7 Bank Runs

"Last year, there was a student sitting at the back – a good student – who exclaimed, 'that's it?'." – Nancy Stokey

7.1 Diamond–Dybvig Model

7.1.1 Setup

There are three periods $t = 0, 1, 2$ and one technology by the bank that can take one unit of resource at date 0 (deposits received) and invest them. If it pulls out the investment at date 1, you get the principal back with no interest; if you pull it out at date 2, you get $R > 1$.

Preferences We have a continuum of households on $[0, 1]$ and they are subject to an i.i.d. preference shock at $\theta \in \{1, 2\}$. The utility function is

$$U(c_1, c_2, \theta) = \begin{cases} u(c_1) & \theta = 1 \\ \rho u(c_1 + c_2) & \theta = 2 \end{cases}$$

and assume $R\rho > 1$. Endowment is 1 in $T = 0$.

Autarky Solution Consider a no-trade solution assuming the individual has the same technology. Denote α the probability that $\theta = 1$. Then:

$$\mathbb{E}u = \alpha u(1) + (1 - \alpha)\rho u(R)$$

The problem with the autakry solution is that the agent has a consumption risk (utility risk) because of the shock. So the market is missing insurance against this shock. Since the shock is i.i.d. across the consumers, if the insurance company could come in, it can insure against it.

Perfect Insurance Market (θ is observable) Suppose the insurance company offers the contract $(-1, c_1, c_2)$ depending on θ. We want the contract to maximize the expected utility of the household:

$$\max_{c_1, c_2} [\alpha u(c_1) + (1 - \alpha)\rho u(c_2)]$$
$$\text{s.t. } \alpha c_1 + (1 - \alpha)\left(\frac{c_2}{R}\right) = 1$$

The FOCs are:

$$\alpha u'(c_1) = \alpha\lambda$$
$$(1 - \alpha)\rho u'(c_2) = (1 - \alpha)\frac{\lambda}{R}$$

Thus it follows that

$$\frac{u'(c_2)}{u'(c_1)} = \frac{1}{\rho R} < 1 \Leftrightarrow c_2^* > c_1^*$$

The next question is whether or not a bank can mimic this.

Introducing a Bank Assume RRA > 1 or $-cu''/u' > 1$. In fact, assume

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

which implies

$$\left(\frac{c_2^*}{c_1^*}\right)^\sigma = \rho R \Rightarrow c_2^* = (\rho R)^{1/\sigma} c_1^*$$

We also want $c_1^* > 1$. Thus from the budget constraint and specializing to the utility function, it is equivalent to

$$\alpha + (1-\alpha)\left(\frac{(\rho R)^{1/\sigma}}{R}\right) < 1$$

Rearranging, we have the following condition that ensures insurance company offering efficient $c_1^* > 1$.

$$(\rho R)^{1/\sigma} < R$$

Why do we need this condition? We are trying to say that a bank can mimic this. We want the bank to say to the depositors that you get some interest in date 1 but less than what you get in date 2. In other words, if the optimal insurance contract has

$$(c_1^*, c_2^*) \text{ with } c_1^* > 1, c_2^* < R$$

then a bank can mimic it. The bank's offer would be: any customer can withdraw at date 1 and get c_1^* or withdraw at date 2 and get c_2^*.

Possible Glitch What would a bank run look like in this setup? What if everyone tries to withdraw at $T = 1$? Then there is not enough to go around – some of them will get rationed. So we need a rationing rule. The D-D rationing rule is the *sequential service contraint:* if fraction f of deposits run, the first in line gets c_1^*; the rest gets 0. So only $1/c^* = f < 1$ can get paid. Is there a way to rule out this bad equilibrium?

1. Deposit insurance from outside
2. Suspension of convertibility: bank pays only the first α of those in line at $T = 1$. This prevents a bank run since the $\theta = 2$ have no incentive to run; if they are patient, they will get paid the full amount.