Mathematical Methods in Economics

Syllabus

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Class Time: Mon-Fri: 10am-12pm; 1pm-3pm. (Except Sep 3rd, Labor Day)

TA Session: Mon-Fri: 3:30pm-4:30pm

Location: SHFE 146, (for Sep 11th and Sep 12th only: SHFE 021)

1 Introduction

The goal of this course—Mathematical Methods in Economics—is to introduce/review the mathematical concepts that will be used in economic studies, especially for the first year Economics PhD sequence. Since the topics are broad and time is limited, our approach will not be as rigorous as textbooks in mathematics: We will prove the most important results and omit some more basic (and sometimes too complicated) proofs. Also, since this is a course introducing mathematical methods, the discussions will be centered around mathematical—instead of economic—topics. Nevertheless, we will use economic examples to illustrate the mathematical tools whenever possible. As the main constituent of the class are Economic PhD students or Master students who will also take the PhD sequence, we

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assume certain level of mathematical background. Specifically, we list the topics that we assume you know below.

- Calculus: Familiar with univariate and multivariate differentiation and Riemann integral, including some basic operational rules (e.g. chain rule, integration by parts etc.).
- Basic Linear Algebra: Understand the notions of vector space, linear transformations, basic matrix algebra, null space, rank, invertibility, determinants and inner product.
- Basic Analysis: Understand basic point set topology, metric space, continuous function, sequence, series and convergences.
- Basic Probability Theory: Understand the terminologies and their meanings, especially random variables, cumulative distribution functions and density functions.
- Proof-based arguments: Familiar with the ways to write/read proofs, basic methods such as induction, contradiction etc.

2 Lectures and TA Sessions

This course will be divided into two parts, the first part will be centered around mathematical methods for microeconomics and probability theory. Topics include basic topology, rigorous treatments of integration and differentiation, optimization methods, measure theory-based probability theory. Kai Hao Yang will be leading this part. The second part of the class will focus on techniques and tools for solving macroeconomic models. We will start the course by introducing some utility and production functions used in macroeconomics and deriving their properties. Then, we will examine dynamic models both in continuous-time and discrete-time environment and look at some economic applications. Kai-Wei Hsu will be leading this part. The class is scheduled to meet every weekdays, Monday to Friday from Sep 4th to Sep 21st (Except Sep 3rd, Labor Day). The lectures will be held from 10am to 12pm and from 1pm to 3pm and the location will be SHFE 146.

There will be TA sessions throughout the course, the TA sessions is for further questions about course materials, problem sets and two sessions of computation that introduces some

commonly used programs in economics. The TA session will NOT be held everyday, we will schedule TA session throughout the course and further announcements will be made. The location will also be SHFE 146.

3 Grading and Problem Sets

There will be exercises for some of the lectures. The problem sets are not compulsory, but may be related to the class material. However, we strongly recommend attending the lectures, reviewing the materials and thinking/writing about the problem sets: If you are not very familiar with all the materials covered in class, these few weeks will be the only weeks that you can have chances to review these tools in details. If you knew these tools but have not been at school for a few months (or years), this will be a chance for you to warm up and pick up some skills that you used to be familiar with. Even if you are completely comfortable with all the materials, working/studying with your cohort during this period can facilitate forming study groups, which will become important during your first year.

A final exam will be held at the end, materials include all topics covered throughout the course.

4 Class Schedule

Tentative schedule is as below:

Part I

- Basic Topology
  - Review of Metric Space and Introduction of Topological Space
  - Function and Correspondence on Topological Space
- Integration and Differentiation
  - Introduction to Lebesgue Measure
  - Lebesgue Integral

  - Differentiation

- Optimization and Comparative Statics
  - First-order Approach
  - Convex Analysis
  - Methods for Comparative Statics

- Introduction to Probability Theory
  - Measurable Space and Probability Measure
  - Random Variable
  - Absolute Continuity and Conditional Expectation
  - Notions of Convergence (if time permits)
  - The Space of Probability Measures and the Weak-* Topology (if time permits)

Part II

- Basics
  - Utility and Production Functions
    - TA session: Epstein-Zin Preferences

- Continuous-Time Models
  - Dynamic Optimization
    - TA session: Perturbation Methods
  - Optimal Control
  - Bellman Equations
  - Stochastic Models: Poisson Process
    - TA session: Brownian Motions

- Discrete-Time Models
  - Dynamic Optimization
    - TA session: Matlab
  - Bellman Equations
    - TA session: Markov Chain, Impulse Response Functions
  - Stochastic Models

References

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