Reputation and Time Consistency

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Recent work has shown that reputational arguments based on trigger strategies can sometimes be used to deal with the problem of time inconsistency in government policy.[1] This idea was exploited by Robert Barro and David Gordon (1983) in a model of inflation and by V. V. Chari and Patrick Kehoe (1988) in a model of capital taxation. Here it is shown that reputational arguments of this sort are widely applicable.[2] A general policy model is developed, based on an economy that is stationary over time, contains identical households, and has no endogenous state variables. It is shown in general that the infinite-horizon version of this model has equilibria in trigger strategies that yield outcomes strictly better than those attainable as equilibria in the one-period model.

I. The Basic Framework

The agents are a continuum of identical households, indexed on the interval $[0,1]$, and the government. Each household must choose an action x from a given set X, and the government must choose a policy y from a given set Y. Since all the households are identical, only situations where they all (or all except one) make the same choice will be considered.[3] Household preferences are represented by a function w defined on $X\times X \times Y$, where $w(x', x, y)$ is the utility of a household that chooses x' if all other households choose x and if the government chooses y.

If x is chosen by all households, y is feasible for the government if the pair (x, y) lies in a given set $Z \subseteq X\times Y$. The government knows that all households are identical, and knows their common preferences and opportunities, the function w and the set X. Moreover, the government is benevolent: its goal is to maximize the utility of the representative household. Thus, $w(x, x, y)$ is the utility of the government if all households choose $x \in X$, the government chooses $y \in Y$, and the feasibility condition $(x, y) \in Z$ is satisfied. The government's utility is not defined if $(x, y) \notin Z$.

A benevolent dictator would simply choose a pair of actions that solves the problem $\max_{(x,y)\in Z} w(x, x, y)$. Any such pair (x^f, y^f) is called a *first-best outcome*. In general such outcomes cannot be reached under a decentralized regime.

A. *The Ramsey Model*

Suppose that the government can commit itself to a policy $y \in Y$ before households make their decisions. Given y, the problem facing the representative household is to choose $x \in X$ to maximize its own utility. Consider values $h(y) \in X$ satisfying

$$(1) \qquad h(y) \in \underset{x\in X}{\operatorname{argmax}}\, w(x, h(y), y).$$

If the government chooses y and if a household expects all others to choose $h(y)$, then its own utility is maximized by choosing $h(y)$. (If w has the additively separable form $w(x', x, y) = u(x', y) + v(x, y)$, as it does in many examples, then (1) has the form $h(y)$

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[1] See James Friedman (1971) for a discussion of trigger strategies. See Finn Kydland and Edward Prescott (1977) for a discussion of time inconsistency.

[2] See Chari et al. (1988) for a related discussion.

[3] The set X may be interpreted as including mixed strategies, however, and if this is done there is no loss in generality in confining attention to symmetric situations.

$\in \operatorname{argmax}_{x \in X} u(x, y)$, and the optimal response $h(y)$ does not depend on the actions of other households.) For simplicity, assume that for each $y \in Y$ there is a unique value $h(y)$ satisfying (1). Then the government expects that all households will respond to its own choice $y \in Y$ by choosing $h(y)$. The feasibility requirement means that it must choose y so that $(h(y), y) \in Z$. Therefore, since it is benevolent it chooses the policy y^r defined by

$$(2) \qquad y^r \in \underset{y \in Y}{\operatorname{argmax}}\, w(h(y), h(y), y)$$

$$\text{subject to} \qquad (h(y), y) \in Z.$$

Any policy y^r satisfying (2) is called a *Ramsey policy*, and $(x^r, y^r) = (h(y^r), y^r)$ is called the associated *Ramsey outcome*. Note that in general $x^f \neq h(y^f)$, so first-best outcomes are not Ramsey outcomes. Note, too, that if there are multiple Ramsey outcomes, they all yield the same utility.

B. *The No-Commitment Model*

Suppose that the government cannot commit itself to a policy, but rather, has the option of revising its choice after households have made their decisions. Then, in essence, the households must choose x before the government chooses y. For each $x \in X$, let $g(x)$ be the optimal response of the government to a situation where all households have chosen x:

$$(3) \qquad g(x) \in \underset{y \in Y}{\operatorname{argmax}}\, w(x, x, y)$$

$$\text{subject to} \qquad (x, y) \in Z.$$

For simplicity assume that for each $x \in X$ there is a unique value $g(x)$ satisfying (3). Since each individual household is infinitesimally small, each perceives, correctly, that its own action has no influence on what is feasible or desirable for the government. Hence it realizes that if all other households choose $\hat{x} \in X$, the government will respond by choosing $g(\hat{x})$. Its own optimal choice is then found by solving $\max_{x \in X} w(x, \hat{x}, g(\hat{x}))$. Thus, we are interested in values $x^n \in X$ such that

$$(4) \qquad x^n \in \underset{x \in X}{\operatorname{argmax}}\, w(x, x^n, g(x^n)).$$

If such a value exists, (x^n, g) is called a *no-commitment equilibrium* and $(x^n, y^n) = (x^n, g(x^n))$ is called the associated *no-commitment outcome*. In any such equilibrium each household expects, correctly, that other households will choose x^n and that the government will choose $g(x^n)$. Hence x^n is its own optimal choice.

