

Assignment 2

(Due Friday, April 19, prior to the start of the Review session)

Problem 1 (JR, Exercise 8.9 - major variation) Analyze the insurance signaling game in JR (ch8) when the coverage B is restricted to being equal to L . Assume that the low risk consumer strictly prefers full insurance at the high-risk competitive price to no insurance. For simplicity, assume that there is a single firm.

- (a). Characterize the unique pure-strategy sequential equilibrium price(s) when attention is restricted to those equilibria in which the insurance company earns zero profits.
- (b). Show that there are pooling sequential equilibria in which the insurance company earns positive profits.
- (c). Characterize a mixed-strategy sequential equilibrium strategy profile in which (1) the insurance company earns zero profits and (2) consumers fully separate. [Hint: assume that there are two offers in equilibrium, and the firm randomizes between “Accept” and “Reject” for one of them. If the firm “rejects”, the game is over and the consumer is uninsured.]

Problem 2 Consider a simple monopoly setting in which a firm sells to a consumer whose unit valuation is distributed uniformly on $[0, 1]$. The firm’s unit costs are zero.

- (a). What is the optimal price for the monopolist? What is the expected consumer surplus?

For the remainder of this problem, suppose that before the monopolist sets the price, the consumer can credibly reveal any closed subset, $M \subseteq [0, 1]$ which contains their valuation. E.g., a consumer of type $\theta = 0.6$ can reveal the *simple* message $\{0.6\}$ to the monopolist, or she could reveal a coarser signal such as “my value is in $[0.5, 0.7] \cup [.8, .9]$ ”, but she cannot lie and say her type is in a set when it is not. A consumer can also send the message $M = [0, 1]$ which conveys no information. Revealing credible information does not have any direct cost to the consumer.

- (b). Show that the original monopoly outcome is an equilibrium outcome in this certified disclosure game. Indicate the equilibrium messages and monopolist’s beliefs (including out-of-equilibrium beliefs) which sustain the equilibrium.
- (c). Show that full revelation with simple messages, followed by prices set to equal valuations, is also an equilibrium and in this equilibrium the monopolist extracts all of the consumer’s surplus. Again, indicate the monopolist’s beliefs that you are using to sustain this equilibrium.
- (d). Construct an equilibrium in which the consumer does better (from an *ex ante* perspective) than the standard monopoly outcome in (a). [Hint: look for a simple 2-step partition, $M_1 = [0, a)$ and $M_2 = [a, 1]$.]

Problem 3 Consider the quadratic Cheap-talk model of Crawford and Sobel (1982), but with θ uniformly distributed on $\Theta = [0, 2]$, with message space $\mathcal{M} = [0, 2]$, and with $b = \frac{1}{8}$. Find the

most informative equilibrium (the one with the most steps), fully characterizing the equilibrium partition.

Problem 4 Consider a setting of career concerns where a manager has one of two types, $1 > \theta_h > \theta_l > 0$, with prior probabilities ϕ and $(1 - \phi)$, respectively. Let $E[\theta] = \phi\theta_h + (1 - \phi)\theta_l$ be the players' prior expectation on θ . Neither the market nor the manager knows the manager's type.

In period 1, the manager may privately learn of an investment project with some probability. An investment project, i , if available, gives a risky return paying y_i with probability θ and 0 with probability $1 - \theta$. A safe asset is always available and pays $r > 0$. Assume that some projects are *ex ante* profitable (i.e., $E[\theta]y_i > r$ for some i and $E[\theta]y_i < r$ for others).

The firm cannot write contracts on investments. Rather, it delegates the investment choice to the manager if a project (known only to the manager) becomes available. The manager therefore may either choose an investment project, or choose the safe risk-free project. The manager is paid w in the current period and will be paid $\text{Prob}[\theta = \theta_h | \cdot]$ after the market observes whether or not she invests and, if she invests, the outcome of the project (success, S , or failure, F).

- (a). Compute the market's inference of the manager's type if she invests in the safe asset?
- (b). Compute the market's inference of the manager's type if she invests in a risky project and it is a success? Compute the market's inference of the manager's type if she invests in a risky project and it is a failure?
- (c). Show that a risk-neutral manager is indifferent between all projects (and the safe asset). Formally, show that the market's expected inference of the manager's type, conditional on investing, is the same as if she invests in the safe asset.
- (d). Argue that if the manager is only slightly risk averse, she will not choose positive NPV projects.

Problem 5 Consider a setting of career concerns where a manager has one of two types, "talented" and "not" with prior probabilities ϕ and $(1 - \phi)$, respectively. Neither the market nor the manager knows the manager's type. The manager is risk neutral.

In period 1, the manager learns of a random investment project that gives a risky return paying either 1 or -1 and a privately observed parameter $\gamma \in (0, 1)$ associated with the project, where γ is distributed uniformly on $[0, 1]$. If the manager is talented, the project pays 1 with probability $\gamma \in (0, 1)$. If the manager is not talented, then the project pays 1 with probability $\frac{1}{2}$ (i.e., the expected value of the project is zero, independent of γ).

Timing: (1) the manager privately observes the project and its γ (γ is private information); (2) the manager decides whether or not to invest in the project; (3) the market observes whether or not investment has taken place and, if an investment has been made, the outcome of the project (success or failure); (4) the manager receives a payment equal to the market's posterior probability that the manager is talented.

Note that the firm cannot write contracts on investments or outcomes. Rather, it delegates the investment choice to the manager who may decide whether or not to proceed with the project. The manager's incentives are entirely determined by the market's posterior of her talent going forward.

- (a). Briefly explain why the firm would like the manager to follow the threshold strategy of invest if and only if $\gamma \geq \frac{1}{2}$.
- (b). Suppose that the market believes that the manager follows a simple threshold strategy of investing if and only if $\gamma \geq \bar{\gamma}$. What are the market's posteriors on the probability of talent, conditional on success, and conditional on failure?
- (c). If the market believes the manager is following the threshold strategy $\bar{\gamma}$, write an expression for the value of $\hat{\gamma}$ that makes the manager indifferent to investing. Show that there is a fixed point $\bar{\gamma} = \hat{\gamma} = \bar{\gamma}^*$ with $\bar{\gamma}^* = 1$. Show that the market's posterior that the manager is talented following investment and failure is 0.
- (d). Is there another equilibrium in which all projects (independent of γ) are pursued and the market makes no inference about the manager following investment success or failure? Why or why not?