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An Empirical Analysis

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# Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis

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This paper investigates the testable restrictions on the time-series behavior of consumption and asset returns implied by a representative agent model in which intertemporal preferences are represented by utility functions that generalize conventional, time-additive, expected utility. The model based on these preferences allows a clearer separation of observable behavior attributable to risk aversion and to intertemporal substitution. Further, it nests the predictions of both the consumption CAPM and the static CAPM, and it allows direct tests of the expected utility hypothesis. We find that the performance of the non-expected utility model and tests of the expected utility hypothesis are sensitive to the choice of both consumption measure and instrumental variables.

## I. Introduction

The precursor to this paper (Epstein and Zin 1989) analyzed a generalization of conventional time-additive, expected utility that built on Kreps and Porteus (1978). When applied to the consumption/portfolio

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lio choice problem of an infinitely lived, representative agent, this utility specification yields testable restrictions on observable variables, namely, real, per capita, consumption growth rates and real asset returns. This paper provides an empirical investigation of these restrictions and, hence, the role played by this form of utility in consumption-based asset pricing models.

An attractive feature of this generalized specification is that intertemporal substitution and risk aversion can be partially disentangled in the sense described in our earlier paper. This added flexibility provides a remedy to a deficiency of existing models noted, for example, by Hall (1987). He observes that, except for the special case of a two-period model, "there does not seem to be a convenient class of utility functions in which the two parameters [the coefficient of risk aversion and the elasticity of substitution] are clearly separated" (p. 26).

The disentangling of risk aversion from the elasticity of substitution is a problem that has been highlighted by the empirical literature on the behavior of asset returns and consumption over time. Representative agent optimizing models have not performed well empirically (see, among others, Hansen and Singleton [1983], Mehra and Prescott [1985], and Grossman, Melino, and Shiller [1987]). One possible explanation for this poor performance is that the maintained specification of preferences is too rigid.<sup>1</sup> Indeed, in the conventional case of an additive and homogeneous von Neumann–Morgenstern intertemporal utility function, the elasticity of substitution and the coefficient of relative risk aversion are constrained to be reciprocals of one another. Hall (1985), Zin (1987), and Attanasio and Weber (1989) have made this observation and they have attempted to remedy the problem by adopting Selden's (1978) ordinal certainty equivalent preferences. But these preferences lead to intertemporal inconsistencies, and the associated Euler equations are applicable only to a naive consumer who continually ignores the fact that plans formulated at any given time will generally not be carried out in the future. In contrast, utility functions considered in this paper are based on a recursive structure and so are intertemporally consistent.

The orthogonality restrictions implied by the Euler equations of the agent's optimization problem are used (as in Hansen and Singleton [1982]) to identify and estimate the parameters of the utility function using the generalized method of moments (GMM). This procedure also allows testing of the expected utility hypothesis, the *goodness of fit* of the general model, as well as its stability over subsamples. Our

<sup>1</sup> Other possible explanations include liquidity constraints (Zeldes 1989), transactions costs (Grossman and Laroque 1990), and incomplete markets with heterogeneous agents (Scheinkman and Weiss 1986).

empirical work uses monthly U.S. data spanning the 1959–86 time period and incorporates a variety of measures for consumption and returns for stocks and bonds.

The paper proceeds as follows: Section II reviews the major features of the model of our earlier paper. Section III details the construction of data, the estimation and testing procedures, and the actual empirical results. Section IV summarizes and concludes the paper.

## II. Utility Functions and Asset Returns

As noted in the Introduction, the model considered in this paper was studied in detail in Epstein and Zin (1989) (to which the reader is referred for a complete analysis). In this section we shall review the major features of the utility specification and derive a set of testable restrictions for consumption and asset returns.

### A. Utility Functions

We consider an infinitely lived representative agent who receives utility from the consumption of a single good. In any period  $t$ , current consumption  $c_t$  is deterministic but future consumption is uncertain. There are two key assumptions underlying our specification of intertemporal utility. First, we assume that the agent forms a *certainty equivalent* of random future utility using his risk preferences. Second, we assume that to obtain current-period lifetime utility, this certainty equivalent is combined with deterministic current consumption via an *aggregator* function. For example, for an agent making a decision in period  $t$ , utility  $\bar{U}_{t+1}$  from period  $t + 1$  onward is random. Its certainty equivalent, given the information available to the agent in the planning period,  $I_t$ , is  $\mu[\bar{U}_{t+1}|I_t]$ . This is combined with current consumption,  $c_t$ , using the aggregator function,  $W$ , so that lifetime utility is given by

$$U_t = W(c_t, \mu[\bar{U}_{t+1}|I_t]). \quad (1)$$

This form of utility generalizes the recursive structure introduced by Koopmans (1960) for deterministic models and also the specifications studied by Kreps and Porteus (1978) in a stochastic setting. Moreover, it implies the intertemporal consistency of preference (in the sense of Johnsen and Donaldson [1985]) and the stationarity of preference (in the sense of Koopmans [1960]). It is shown in Epstein and Zin (1989) that this class of preferences allows a separation of risk aversion from substitution that is not possible in the expected utility framework. Roughly speaking, intertemporal substitution is encoded in  $W$  while the certainty equivalent function  $\mu$  reflects the degree of

risk aversion. Expected utility is a special case of (1), as will be shown below.