Note that in general x^r does not satisfy (4). That is, if households (expecting the government to implement the Ramsey policy) choose x^r, then it is *not* optimal for the government to choose y^r. Formally $g(x^r) \neq y^r$, as can be seen by comparing (2) and (3). The reason is that if the government moves first, it takes into account the influence that its own action has on the decisions of households (through the function h), while if it moves second, the decisions of households have already been made.

Note, too, that if the government moves first it has the option of choosing y^n. Households would then respond by choosing x^n, producing the no-commitment outcome. Formally, $x^n = h(y^n)$, as can be seen by comparing (1) and (4). Hence the (common) utility of all Ramsey outcomes is at least as great as the utility of any no-commitment outcome.

II. Examples

Before proceeding, it is useful to examine two concrete examples. The first is a simple version of the capital levy problem. Each household receives an endowment of ω units of goods, that can be invested or hoarded in any proportions. The rate of return on capital (invested goods) is $i > 0$; hoarded goods earn no return. The government purchases public goods with revenue raised from a flat-rate tax on capital and its returns; hoarded goods cannot be taxed. The household consumes as private goods its hoarded goods and its net-of-tax capital and returns, and in addition consumes all of the public good that is produced. Its utility over private consumption c and public goods g is given by $W(c,g)$.

In this example each household must choose a level of investment x from the set $X=[0,\omega]$, and the government must choose a tax rate y from the set $Y=[0,1]$. Since the government simply spends on public goods whatever revenues it collects, there are no feasibility restrictions: $Z=X\times Y$. The function w is defined in terms of W:

$$w(x',x,y)=W(\omega-x'+(1-y)(1+i)x',\; y(1+i)x).$$

Suppose that W is strictly increasing, strictly concave, and once continuously differentiable, with $W_1(\omega,i\omega)<W_2(\omega,i\omega)$. That is, if all households invest their endowments, and if the principal is used for private consumption and the interest for public goods, then the marginal utility of public goods exceeds the marginal utility of private goods. Under this assumption it is easy to show that the first-best outcome has $x^f=\omega$ and $y^f>i/(1+i)$; that the Ramsey outcome is $x^r=\omega$ and $y^r=i/(1+i)$; and that all no-commitment outcomes have $x^n=0$ and $y^n\geq i/(1+i)$.

The second example is an economy in which each household is both a potential inventor and a potential consumer of the inventions of others. Each household must choose whether to incur the cost necessary to develop its invention, and the government must choose whether to grant patent protection to inventions. Assume that the cost of developing each invention is \$1. Assume further that if patent protection is granted, then the inventor's net profit is $a-1>0$ and consumers' surplus is $b>0$; while if patent protection is not granted, the inventor's net profit is -1 and consumers' surplus is $c>a+b$. If the invention is not developed, there are no costs or benefits to anyone.

Each household must decide whether or not to develop its inventions, so $X=\{D,\sim D\}$. The government must decide whether or not to grant patent protection to inventors, so $Y=\{P,\sim P\}$. All combinations of actions are feasible, so $Z=X\times Y$. The utility function in this case has the additively separable form $u(x',y)+v(x,y)$, where the two terms represent the household's roles as inventor and as consumer: $u=-1,0$, or $a-1$ and $v=0$, b, or c. Clearly the first-best outcome is $(D,\sim P)$; the Ramsey outcome is (D,P); and the no-commitment outcome is $(\sim D,\sim P)$.

III. The Repeated No-Commitment Model

Let (X,Y,Z,w) be given, and assume there is a unique no-commitment equilibrium (x^n,g). Assume that there is a unique Ramsey outcome (x^r,y^r) and that it is strictly better than the no-commitment outcome: $w(x^r,x^r,y^r)>w(x^n,x^n,y^n)$.

