Price Theory III Problem Set 5 Solution

May 19, 2019

Problem 1

1a)

For each θ we set

$$e^{FB}(\theta) = \underset{e}{\operatorname{argmax}}\, \pi(e) - g(e, \theta),$$

so first order condition is

$$\pi'(e^{FB}(\theta)) = g_e(e^{FB}(\theta), \theta).$$

Second order condition is satisfied because $\pi'' < 0, g_{ee} > 0$.

1b)

An IC direct mechanism is such that

$$U(\theta) = w(\theta) - g(e(\theta), \theta) \geq w(\theta') - g(e(\theta'), \theta)$$

for all θ, θ'.

1c)

Following the lecture note, from IC constraints for θ and θ', we get

$$U(\theta) \geq U(\theta') - g(e(\theta'), \theta) + g(e(\theta'), \theta')$$

and

$$U(\theta') \geq U(\theta) - g(e(\theta), \theta') + g(e(\theta), \theta).$$

Combining those, we get

$$-(g(e(\theta), \theta') - g(e(\theta), \theta)) \leq U(\theta') - U(\theta) \leq -(g(e(\theta'), \theta') - g(e(\theta'), \theta)).$$

By single crossing property $g_{e\theta} > 0$, we get that e is nondecreasing, and by dividing the whole inequalities by $\theta' - \theta$, and letting $\theta' \to \theta$, we get $U' = -g_\theta$ almost everywhere. From there we conclude

$$U(\theta) = U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} g_\theta(e(s), s) ds$$

(we could reverse the way in which we integrate i.e. $U(\theta) = U(\bar{\theta}) + \int_\theta^{\bar{\theta}} g_\theta e(s), s) ds$ but then IR condition will be a bit harder to come up with.)

1d)

Now the firm's objective is

$$\max_{e(\theta)} E[\pi(e(\theta)) - w(\theta)] = E[\pi(e(\theta)) - (U(\theta) + g(e(\theta), \theta))]$$

such that e is nondecreasing. As usual, using the condition for ICIR mechanisms, rewrite the above equation as

$$\int_{\underline{\theta}}^{\bar{\theta}} \pi(e(\theta)) - g(e(\theta), \theta) f(\theta) d\theta - U(\underline{\theta}) + \int_{\theta=\underline{\theta}}^{\bar{\theta}} \int_{s=\underline{\theta}}^{\theta} g_\theta(e(s), s) ds f(\theta) d\theta.$$

The second term, $U(\underline{\theta})$ can be set 0 and still satisfies IR, so we set it to 0 .

The third term can be written as (because the limit of integral is $\underline{\theta} \leq s \leq \theta \leq \bar{\theta}$)

$$\begin{aligned}\int_{s=\underline{\theta}}^{\bar{\theta}} \int_{\theta=s}^{\bar{\theta}} g_\theta(e(s), s) f(\theta) d\theta ds &= \int_{s=\underline{\theta}}^{\bar{\theta}} g_\theta(e(s), s) [\int_{\theta=s}^{\bar{\theta}} f(\theta) d\theta] ds = \int_{s=\underline{\theta}}^{\bar{\theta}} g_\theta(e(s), s)(1 - F(s)) ds \\ &= E[g_\theta(e(s), s) \frac{(1 - F(s))}{f(s)}].\end{aligned}$$

Therefore the objective is

$$\int_{\underline{\theta}}^{\bar{\theta}} \pi(e(\theta)) - g(e(\theta), \theta)) + g_\theta(e(\theta), \theta) \frac{(1 - F(\theta))}{f(\theta)}] f(\theta) d\theta$$

such that e is nondecreasing. We optimize this pointwise (for each θ), and check if we have a nondecreasing e.

Fixing θ, the first order condition for $e(\theta)$ is

$$\pi'(e(\theta)) - g_e(e(\theta), \theta)) + g_{\theta e}(e(\theta), \theta) \frac{(1 - F(\theta))}{f(\theta)} = 0.$$

Also the second order condition is

$$\pi''(e(\theta)) - g_{ee}(e(\theta),\theta) + g_{\theta ee}(e(\theta),\theta)\frac{(1-F(\theta))}{f(\theta)} < 0,$$

since we know $\pi'' < 0, g_{ee} > 0$, a sufficient condition for the second order condition is $g_{\theta ee} < 0$.

