Price Theory
Proposed Solution to Problem Set 3, Question 1

October 28, 2018

A business owner needs to hire a manager for his business. The true profits of the business $T + u$ are increasing in the manager's talent T, and are divided between manager and owner. The manager can expend effort to appropriate some of the profit from the owner, which the manager keeps for himself in addition to the share of reported profit that he gets pursuant to his contract with the owner.

The owner is choosing between less talented family members and more talented people who are unrelated (talent is observed by all). The relationship between the owner and a blood relative is altruistic in the sense that either owner or manager maximizes a weighted sum of his own "personal" utility and the utility of the other person. But the relationship leans toward selfish in that the weight on own utility exceeds one half.

1 *Setting*

We have two agents in this world: the owner and the manager. While talent T is observed by everyone, u is in principle only observed by the manager. Thus, true profits are not observable by the owner.

Regarding preferences, we will assume that every selfish person has the same preferences.[1] Since we are not interested in thinking about the underlying consumption problem (which could involve many consumption goods) we will directly model the indirect utility function. Moreover, we will assume that the egoist utility function is such that the indirect utility function (or value function) is $V(M) = M$, where M denotes the person's income.[2]

Regarding altruistic preferences, note that someone can be altruistic towards another, even if this other person is selfish. This gives rise to three forms of altruism in any given couple. Consider first the case where both of them are altruistic. In this case, the value to each one given income levels is

$$V_{self}(M_{self}, M_{other}) = \beta M_{self} + (1-\beta)V_{other}(M_{other}, M_{self}),$$

where $V_{self} = V_{other}$. To solve for the value function, notice that this

[1] This is what will be meant by "personal" utility function.

[2] This is actually not very restrictive. Under preferences that feature a constant elasticity of subsitution (CES), the value function is proportional to income, where the scaling constant is a function of prices.

system of equations yields a geometric series:

$$\begin{aligned} V_{self}(M_{self}, M_{other}) = \; & \beta M_{self}\left[1 + (1-\beta)^2 + (1-\beta)^4 + \dots\right] \\ & + (1-\beta)\beta M_{other}\left[1 + (1-\beta)^2 + (1-\beta)^4 + \dots\right] \\ = \; & \frac{\beta}{1-(1-\beta)^2}\left[M_{self} + (1-\beta)M_{other}\right] \\ \propto \; & M_{self} + (1-\beta)M_{other}. \end{aligned}$$

If the other individual is not altruistic, then

$$V_{self}(M_{self}, M_{other}) = \beta M_{self} + (1-\beta)M_{other}.$$

Manager's Decisions

By specifying profits to be $T + u$, we are abstracting away from the decisions the manager can take to configure profits. Instead, we focus on how many profits to steal from the owner. Let s be the share of actual profits that the manager appropriates. We assume that stealing a fraction s has an associated cost of effort denominated in dollars of $e(s)$. Further, we take e to be increasing and convex, with $e(0) = 0$. Thus, stealing becomes increasingly harder as the amount stolen increases.

Now, how much to steal will depend on the type of contract the manager signs. Let θ denote the contract-specified share of *reported* profits that is kept by the manager. We will consider the types of contracts described in the question, i.e. if the manager reports x profits, he gets θx dollars.

Owner's Decisions

Notice that allowing the owner to choose θ implies that the solution to this question requires game theory tools. Instead, we think of θ as being an equilibrium price that the owner takes as given. The decision we model is then which manager to hire.

a.

Under what conditions can the owner prevent the unrelated manager from hiding any profit?

Notice first that, the manager's payoff from stealing s is

$$s(u+T) + (1-s)\theta(u+T) - e(s).$$

Therefore, if he were to steal a positive amount, he would choose a stealing level s^* that equates the marginal benefit of stealing to its marginal cost:[3]

$$(1-\theta)(u+T) = e'(s^*). \tag{1}$$

[3] Note that an interior solution – $s^* \in (0,1)$ – will be achieved as long as stealing large shares of profits requires a lot of effort (e.g. $e(s) \to \infty$ as $s \to 1$) and stealing a bit more when one is barely stealing is easy (e.g. $e'(0) = 0$).

