Summary of Market Insurance, Self-Insurance, and Self-Protection

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1 Market Insurance

- An individual is faced with two states (0,1) with probability(w/p) p and 1-p respectively
- His real world income endowment in each state is given at certainty by $I_0^e$and I_1^erespectively where I_0^e-I_1^e is prospective loss if state 0 occurs. If income in state 1 can be exchange for income in state 0 at the fixed rate

$$-\frac{dI_1}{dI_0} = \pi \tag{1}$$

- $\pi =' Price$ of insurance' measured in terms of income in state 1
- The amount of insurance purchased in state 0 can be defined as difference between the actual and endowed incomes:

$$s = I_0 - I_0^e \tag{2}$$

- The expenditure on insurance measured in terms of state 1's income is

$$b = I_1^e - I_1 = s\pi \tag{3}$$

- Substituting (2) into (3) gives the opportunity boundary or the budget line AB in Figure 1

$$I_1^e - I_1 = \pi(I_0 - I_0^e) \tag{4}$$

- Individual choose optimal income in states 1 and 0 by maximizing the expected utility of the income prospect,

$$U* = (1-p)U(I_1) + pU(I_0) \tag{5}$$

subject to the contraint given by the opportunity constraint (4)

- The first order optimality condition condition is

$$\frac{U^*_{I_0}}{U^*_{I_1}} = \frac{pU(I_0)'}{(1-p)U(I_1)'} = \pi \tag{6}$$

where $\frac{pU(I_0)'}{(1-p)U(I_1)'}$is the slope of the indifference cruve, and $\pi$is the slope of the budget line. In equilibrium, they must be equal (See point P)

- One can seperate taste from environmental factors by dividing p/(1-p) through in (6) to obtain

$$\bar{\pi} = \frac{1-p}{p}\pi = \frac{U(I_0)'}{U(I_1)'} \tag{7}$$

- $\bar{\pi}$ =Price of insurance deflated by the actuarially fair price p/(1-p), is a measure of the 'real' price of insurance because a fair price is costless to individual. Equation (7) implies that the real price of insurance equals to the ratio of the marginal utility of $I_0 to$ that of I_1.

- Imposing marginal utility of income is strictly declining makes the indifference curve convex from the origin

$$D = -pU_0^{''} - \pi^2(1-p)U_1^{''} > 0 \tag{8}$$

- If the slope of indifference curve exceeded the price of insurance at endowment point E, some insurance would be demanded-some I_1would be traded for I_0 :

$$\bar{\pi} < \frac{U(I_0)'}{U(I_1)'} \tag{9}$$

If the opposite was true, "gambling would occured" (Figure 2) i.e. Individual would give up $I_0$for I_1.

- Note that inferences about attitudes toward risk cannot be made independently of existing market opportunities; a person may appear to be a 'risk avoider' under one combination of prices and potential loss and a 'risk taker' on another. (Compare Figure 1 and Figure 2)

- If the insurance were actuarially fair, equation 7 would reduce to 1=$U(I_0)'/U(I_1)'$; incomes would equalized in both states of the world.

2 Substitution Effects

Find the effect of an exogenous increase in price of insurance. Differentiate the FOC's and use Cramer's rule, we have:

$$\frac{dI_0}{d\pi} = \frac{1}{D}[-(1-p)U_1^{'} + (I_0 - I_0^e)\pi(1-p)U_1^{''}] \tag{10}$$

The denominator D is postive. From (10), an increase in the relative cost of insurance would necessarily decrease the demand for I_0 in state 0 if we assume$I_0 - I_0^e > 0$. Moreove, it also reduces the amount of insurance purchased, since I_0^eremains the same: ds/d$\pi = dI_0 - dI_0^e/d\pi = dI_0/d\pi$

Similarly, the effect of increase in πon $I_1$and thus the amount spend on insurance is

$$\frac{dI_1}{d\pi} = \frac{1}{D}[(1-p)U_1^{'}\pi + (I_0 - I_0^e)pU_1^{''}] \tag{11}$$

Here, the result is ambigous since $U_1^{'}$is positve whereas $(I_0 - I_0^e)pU_1^{''} = spU_0^{''}$is negative if $U_0^{''} < 0$ and $s > 0$. The result is ambigious because, although an increase in πreduces the amount of insurance purcahsed, each unit become more expesnive. Consequently, the amount spent on insurance would decline only if the price elasticity of demand for insurance exceeded unity. If both $I_1$and $I_0$are superior goods(income lasticity greater than 1), the income and substitution effects reduce the demand of I_1wherease they have opposite effects on demand for I_1. Note that even if I_0 is inferior good, however, an expenditure compensated increase in π must always reduce the demand for $I_0$and increase I_1.