Suppose that the no-commitment model is repeated infinitely often and that all agents discount future utilities by the common factor $\beta\in(0,1)$. The question then arises: Are there equilibria of the infinite-horizon model that have outcomes strictly better than (x^n,y^n)? Specifically, if the government selects an outcome, the Ramsey outcome or any other feasible outcome (x^0,y^0) with $w(x^0,x^0,y^0)>w(x^n,x^n,y^n)$, is there an equilibrium of the infinite-horizon model in which the outcome is (x^0,y^0) every period? This question can be answered by constructing trigger strategies for each agent that are stationary functions of a low-dimensional state variable, and then checking whether the required equilibrium conditions hold.

Suppose the state space is S. Each period, households observe the current state before they choose their actions, so a stationary strategy for a household is a function $\sigma\colon S\to X$. Each period the government observes both the state and the actions of households before it chooses its policy, so a stationary strategy for the government is a function $\gamma\colon S\times X\to Y$ with $(x,\gamma(s,x))\in Z$, all $x\in X$, all $s\in S$. The interpretation is that $\sigma(s)$ is the action taken by the household if the current state is s, and $\gamma(s,x)$ is the policy chosen by the government if the current state is s and all households choose x. The law of motion for the state variable is a function $f\colon S\times Z\to S$ describing next period's state as a function of the current state and current outcome.

Since each agent takes as given the strategies of all other agents and the law of motion for the state variable, the problem any agent faces is a stationary dynamic programming problem. If the strategy specified for each agent is an optimal decision rule, then the specified strategies form a *stationary perfect equilibrium* (SPE). To determine the equilibrium outcome path, an initial value for the state variable must also be specified.

In general, policy models of the type described above have many SPEs (and many nonstationary equilibria as well). Equilibria of two types will be considered here. In the first, a "deviation" by the government causes households to mistrust it forever. The state is then simply a zero-one indicator for whether the government has ever deviated. In the second case, a deviation causes households to mistrust the government for a specified, finite number of periods. The state then indicates for how many more periods households will mistrust the government.

A. *Infinite-Reversion Trigger Strategies*

Let $(x^0, y^0) \in Z$ with $w(x^0, x^0, y^0) > w(x^n, x^n, y^n)$ be the outcome of interest. Let the state space be $S = \{0,1\}$, let the initial state be $s = 0$, and let the law of motion for the state be $f(0, x^0, y) = 1$ if $y \neq y^0$ and $f(s, x, y) = s$ otherwise. If in any period all households choose x^0 and the government chooses anything other than y^0, then the government has deviated. As long as no deviation has occurred, the value of the state variable is zero. The first time a deviation occurs, the value of the state variable changes from zero to one, and it then remains one forever.

Let the strategy for each household be $\sigma(0) = x^0$ and $\sigma(1) = x^n$, and let the strategy for the government be $\gamma(0, x^0) = y^0$ and $\gamma(s, x) = g(x)$, otherwise. That is, households choose the action x^0 or x^n, depending on whether a deviation has not or has occurred. The government chooses the policy y^0 if no deviation has occurred and if all households have chosen x^0; otherwise the government chooses the policy $y = g(x)$.

Do these strategies constitute an equilibrium? Since no household can affect the evolution of the state variable, the policy of the government, or the actions of other households, its future payoffs are independent of its own current action. Hence, for the household, it suffices to check whether its current action maximizes its current utility, assuming that all other households adopt the strategy σ and the government adopts the strategy γ. For $s = 0$ and $s = 1$, respectively, the required conditions are

$$\text{(5a)} \quad w(x^0, x^0, y^0) \geq w(x, x^0, y^0), \quad \text{all } x \in X;$$

$$\text{(5b)} \quad w(x^n, x^n, y^n) \geq w(x, x^n, y^n), \quad \text{all } x \in X.$$

It follows immediately from the definition of (x^n, y^n) that (5b) holds, while (5a) holds if and only if $x^0 = h(y^0)$.

If $s = 1$, or if $s = 0$ and $x \neq x^0$, then the government cannot affect the evolution of the state variable. In these cases, it suffices to check whether the government's strategy maximizes its current utility. Since g satisfies (3), the required condition holds. If $s = 0$ and $x = x^0$, the government *can* affect the evolution of the state variable, so its future utilities depend on its current policy. Hence the required condition is

$$\text{(6)} \quad \frac{1}{1-\beta} w(x^0, x^0, y^0) \geq w(x^0, x^0, y) + \frac{\beta}{1-\beta} w(x^n, x^n, y^n),$$
$$\text{all } y \neq y^0 \text{ with } (x^0, y) \in Z.$$

The left side of (6) is the total discounted utility of the government if it chooses the policy y^0 in the current period, under the assumption that households continue to choose x^0 in subsequent periods and the government continues to choose y^0 in subsequent periods. The right side is its utility if it chooses the policy y today, and in all subsequent periods chooses y^n in response to the choice x^n that households adopt. With terms rearranged, (6) can be written as

$$(7)\quad \frac{\beta}{1-\beta}\left[w(x^0,x^0,y^0)-w(x^n,x^n,y^n)\right] \geq w(x^0,x^0,y)-w(x^0,x^0,y^0), \quad \text{all } y\neq y^0 \text{ with } (x^0,y)\in Z.$$

The current gain from deviating, the term on the right, must be less than the total discounted losses from having the no-commitment outcome (x^n, y^n) rather than the outcome (x^0, y^0) in all future periods, the term on the left.