Before we examine the properties of (1) and the agent's optimization problem in greater detail, it is convenient to make explicit functional form assumptions for  $W$  and  $\mu$ . This also facilitates the empirical work of Section III. First, assume that the aggregator function has the form

$$\begin{aligned} W(c, z) &= [(1 - \beta)c^\rho + \beta z^\rho]^{1/\rho}, & 0 \neq \rho < 1, \\ W(c, z) &= (1 - \beta)\log(c) + \beta \log(z), & \rho = 0, \end{aligned} \quad (2)$$

where  $c, z \geq 0$  and  $\beta = 1/(1 + \delta)$ ,  $\delta > 0$ . When future consumption in (2) is deterministic, this aggregator function results in an intertemporal constant elasticity of substitution utility function with elasticity of substitution  $\sigma = 1/(1 - \rho)$  and rate of time preference  $\delta$ . Thus the parameter  $\rho$  is interpreted as reflecting substitution. We consider an  $\alpha$ -mean (or constant relative risk aversion expected utility) specification for the certainty equivalent. For a random variable  $\tilde{x}$ , the  $\alpha$ -mean specification for  $\mu$  is given by

$$\begin{aligned} \mu[\tilde{x}] &= [E\tilde{x}^\alpha]^{1/\alpha}, & 0 \neq \alpha < 1, \\ \log(\mu) &= E \log(\tilde{x}), & \alpha = 0, \end{aligned} \quad (3)$$

where  $E$  is the expectation operator. This leads to the recursive structure for intertemporal utility (if  $\alpha \neq 0$  and  $\rho \neq 0$ ) given by

$$U_t = [(1 - \beta)c_t^\rho + \beta(E_t \tilde{U}_{t+1}^\alpha)^{\rho/\alpha}]^{1/\rho}, \quad (4)$$

where  $E_t$  is the conditional expectation operator given  $I_t$ .

Epstein and Zin (1989) show that  $\alpha$  may be interpreted as a (relative) risk aversion parameter with the degree of risk aversion increasing as  $\alpha$  falls. When  $\alpha = \rho$ , (4) specializes to the familiar expected utility specification

$$U_t = \left[ (1 - \beta)E_t \sum_{j=0}^{\infty} \beta^j \tilde{c}_{t+j}^\alpha \right]^{1/\alpha}. \quad (5)$$

In this case, there is indifference to the way in which uncertainty about consumption is resolved over time (in the sense of Kreps and Porteus [1978]). For the more general (4), early (late) resolution of uncertainty is preferred if  $\alpha < (>) \rho$ .

### B. Euler Equations

The representative agent is endowed with an initial stock of the consumption good,  $A_0$ , which can be either consumed or invested in assets traded on competitive markets. The gross, real return on an

asset held throughout period  $t$  is given by  $\mathbf{R}_t$ . When there are many assets available to the agent, say  $N$ ,  $\mathbf{R}_t$  is an  $N$ -vector of returns on individual assets with typical element  $R_{j,t}$ . The fraction of the agent's total wealth held in the  $j$ th asset in period  $t$  is denoted  $\omega_{j,t}$ , and the  $N$ -vector of portfolio weights is denoted by  $\boldsymbol{\omega}_t$ . There are only  $N - 1$  independent elements in  $\boldsymbol{\omega}_t$  since the constraint

$$\sum_{j=1}^N \omega_{j,t} = 1 \quad (6)$$

holds for all  $t$ . Typically, the return on an asset is uncertain during the period in which the asset is held,<sup>2</sup> which implies that future wealth and consumption are random variables. In fact, the agent's wealth evolves according to the random process

$$A_{t+1} = (A_t - c_t)\boldsymbol{\omega}_t^T \tilde{\mathbf{R}}_t, \quad t > 0. \quad (7)$$

The agent is thereby able to affect future consumption flows by trading in the risky financial assets.<sup>3</sup> Naturally, the allocation of consumption over time is chosen so as to maximize the lifetime utility of the agent.

It is straightforward to show (as in Epstein and Zin [1989]) that the recursive structure of preferences along with the homogeneous functional forms and the linear budget constraint result in a set of necessary conditions for the joint consumption allocation and portfolio choice problem that can be written in terms of observable variables. We begin by defining the optimal value of utility in (4) as a function  $J$  of current wealth and current information. The Bellman equation takes the form

$$J(A_t, I_t) \equiv \max_{c_t, \boldsymbol{\omega}_t} \{ (1 - \beta)c_t^\rho + \beta[E_t J(\tilde{A}_{t+1}, \tilde{I}_{t+1})^\alpha]^{p/\alpha} \}^{1/\rho}. \quad (8)$$

By the homogeneity of the planning problem, this optimal value is proportional to wealth, that is,

$$J(A_t, I_t) = \phi(I_t)A_t \equiv \phi_t A_t. \quad (9)$$

The maximization with respect to consumption on the right-hand

<sup>2</sup> When emphasis is required, a tilde is used to denote a random variable.

