Price Theory I
Problem Set 6, Question 1

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a.

Multiple equilibria is essentially a question of the sign of the labor supply slope. If demand is perfectly elastic, or downward sloping, a necessary condition for multiple equilibria is that supply slope changes sign at some point. Since supply is typically upward sloping, we want to find conditions for it to be downward sloping over some range. Let $L = M + F$ denote aggregate labor supply, where M and F denote male and female labor supply, respectively. Differentiate this expression with respect to w, and rewrite in terms of elasticities to get: $\varepsilon_L^w = \varepsilon_F^w f + \varepsilon_M^w m$, where f and m are the female and male labor share, respectively, and ε_x^y denotes the elasticity of x with respect to y. If men do not care about the gender market share, ε_M^w will be positive (typical supply) and thus we need female supply to be sufficiently downward sloping over some range; that is, $\varepsilon_F^w < -\varepsilon_M^w \frac{M}{F}$.

Intuitively, this can happen when female labor share is very low: the marginal benefit of having more women in the profession is enough compensation for work, so women are willing to work for lower wages as female share increases. As the fraction of women increases, the marginal benefit decreases, so women need to get compensated with higher salary—thus returning to the usual upwards-slope supply. Figure 1 presents an example of multiple equilibria. If female market share increases with L, for low levels of labor, an increase in L is associated

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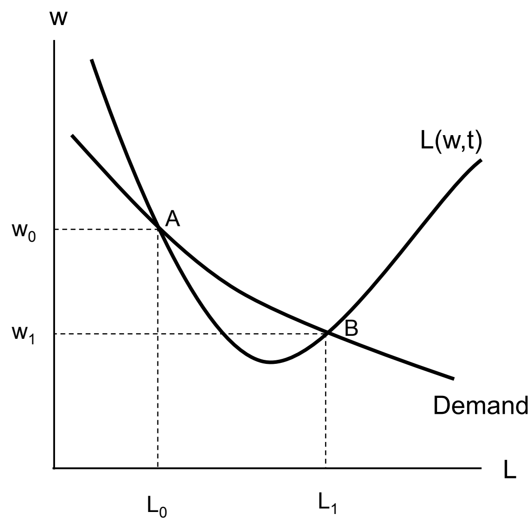
with a big enough increase in f such that the network externality lowers down the wages demanded by female workers.

Figure 1: **Market equilibria with network externalities**



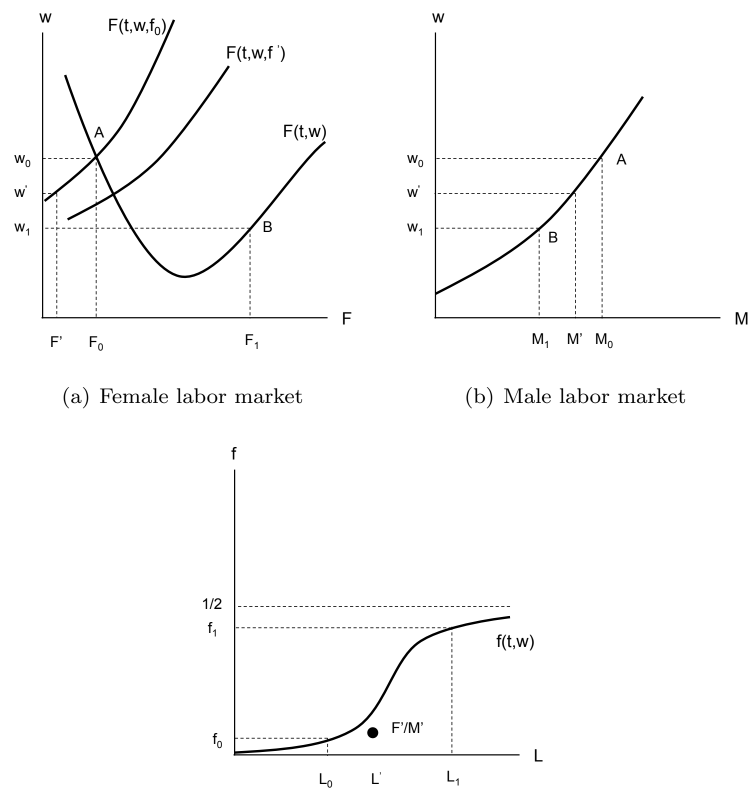
How can we obtain such figure for the labor supply? I assume female workers make their decisions based on their beliefs (expectation) about the equilibrium female share, f^e, which will turn out to be correct (rational expectations). Given these expectations, the conditional female labor supply is $F(t, w, f^e)$, which is increasing in w, decreasing in t and increasing in f^e. Figure 2(a) pictures two of these curves for different beliefs f_0 and f. Note that, given fixed beliefs, female supply is increasing; and an increase in beliefs shifts this conditional female supply to the right (from $F(t, w, f_0)$ to $F(t, w, f)$). To find the unconditional female supply, we solve the following fixed point problem:

$$f^* = \frac{F(t, w, f^*)}{F(t, w, f^*) + M(t, w)} \tag{1}$$

The solution to this problem is a function $f(t, w)$—the 'rational expectations' female share.

