Price Theory I
Problem Set 3, Question 2

Rafael Jiménez*

Fall 2018

a.

Consider a representative agent in this economy. Call $x^{pr}(t^{pr}, t^{pu}, p, m)$ and $x^{pu}(t^{pr}, t^{pu}, p, m)$ the demands for private and public education, respectively. $t^i, i \in \{pr, pu\}$ denotes tuition for private and public colleges, p denotes the price of a generic consumption good y and m denotes income.

The relative demand for private versus public universities has been observed to depend on the inflation-adjusted difference between tuition rates, rather than the tuition ratio, which can be expressed as:

$$\frac{x^{pr}(t^{pr}, t^{pu}, p, m)}{x^{pu}(t^{pr}, t^{pu}, p, m)} = F\left(t^{pr} - t^{pu}, p, m\right)$$

for some function F. You could also write this as $F\left(\frac{t^{pr}-t^{pu}}{p}, \frac{m}{p}\right)$, and this case is covered in the more general expression above.

Write the market share (in terms of revenue) as:

$$\frac{t^{pu}x^{pu}}{t^{pu}x^{pu} + t^{pr}x^{pr}} = \frac{t^{pu}}{t^{pu} + t^{pr}F(t^{pr} - t^{pu}, p, m)} = \frac{1}{1 + \frac{t^{pr}}{t^{pu}}F(t^{pr} - t^{pu}, p, m)},$$

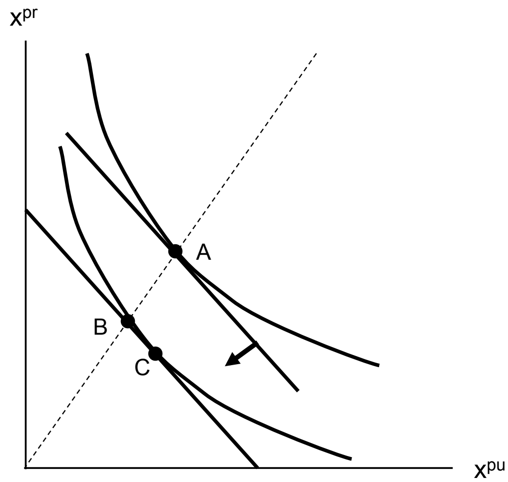
which is obtained by dividing over x^{pu} and t^{pu} in both the numerator and denominator. Notice that a proportional increase in tuition (i.e., λt^i for $\lambda > 1$) affects the market share only via the relative demand for private vs public universities—which makes sense, since the relative tuition stays constant. We

*These answers benefited from discussion with Casey Mulligan, Ivan Kirov and Francisco del Villar. All errors are my own.

expect this relative demand to be decreasing on $t^{pr} - t^{pu}$, which means that F_1 (the first derivative of F) is negative and thus the market share of public schools increases.

Graphically, what happens is a parallel shift of the budget constraint towards the origin, pictured in Figure 1. If preferences were homothetic, the ratio of private vs public would remain unchanged (movement from A to B, along the dashed ray from the origin); however, the assumption made in this question tells us that the new equilibrium will be located in point C, with a relatively higher demand for public universities (located over a ray from the origin that is flatter than the one that passed through A).

Figure 1: **Proportional increase in tuition rates**



To see why this is the case, split the consumer's problem into an 'outer' maximization problem consisting on choosing x and an 'inner' maximization problem over college enrollment. The budget constraint of this problem is $t^{pu}x^{pu} + t^{pr}x^{pr} = \widetilde{m}$, where $\widetilde{m} = m - py$ is the income available to spend on college education. The proportional increase in tuition rates is thus equivalent to a reduction of this income to $\widetilde{m}/\lambda < \widetilde{m}$.

b.

Marshallian demand is homogeneous of degree zero on prices and income. In the previous part the only thing that changed were tuition rates; the price of other goods and income remained the same. Thus, we cannot use this substitution effect (increase in market share of public colleges) to argue inconsistency with homogeneity of demand. This is still true when we also increase household incomes in the same proportion as tuition rates; the price of other consumption goods—indexed here by p—remains unchanged.

c.

Before taking first-order conditions, inspect $u(g,s)$ and notice that it is not the 'standard' utility function in the sense that prices t^i appear there. To see why this is the case, recall that quasilinear functions are given by $u(x,y) = v(x) + y$, where v is concave and y often denotes 'general consumption'—as in this case, we want to analyze college decisions by grouping everything else into this broad category. Quasilinear functions are commonly used to analyze the demand for goods x that have a small budget share and thus have negligible income effects.[1]

The consumer's problem would be:

$$\max_{x^{pr},x^{pu},y} v(x^{pr}, x^{pu}) + y$$
$$\text{s.t. } t^{pu}x^{pu} + t^{pr}x^{pr} + py = m.$$

Solve for y in the budget constraint and plug back into the objective function to get an equivalent problem:

$$\max_{x^{pr},x^{pu}} v(x^{pr}, x^{pu}) + \widehat{m} - t_g x^{pu} - t_v x^{pr}.$$

where I have 1) ignored corner solutions and 2) defined inflation-adjusted tuition rates (i.e., expressed in terms of the consumption good) as: $t_g = t^{pu}/p$, $t_v = t^{pr}/p$ and inflation-adjusted income as $\widehat{m}/p$. From the definitions given in the PS, it's clear that $s = x^{pu} + x^{pr}$ is total college enrollment and $gs = x^{pu}$, with g the fraction of students enrolled in a public college. The problem can then be redefined in terms of s and g as:

$$\max_{s,g} \underbrace{v(s(1-g), gs) - t_g gs - t_v s(1-g)}_{u(g,s)}.$$

Now you understand why the function $u(g,s)$ has college tuition rates there and why you can take FOC with respect to the utility function alone (i.e., why we're