Thus, (x^0, y^0) can be supported as an equilibrium outcome by the strategies described above if $x^0 = h(y^0)$ and if (7) holds. Several conclusions should be noted. First, the Ramsey outcome is sustainable in equilibrium if and only if agents do not discount the future too much. By definition the Ramsey outcome satisfies $x^r = h(y^r)$, and it satisfies (7) if and only if β is sufficiently close to one. Second, if the Ramsey outcome does not satisfy (7), the two conditions may be used to determine the best outcome that can be sustained in equilibrium. Third, nothing better than the Ramsey outcome can be sustained in equilibrium. Since the Ramsey outcome satisfies (2), there is no outcome with higher utility that satisfies $x^0 = h(y^0)$. Finally, the equilibria described here do not require households to collude. The action taken by any household in any period is simply an optimal decision given its (correct) expectations about the actions that will be taken by other households and by the government. This fact is reflected in (5).

B. *Finite-Reversion Trigger Strategies*

Suppose that a deviation causes households to mistrust the government for only a finite number of periods T. Can outcomes better than (x^n, y^n) be sustained as equilibria in such a world? Let $(x^0, y^0) \in Z$ be the outcome of interest, let $S = \{0, 1, \dots, T\}$ be the state space, and let the initial state be $s = 0$. Let the law of motion be $f(0, x^0, y^0) = 0$; $f(0, x^0, y) = T$, if $y \neq y^0$; and $f(s, x, y) = s - 1$, if $s \neq 0$. As before, if $s = 0$, if all households choose x^0, and if the government chooses any policy $y \neq y^0$, then the government has deviated. If the government deviates, the state jumps from zero to T and then counts down from T to zero.

Let the strategy for each household be $\sigma(0) = x^0$ and $\sigma(s) = x^n$, $s \neq 0$, and let the strategy for the government be $\gamma(0, x^0) = y^0$ and $\gamma(s, x) = g(x)$, otherwise. As before, the strategy σ is optimal for each household if and only if (5a) and (5b) hold. For the government, the strategy γ is optimal if and only if

$$(8)\quad \frac{\beta(1-\beta^T)}{1-\beta}\left[w(x^0,x^0,y^0) - w(x^n,x^n,y^n)\right] \geq w(x^0,x^0,y)-w(x^0,x^0,y^0), \quad \text{all } y\neq y^0 \text{ with } (x^0,y)\in Z.$$

Condition (8), which replaces (7), reflects the fact that a deviation is now followed by a reversion to the no-commitment outcome for only a finite number of periods. Thus, (x^0, y^0) can be supported as an equilibrium outcome with a T-period reversion if $x^0 = h(y^0)$ and if (8) holds.

Note that if (8) holds, then (7) holds. More generally, if (8) holds for T, then it also holds for all $T' > T$. That is, longer reversions improve the possibilities for sustaining outcomes as equilibria. Note that T should not be viewed as a policy parameter, however; it is an exogenous parameter describing a characteristic of households. A deviation by the government makes households mistrustful, and T simply describes how long they remain mistrustful. Thus, it may be related to psychological factors or to how long the current government is expected to remain in office.

IV. Remarks

The arguments in Dilip Abreu (1988) suggest that it is easier to sustain the Ramsey outcome or others if a reversion that yields lower utility than the no-commitment outcome is used. But in many contexts, including the two examples above, it can be shown that no lower-utility reversion satisfies the required equilibrium conditions. Moreover, it seems doubtful that equilibria involving such reversions (if any exist) will describe observed outcomes in policy contexts: they require a degree of coordination and sophistication on the part of households that seems highly implausible.

A very different model of reputation, applicable in finite-horizon contexts, is provided in David Kreps et al. (1982), and could have been applied here. As shown by Drew Fudenberg and Eric Maskin (1986), however, the equilibrium outcomes for the two approaches coincide if the finite horizon is sufficiently long. Therefore, since the approach here provides a much simpler set of equilibrium conditions, it is a useful way of characterizing equilibrium outcomes, even if one prefers the interpretation in Kreps et al.

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