3 Income Effects

Equation (4) can be written as

$$I_1^e + \pi I_0^e = W = I_1 + \pi I_0 \tag{12}$$

where W is a measure of the total opportunities available. Differentiate FOC to get

$$\begin{aligned}\frac{dI_0}{dW} = \frac{dI_0}{dI_0^e} = \frac{dI_0}{dI_1^e}\frac{dI_0^e}{dW} &= -\frac{D_{31}}{D} \\ \frac{dI_1}{dW} = \frac{dI_1}{dI_0^e} = \frac{dI_1}{dI_1^e}\frac{dI_0^e}{dW} &= -\frac{D_{32}}{D}\end{aligned} \tag{13}$$

where $D_{31} = \pi(1-p)U_0^{''}$and $D_{32} = pU_1^{''}$. The income demanded in each state necessarily increases with opportunities if the marginal utility of income is falling. Hence, an increase in each state's endowment increases the demand for income in other state as well.

However, effects on the demand for insurance differ depending on how different endowments change. If I_1^ealone increased,

$$\frac{ds}{dI_1^e} = \frac{dI_0}{dI_1^e} > 0 \tag{14}$$

and demand for insurance would increase. Similarly, if I_1^ealone increased,

$$\frac{ds}{dI_0^e} = \frac{dI_0}{dI_0^e} - 1 < 0 \tag{15}$$

and the demand for insurance would decreased. Equations (14) and (15) imply that if the difference in endowed income-the endowed loss from hazard-increased either because I_0^edecreased or I_1^eincreased, the demand of insurance would increase i.e. a person is likely to insure large rather than small loss.

If both edwoments are change by the same percentage,

$$(\epsilon_s W - 1) = I_0(\eta_0 - 1) \tag{16}$$

where $\epsilon_{sw} = ds/dW * W/s$ and $\eta_0 = dI_0/dW * W/I_0$are the opportunity elasticities for s and I_0,respectively. From (16), we have two cases to consider :

- If the slopes of the indifference curves are constant along a given ray from origin-there is constant risk aversion-$\eta_0 = \eta_1 = 1$. An increase endowments would then increase the demand of insurance by the same proportion

- If there is increasing relative risk aversion, $\eta_0$and thus ϵ_{sW} would exceed unity. Increasing relative risk aversion implies that elasticities of substitution between $I_1$and I_0tends to decline as opportunities increase. However, relative risk aversion remains constant about the certainty line.

4 Rare losses

Changes in p, the probability of loss do not affect the incentive to insure as long as the real price of insurance is independent of p. Note that if insurance is actuarially fair, the real price would always equal to unity, and hence independent of p. Now, introduce loading

$$\pi = (1+\lambda)p/(1-p) \tag{17}$$

where $\lambda$is the “loading factor”. If λwere independent of p, so would also be the real price of insurance and p would no effect on the incentive to insure. However, since rare losses are frequently insured, $\lambda$is presumably positively related to p, perhaps because processing and investigating cost increases as p increases. Even if incentive to insure is independent of p, the amount insured would decline and the expenditure on insurance would increase as p increase.

5 Self-Insurance and Self-Protection

Introduction: It is somewhat artificial to distinguish behaviour that reduces the probability of a loss from behaviour that reduces the size of a loss, since many actions do both. Neverthelss, Becker do that.

5.1 Self-Insurance

Assume that market insurance is unavailable. Loss to a person is $L = L(L^e, c)$,where $L^e = I_1^e - I_0^e$is the endowed loss, c is the expenditure on self-insurance, and $\frac{dL}{dc} = L'(c) \leq 0$.The expected utility is

$$U* = (1-p)U(I_1^e - c) + pU(I_1^e - L(L^e, c) - c) \tag{18}$$

The value c that maximize equation (18), c^0,satistifes the following FOC

$$\frac{-1}{L'(c^0)+1} = \frac{pU_0'}{(1-p)U_1'} \tag{19}$$

This maximizes expected utility if the marginal utility of income and marginal productivity of self-insurance are decreasing-the indifference cruves are convex and the production transformation curve between income in states 1 and 0 is concae to the origin. A neccessary condtion for a positive self-insurance is $-L'(c^0) > 0$ or that there be a net addition to income in state 0. A sufficient condtion is that

