Price Theory I
Problem Set 7, Question 1

Rafael Jiménez*

Fall 2018

This question is about household decision-making. Prior to Becker's work (Becker, 1981), Economics treated households as individuals, with a single utility function and a pooled budget constraint. During the 80s and 90s the literature debated about how to model household choice (Pollak, 2002). Several alternatives (Nash bargaining, collective models) have been proposed. Bottom line: different assumptions will give you different implications in terms of the neutrality of income distribution between mother and father (with cash transfers being an example that affects income distribution).

a.

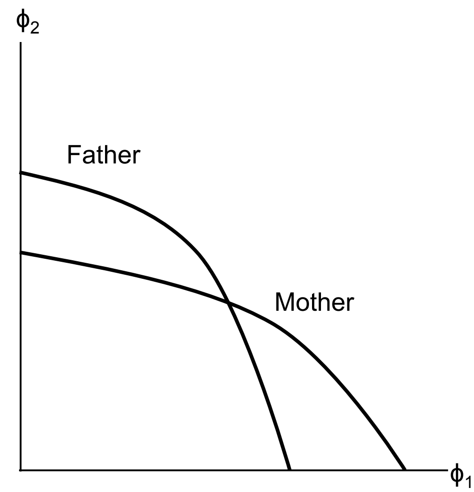
Father and mother derive utility from their personal (rival) consumption c^f, c^m, respectively, and from the public good j, which is produced using two tasks as inputs, $j = F(\phi_1, \phi_2)$. They both have an endowment of one unit of time which they can allocate to market production, which pays wage w, and/or to both tasks. The total amount of task $k \in \{1, 2\}$ available is a function of the amount of time devoted by mother and father; $\phi_k(t_k^f, t_k^m)$. Parents differ in terms of how productive their time is when performing the tasks; devoting time to one task takes time away from the other task, and they differ in the magnitude of this opportunity cost. We say father has a comparative advantage in task 2 if:

$$\left| \frac{\partial \phi_2 / \partial t_2^f}{\partial \phi_1 / \partial t_1^f} \right| > \left| \frac{\partial \phi_2 / \partial t_2^m}{\partial \phi_1 / \partial t_1^m} \right|$$

*These answers benefited from discussion with Casey Mulligan.

To see why this is the case, take a look at Figure 1. This picture plots the production possibility frontiers for each parent; that is, the combinations of task 1 and 2 they can produce using some amount of time $\bar{t} \leq 1$. We say that father has a comparative advantage in task 2 if the slope of his production possibility frontier is steeper than the mother's (i.e., sacrifice less of task 1 to produce an extra unit of task 2). Since $t_1^i + t_2^i = \bar{t}$, $dt_1^i = -dt_2^i$, we can compute the slope of fathers PPF as: $d\phi_2/d\phi_1 = \frac{d\phi_2}{dt_2^f}\frac{dt_2^f}{d\phi_1} = -\frac{\partial\phi_2/\partial t_2^f}{\partial\phi_1/\partial t_1^f}$.[1]

Figure 1: **Production possibility frontiers**



Since 'the father makes all decisions', I assume that he confiscates mother's time endowment, decides the optimal allocation and compensates her appropriately. This part is where you can make different assumptions (e.g., collective models, Nash Bargaining, etc.) which will give you different results. The father's budget constraint is thus:

$$c^f = w(1 - t_1^f - t_2^f) + w(1 - t_1^m - t_2^m) + y,$$

where y is the compensation to the mother. The marriage market (i.e., outside option) pins down what this compensation should be equal to (Becker, 1981, Chapter 8). Moreover, comparative advantage leads to gains from trade (i.e.,

[1]Note, first, that this slope is negative (it has to be, since both tasks require time and time is a scarce resource). Moreover, it is possible to show that the PPF is concave under some assumptions on the concavity of ϕ_k.

