Positive Long Run Capital Taxation: Chamley-Judd Revisited
(Straub and Werning)

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Theory of Income II 2019
(Based on Werning's presentation)

Today

- Straub & Werning (2018), *Positive Long Run Capital Taxation: Cahmley-Judd Revisited*. (Working paper).
- How to compute the stationary distribution with the computer?

Motivation

- Classic question in public finance: **Should we tax capital income?**
- Two common rationales:
  - reduce distortionary labor taxes.
  - redistribution
- Potentially large efficiency costs

Motivation (cont.)

Two benchmark models to think about capital income tax:

- **Chamley (1986):**
  - Representative agent model.
  - Trade-off: lower labor taxes vs. efficiency
- **Judd (1985)**
  - Two class model: worker vs capitalist
  - Trade-off: redistribution vs. efficiency
- **Both models:** zero tax on capital in the steady state.
- **Intuition:** not clear, the result surprise many economist at the time (today I don't know it either).

This paper

- Revisit the Chamley-Judd results using their own models.
  - Show their results hold under certain assumptions on preferences (not explicit in their original work)
  - Particularly important the IES. Overturn conclusions when IES<1 (Judd model, Chamley requires something else).
- What went wrong in their proof?
- Their results require convergence to interior steady state for quantities and multipliers. Under certain assumptions:
  - Judd (1985): possibly non-interior steady state for quantities
  - Chamley (1986): possibly non-interior steady state for multipliers

Road map

- Judd (1985): Capital taxation and redistribution.
  - Model
  - Review of Judd (1985) result
  - How to overturn the result

Judd (1985)

- Two agents: Capitalists and workers with preference over consumption
- **Capitalists**
  - only capital income
  - own initial capital stock k_0
- **Workers**
  - only labor income, inelastic labor supply of 1
  - don't have access to the saving market (consume hand-to-mouth)
- **Policy instruments**
  - capital taxes, lump-sum transfers to workers
  - no government bonds, no consumption taxes
  - full ex-ante commitment to tax policy

Environment

- **Workers:**

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t), \quad c_t = w_t + T_t$$

- **Capitalists:**

$$\max \sum_{t=0}^{\infty} \beta^t \frac{C^{1-\sigma}}{1-\sigma},$$

$$C_t + a_{t+1} = R_t a_t, \quad a_{t+1} \geq 0$$

R_t =after-tax interest rate on capital.

- **Firm:**

$$R_t^* = f'(k_t) + 1 - \delta, \quad \boxed{w_t = f(k_t) - f'(k_t)}$$

- **Market clearing:**

$$c_t + C_t + g + k_{t+1} = f(k_t) + (1-\delta) k_t, \qquad a_t = k_t$$

- **Balanced budget:**

$$g + T_t = (R_t^* - R_t) k_t$$

Optimality + Market Clearing

- **Workers:**

$$c_t = f(k_t) - f'(k_t) + T_t$$

- **Capitalists:**

$$\beta U'(C_t)(C_t + k_{t+1}) = U'(C_{t-1}) k_t$$
$$\lim_{t\to\infty} \beta^t U'(C_t) k_{t+1} = 0$$

- **Market clearing:**

$$c_t + C_t + g + k_{t+1} = f(k_t) + (1-\delta) k_t, \qquad a_t = k_t$$

Ramsey Problem

- Planner maximizes weighted sum of utilities

$$
\begin{aligned}
\max \sum_{t=0}^{\infty} \beta^{t}\left\{u\left(c_t\right)+\gamma U\left(C_t\right)\right\} \\
s.t.\ c_t+C_t+g+k_{t+1}=f\left(k_t\right)+(1-\delta)\, k_t, \\
\beta U'\left(C_t\right)\left(C_t+k_{t+1}\right)=U'\left(C_{t-1}\right) k_t, \\
\lim_{t\to\infty} \beta^{t} U'\left(C_t\right) k_{t+1}=0.
\end{aligned}
$$

- Focus on the case where redistribution goes from **capitalists**$\rightarrow$**workers.**
    - requires sufficiently low γ.
    - for simplicity let $\gamma=0$

Ramsey Problem: FOC

- Let μ_t be the multiplier for the Implementability constraint and λ_t the multiplier for the resource constraint.
- Define $\kappa_t = k_t / C_{t-1}$, $v_t = U'(C_t)/u'(c_t)$.

$$\mu_0 = 0 \tag{1}$$

$$\lambda_t = u'(c_t) \tag{2}$$

$$\mu_{t+1} = \mu_t \left(\frac{\sigma - 1}{\sigma \kappa_{t+1}} + 1 \right) + \frac{1}{\beta \sigma \kappa_{t+1} v_t} \tag{3}$$

$$\frac{u'(c_{t+1})}{u'(c_t)} \left[f'(k_{t+1}) + 1 - \delta \right] = \frac{1}{\beta} + v_t (\mu_{t+1} - \mu_t) \tag{4}$$

Ramsey Problem: Solution

Theorem

(Judd, 1985 - adjusted) Suppose quantities and multipliers converge to an interior steady state. Then, the tax on capital is zero in the limit: $\mathscr{T}_t = 1 - \frac{R_t}{R_t^*} \to 0$.

Proof.