<sup>3</sup> Note that a term explicitly measuring labor income is not present in this wealth constraint. If labor income is nonstochastic and there is a riskless asset, then the sequence of incomes can be discounted back to period 0 and treated as part of the initial endowment. If labor income is stochastic, then (7) is still applicable provided that the wealth measure is reinterpreted. That is, labor income can be treated as a stochastic dividend for a nontraded asset, say human capital. A shadow price and a shadow return can be computed for this asset in equilibrium as in Epstein (1988). In this way the portfolio is extended by one asset and wealth now measures the *full* wealth (human and nonhuman) of the agent. The problem posed by stochastic labor income is, therefore, a problem in the measurement of the return on the wealth portfolio.

side of (8) and the budget constraint (7) together imply

$$c_t^{\rho-1} = \beta(A_t - c_t)^{\rho-1} \mu^{*\rho}, \quad (10)$$

where  $\mu^* = [E_t(\bar{\Phi}_{t+1}^\alpha \bar{M}_t^\alpha)]^{1/\alpha}$ , and  $M_t$  is the gross return on the optimal portfolio. Given the structure of the problem, consumption is also proportional to wealth, so that we can rewrite (10) as

$$\psi_t^{\rho-1} = \beta(1 - \psi_t)^{\rho-1} \mu^{*\rho}, \quad (11)$$

where  $c_t = \psi(I_t)A_t \equiv \psi_t A_t$ . Solve for  $\mu^*$  (assuming that  $\rho \neq 0$ ) and substitute into the definition of the value function in equations (8) and (9) to deduce that

$$\phi_t = \left(\frac{c_t}{A_t}\right)^{-1/\rho} \quad \text{for all } t. \quad (12)$$

Substitute this expression into (11) to obtain the Euler equation for optimal consumption decisions (when  $\alpha \neq 0$  and  $\rho \neq 0$ )

$$E_t \left[ \beta \left( \frac{\bar{c}_{t+1}}{c_t} \right)^{\rho-1} \bar{M}_t \right]^\gamma = 1, \quad (13)$$

for all  $t$ , where  $\gamma = \alpha/\rho$ .

Turn now to the restrictions implied by optimal portfolio selection. The maximization with respect to  $\omega_t$  on the right-hand side of (8) is equivalent to the problem

$$\max_{\omega_t} [E_t(\bar{\Phi}_{t+1} \omega_t^T \bar{\mathbf{R}}_t)^\alpha]^{1/\alpha}, \quad (14)$$

subject to the restriction in (6). After substituting (12), we obtain the following necessary conditions:

$$E_t \left[ \left( \frac{\bar{c}_{t+1}}{c_t} \right)^{\gamma(\rho-1)} \bar{M}_t^{\gamma-1} (\bar{R}_{j,t} - \bar{R}_{1,t}) \right] = 0, \quad j = 2, \dots, N, \quad (15)$$

for all  $t$ .

Equations (13) and (15) when taken together represent the Euler equations of the problem defined in (8). They can be combined to yield a set of  $N$  equations that allow a more direct comparison with the typical expected utility Euler equations. Multiply (15) by  $\omega_{j,t}$ , sum over  $j$ , and substitute from (13) to derive

$$E_t \left[ \beta^\gamma \left( \frac{\bar{c}_{t+1}}{c_t} \right)^{\gamma(\rho-1)} \bar{M}_t^{\gamma-1} \bar{R}_{j,t} \right] = 1, \quad j = 1, \dots, N. \quad (16)$$

When  $\gamma = 1$ , the Euler equations of the expected utility model are obtained.

Another specialization of this model that is of interest is the case

of logarithmic risk preferences,  $\alpha = 0$  ( $\rho \neq 0$ ). Then the counterpart to equations (16) is

$$E_t[\tilde{M}_t^{-1} \tilde{R}_{j,t}] = 1, \quad j = 1, \dots, N, \quad (17)$$

which is a system of  $N - 1$  independent equations that impose the same restriction as those implied by the expected utility problem with logarithmic preferences. Moreover, the parameter  $\rho$  governing intertemporal substitutability cannot be identified from these equations. The distinction between the non-expected utility model with logarithmic risk preferences and the expected utility model with logarithmic preferences lies in the counterpart to equation (13). To obtain the logarithmic specialization of this equation, divide both sides by  $\gamma$  and rearrange terms to get

$$E_t \left[ \frac{[\beta(\tilde{c}_{t+1}/c_t)^{\rho-1} \tilde{M}_t]^\gamma - 1}{\gamma} \right] = 0. \quad (18)$$

As  $\alpha$  approaches zero,  $\gamma$  approaches zero and (18) converges to

$$\log \beta + (\rho - 1)E_t \log \left( \frac{\tilde{c}_{t+1}}{c_t} \right) + E_t \log(\tilde{M}_t) = 0. \quad (19)$$

Clearly, this equation permits discrimination between the logarithmic expected utility model ( $\alpha = \rho = 0$ ) and the non-expected utility model with logarithmic risk preferences ( $\alpha = 0$ ,  $\rho \neq 0$ ).

