Koyck (1954): Short Run vs. Long Run Response

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$$y^* = \text{optimal level (Long Run)}$$
$$y_t = \text{actual level}$$
$$y_{t-1} = \text{previous level (already realized)}$$

Pick current y_t to minimize the following loss function

$$\underbrace{\frac{1}{2}\phi(y^* - y_t)^2}_{\substack{\text{cost of being away from optimum} \\ \theta>0}} \qquad + \underbrace{\frac{1}{2}(y_t - y_{t-1})^2\eta}_{\substack{\text{cost of adjustment}^2 \\ \eta>0}}$$

Mini costs:

$$\text{FOC:} -\phi(y^* - y_t) + \eta(y_t - y_{t-1}) = 0$$

$$\text{FOC}: y_t = \frac{\phi}{\phi+\eta}y^* + \frac{\eta y_{t-1}}{\phi+\eta}$$

Partial adjustment model

- Let $y^* = \tau(x)$

$$\frac{\partial y_t}{\partial x} = \underbrace{\frac{\phi}{\phi + \eta}}_{<1} \frac{\partial y^*}{\partial x} = \frac{\phi}{\phi + \eta} \frac{\partial \tau(x)}{\partial x}$$

- Suppose $y^*_{i,t} = X_i \beta + U_{i,t}$
- $U_{it} = \rho U_{i,t-1} + \varepsilon_{it}$
- ε_{it} mutually uncorrelated
- **Question: Is the model identified?** Consider three cases
  - a. $X_{it} = \bar{X}$: a constant for all $i + t$
  - b. X_{it} varies over people and time
  - c. $X_{it} = X_t$ is the same variable over persons and time
- You observe Y_{it}, $Y_{i,t-1}$ and the specified X values