Now to check e nondecreasing, we check whether $e' \geq 0$. Differentiating the first order condition with respect to θ, we get

$$\pi'' e' - (g_{ee}e' + g_{e\theta}) + (g_{\theta ee}e' + g_{e\theta\theta})\frac{(1-F)}{f} + g_{\theta e}\frac{d}{d\theta}(\frac{(1-F)}{f}) = 0,$$

where functions are evaluated at θ and $e(\theta)$. This can be rearranged to obtain

$$e' = \frac{g_{e\theta}(1 - \frac{d}{d\theta}(\frac{1-F}{f})) - g_{e\theta\theta}}{\pi'' - g_{ee} + g_{\theta ee}}.$$

We concluded that $g_{\theta ee} < 0$, so denominator is negative. So the only way that this expression is positive is when the numerator is also negative. Since we assumed monotone hazard rate, $\frac{d}{d\theta}(\frac{1-F}{f}) \geq 0$, and it follows that we need $g_{e\theta\theta} > 0$ and $\frac{d}{d\theta}(\frac{1-F}{f}) \leq 1$ for sufficiency.

Problem 2

This is a problem of monopolistic screening, and the firm will make positive profits in contrast to the situation in competitive screening described in the competitive-screening exercise (MWG 13.D.1). The monopolist's problem is

$$\max_{\{(w_H,t_H),(w_L,t_L)\}} \phi(\theta_H(1+t_H) - w_H) + (1-\phi)(\theta_L(1+t_L) - w_L),$$

subject to

$$w_L - c(t_L,\theta_L) \geq 0 \quad (IR_L)$$

$$w_H - c(t_H,\theta_H) \geq 0 \quad (IR_H)$$

$$w_H - c(t_H,\theta_H) \geq w_L - c(t_L,\theta_H) \quad (IC_H)$$

$$w_L - c(t_L,\theta_L) \geq w_H - c(t_H,\theta_L) \quad (IC_L).$$

Note that (IR_L) and (IC_H) (together with the assumption that $c_{t\theta}(t,\theta) < 0$) imply that (IR_H) is satisfied. Furthermore, (IC_H) must bind. Suppose otherwise that it was slack. Then w_H could be lowered without violating the remaining constraints, which would increase profits.

Also note that (IC_H) and (IC_L) imply that $t_H \geq t_L$. This condition, together with (IC_H) binding, implies that (IC_L) is slack. Thus, we can restate the monopolist's program as

$$\max_{\{(w_H,t_H),(w_L,t_L)\}} \phi(\theta_H(1+t_H)-w_H)+(1-\phi)(\theta_L(1+t_L)-w_L),$$

subject to

$$w_L - c(t_L,\theta_L) \geq 0 \quad (IR_L)$$

$$w_H - c(t_H,\theta_H) \geq w_L - c(t_L,\theta_H) \quad (IC_H)$$

$$t_H \geq t_L.$$

The participation constrain of the lowest θ_L must bind at the optimum (profits could be increased by lowering w_L without violating the (IC_H)). Given that (IR_L) and (IC_H) bind, we can substitute for wages in the program and obtain

$$\max_{\{t_H,t_L)\}} \phi(\theta_H(1+t_H)-c(t_H,\theta_H)+c(t_L,\theta_L)-c(t_L,\theta_H))+(1-\phi)(\theta_L(1+t_L)-c(t_L,\theta_L)),$$

subject to $t_H \geq t_L$. The first-order conditions for t_H and t_L are

$$\theta_H = c_t(t_H,\theta_H),$$

$$\theta_L = c_t(t_L,\theta_L) - \frac{\phi}{1-\phi}\left(c_t(t_L,\theta_L)-c_t(t_L,\theta_H)\right).$$

Thus, t_H is set to the first best level and t_L is distorted downwards below the first-best level. (Which also implies $t_H \geq t_L$.)

Several contrasts appear with respect to the competitive screening model. (1) There is always an equilibrium in the monopoly model. (2) The firm makes positive profits unlike the competitive screening game. (2) The monopolist makes the θ_H type indifferent between the two allocations (in the competitive screening model, the θ_L type is generally indifferent). (3) The monopolist will distort the θ_L's task downwards but always implement the efficient outcome for θ_H, unlike the competitive screening model in which the θ_L type always chooses the efficient allocation and t_H is equal to or exceeds the first-best level.