[1]Recall that the Slutsky equation can be written in terms of elasticities and the budget share; when this share is small, the income effect is zero. To see why quasilinear demand for x does not depend on income, take FOC and note that in an interior solution, $u'(x) = p_x/p_y$, so x depends entirely on the price ratio.

solving an unconstrained problem).[2]

Take first-order conditions with respect to g:

$$\begin{aligned}\delta s - s\ln(sg) - s + s\ln[(1-g)s] + s - t_g s + t_v s = 0,\\ \delta s - s\ln g + s\ln(1-g) - t_g s + t_v s = 0,\end{aligned}$$

factor out s (total college enrollment cannot be zero) and rearrange:

$$\ln\left(\frac{1-g}{g}\right) = t_g - t_v - \delta. \tag{1}$$

Notice that $x^{pr}/x^{pu} = (1-g)/g = \kappa\exp\left[-(t_v - t_g)\right]$, where $\kappa \equiv \exp(-\delta)$. In words, this functional form yields relative demand for private vs public colleges that depends on the inflation-adjusted difference between tuition rates, rather than the tuition ratio. Note it was not necessary to take FOC with respect to s for this answer.

d.

Solve for g from Equation (1):

$$g = \frac{\kappa\exp(t_v - t_g)}{1 + \kappa\exp(t_v - t_g)}.$$

In words, the fraction of college students in a public college is given by a logistic function of the inflation-adjusted difference between tuition rates. It is not necessary for this question to have prior knowledge of the logit demand model (LDM) or to derive this model in full (the classic reference is Nevo (2000)). The takeaway is that LDM, despite having different assumptions, gives market shares identical to what we derived for g here. The simplest form of LDM is a discrete-choice model; individual i has to choose one alternative (product) $j \in \{1, \ldots, J\}$. The indirect utility of choosing alternative j is given by:

$$u_{ij} = X_{ij}'\beta + \varepsilon_{ij},$$

where X_{ij} is a vector of characteristics (of the individual, the product, and the individual interacted with the product), β_i is a vector of coefficients and ε_{ij} is

[2] Note I did not include income since this is an affine transformation; x^{pr} and x^{pu} do not depend on income as argued above.

a stochastic term. The individual's problem is thus $\max\{u_{i1}, \ldots, u_{iJ}\}$ and the probability that she chooses product j is:

$$\Pr(u_{ij} \geq u_{ik}\ \forall k | X_{ij}, \beta)$$

Under some distributional assumptions for ε_{ij}, it can be shown that the probability that the individual chooses i is given by a logistic function; $\exp(X_{ij}'\beta)/[1+\sum_k \exp(X_{ik}'\beta)]$. When sample size is big, we can approximate the fraction of people that choose j by $\Pr(i \text{ chooses } j)$.

e.

We can think that the tuition difference $t_v - t_g$ might difference out some common component in both private and public universities—say, knowledge—and capture an additional good provided by private universities—say, "reputation". Suppose that utility is defined not over private and public education but over some characteristics (remember the household production model?) such as knowledge k, reputation r and some other consumption good y. Individuals obtain (produce) knowledge by going to private or public universities, $k = \alpha(x^{pr} + x^{pu})$, but obtain reputation only by going to private universities, $r = \beta x^{pr}$.[3] Individuals have to choose college enrollment and consumption of the private good to solve:

$$\begin{aligned} \max_{x^{pr}, x^{pu}, y} & \; u(k, r, y) \\ \text{s.t. } & t^{pu} x^{pu} + t^{pr} x^{pr} + py = m \\ & k = \alpha(x^{pr} + x^{pu}) \\ & r = \beta x^{pr}. \end{aligned}$$

denote by u_i the ith derivative of u. Take FOC:

$$\begin{aligned} u_1\alpha + u_2\beta &= \lambda t^{pr} \\ u_1\alpha &= \lambda t^{pu} \\ u_3 &= \lambda p \end{aligned}$$

[3] I used these functional forms for simplicity, but we can easily generalize it to some non-linear forms, as long as private and public schools are perfect substitutes on knowledge.

Reorganize the system to get: $\frac{\beta u_2}{u_3} = t_v - t_g$. A sufficient condition would be that $\frac{u_2}{u_3}$ is a function of k/c, for example.[4]

f.

We have seen that a proportional increase in tuition decreases the market share of private universities. Hence, a proportional subsidy to tuition would increase their share. Nevertheless, a constant subsidy increases the relative tuition of private vs public schools:

$$\underbrace{\frac{t^{pr} - \lambda}{t^{pu} - \lambda}}_{\text{New tuition ratio}} > \underbrace{\frac{t^{pr}}{t^{pu}}}_{\text{Previous tuition ratio}}$$

Going back to our expression of public school market share in part a), tuition differences remain the same (so the ratio x^{pr}/x^{pu} remains unchanged), but the market share of private schools increases as well.

However, private schools need not care about maximizing their market share or even their profits (there are many non-profit private universities).[5] Private universities might care about their share in enrollment, in which case they would likely prefer the proportional subsidy since the constant subsidy does not alter the relative demand (part a).

References

Nevo, A. (2000). A practitioner's guide to estimation of random-coefficients logit models of demand. *Journal of Economics & Management Strategy*.

[4]It is not crucial to provide an example of a functional form that satisfies this; the mere intuition of $t_v - t_g$ cancelling a common component and capturing an additional component of private schools would be enough. However, if you're curious about what functional form might satisfy this, consider $u(k, r, y) = k^\gamma r^{1-\gamma} + f(y)$ for some increasing concave function f and $\gamma \in (0, 1)$.

[5]If you're curious about a discussion of whether profit/non-profit schools are better from a social point of view, read the "For profit colleges" in the Becker-Posner book.