If we plug $f(t, w)$ back into the conditional female labor supply—since we have rational expectations, so beliefs are correct, $f^e = f(t, w)$— we get the unconditional female labor supply $F(t, w)$; that is, the curve that incorporates the network effects as wages change. The aggregate labor supply is thus the hori-

Figure 2: **Market equilibria with network externalities**



(a) Female labor market

(b) Male labor market

(c) Female share

zontal sum (i.e., for a fixed wage w and fixed t) of $L(t,w) = F(t,w) + M(t,w)$. To see why labor supply can slope downwards, differentiate with respect to w:

$$\frac{\partial L(t,w)}{\partial w} = \frac{\partial F(t,w,f(t,w))}{\partial w} + \frac{\partial F(t,w,f(t,w))}{\partial f}\frac{\partial f(t,w)}{\partial w} + \frac{\partial M(t,w)}{\partial w}$$

Note that the only term that could be negative is $\partial f(t,w)/\partial w$. To see more intuitively the implications of this term being negative, break it down as $\partial f/\partial w = \frac{df}{dL}\frac{\partial L}{\partial w}$, plug back in the above problem and solve:

$$\frac{\partial L(t,w)}{\partial w} = \frac{\frac{\partial F(t,w,f(t,w))}{\partial w} + \frac{\partial M(t,w)}{\partial w}}{1 - \frac{\partial F(t,w,f(t,w))}{\partial f}\frac{df(t,w)}{dL}}$$

Note that, if female share raises as aggregate labor raises (Figure 2c), the denominator has a positive term (1) and a negative term (network externalities). If the network externalities are big enough, the denominator is negative and thus supply slopes downwards. Why is $df/dL > 0$ consistent with our Figure 2? Observe point A in panel a. Ratio $f_0 = F_0/(F_0 + M_0)$ solves the fixed point problem (1) when $w = w_0$. Now suppose wages move to w'. If beliefs stayed fixed at f_0, female labor supply would move down to F' (along the curve $F(t, w, f_0)$) which would give some share $F'/(F' + M') \neq f_0$ as a result. The problem is that this share is not equal to beliefs f_0 (i.e., does not solve the fixed point problem (1), and thus does not lie along the curve $f(t, w)$ of Figure 2c). This means that expectations have to adjust, up to f', such that female labor expands to a new point that is consistent with the fixed-point problem.

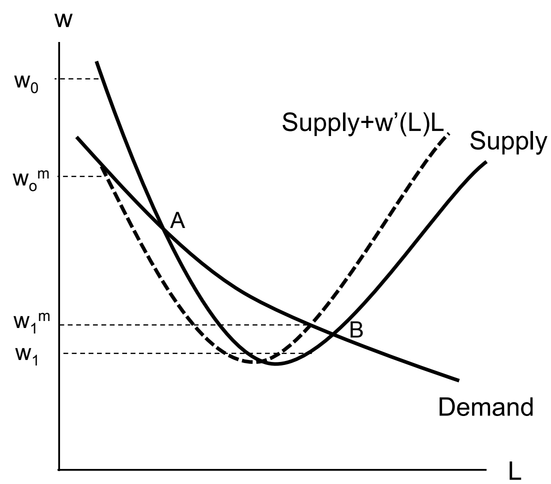
b.

The monopsonist's problem consists on choosing labor supply L to maximize $B(L) - w(L)L$, where B denotes benefit (revenue) and $w(L)$ denotes aggregate labor supply. Figure 2 gives the equilibrium with monopsony and the comparison with perfect competitive equilibria. The monopsonist's first order condition consists of equation $B'(L) = w'(L)L + w(L)$ instead of $B'(L) = w(L)$. When supply is upwards sloping, the marginal revenue is above the supply curve, but it is below when supply is downwards sloping ($w'(L) < 0$). This means that the effective wage that the monopsonist is willing to pay is above the wage that workers demand when supply is upward sloping, but below when it is downward sloping. For this reason, there can only be one equilibrium in this case, located in the upward sloping range. Notice that it might be the case that the monopsonist equilibrium provides higher labor that the competitive equilibrium A, as opposed to the traditional monopsony case.

c.