Interior steady state implies: $c_t = c > 0$, $C_t = C > 0$, $k_t = k > 0$ and $\mu_t = \mu$. Using the previous equations:

$$R^* = \frac{1}{\beta}$$

$$R = \frac{1}{\beta}$$

Hence $\mathscr{T}_t \to 0$. $\square$

Ramsey Problem: Solution

- $\cdots$ **or not?**

Ramsey Problem (Lansing,1999 and Reinhorn, 2002 and 2013)

- Simple case: $\sigma = 1 \implies U(C) = \log C$.Optimality in capitalist's problem:

$$C_t = (1-\beta) R_t k_t$$
$$k_{t+1} = \beta R_t k_t$$

- Substitute into Planner's problem ($\gamma = 0$):

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$s.t.\ c_t + \frac{1}{\beta} k_{t+1} + g \leq f(k_t) + (1-\delta) k_t$$

Neoclassical growth model with higher cost of capital!

- Steady State implies $R^* = 1/\beta^2$ and $R = 1/\beta$. The tax is then $\mathscr{T} = 1-\beta$.

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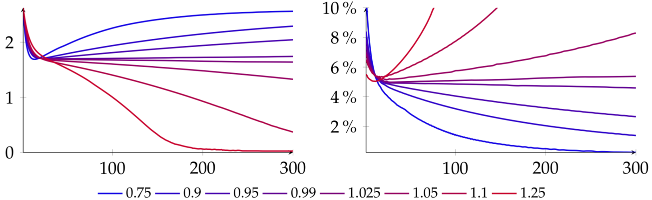
Straub and Werning on Judd result

- The previous example showed a case where an interior steady state for quantities exists however tax on capital are positive.
- Why? Multipliers do not converge to an interior solution! (Reinhorn 2002).
- The result is not specific to log-preferences.
- When $\sigma > 1$ taxes are positive in the long run but non-interior steady state $c_t = 0$ $k = k_g$.
  - Haven't got through the entire proof but, workers own $n = 1$, $k = k_g \implies w = f(k_g) - f'(k_g) \implies T = -[f(k_g) - f'(k_g)]$.
  - What happened with redistribute towards workers???

The result in a graph

Figure 1: Optimal time paths for capital (left) and wealth taxes (right).



Note. This figure shows the optimal time paths of capital k_t (left panel) and wealth taxes $\mathcal{T}_t$ (right panel) for various values of the inverse IES σ.

Stationary distribution

Fix notation:

- Aiyagari type model. Assets are denote with a, labor shock with ℓ.
- Policy function for assets tomorrow $g(a, \ell)$.
- Distribution of assets and labor productivity **today** $\phi_t(a, \ell)$ where $(a, \ell) \in \mathscr{A} \times \mathscr{L}$.
- Distribution of assets and labor productivity **tomorrow** $\phi_{t+1}(a, \ell)$ where $(a, \ell) \in \mathscr{A} \times \mathscr{L}$.
- In what follow we let $\mathscr{L} = \{\ell_L, \ell_H\}$ and $\mathscr{A} = \{a_1, a_2, \cdots, a_n\}$.
- ℓ is first order Markov. Transition matrix Π.

What to do?

- Construct the "Law of Motion" for ϕ_t.
- Translate that into the computer.

Stationary distribution

- I want to know the proportion of agents with state (a_k, ℓ_j) tomorrow given ϕ_t. That is given by:

$$\phi_{t+1}(a_k, \ell_j) = \sum_{m=1}^{2} \left\{ \sum_{i=1}^{n} \mathbf{1}\{g(a_i, \ell_m) = a_k\} \phi_t(a_i, \ell_m) \Pi_{mj} \right\}.$$

- Decompose the sum to understand the logic:
  - $\mathbf{1}\{g(a_i, \ell_m) = a_k\} = 1$ if agent with state (a_i, ℓ_m) chooses a_k for tomorrow.
  - $\mathbf{1}\{g(a_i, \ell_m) = a_k\} \phi_t(a_i, \ell_m)$ is the proportion of agents today with state (a_i, ℓ_m) that choose a_k for tomorrow.
  - $\mathbf{1}\{g(a_i, \ell_m) = a_k\} \phi_t(a_i, \ell_m) \Pi_{mj}$ from those only Π_{mj} will have ℓ_j tomorrow.
  - the sums just account for the many possibilities to get to (a_k, ℓ_j).

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Stationary distribution

- The stationary distribution solves:

$$\phi\left(a_k,\ell_j\right)=\sum_{m=1}^{2}\left\{\sum_{i=1}^{n}\mathbf{1}\left\{g\left(a_i,\ell_m\right)=a_k\right\}\phi\left(a_i,\ell_m\right)\Pi_{mj}\right\}$$

- Suggest how to obtain ϕ:
  1. Star with a guess for ϕ: $\phi_0\left(a,\ell\right)$
  2. Use the above equation to compute $\phi_1\left(a,\ell\right)$.
  3. Check the difference between the two: $\max_{\{a,\ell\}}\left|\phi_1-\phi_0\right|$. If less than tolerance error stop. Otherwise set $\phi_0=\phi_1$ an repeat.

Remark

It is possible to state the equation above in terms of matrices and then just solve the system:

$$\left(\mathbb{I}-\mathbb{T}\right)\phi=0.$$

That is ϕ is the eigenvector associated with the unit eigenvalue of the matrix $\mathbb{T}$.