Problem 3

3a)

Indifference curves and isoprofit line will look like this:

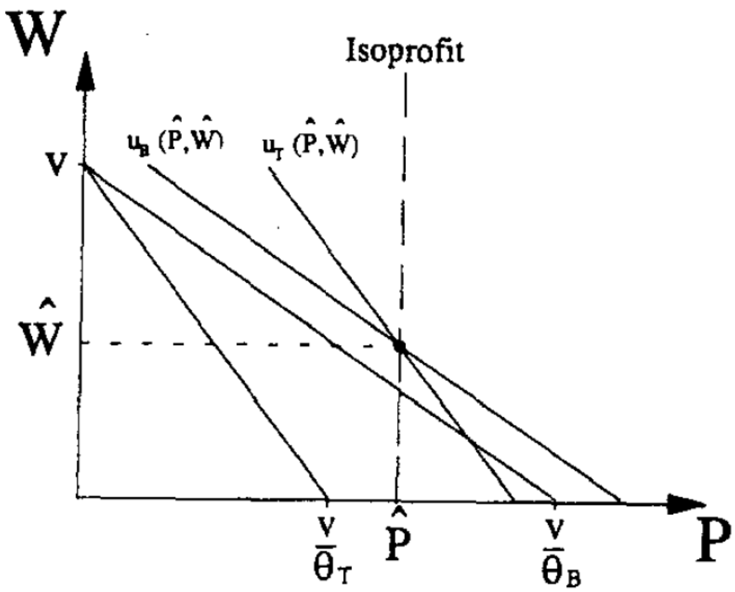


Figure 14.C.8

3b)

(Assuming they want to sell to both type) Air Shangri-la wants to solve

$$\max_{p_b,p_t,w_b,w_t} \phi p_b + (1-\phi)p_t$$
$$s.t. v - \theta_b p_b - w_b \geq 0 \text{ (IRb)}$$
$$v - \theta_t p_t - w_t \geq 0 \text{ (IRt)}$$
$$v - \theta_b p_b - w_b \geq v - \theta_b p_t - w_t \text{ (ICb)}$$
$$v - \theta_t p_t - w_t \geq v - \theta_t p_b - w_b \text{ (ICt)}$$
$$p_b, p_t, w_b, w_t \geq 0.$$

From (IRt) (ICb), and $\theta_t > \theta_b$, we get

$$v - \theta_b p_b - w_b \geq v - \theta_b p_t - w_t > v - \theta_t p_t - w_t \geq 0$$

which means (IRb) is not binding. This implies that (IRt) must bind, because otherwise we could set

$p_b' = p_b + \epsilon$ and $p_t' = p_t + \epsilon$ and satisfy all the constraints while strictly increasing the profit (note that IC won't change, therefore (IRb) doesn't bind whenever (IRt) is satisfied).

3c)

Suppose otherwise that $w_b = c > 0$. Consider $w_b' = 0$, $p_b' = p_b + \frac{c}{\theta_b}$, which keeps the utility level of business travelers while earning a strictly higher profit. Since $\theta_t > \theta_b$, $c < \theta_t \frac{c}{\theta_b}$ so (ICt) is still satisfied, contradicting to our assumption that original scheme is optimal. If (ICb) is not binding, we could raise p_b to the level where (ICb) is binding, since (IRt) and (ICb) imply (IRb), and (IRt) is not affected by this operation.

3d and 3e)

What we know so far: $w_b = 0$, (ICb) and (IRt) bind, and (IRb) can be ignored.

Using (IRt) we get $w_t = v - \theta_t p_t$. Substituting this into (ICb) yields

$$p_b = p_t \frac{\theta_b - \theta_t}{\theta_b} + \frac{v}{\theta_b}.$$

Now the objective function is

$$\phi[p_t \frac{\theta_b - \theta_t}{\theta_b} + \frac{v}{\theta_b}] + (1 - \phi) p_t,$$

which is linear in p_t. Taking FOC, we see that if $\frac{\phi}{1-\phi} > \frac{\theta_t - \theta_b}{\theta_b}$ we want to increase p_t as much as possible, and otherwise $p_t = 0$. Now because (IRt) binds we see that setting $(p_t, w_t) = (\frac{v}{\theta_t}, 0)$ gives the highest p_t in this problem. For business travellers, because (ICb) must bind, and $w_b = 0$, we set $(p_b, w_b) = (p_t, w_t)$ (i.e. pooling).