Suppose tuition increases. Male supply contracts, since they require higher wages to compensate for the higher tuition costs. Female individual supply has two effects: on the one hand, they demand higher wages to compensate for the higher tuition, but on the other hand the female labor share is potentially changing. To see this, fix w and differentiate L with respect to t. We do the

Figure 3: **Equilibrium with monopsony**



Notes: The dashed line represents the derivative of $w(L)L$, where $w(L)$ is labor supply. w denotes wages paid to workers and w^m denotes the wage that the monopolist would be willing to pay for the corresponding amount of labor. Since $w_0 > w_0^m$, this cannot be a monopsonist equilibrium. A and B denote equilibria under perfect competition.

same trick as above to get:

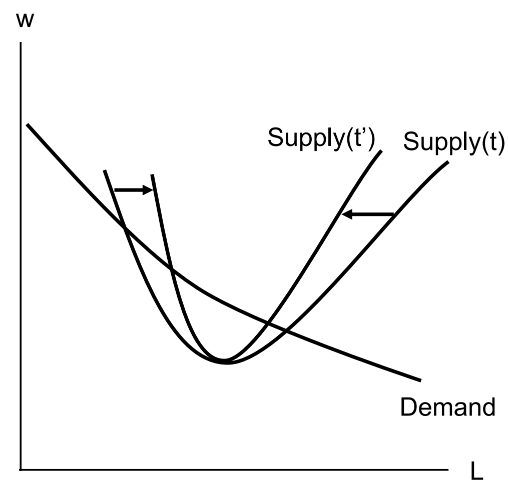
$$\frac{\partial L(t,w)}{\partial t} = \frac{\frac{\partial F(t,w,f(t,w))}{\partial t} + \frac{\partial M(t,w)}{\partial t}}{1 - \frac{\partial F(t,w,f(t,w))}{\partial f}\frac{df(t,w)}{dL}}$$

Note that the numerator is negative, and the denominator is the same as above. This means that the denominator is negative in the area where supply is downward sloping and is positive where supply is upward sloping.

The new equilibria look as in Figure 4; the low-level employment increases and the high level employment equilibrium decreases.

Of course it matters for an individual's supply whether only she pays more tuition, or all students do. When only she pays more tuition, the only change is that the individual has a higher reservation wage, but equilibrium wages and female labor share remain constant. In this case, she is unambiguously worse-off, as her surplus shrinks.

Figure 4: **Equilibria with higher tuition**



d.

For part a), if demand is inelastic enough, we might have only one equilibrium (think of the liming case when demand is perfectly inelastic). In particular, if the slope of demand is 'more negative' than the slope of supply everywhere, there is only one equilibrium. Moreover, if demand is very elastic (think of the limiting case when it is perfectly elastic), then female workers are unambiguously better off in the equilibrium with high level of female employment: wages do not decrease much but reservation wages are smaller (and a higher fraction of women are working). For part b), the difference in employment between competitive equilibria and monopsonist equilibria is higher when demand is more elastic (i.e., wages adjust less). For part c), as demand is more inelastic, wages absorb more of the change in tuition. If you think of the limiting case where demand is perfectly inelastic, wages increase enough to compensate for the higher tuition costs, without labor and labor shares changing.

e.

The dramatic increase of female enrollment in law and medical school might be due to a change from one equilibrium, with low female labor participation, to another one with high female labor share. What causes the shift from one equilibrium to another? Notice that up to know we have assumed implicitly that individuals have rational expectations; they guess the female employment share and this guess turns out to be correct. Many things could have caused the shift in expectations: the second feminist wave at the end of the 60s or the end of the Vietnam war which, as Card and Lemieux (2001) argued, caused a sudden drop in male college enrollment (which had increased in the previous years to avoid being drafted).

f.

Gender composition need not enter the utility function directly. It might be the case that some other 'good' enters the utility function, which is a 'complement' of gender composition (e.g., the likelihood of sexual harassment in the workplace is likely to be decreasing with female labor share). Moreover, utility functions do not necessarily need to be involved at all: it might be the case that women productivity (either in the workplace or in school) has network externalities (e.g., labor literature has highlighted the rise of the importance of 'soft skills' in production).