satisfied. Describe p^e_{N+1} as precisely as you can.

$p^e_{N+1} = \Lambda P^e$

What in matrix:

If $y_{n+1,j} = \sum_i \lambda_k y_{k,j}$ for each j

$p^e_{n+1,j} = \sum_i \lambda_k p^e_{k,j}$ for all j

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c. Suppose the condition(s) in (a) are **not** satisfied. Precisely describe a theoretical method for characterizing p^e_{N+1} in terms of the primitives ρ, u, Q, Y.

Then we have to use the formula. If the quantity of asset is very small, then the marginal utilities remain the same in all states.

$$u'\left(\sum_n y_{nj}\right) p^e_{n+1,j} = \frac{1}{1+\rho} \sum_{k=1}^{J} q_{jk}\, u'\left(\sum_m y_{mk}\right)\left(y_{n+1,k} + p^e_{n+1,k}\right)$$

~~If we want a matrix expression, then define~~

~~An~~ ~~[illegible]~~

In matrices

$$A_{J\times J} = \begin{pmatrix} u'\left(\sum_i y_{n1}\right) & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & \ddots & \\ 0 & & & u'\left(\sum_i n_J\right) \end{pmatrix}$$

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$$A\, p^e_{n+i} = \frac{1}{1+\rho} Q A \left[Y_{n+1} + P^e_{n+1} \right]$$

3. Financing debt redemption (25 points) 23/

Consider the Lucas-Stokey (1983) model, specialized to quasilinear utility. There is one good, which is nonstorable, produced using labor as the only input. Assume there is no government spending in any period, so the resource constraint is

$$c_t + x_t \leq 1, \quad \text{all } t,$$

where c_t is private consumption and x_t is leisure.

The preferences of the representative household are additively separable over time, linear in consumption, and quadratic in labor supply

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \left[c_t - \frac{1}{2} \alpha \left(1 - x_t\right)^2 \right],$$

where $\rho > 0$ and $\boxed{\alpha > 1}$. The initial government debt has a maturity structure with a 2-period cycle. In particular, the debt outstanding at $t = 0$ is

$$_0b_t = \begin{cases} \hat{b}, & t = 1, 3, 5, \ldots, \\ 0, & \text{otherwise,} \end{cases}$$

where $\hat{b} > 0$ is not too large.

The government must raise revenue from a tax on labor income. In particular, in each period a flat-rate tax can be levied, and the rate can change from period to period. Let τ_t denote the tax rate in period t, so $\tau_t(1 - x_t)$ is tax revenue in period t.

Let p_t denote the price of goods in period t.

a. Write the budget constraint and first order conditions for the household. [Turn the page for more space.]

Budget: $\sum_{t=0}^{\infty} p_t \left[c_t - (1-\tau_t)(1-x_t) - {}_0b_t \right] \leq 0$

FOC:

$$\frac{\partial}{\partial C_t}: \left(\frac{1}{1+\rho}\right)^t - \lambda p_t = 0$$

$$\frac{\partial}{\partial X_t}: \left(\frac{1}{1+\rho}\right)^t \cdot \alpha(1-X_t) - \lambda p_t(1-\tau_t) = 0$$

$$p_t = \left(\frac{1}{1+\rho}\right)^t \cdot \frac{1}{\lambda}$$

$$\left(\frac{1}{1+\rho}\right)^t \alpha(1-X_t) - \left(\frac{1}{1+\rho}\right)^t (1-\tau_t) = 0$$

$$\boxed{\alpha(1-X_t) = (1-\tau_t)}$$

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b. What is the implementability constraint for the government? Does the government face any other constraints?

$p_t = \frac{1}{\lambda}\left(\frac{1}{1+\rho}\right)^t \Rightarrow \frac{p_t}{p_0} = \left(\frac{1}{1+\rho}\right)^t$.

Normalize $p_0 = 1$.

Plug into the budget everything derived in FOC:

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \left[C_t - {}_0B_t - \alpha(1-X_t)^2\right] = 0$$

This is the implementability constraint.