### C. Discussion

It is instructive to analyze the Euler equations in terms of a general model of asset pricing as in Hansen and Jagannathan (this issue). In their terminology, the intertemporal marginal rate of substitution (IMRS) is used by agents to discount future payoffs to determine current asset prices. Equations (16) imply an IMRS equal to

$$\left[ \beta \left( \frac{\tilde{c}_{t+1}}{c_t} \right)^{\rho-1} \right]^\gamma \left( \frac{1}{\tilde{M}_t} \right)^{1-\gamma}, \quad (20)$$

that is, a geometric weighted average of the IMRS from the standard expected utility model and the IMRS from the logarithmic expected utility model. The weights attached to each IMRS are determined by the parameter  $\gamma$ . When  $\gamma = 1$ , consumption growth is sufficient for discounting asset payoffs as in the intertemporal (or consumption) capital asset pricing model (CAPM). When  $\gamma = 0$ , the market return is sufficient for discounting individual asset payoffs as in the simple (or static) CAPM. For all other values of  $\gamma$ , both consumption growth and the market return are necessary for determining the IMRS.



To gain insight into the comparative empirical predictions of the expected utility model and the model of (16), consider a linear approximation to the geometric weighted average for the IMRS in (20). Although approximating a geometric average with an arithmetic average may be imprecise for some values of the variables and the parameters, it is useful for gaining insight into the first-order properties of these restrictions.<sup>4</sup> Substitution of the approximate IMRS,

$$\gamma \left[ \beta \left( \frac{\tilde{c}_{t+1}}{c_t} \right)^{\rho-1} \right] + (1 - \gamma) \left( \frac{1}{\tilde{M}_t} \right), \quad (21)$$

into the Euler equation for the portfolio return (13) implies the approximate restriction

$$E_t \left[ \beta \left( \frac{\tilde{c}_{t+1}}{c_t} \right)^{\rho-1} \tilde{M}_t \right] \approx 1, \quad (22)$$

which is precisely the restriction implied by the expected utility problem analyzed by Hansen and Singleton (1982, 1983). However, if this linear approximation is substituted into the Euler equation (16) for an arbitrary asset return, it is clear that a restriction like (22) is not likely to hold. Rather, the following condition is predicted to be approximately satisfied:

$$\gamma E_t \left[ \beta \left( \frac{\tilde{c}_{t+1}}{c_t} \right)^{\rho-1} \tilde{R}_{j,t} \right] + (1 - \gamma) E_t [\tilde{M}_t^{-1} \tilde{R}_{j,t}] \approx 1. \quad (23)$$

Therefore, it is evident that distinguishing between the empirical predictions of these two models is likely to require the use of other assets in addition to the market portfolio. Further, from the perspective of the non-expected utility model, the predictions of the expected utility model should be difficult to reject using the market portfolio but should be rejectable using other assets. This is, in fact, the pattern that emerges from the empirical work in Hansen and Singleton (1982, 1983).

Equation (23) is also useful for gaining insight into the static versus consumption CAPM debate. Heuristically, the static CAPM (surveyed in Jensen [1972]) measures the riskiness of an asset by means of the covariance of its return with the return on the market portfolio. In the intertemporal CAPM (see Merton 1973; Breeden 1979), the riskiness of an asset is measured by the covariance of its return with the marginal rate of substitution of consumption over time (most

<sup>4</sup> Alternatively, we could make the same point by assuming that consumption growth and asset returns are jointly lognormally distributed and then approximating the conditional expectations in (12) and (15) with closed-form expressions; see eq. (24) below.

commonly specified as a function of the growth rate of consumption). It is evident from (23) that in the non-expected utility model considered here, the riskiness of an asset is related to both the covariance of its return with the market portfolio (as indicated by the second term in [23]) and the covariance of its return with the growth rate of consumption (as indicated by the first term in [23]). This is even clearer if we assume that consumption growth and the vector of asset returns are jointly lognormally distributed. In this case we obtain

$$E_t[\tilde{R}_{j,t}^* - \tilde{R}_{i,t}^*] \approx \frac{\Sigma_{i,t} - \Sigma_{j,t}}{2} + (1 - \gamma)\text{cov}_t(\tilde{M}_t^*, \tilde{R}_{j,t}^* - \tilde{R}_{i,t}^*) \\ - \gamma(\rho - 1)\text{cov}_t(\tilde{X}_{t+1}^*, \tilde{R}_{j,t}^* - \tilde{R}_{i,t}^*), \quad (24)$$

where asterisks denote logarithms,  $\Sigma_{i,t}$  is the conditional variance of  $\tilde{R}_{i,t}^*$  given  $I_t$ , and  $\text{cov}_t$  is the conditional covariance operator given  $I_t$ .<sup>5</sup> The prediction of the static CAPM regarding the appropriate measure of risk obtains when  $\gamma = 0$ , that is, logarithmic risk preferences. The prediction of the intertemporal CAPM regarding the appropriate measure of risk results under the expected utility restriction,  $\gamma = 1$ . It is clear from (24) that, in general, neither of these alone will suffice for measuring risk.