They only serve business travellers when $\frac{\phi}{1-\phi} < \frac{\theta_t - \theta_b}{\theta_b}$, in which case $p_t = 0$. Therefore we have $(p_t, w_t) = (0, v)$ from (IRt), and $(p_b, w_b) = (\frac{v}{\theta_b}, 0)$ from (ICb).

Now we need to consider another free parameter, cost. If the cost is below $\frac{v}{\theta_t}$ as in (a)-(c) then we are solving the optimization problem as before. If the cost is between $\frac{v}{\theta_t}$ and $\frac{v}{\theta_b}$ then selling to both type is not profitable, therefore Air Shangri-la wants to only sell to business customers even if $\frac{\phi}{1-\phi} > \frac{\theta_t - \theta_b}{\theta_b}$ (pooling case). If the cost is higher than $\frac{v}{\theta_b}$ then they will always not operate.

Problem 4

4a)

The monopolist sets the full insurance contract so that customers' utility is the certainty equivalence for each type.

4b)

Their problem is

$$\min_{c^L,c^H} \phi[\pi_L c_2^L + (1-\pi_L)c_1^L] + (1-\phi)[\pi_H c_2^H + (1-\pi_H)c_1^H]$$

$$s.t. \pi_L u(c_2^L) + (1-\pi_L)u(c_1^L) \geq \pi_L u(y-L) + (1-\pi_L)u(y) \ (IRL)$$

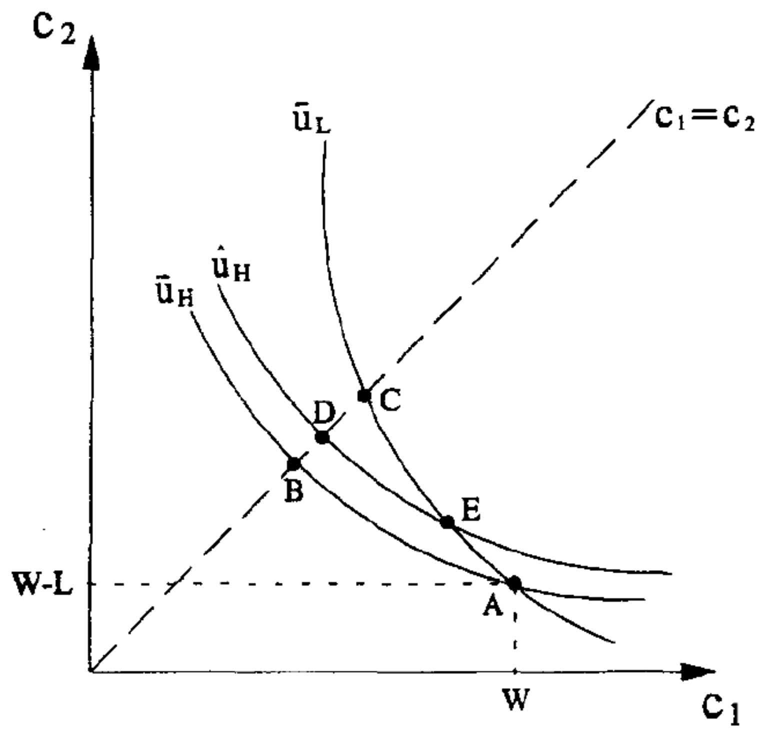
$$\pi_H u(c_2^H) + (1-\pi_H)u(c_1^H) \geq \pi_H u(y-L) + (1-\pi_H)u(y) \ (IRH)$$

$$\pi_L u(c_2^L) + (1-\pi_L)u(c_1^L) \geq \pi_L u(c_2^H) + (1-\pi_L)u(c_1^H) \ (ICL)$$

$$\pi_H u(c_2^H) + (1-\pi_H)u(c_1^H) \geq \pi_H u(c_2^L) + (1-\pi_H)u(c_1^L) \ (ICH).$$

We present two solutions, first by graphics, and second by solving this using Kuhn-Tucker condition. (We thank Elena Istomina, Aleksei Oskolkov, and Lillian Rusk for this.)