$$\frac{-1}{L'(L^e, 0)+1} < \frac{pU_0'}{(1-p)U_1'} \tag{20}$$

An increase in unit cost of self-insurance, measured by the marginal productivity of self-insurance, would reduce the demand of self-insuranc, measured by c^0:

$$\frac{dc^0}{d\alpha} \quad < \quad 0 \tag{21}$$

where α is a parameter that reduces the absolute value of L' given c. Similary, a reduction in I_0^ewould increase the demand for self-insurance:

$$-\frac{dc^0}{dI_0^e} \quad > \quad 0 \tag{22}$$

From equation (20), the incentive to self-insure is smaller for rare losses compare to market for insurance. The reason is theat the loading factor of self-insurance is larger for rare losses because its price can be presumed to be independent of the probability of loss. Note that the effect on self-insurance from a change in income has to be seperated from the effect of the associated change in marginal productivity.

If market and self-insurance were both available , then

$$U* \quad = \quad (1-p)U(I_1^e - c - s\pi) + pU(I_1^e - L(L^e, c) - c + s) \tag{23}$$

If the price of insurance were independent of the amount of self-insurance, FOC is

$$\begin{aligned} -(1-p) \quad U_1'\pi + pU_0' &= 0 \\ -(1-p)U_1' - U_0'[L'(c)+1] \quad &= \quad 0 \end{aligned} \tag{24}$$

By combining (24), we have

$$\pi \quad = \quad -\frac{1}{L'(c)+1} \tag{25}$$

In equilibrium, “shadow price” of self-insurance would be equal to market price of insurance. Can infer that market insurance and self-insurance are substitutes in the sense that an increase in π,the probability of loss being the same, would decrease the demand for market insurance and increase the demand for insurance. Similarly, the purchase of market insurance would reduce the demand for self-insurance. When market insurance is available at fair price, (25) become

$$-L'(c) \quad = \quad \frac{1}{p} \tag{26}$$

precisely the condition that maximizes expected income. The following are the important points from these equations:

- Even if the price of insurance were not fair, the optimal value of self-insurance would maximize the market value of income regardless of the shape of indifference curve or the probability distribution of states
- Effect of specific parameters on the demand for market and self-insurance when market or self insurance when both are available are different when they are alone available. For instance, an increase in endowed losses increases the demand for self- or market insurance when either is alone. However, when both are available, the indirects effect and offset the direct effect. Similarly, because a decrease in probability of loss reduces the demand for self insurance, it increases the demand of market insurance. Therefore, people more likely to use the market to insure rare losses because of a substitution between market and self-insurance.

5.2 Self-Protection, Subjective Probabilities and Moral Hazard

Self-insurance and martket insurance both redistribute income towar hazardous states, whereas self-protection reduces the probabilities of these states. Unlike insurance, self-protection does not redistribute income, because the amount spent reducing the probability of a loss decreases income in all states, leaving unchanged the absolute size of the loss.

Although an appropriate definition of states would produce state probabilities that are independent of human actions, it would not produce distribution of outcomes-relevant probability distribution- that is independent of these actions i.e. the human can't change the probability of lightning storm but can change the probability of loss occur due to lightning storm.

Look at Figure 6. Let's say we are endowed with outcomes given by AB. Self-insurance(CD) lowers the probability of low and high outcomes. Self-protection, shift the distribution to the left, say EF, reduces the probability of low outcomes and raises the probability of high one.

5.2.1 Model for self-protection without market insurance

$$U* = [1 - p(p^e, r)]U(I_1^e - r) + p(p^e, r)U(I_0^e - r) \tag{27}$$

$$-p'(r^0)(U_1 - U_0) = (1 - p)U_1' + pU_0'(\text{Optimal Condition}) \tag{28}$$

Let assume probability of hazardous sate can be reduced by appropriate expenditure: $p = p(p^e, r)$, where $p^e$is the endowment probability of hazard and r is the expenditure on self protection, and dp/dr = p'(r) $\leq$0. In equation (28), LHS is marginal gain from reduction in p;RHS is decline in utility due to decline in both income, is the marginal cost. The second order condition is

$$U*_{rr} = -p''(r^0)(U_1 - U_0) + 2p'(r^0)(U_1' - U_0') + (1 - p)U_1'' + pU_0'' < 0 \tag{29}$$