Whether this is the actual utility-maximizing behavior will depend on how stealing s^* compares against not stealing at all. In this respect, it is worth noting that $e()$ is likely not continuous around 0. In other words, stealing a bit, even if it is a low amount, already requires making up lies and carries a moral stigma. How large this cost of stealing any amount is determines whether one will decide to go for s^* or stick to no stealing at all. In math, the manager will decide to steal iff

$$\begin{aligned} &\theta(u+T) < s^*(u+T) + (1-s^*)\theta(u+T) - e(s^*) \\ \iff &e(s^*) < s^*(1-\theta). \end{aligned} \tag{2}$$

In the case in which the manager does prefer to steal s^*, stealing could be prevented if we change our setting and allow the owner to observe u. In this case, he could write down a contract where, given the reported profits π and the manager's talent T, the manager gets $\theta\pi$ if $\pi = T + u$ and zero otherwise. In other words, the owner has zero tolerance for stealing.

b.

Hereafter, assume that the owner cannot prevent the unrelated manager from hiding profits. Do you expect the business owner's son to hide profits? Does it matter whether it is the owner or the son who is altruistic?

We are back to the case where the owner does not observe u and considers contracts whereby the manager gets a fixed share of reported profits.

The relationship between father and son can be one where both are altruistic towards the other, or only one of them is altruistic.

The natural starting point is the case where both are altruistic. In this case, the manager's problem is to maximize her value, which for a given choice of s is proportional to:[4]

[4] Refer to the setup for the value of the son.

$$s(u+T) + (1-s)\theta(u+T) - e(s) + (1-\beta)(1-s)(1-\theta)[u+T].$$

As usual, the optimal amount of stealing in the event of stealing is determined by equating the marginal benefit of stealing to its marginal cost:

$$(1-\theta)\beta(u+T) = e'(s^*).$$

Note that the marginal benefit of stealing is increasing in the selfishness parameter, β. Thus, if he were to steal he would steal less than the selfish manager. Moreover, because of (2), he is also less likely to steal than the selfish manager.

This qualitative conclusion continues to hold when the son is altruistic towards the owner, but the owner is not altruistic towards his son. In this case, stealing s yields value

$$\beta\left[s(u+T)+(1-s)\theta(u+T)-e(s)\right]+(1-\beta)(1-s)(1-\theta)[u+T]$$

and his optimal stealing choice in the event of stealing satisfies

$$(1-\theta)\frac{2\beta-1}{\beta}(u+T)=e'(s^*).$$

Note that $\frac{2\beta-1}{\beta}<\beta$ so that the son steals less than in the case with double-sided altruism. This is counter-intuitive only on the surface: when both parent and son are altruist, the son cares about his own income directly *and* indirectly, since the owner is happy to see a happy son. Again, because of (2), he is also less likely to steal than the son with an altruistic parent.

Finally, notice that when only the parent is altruistic, the son solves the problem in question *a*.

c.

Is your answer different if the manager is the owner's son-in-law, who has an altruistic relationship with his wife (the owner's daughter)?

We will proceed with the following altruistic structure: the manager and his wife are both altruistic among them; the daughter and the owner are also altruistic among them; and the owner and the manager are not altruistic among them. Furthermore, we have to say something about the utility of the wife. We will abstract from the intra-household allocation problem and simply assume that the wife cares only about her husband's income if she were selfish. In addition, we will assume that the wife cares about the husband and owner equally.

Thus, value functions (or indirect utility functions) are as follows:[5]

$$\begin{aligned}V_{manager}&=\beta M_{manager}+(1-\beta)V_{wife}\\V_{wife}&=\beta M_{manager}+\frac{1-\beta}{2}[V_{manager}+V_{owner}]\\V_{owner}&=\beta M_{owner}+(1-\beta)V_{wife}.\end{aligned}$$

[5] We omit the arguments of the value functions – $M_{manager}$ and M_{owner} – for the sake of readability.