Graphical intuition comes from the typical two-types monoplistic screening models: IRL and ICH bind (notice that it's not a typical screening problem because the outside option is type-dependent). If we believe in the structure of the problem the optimal solution would look like this:



We start from contracts A (for L, no insurance at all) and B (for H). Along the $\bar{u}_L$ line we can't go to the right of A because ICH must bind, that would in turn violate IRH. To the left we have the trade off: moving leftward earns some profit from the low type (remember firm's indifference curve is negative 45 degree line) but ICH binding means firm is losing (compared to situation B) from the high type. Therefore where the firm ends up depends on how many low types there are (ϕ). If there are many low types it will be beneficial for the firm to move to the left (to contracts like point D and E). In any cases, we have full insurance for high type because that's the firm's best way to sastisfy ICH. However, unless there are only low types (in which case we are back to (a)), low types will be partially uninsured.

Let's formally confirm this with Lagrangian. Let λ_i correspond to type i's IC and μ_i to type i's IR constraint. Then, the FOCs are:

$$\begin{aligned}
[c_1^L] :& \phi - u'(c_1^L)\left[\lambda_L + \mu_L - \lambda_H \frac{1-\pi_H}{1-\pi_L}\right] = 0 \\
[c_1^H] :& (1-\phi) - u'(c_1^H)\left[\lambda_H + \mu_H - \lambda_L \frac{1-\pi_L}{1-\pi_H}\right] = 0 \\
[c_2^L] :& \phi - u'(c_2^L)\left[\lambda_L + \mu_L - \lambda_H \frac{\pi_H}{\pi_L}\right] = 0 \\
[c_2^H] :& (1-\phi) - u'(c_2^H)\left[\lambda_H + \mu_H - \lambda_L \frac{\pi_L}{\pi_H}\right] = 0.
\end{aligned}$$

Observation 1: Since the utility function is strictly increasing, for each type, at least one of the constraints (IR or IC) must bind. Otherwise we would decrease c_j^i for some j to hit IR or IC.

Observation 2: Consider the IC constraints. If IC for high type binds, then

$$\frac{u(c_1^L) - u(c_1^H)}{u(c_2^H) - u(c_2^L)} = \frac{\pi_H}{1-\pi_H}.$$

If IC for low type binds, then

$$\frac{u(c_1^L) - u(c_1^H)}{u(c_2^H) - u(c_2^L)} = \frac{\pi_L}{1-\pi_L},$$

which means at least one of the λ_i is 0 (2 ICs can't bind).

Claim 1: $\lambda_H > 0, \lambda_L = 0, \mu_L > 0$.

Suppose otherwise that $\lambda_H = 0$ (ICH is slack). Then, from Observation 1, $\mu_H > 0$ (IRH binds). From FOCs $[c_1^L]$ and $[c_2^L]$, $c_1^L = c_2^L$. By IRL we have

$$\begin{aligned}
\pi_L u(c_2^L) + (1-\pi_L)u(c_1^L) = u(c_1^L)& \\
&\geq (1-\pi_L)u(y) + \pi_L u(y-L) \\
&> (1-\pi_H)u(y) + \pi_H u(y-L)
\end{aligned}$$

because $\pi_H > \pi_L$.

Now, from ICH, $(1-\pi_H)u(c_1^H) + \pi_H u(c_2^H) \geq u(c_1^L) > (1-\pi_H)u(y) + \pi_H u(y-L)$, which means that the

IRH is not binding. But we claimed $\mu_H > 0$. Contradiction.

Therefore, ICH must bind in optimal. Then, by Observation 2, $\lambda_L = 0$ so from FOCs $[c_1^H]$ and $[c_2^H]$, $c_1^H = c_2^H$. The high-risk type is fully insured. Because of Observation 2, we know that ICL cannot bind, and by Observation 1 we see that IRL must bind.

Claim 2: $\mu_H = 0$.