The government also faces a resource constraint.

(this is enough, as consumer's budget + RC ⇒ government budget and implementability takes care of the c. budget)

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Time-consistency may be another constraint, but let us assume the maturity structure is rich enough, so that the ~~the~~ perfect commitment allocation is achievable.

c. What are the first order conditions for the government's problem.

$$\mathcal{L} = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left[c_t - \frac{1}{2}\alpha(1-x_t)^2\right] - \lambda\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t\left(c_t - \circ b_t - \alpha(1-x_t)\right)\right] - + \sum_{t=0}^{\infty} \mu_t \left(1 - c_t - x_t\right)$$

$$\text{FOC: } \frac{\partial}{\partial c_t}: \left(\frac{1}{1+\rho}\right)^t - \left(\frac{1}{1+\rho}\right)^t \lambda - \mu_t = 0$$

$$\frac{\partial}{\partial x_t}: \frac{1}{1+\rho^t}\left[\alpha(1-x_t) - \lambda\left(2\alpha(1-x_t)\right)\right] - \mu_t = 0$$

$$\Rightarrow \alpha(1-x_t) - \lambda\left(2\alpha(1-x_t)\right) = (1-\lambda)$$

$\Rightarrow$ x_t is constant over time

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d. Solve for the Ramsey allocation as explicitly as you can.

x_t is constant. We can now use the two constraints.

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \left[(1-x) - \circ b_t - \alpha(1-x)^2\right] = 0$$

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t\left[(1-x) - \alpha(1-x)^2\right] = (1+\rho)\frac{\left[(1-x) - \alpha(1-x^2)\right]}{\rho}$$

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \circ b_t = 0 + \frac{1}{1+\rho}\hat{b} + 0 + \left(\frac{1}{1+\rho}\right)^2\hat{b} + \dots =$$

$$= \frac{\hat{b}}{1+\rho}\frac{1}{1-\left(\frac{1}{1+\rho}\right)^2} = \frac{\hat{b}}{(1+\rho)}\frac{(1+\rho)^2}{\rho(\rho+2)} = \hat{b}\frac{1+\rho}{\rho(\rho+2)}$$

From this:

$$\cancel{(1+\rho)}\frac{(1-x) - \alpha(1-x)^2}{\cancel{\rho}} = \hat{b}\frac{\cancel{1+\rho}}{\cancel{\rho}(\rho+2)}$$

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consumption and multiplier!

$$\alpha(1-x)^2 - (1-x) + \frac{\hat{b}}{\rho+2} = 0$$

$$\alpha x^2 - 2\alpha x + \cancel{1} - \cancel{1} + x + \frac{\hat{b}}{\rho+2} = 0$$

$$\alpha x^2 + (1-2\alpha)x + \frac{\hat{b}}{\rho+2} = 0$$

$$x_{1,2} = \frac{(2\alpha - 1) \pm \sqrt{(1-2\alpha)^2 - 4\alpha\frac{\hat{b}}{\rho+2}}}{2\alpha}$$

0

We need to choose out of this the one that gives higher utility.

e. Are consumption and labor supply higher or lower in the periods when debt is due? Are the associated tax rates higher or lower?

The labor supply is constant over ~~[illegible]~~ time. Given that in all periods $c_t = 1 - x_t$ and $x_t = x$, consumption is constant over time, as well.

$$\tau_t = \alpha(1-x_t)^2 = \alpha(1-x)^2 = \text{const}$$

So everything is constant over time, does not change with periods in which the maturity comes.

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f. What additional condition is needed to determine the levels for consumption, labor supply, and the tax rates?

We need to know more $\hat{\delta}$. The efficient $x \overset{ef.}{=} \frac{\alpha - 1}{\alpha}$.

We need to choose the one out of $x_{1,2}$ that is closer to this number.

Then, $c = 1 - x$

$\tau = \alpha(1-x)$

So as long as we know which x is better out of $x_{1,2}$, we know everything asked here

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