Equation (24) also provides a structural model for interpreting the empirical exercise of Mankiw and Shapiro (1986). They try to select the *better* model on the basis of the predictive power of each of these covariances. Their evidence shows that the covariance of an asset's return with the growth rate in consumption does not contribute much to the prediction of excess returns when the covariance of the asset's return with the market portfolio is controlled for. This can be viewed, given equation (24), not as a test of the static CAPM against the intertemporal CAPM, but rather as a test of the expected utility hypothesis. That is, if expected utility held, (24) predicts that they would reach the opposite conclusion. Their evidence is consistent with (24) when  $\gamma = 0$ . On the other hand, when other econometric techniques are applied to the static CAPM, as in Bollerslev, Engle, and Wooldridge (1988), growth rates of consumption still have some predictive power for excess returns. Therefore, given these conflicting findings, the appropriate value for  $\gamma$  cannot be determined *ex ante*.

It is clear that the model we have described in this section can be

<sup>5</sup> See Hansen and Singleton (1983), Hall (1985), or Zin (1987) for a detailed account of how homothetic utility and lognormality yield such closed-form expressions. Equation (24) is an approximation since we require that individual returns and a linear combination of these returns, i.e., the portfolio return, are jointly lognormally distributed. See Duffie and Epstein (1990) for the derivation of the continuous-time counterpart of (24), which holds exactly.

used to address numerous outstanding empirical issues in macroeconomics and finance. For example, the equity premium puzzle of Mehra and Prescott (1985), the term structure puzzles studied in Backus, Gregory, and Zin (1989), and the price/dividend variance inequalities in Grossman and Shiller (1981) can all be reassessed from the perspective of non-expected utility models.<sup>6</sup> This is left for future work. We now turn to a direct examination of the empirical predictions of our model.

### III. Data, Estimation, and Testing

#### A. Data

We have constructed a number of monthly data sets (for the United States) differing in the measurement of consumption (and, hence, prices), asset returns, and the time period. Monthly data have been used in previous studies (e.g., Hansen and Singleton 1982, 1983), so that our results will allow a direct comparison with the literature. Further, the frequency of observations of a month (as compared to a quarter or a year) may correspond more closely to the relevant timing of actual decisions of individuals in the economy. Examining different consumption measures and time periods should provide some insight into the robustness of our empirical model to deviations from the underlying assumptions.

The economy under consideration has only a single consumption good. In the expected utility model, it is frequently assumed that there are a number of different goods, say durables and nondurables, but utility is additively separable across these goods. Thus use of nondurable consumption alone is theoretically justifiable. A similar argument is not applicable here, however. It is necessary for us to assume that the service flow from *total* consumption is a constant proportion of the service flow from *measured* consumption. Given the difficulty in measuring the service flows generated from durable goods, this is not a directly testable assumption. It is, therefore, on the same uncertain ground as the separability assumptions made in the expected utility literature, since a test of those would also require a measure of the service flows (rather than the stocks) of all goods (including durables). Recent work by Heaton (1990) provides an anal-

<sup>6</sup> Recent work by Weil (1989) on the Mehra-Prescott puzzle using the model in this section is not encouraging. Hansen and Jagannathan (this issue), however, show that the logarithmic specialization of our model comes closer to satisfying moment restrictions implied by asset data than the expected utility model with moderate risk aversion. Epstein and Zin (1989, p. 959) demonstrate how the model in this section can be used to derive present-value stock price representations that are analogous to those used by Grossman and Shiller (1981).

ysis of a model in which both time aggregation and durable goods are treated explicitly in a linear-quadratic expected utility framework. Incorporating these features into our model, however, is beyond the scope of this paper and is not attempted.

The model's restrictions apply to average behavior if wealth, but not preferences, varies across consumers. That is, we adopt the common (though restrictive) assumption of identical and homothetic preferences to justify aggregation over consumers and we apply the model to data on per capita consumption.

To check the robustness of our results, we use four different measures of per capita consumption. The first is expenditures on nondurable goods. To obtain the second measure of consumption, we remove expenditures on clothing and shoes from nondurables since the nondurability assumption for these goods may be questionable. The third is expenditures on nondurable goods and services. This provides a broader measure of consumption. However, since services contain some expenditures with durable components, we construct a fourth measure of consumption by removing clothing, shoes, and medical expenditures from nondurables and services.<sup>7</sup>

We turn now to the measurement of asset returns. The nominal return on the optimal portfolio is measured with the value-weighted index of shares traded on the New York Stock Exchange. A number of issues arise from the use of this measure, but the primary concern for our purposes is whether it is sufficiently broad to capture the relevant part of actual holdings of wealth; that is, Roll's (1977) critique of CAPM is relevant here. If stochastic wages are a large factor in the wealth constraint of the typical agent, then, as discussed in Section II, the return on the optimal portfolio of the agent should reflect the shadow return of the agent's human capital. Rather than attempt a lengthy analysis of this issue at this time, we shall simply assume that factors that may not be properly measured by the value-weighted index of stock returns do not affect the empirical analysis in an appreciable way. The appropriateness of this assumption, vis-à-vis the empirical results below, remains an open question.

We test the model using asset returns corresponding to both government debt and corporate equity. We view the market portfolio as consisting of five individual stock return indices that are value-weighted returns for broad groups of the standard industrial classification (SIC) of individual firms. Since we shall always include an

<sup>7</sup> The data for constructing these consumption series were taken from Citibase. Prices are measured with the implicit deflators corresponding to the definition of consumption adopted. These prices are used to deflate nominal asset returns. Real consumption variables are put in per capita terms using total civilian population.