Suppose otherwise that for high type both IC and IR bind. In this case, we have high type is fully insured, and by ICH

$$u(c^H) = (1-\pi_H)u(y) + \pi_H u(y-L) = (1-\pi_H)u(c_1^L) + \pi_H u(c_2^L).$$

Rearranging the second equation,

$$\frac{u(y) - u(c_1^L)}{u(c_2^L) - u(y-L)} = \frac{\pi_H}{1-\pi_H}.$$

Because IRL binds, we also have

$$\frac{u(y) - u(c_1^L)}{u(c_2^L) - u(y-L)} = \frac{\pi_L}{1-\pi_L},$$

which is a contradiction.

Combining those two claims, if an optimal mechanism exists with two types being served, it must satisfy the following set of equations [1]:

$$\begin{aligned}
[c_1^L] :& \phi - u'(c_1^L)\left[\mu_L - \lambda_H \frac{1-\pi_H}{1-\pi_L}\right] = 0 \\
[c_1^H, c_2^H] :& (1-\phi) - u'(c_1^H)\lambda_H = 0 \\
[c_2^L] :& \phi - u'(c_2^L)\left[\mu_L - \lambda_H \frac{\pi_H}{\pi_L}\right] = 0 \\
[ICH] :& u(c_1^H) = (1-\pi_H)u(c_1^L) + \pi_H u(c_2^L) \\
[IRL] :& (1-\pi_L)u(c_1^L) + \pi_L u(c_2^L) = (1-\pi_L)u(y) + \pi_L u(y-L).
\end{aligned}$$

Now use $[c^H]$ to get $\lambda_H = \frac{1-\phi}{u'(c_1^H)}$, then solve for μ_L using either $[c_1^L]$ or $[c_2^L]$. We get the following set of equations:

$$\begin{aligned}
[FOC] :& 1 = \frac{u'(c_2^L)}{u'(c_1^L)} - \frac{1-\phi}{\phi}\frac{u'(c_2^L)}{u'(c_2^H)}\left[\frac{\pi_H}{\pi_L} - \frac{1-\pi_H}{1-\pi_L}\right] \\
[ICH] :& u(c_2^H) = (1-\pi_H)u(c_1^L) + \pi_H u(c_2^L) \\
[IRL] :& (1-\pi_L)u(c_1^L) + \pi_L u(c_2^L) = (1-\pi_L)u(y) + \pi_L u(y-L).
\end{aligned}$$

[1] After we solve the equations, we need to check the constraints we dropped. We only ruled out the possibility of them binding, but not that they hold necessarily. In fact, the conditions required for the dropped constraints to hold are $c_1^L \geq c_1^H$, $c_1^L \leq y$.

The second term in [FOC] is positive because $\left[\dfrac{\pi_H}{\pi_L} - \dfrac{1-\pi_H}{1-\pi_L}\right] > 0$. So we have $u'(c_2^L) > u'(c_1^L)$, the low type is not fully insured anymore.

This rationing occurs because of the single crossing. If the monopolist were to offer full insurance to both types, high-risk type would always pretend to be low-risk. The only way the monopolist can sort the two types out is by leaving the low risk underinsured (as for high-risk type this is more costly).

4c)

The difference is surplus allocation: in competitive screening model, price is driven down to zero-profit level and consumers get positive (compared with the outside option) utility. However in this case, we have IRL binding, that means that the surplus is taken by the monopolist.

Problem 5

5a)

For type θ_2,

$$\begin{aligned} \theta_2 q_2 - t_2 &\geq \theta_2 q_1 - t_1 && \text{by IC21} \\ &\geq \theta_1 q_1 - t_1 && \text{because } \theta_2 > \theta_1 \\ &\geq 0 && \text{by IR1,} \end{aligned}$$

which yields IR2. Indeed, if $q_1 > 0$, then IR2 must be slack. A similar argument for θ_3 establishes IR3. Again, if $q_2 > 0$, then IR3 must be slack.

5b)

IC21 and IC12 imply

$$\theta_2(q_2 - q_1) \geq t_2 - t_1 \geq \theta_1(q_2 - q_1).$$

Hence, $q_2 \geq q_1$. A similar argument shows IC32 and IC23 imply $q_3 \geq q_2$.