incentives to get married). Assume for simplicity that the imputed income from the marriage market; that is, the minimum compensation that mothers accept is w.[2] Note that the father faces the opportunity cost of mother's labor; the time he decides the mother should spend working at home is time he could have allocated to the market. Mother's budget constraint is $c^m = y$, so the father's compensation decision is equivalent to choosing mother's consumption. The problem he faces is:

$$
\begin{aligned}
\max_{\{t_k^i, c^f, c^m\}} \; & u^f(c^f, j) \\
\text{s.t. } & c^f + c^m + w(t_1^f + t_2^f + t_1^m + t_2^m) = 2w \\
& j = F(\phi_1(t_1^f, t_1^m), \phi_2(t_2^f, t_2^m)) \\
& t_k^i \geq 0, \; i \in \{m, f\}, \; k \in \{1, 2\} \\
& t_1^i + t_2^i \leq 1, \; i \in \{m, f\} \\
& c^m \geq w
\end{aligned}
$$

Note that the father is effectively facing the household's budget constraint, not his individual budget constraint. In this sense, this model is 'unitary' due to the dictatorial power of the husband. The father will set $c^m = w$ and mother's preferences do not play a role at all in consumption and production decisions.

Before the altruistic case, let's see this problem graphically. Note that we can break down the maximization in two steps. First, an inner minimization problem in which the father chooses time allocation to minimize the cost of j, for a fixed j. This step is analogous to the cost minimization problem of a firm. Second, the father chooses c^f and j to maximize his utility subject to the budget constraint. This problem is the standard consumer problem.

Since all inputs have the same opportunity cost (lost market wages), the

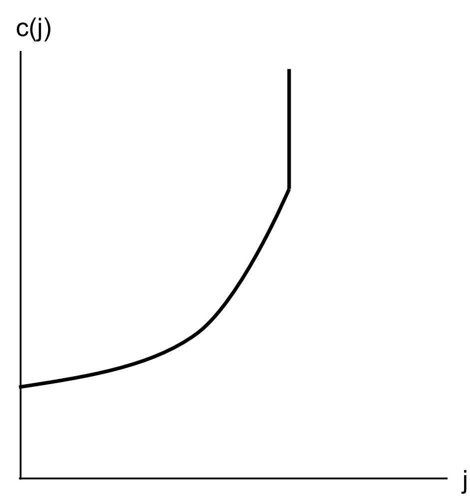
[2] Otherwise, the marriage market leaves, after transfers m between mother and father, incomes $I^f = w - m$ and $I^m = w + m$, and m will cancel out eventually when we plug in the mother's budget constraint into the father's budget constraint.

inner minimization problem is, for a fixed j:

$$\min_{\{t_k^i\}} w(t_1^f + t_2^f + t_1^m + t_2^m)$$
$$\text{s.t. } j = F(\phi_1(t_1^f, t_1^m), \phi_2(t_2^f, t_2^m))$$
$$t_k^i \geq 0,\ i \in \{m, f\},\ k \in \{1, 2\}$$
$$t_1^i + t_2^i \leq 1,\ i \in \{m, f\}$$

This minimization problem yields the cost function $c(j)$, which looks roughly like Figure 2. The slope of the cost is infinite when both mother and father are working full time in the production of j. There might be some other kink to the left of this point, if one parent hits their budget constraint. If we were to assume that the time constraint is non-binding, the slope (marginal cost) would be equal to $w/(\partial F/\partial t_k^i)$, where $\partial F/\partial t_k^f = \partial F/\partial t_j^m$ (which is increasing under concavity of F and ϕ_k). With this in mind, the father's outer maximization

Figure 2: **Cost of joint output**

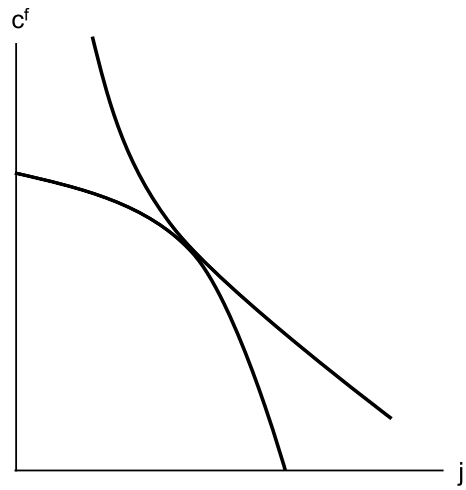


problem is just:

$$\max_{\{c^f, j\}} u^f(c^f, j)$$
$$\text{s.t. } c^f + c(j) = 2w$$

We can see the solution to this problem in Figure 3. Note that the budget constraint is no longer a flat line since the cost of j is not constant: differentiating the budget constraint we get $dc^f/dj = -c'(j)$, which is negative and has a negative derivative.