TABLE 1  
DESCRIPTIVE STATISTICS

VARIABLE	MEAN		
	1959:4–1986:12	1959:4–1978:12	1979:1–1986:12
1. Nondurables			
$c_{t+1}/c_t$	1.0010 (.0078)	1.0012 (.0081)	1.0006 (.0070)
$M_t$	1.0052 (.0427)	1.0028 (.0423)	1.0114 (.0431)
$B_t$	1.0013 (.0047)	1.0003 (.0042)	1.0038 (.0050)
$R1_t$	1.0055 (.0539)	1.0043 (.0472)	1.0086 (.0680)
$R2_t$	1.0050 (.0377)	1.0024 (.0383)	1.0115 (.0355)
$R3_t$	1.0065 (.0540)	1.0030 (.0534)	1.0151 (.0549)
$R4_t$	1.0061 (.0484)	1.0033 (.0482)	1.0130 (.0485)
2. Net Nondurables			
$c_{t+1}/c_t$	1.0008 (.0076)	1.0011 (.0079)	1.0001 (.0066)
$M_t$	1.0050 (.0428)	1.0026 (.0425)	1.0110 (.0433)
$B_t$	1.0011 (.0051)	1.0001 (.0045)	1.0034 (.0058)
$R1_t$	1.0053 (.0539)	1.0041 (.0472)	1.0082 (.0679)
$R2_t$	1.0049 (.0378)	1.0022 (.0383)	1.0112 (.0359)
$R3_t$	1.0063 (.0542)	1.0028 (.0535)	1.0147 (.0551)
$R4_t$	1.0059 (.0485)	1.0031 (.0483)	1.0126 (.0486)
3. Nondurables and Services			
$c_{t+1}/c_t$	1.0017 (.0044)	1.0019 (.0045)	1.0012 (.0042)
$M_t$	1.0050 (.0428)	1.0026 (.0425)	1.0110 (.0433)
$B_t$	1.0009 (.0029)	1.0001 (.0024)	1.0027 (.0030)
$R1_t$	1.0051 (.0540)	1.0041 (.0471)	1.0076 (.0683)
$R2_t$	1.0046 (.0372)	1.0022 (.0379)	1.0105 (.0350)
$R3_t$	1.0060 (.0536)	1.0028 (.0530)	1.0140 (.0545)
$R4_t$	1.0057 (.0481)	1.0031 (.0479)	1.0119 (.0483)

TABLE 1 (Continued)

VARIABLE	MEAN		
	1959:4–1986:12	1959:4–1978:12	1979:1–1986:12
4. Net Nondurables and Services			
$c_{t+1}/c_t$	1.0014 (.0047)	1.0016 (.0046)	1.0009 (.0048)
$M_t$	1.0048 (.0424)	1.0026 (.0420)	1.0102 (.0429)
$B_t$	1.0008 (.0031)	1.0001 (.0026)	1.0027 (.0035)
$R1_t$	1.0051 (.0539)	1.0041 (.0471)	1.0075 (.0682)
$R2_t$	1.0046 (.0372)	1.0022 (.0379)	1.0104 (.0351)
$R3_t$	1.0060 (.0537)	1.0028 (.0531)	1.0140 (.0545)
$R4_t$	1.0057 (.0481)	1.0031 (.0479)	1.0119 (.0483)

NOTE.— $M_t$  is the market return;  $B_t$  is the bond return; and  $R1_t$ ,  $R2_t$ ,  $R3_t$ , and  $R4_t$  are returns on stock portfolios. Net nondurables excludes expenditures on clothing and shoes. In addition to clothing and shoes, net nondurables and services excludes medical expenditures. Standard errors are in parentheses.

equation for the market return, one of these five individual returns will be redundant. We therefore omit group D, manufacturing, from the analysis, which leaves us with four equity returns.<sup>8</sup> The first equity return is a value-weighted index of stocks in the broad groups A, B, and C of the SIC code (i.e., agriculture, forestry, fishing, mining, and construction). Similarly, the second asset return is a value-weighted index of stocks in category E of the SIC code (transportation and public utilities). The third asset return comprises groups F and G (wholesale trade and retail trade), and finally, the fourth asset return comprises groups H and I (finance, insurance, real estate, and services). Along with these monthly stock returns we also use a 30-day U.S. Treasury bill return<sup>9</sup> to test the restrictions of the theory discussed in Section II. All nominal asset returns are converted to real returns using the appropriate consumption deflator.

The sample period extends from April 1959 to December 1986. We also consider a subsample roughly corresponding to the sample period used by Hansen and Singleton ending in December 1978. This division will allow direct comparison with the related work of Hansen and Singleton as well as a test of the structural stability of our model.

Descriptive statistics for our data are displayed in table 1. It is worth

<sup>8</sup> Stock returns data were taken from the monthly tape of the Center for Research in Security Prices (CRSP) of the University of Chicago.