5c)

IC12 can be rewritten as $t_2 - t_1 \geq \theta_1(q_2 - q_1)$. IC23 can be written as $t_3 - t_2 \geq \theta_2(q_3 - q_2)$. Summing these inequalities yields

$$t_3 - t_1 \geq \theta_1(q_2 - q_1) + \theta_2(q_3 - q_2) = \theta_1(q_3 - q_1) + (\theta_2 - \theta_1)(q_3 - q_2) \geq \theta_1(q_3 - q_1),$$

where the last inequality follows from monotonicity $q_3 \geq q_2$. Hence, IC12 and IC23 imply IC13. Similarly, IC21 can be written as $t_2 - t_1 \leq \theta_2(q_2 - q_1)$ and IC32 can be written $t_3 - t_2 \leq \theta_3(q_3 - q_2)$. Summing we obtain

$$t_3 - t_1 \leq \theta_2(q_2 - q_1) + \theta_3(q_3 - q_2) = \theta_3(q_3 - q_1) - (\theta_3 - \theta_2)(q_2 - q_1) \leq \theta_3(q_3 - q_1),$$

where the last inequality makes use of the assumption $q_3 \geq q_2$. Hence, IC21 and IC32 (with monotonicity) imply IC31.

5d)

Suppose that IR1 does not bind so that $\theta q_1 - t_1 \geq \varepsilon > 0$. It must be that $q_1 > 0$, and thus IR2 is slack. (See answer to (a).) By monotonicity, $q_2 \geq q_1 > 0$, so IR3 is slack. In this case, all transfers can be raised by ε and all of the IC and IR constraints are all satisfied. Hence, the original transfers were not optimal.

5e)

Suppose IC21 does not bind, i.e. $\theta_2 q_2 - t_2 \geq \theta_2 q_1 - t_1 + \varepsilon$. Then increase t_2 and t_3 by ε. IC12 is relaxed, while (IC32) and (IC23) and unaffected.

5f)

Suppose (IC32) does not bind, i.e. $\theta_3 q_3 - t_3 \geq \theta_3 q_2 - t_2 + \varepsilon$. Then increase t_3 by ε. (IC23) is relaxed, while (IC21) and (IC12) and unaffected.

5g)

Because IC21 binds (e) and $q_2 \geq q_1$, we have $t_2 - t_1 = \theta_2(q_2 - q_1) \geq \theta_1(q_2 - q_1)$, which implies (IC12). The proof that (IC23) can be ignored is similar.

5h)

After our work above in (a)-(g), we can conclude that the solution to the firm's program maximizes the firm's profit subject to the participation constraint of the lowest type, (IR1), the downward local incentive constraints (i.e., (IC32), (IC21)) and monotonicity (i.e., $q_3 \geq q_2 \geq q_1$).

We'll solve the relaxed program (ignoring monotonicity) and check ex post that it is satisfied. Because we know IR1 is binding, we have $t_1 = \theta_1 q_1$. Because we know IC21 is binding, we have $t_2 - t_1 = \theta_2(q_2 - q_1)$, and so

$$t_2 = \theta_1 q_1 + \theta_2(q_2 - q_1) = \theta_2 q_2 - q_1(\theta_2 - \theta_1).$$

Because we know IC32 is binding, we know $t_3 = t_2 + \theta_3(q_3 - q_2)$, and so

$$t_3 = \theta_2 q_2 - q_1(\theta_2 - \theta_1) + \theta_3(q_3 - q_2) = \theta_3 q_3 - (\theta_3 - \theta_2)q_2 - (\theta_2 - \theta_1)q_1.$$

Plugging these values for the transfers into the profit function, we obtain

$$\Pi = \phi_1 \left(\theta_1 q_1 - \frac{1}{2} q_1^2 \right) + \phi_2 \left(\theta_2 q_2 - (\theta_2 - \theta_1) q_1 - \frac{1}{2} q_2^2 \right) + \phi_3 \left(\theta_3 q_3 - (\theta_3 - \theta_2) q_2 - (\theta_2 - \theta_1) q_1 - \frac{1}{2} q_1^2 \right).$$

Differentiating with respect to quantities yields three first-order conditions:

$$\phi_1(\theta_1 - q_1) = (\phi_2 + \phi_3)(\theta_2 - \theta_1),$$

$$\phi_2(\theta_2 - q_2) = \phi_3(\theta_3 - \theta_2),$$

$$\phi_3(\theta_3 - q_3) = 0.$$

Using the given parameters, we have $q_1^* = 2$, $q_2^* = 4$ and $q_3^* = 6$.