We can now substitute the wife's value function in the other two to get the following:

$$V_{manager} \propto \beta M_{manager} + (1-\beta)\left[\frac{1-\beta}{2(2-\beta)} V_{owner}\right]$$
$$V_{owner} \propto \beta M_{owner} + (1-\beta)\left[\beta M_{manager} + \frac{1-\beta}{2} V_{manager}\right].$$

Notice that these are very similar to the standard value functions in the case that the manager and owner are in a reciprocal altruistic relationship. From the point of view of the manager, the difference now is that the weight on the owner's value is lower.[6] Therefore, the manager will steal more than in the direct altruism case, but less than in the selfish case.

[6] This follows since $\frac{1-\beta}{2(2-\beta)} < 1$.

Note that this is closely related to Becker's 'Rotten Kid Theorem': namely, family members will act to maximize their joint surplus, even if they are selfish, providing that their incentives are aligned. In this case, all actors (wife, son-in-law, and owner) have their payoffs depending on the actual profits, which in turn depend on T. Therefore, they all act to maximize these profits. This *joint-surplus maximization* is an important economic concept.

d.

Is your answer to (b) different if the family is not altruistic, but the owner brought up his son to "feel" guilt, which you can interpret as a greater disutility of effort towards stealing.

In our setting, we are interested in understanding a change in the effort function, from e to $\tilde{e}$. The new function $\tilde{e}$ incorporates the guilt of stealing which the parent has instilled in their son.

In principle, we can think of $\tilde{e}$ as differing from e in level, slope, or both. It's important to note that **if $\tilde{e}$ only differs in level from e, the son will not steal any less** in the event of stealing. That's because he compares the **marginal** benefit of stealing to the **marginal** cost of stealing. Changing the *level* of e by instilling guilt will only affect the son's decision to steal or not. In other words, level changes in $e()$ affect the decision to steal purely on the extensive margin, whereas slope changes in $e()$ affect stealing decisions on the intensive margin, in the event of stealing. Notice that, by changing the optimal stealing amount, slope changes in $e()$ can also influence the decision to steal or not.[7]

[7] See (2).

e.

Under what conditions does the owner prefer to hire his less talented son, rather than a more talented unrelated manager? Refer to your setups for (b) and (d).

We consider now two talent levels, T_{son} and T_{other}, and a selfish owner for simplicity.

The tradeoff facing the owner is the following: let his/her son manage the business and steal (with the owner getting some utility from this), or let a professional manager manage the business and steal more, but also produce more.

Note first that if $T_{son} = T_{other}$ then the owner prefers to hire the son: (i) if neither of them were to steal, he would be indifferent, (ii) if both steal, the son steals less (recall question *b*., and (iii) If one steals and the other does not, it has to be that the son is the one not stealing.

In order to continue, we have to make further assumptions on our setting. As it stands, the owner's profits could be increasing or decreasing in the talent of unrelated managers, depending on whether the amount stolen increases a lot with talent or not. Concretely,

$$\frac{\partial M_{owner}}{\partial T} = (1-\theta)\left[(1-s^*(T)) - \frac{ds^*(T)}{dT}(u+T)\right],$$

where, differentiating (1) w.r.t. T[8]

$$\frac{ds^*(T)}{dT} = \frac{1-\theta}{e''(s^*(T))}.$$

[8] Note this equation holds in a neighborhood of T!

To recap, the math is saying that the owner's profits are increasing in talent only if stealing does not increase too much with talent, and this is the case if the marginal cost of stealing rises very fast at low levels of stealing.

We assume the effort function satisfies this condition, guided by the idea that profits increasing with manager's talent is verified in real life.

What we can take away from this argument is that there should be a *cutoff* level of talent T^* that makes the owner indifferent between hiring the son and the unrelated manager. At unrelated-manager talents $T_{other} < T^*$, the owner prefers the son. At unrelated-manager talents $T_{other} > T^*$, the owner prefers the unrelated manager.

f.

Using this model, what can you say about the productivity effects of social institutions that encourage honesty among unrelated people?

Note that the efficient outcome in this world occurs when total profits are maximized, and this is the case when the most able managers are hired. It makes no difference who gets which slice of this pie: for efficiency, we want to make the pie as big as possible. But because of moral hazard problems, the owner may not find it worthwhile to employ the most talented manager. If we had social institutions that shift the level of the cost of effort up, we would observe more able managers directing companies and total surplus would increase.