<sup>9</sup> Data for this return were taken from the Fama term structure file of the monthly CRSP bond tape.

stopping to take note of some of the patterns that emerge from these simple statistics. First, notice that the variance of asset returns is always substantially greater than the variance of the consumption growth rate. This is true across consumption measures and across time periods. Second, notice that for the time period beginning in 1979, average returns are substantially larger than in the earlier period: the market rate is about four times larger, the bond rate is about 30 times larger, and the rates on the other stock portfolios range from two to five times larger. Further, the average growth rate in consumption is much less for the latter part of the sample: roughly one-half as large for nondurables, nondurables and services, and net nondurables and services, and one-tenth as large for net nondurables. Though this analysis is very casual, it seems to be indicating that the data after 1979 are qualitatively different from the data before 1979. This issue will be investigated in greater detail in the context of formal estimation and hypothesis testing below. These particular subsamples, although ad hoc, are convenient because the first corresponds with the sample used by Hansen and Singleton and also because the division corresponds roughly with the change in the operating procedures of the Federal Reserve in the latter part of 1979. This event is considered to represent a large structural change in the Fed's behavior that could conceivably affect economic activity, especially capital markets, in a detectable way. Finally, note that netting out clothing, shoes, and medical expenditures from our measures of consumption reduces the average growth rate but does not have much of an effect on the variability. This is in contrast to the inclusion of services, which results in a higher average growth rate and a smaller standard error. Real returns appear to be relatively stable when services are added to the consumption measure, indicating that the implicit prices do not differ much across these definitions of consumption.

### *B. Estimation and Testing Procedures*

The econometric procedures detailed in Hansen and Singleton (1982) are well suited for estimating Euler equations of the type presented in Section II since these equations imply, ex post, an additive forecast error. The large sample properties of these GMM estimators and tests have been thoroughly documented,<sup>10</sup> so we shall only briefly outline their application to our model.

We shall typically work with a set of five of the equations derived

<sup>10</sup> In addition to Hansen and Singleton (1982), see Hansen (1982, 1985), Bates and White (1985), and Newey (1985).

TABLE 2  
GMM RESULTS FOR STOCKS AND NONDURABLES

	1959:4–1986:12			1959:4–1978:12		
	INST1	INST2	INST3	INST1	INST2	INST3
Nondurables						
$\delta$	.0033 (.0018)	.0033 (.0018)	-.0015 (.0036)	.0006 (.0031)	.0006 (.0031)	-.0040 (.0031)
$\gamma$	-.0108 (.0564)	-.0146 (.0564)	-.0235 (.0746)	-.0297 (.0659)	-.0122 (.0644)	-.0065 (.0889)
$\sigma$	.8652 (.5422)	.8158 (.5021)	.1754 (.0728)	.8499 (.6331)	.7973 (.5948)	.2392 (.0950)
$\alpha$	.0016 (.0127)	.0033 (.0182)	.1106 (.3402)	.0052 (.0307)	.0031 (.0215)	.0207 (.2790)
$J(12)$	19.04 [.088]	24.20 [.019]	8.054 [.781]	18.54 [.100]	18.69 [.096]	10.22 [.597]
$J(13) - J(12)$	157.17 [.000]	165.73 [.000]	7.942 [.005]	141.46 [.000]	148.39 [.000]	6.760 [.009]
GH(15)				16.40 [.356]	15.40 [.423]	17.94 [.266]
Net Nondurables						
$\delta$	.0036 (.0018)	.0037 (.0018)	-.0005 (.0036)	.0002 (.0031)	.0003 (.0031)	-.0037 (.0031)
$\gamma$	.0087 (.0619)	.0235 (.0637)	-.0233 (.0801)	-.0181 (.0720)	.0126 (.0705)	-.0063 (.0797)
$\sigma$	.7388 (.4548)	.7826 (.5167)	.2000 (.0873)	.8042 (.6025)	.7815 (.5856)	.2617 (.1104)
$\alpha$	-.0031 (.0218)	-.0065 (.0237)	.0932 (.3147)	.0044 (.0276)	.0035 (.0199)	.0176 (.2223)
$J(12)$	16.75 [.159]	23.00 [.028]	8.09 [.778]	17.39 [.136]	18.12 [.112]	13.22 [.353]
$J(13) - J(12)$	115.46 [.000]	121.83 [.000]	8.336 [.004]	111.94 [.000]	118.59 [.000]	7.099 [.008]
GH(15)				15.77 [.398]	15.53 [.414]	19.61 [.187]

NOTE.—INST1 =  $\{1, c_t/c_{t-1}, c_{t-1}/c_{t-2}\}$ , INST2 =  $\{1, c_t/c_{t-1}, M_{t-1}\}$ , and INST3 = INST2 $_{-1}$ .  $J(n)$  is Hansen's test of the overidentifying restrictions, GH(n) is the Ghyssels-Hall test for structural stability (both have limiting  $\chi^2(n)$  distributions), and  $J(n+1) - J(n)$  is a likelihood ratio-type test of the  $\gamma = 1$  restriction (distributed as  $\chi^2(1)$  in the large samples). Asymptotic standard errors are in parentheses, and asymptotic  $p$ -values are in brackets.