Figure 3: **Maximization problem**



Now suppose the father is altruistic, so his utility is given by $u^f(c^f, j, u^m)$. The mother's preferences appear in both the budget constraint in Figure 3 and the indifference curve. To see this, solve an intermediate maximization problem, given c^f and j, where the father chooses c^m. The FOC give us $u_3^f(c^f, j, u^m)u_1^m(c^m, j) = u_1^f(c^f, j, u^m)$, which defines implicitly $c^m(c^f, j)$. The budget constraint in the outer maximization problem is thus $c^f + c^m(c^f, j) + c(j) = 2w$. The derivative of the budget constraint has now some additional terms $\partial c^m/\partial j$ and $\partial c^m/\partial c^f$, which depend on the mother's preferences over the joint output, unless $\partial u^m/\partial c^m$ does not depend on j (separability). Moreover, indifference curves now have these additional terms too; the marginal rate of substitution is now:

$$\frac{\frac{\partial u^f}{\partial c^f} + \frac{\partial u^f}{\partial u^m}\frac{\partial c^m}{\partial c^f}}{\frac{\partial u^f}{\partial j} + \frac{\partial u^m}{\partial j} + \frac{\partial u^m}{\partial c^m}\frac{\partial c^m}{\partial j}}$$

Note that, even if we have separability between mother's utility from the joint output and consumption, we still have the term $\frac{\partial u^m}{\partial j}$ in the denominator, so in this case the mother's preference for the joint production output affects the amount that is produced.

b.

The father faces the household's budget constraint (there is income pooling) so the distribution of income—in particular, who gets the cash transfer— is irrelevant for decisions. This is true both when the father is egoistic and when he is altruistic. Note that this is not true in general; some collective models break this property (see Browning et al. (2004) for a general discussion and Doepke and Tertilt (2014) for an application to cash transfers).

c.

I take 'efficient' to mean Pareto-optimal. These allocations are achieved by means of a central planner solving, for some $\lambda \in [0,1]$:

$$\begin{aligned}
\max_{\{t_k^i, c^i\}} \ & \lambda u^f(c^f, j) + (1-\lambda) u^m(c^m, j) \\
\text{s.t. } & c^f + c^m + w(t_1^f + t_2^f + t_1^m + t_2^m) = w + \tau \\
& j = F(t_1^f, t_2^f, t_1^m, t_2^m) \\
& t_k^i \geq 0, \ i \in \{m, f\}, \ k \in \{1, 2\} \\
& t_1^i + t_2^i \leq 1, \ i \in \{m, f\}
\end{aligned}$$

If the λ weights are constant, the allocations do not depend on who receives the cash transfer τ; there is income pooling, so all it matters is the total income received by the household—disregarding the altruistic preferences of the father. Note that our model in a) and b) corresponds to the case where $\lambda = 1$ with the additional restriction that $c^m \geq w$.

In collective models, however, Pareto weights are allowed to depend on many variables (Browning et al., 2004). Perhaps as a tautology, we can allow these weights to depend on the distribution of income between mother and father, in which case the target of the cash transfer matters by construction.

d.

Not necessarily. First, it could be the case that education/health investment in a given child yields lower returns than some alternative investment (e.g., simply putting the money in the bank). If the parents are inefficient in their investment, or if the child is inefficient in capitalizing investment as an adult, then the returns form investing in the bank might be higher than investing in the child (so the child would have a higher income in the future if his/her parents invest in the bank instead of in education/health).

Additionally, we have to consider what ‘living standards’ mean. For example, the children of very strict parents—who invest significantly in their kids’ health and education— need not be happier adults. If, say, individual utility depends on the stock of ‘fun’, and if investment in health and education crowds out parental investment in their kids’ ‘fun’, then these kids will have a lower stock of this in the future, despite having higher income and health.

References

Becker, G. S. (1981). *A treatise on the family.* Harvard University Press.

Browning, M., P.-A. Chiappori, and V. Lechene (2004). Collective and unitary models: a clarification. *Working Paper*.

Doepke, M. and M. Tertilt (2014). Does female empowerment promote economic development? *NBER Working Paper No. 19888*.

Pollak, R. A. (2002). Gary Becker’s contributions to family and household economics. *NBER Working Paper No. 9232*.