Decreasing marginal utility of income is not a sufficient nor neccessary condition. If $p''(r^0) > 0$,equation (29) is always satisfied if the MU is constant but not if MU is decreasing or increasing. Models implications

- Incentive to self-protect is not so dependent on attitudes toward risk, and could be as strong for risk preferres as for risk avoiders.
- A decline on I_0^ealone might not increase the demand for self protection, even if the marginal utility of income were falling because a decline in I_0^e would increase the marginal cost of protection
- A decline in marginal productivity of protection-an increase in the shadow price of protection-always decreases the demand for self-protection regardless of attitudes towards risks. Therefore, if the endowed probabilities and income are the same, more efficient providers of self-protection would have lower equilibrium probabilities of hazard.

5.2.2 Model for self-protection with market insurance

The function

$$U* = [1 - p(p^e, r)]U(I_1^e - r) + p(p^e, r)U(I_0^e - r) \tag{30}$$

would be maximized w.r.t. r and s. FOC are

$$\begin{aligned} -(1-p)'_1\pi + pU'_0 &= 0 \qquad (31) \\ -p'(r*) * (U_1 - U_0) - (1-p)U'_1[1 + s * \pi'(r*)] - pU'_0 &= 0 \qquad (32) \end{aligned}$$

The term $\pi'(r)$measures the effect of change in self-protection on the price of market insurance through its effect on p and loading factionλ

$$\frac{d\pi}{dr} = \pi'(p)p'(r) + \pi'(\lambda)\lambda'(r) \qquad (33)$$

The effect of market insurance on the demand for self-protection is called "Moral Hazard". In particular, moral hazard refers to an alleged detterrent effect of market insurance on self-protection. Market insurance has two opposite effects on self-protection:

- Self protection is discourage because marginal gain is reduced by the reduction of the difference between the incomes, and thus the utilities in different states(Effect 2)
- It is encourage if the price of market insurance is negatively related to to the amount spent on protection through the effect of these expenditures on probabilities(Effect 1)

Effect 1

Consider equation (31). $\pi = p/(1-p)$ implies that

$$\pi'(r) = p'(r)/(1-p)^2 \qquad (34)$$

Substiting $U_1 = U_0$and $U'_1 = U_0$into equation (32) and using (33a), we have

$$p'(r*) = -(1-p)/s = \frac{-1}{I^e_1 - I^e_0} \qquad (35)$$

By (33a), there is incentive to spend on protection since $\pi$is negatively related to r. By (34), fair price market insurance encourages an expenditure on self-protection that maximizes expected income. Hence, no moral hazard. The optimal self-protection, r*, can be larger than the amount spent in the absence of market insurance, r^0. By (34) and (38), and the condition p'(r)<0, r* would be larger than r^0if

$$\frac{U(I^e_1 - r^0) - U(I^e_0 - r^0)}{I^e_1 - I^e_0} = \overline{U}' < (1-p)U'(I^e_1 - r^0) + pU'(I^e_0 - r^0) \qquad (36)$$

which is likel,y provided p is not very small and U is concave. Implies that market insurance and self-protection are complements in the sense that:

- Availability of the former could increase demand for the latter
- Increase in productivity of self-protection or a decrease in the real cost of market insurance would increase demand for both

Effect 2

The price of market insurance was indepenent of expenditures on self-protection - the loading factor increased sufficiently to offset exactly the reduction in the probability of loss. Since the demand for market insurance is negatively related to the loadings, it would be negatively related to expenditures of on self-protection. Implications: For those market insurance whose prices that are largely independent of expenditures on self-protection, one should either observed a large demand for insurance and a small demand for self-protection, or the converse.

Misc

- Since the price of self-insurance is independent of probability of hazard, and thus expenditures on self-protection, our analysis of market insurance implies that self-insurance is likely to create moral hazard.
- Analysis of moral hazard can be used not only on the relation between self-protection and insurance, but also to the relation between protection and insurance for all uncertain events that can be influenced by human actions.
- Case study 1: Does unemployment compensation relief or encouraged someone becomes unemployed? If cost in time and embarassment of applying for compensation relief is sufficiently positively correlated with its frequency, the availability of insurance might encourage insured to make his own effort.
- Case study 2: Seat belts are ways to self-insure and have cost essentially unrelated to the probability to automobile accident. Hence, create moral hazard.