in Section II. To maintain the identifiability of the parameter  $\sigma = (1 - \rho)^{-1}$ , we shall always include equation (18) linking the market return with consumption growth in a form that allows for the logarithmic-risk special case. We then include four other asset return equations of the form (16). Tables 2 and 3 contain results from using the four stock return equations. Tables 4 and 5 contain results obtained by using the first three stock return equations and replacing the fourth stock return equation with the Treasury bill return equation. The existing evidence (e.g., Hansen and Singleton 1982, 1983;



TABLE 3

GMM RESULTS FOR STOCKS AND NONDURABLES AND SERVICES

	1959:4–1986:12			1959:4–1978:12		
	INST1	INST2	INST3	INST1	INST2	INST3
Nondurables and Services						
$\delta$	-.0022 (.0036)	-.0020 (.0036)	-.0029 (.0055)	-.0024 (.0061)	-.0043 (.0061)	-.0073 (.0123)
$\gamma$	-.0043 (.0473)	.0355 (.0509)	-.0609 (.4876)	.0981 (.0598)	.0365 (.0613)	-.0095 (.2131)
$\sigma$	.2486 (.1455)	.2616 (.1528)	.2539 (.5622)	.3546 (.3480)	.3028 (.2437)	.2082 (.2744)
$\alpha$	.0131 (.1474)	-.1001 (.1510)	.1790 (.1576)	-.1785 (.2698)	-.0840 (.1579)	.0360 (.8155)
$J(12)$	26.58 [.009]	30.12 [.003]	7.81 [.800]	26.38 [.009]	21.24 [.047]	11.90 [.454]
$J(13) - J(12)$	210.70 [.000]	175.87 [.000]	2.73 [.098]	163.82 [.000]	153.69 [.000]	7.73 [.009]
GH(15)				17.56 [.287]	19.60 [.188]	15.41 [.422]
Net Nondurables and Services						
$\delta$	-.0019 (.0036)	-.0019 (.0036)	-.0021 (.0091)	-.0044 (.0061)	-.0044 (.0046)	-.0074 (.0092)
$\gamma$	.0898 (.0528)	.0844 (.0528)	-.0535 (.5112)	-.0067 (.0629)	-.0057 (.0644)	-.0087 (.1723)
$\sigma$	.2419 (.1383)	.2454 (.1383)	.2609 (.4621)	.2572 (.2024)	.2588 (.1947)	.1919 (.1794)
$\alpha$	-.2816 (.2438)	-.2596 (.2274)	.1517 (1.441)	.0194 (.1855)	.0162 (.1886)	.0365 (.7266)
$J(12)$	24.65 [.017]	28.95 [.004]	7.99 [.786]	25.57 [.012]	19.52 [.077]	13.40 [.341]
$J(13) - J(12)$	138.02 [.000]	134.54 [.000]	.846 [.358]	148.99 [.000]	133.41 [.000]	1.36 [.243]
GH(15)				22.30 [.100]	23.53 [.074]	17.91 [.267]

NOTE.—See table 2.

Mehra and Prescott 1985) suggests that explaining bond and stock returns jointly poses a problem for the expected utility model. Including bond returns in our empirical analysis will allow us to investigate whether our non-expected utility specification provides a better empirical model of the equity premium.

To implement GMM estimation, it is first necessary to identify a set of *instruments*. The estimator is based on the fact that the forecast error associated with Euler equations (such as those derived in Sec. II) is additive and is uncorrelated with any information available to agents during the planning period. We can therefore generate arbitrarily many orthogonality restrictions if we can identify variables that

TABLE 4  
GMM RESULTS FOR BONDS, STOCKS, AND NONDURABLES

	1959:4–1986:12			1959:4–1978:12		
	INST1	INST2	INST3	INST1	INST2	INST3
Nondurables						
$\delta$	.0016 (.0007)	.0017 (.0007)	–.0010 (.0036)	.0000 (.0015)	–.0002 (.0015)	–.0031 (.0015)
$\gamma$	.0509 (.0600)	.0595 (.0564)	–.0010 (.0873)	.0818 (.0659)	.0799 (.0567)	.1093 (.0812)
$\sigma$	.7526 (.1838)	.7183 (.1637)	.1931 (.0764)	.7149 (.2146)	.6665 (.1794)	.2423 (.0874)
$\alpha$	–.0167 (.0091)	–.0233 (.0182)	.0042 (.3693)	–.0326 (.0230)	–.0400 (.0245)	–.3418 (.2882)
$J(12)$	30.71 [.002]	37.91 [.000]	9.55 [.655]	28.29 [.005]	28.03 [.005]	13.87 [.309]
$J(13) - J(12)$	214.85 [.000]	212.48 [.000]	7.47 [.006]	181.66 [.000]	221.94 [.000]	2.05 [.152]
GH(15)				25.50 [.044]	26.68 [.031]	22.77 [.089]
Net Nondurables						
$\delta$	.0017 (.0005)	.0019 (.0007)	–.0008 (.0018)	.0001 (.0015)	–.0001 (.0015)	–.0027 (.0015)
$\gamma$	.0427 (.0509)	.0571 (.0637)	.0050 (.0910)	.0807 (.0751)	–.0800 (.0629)	–.0006 (.0766)
$\sigma$	.7336 (.1910)	.7125 (.1728)	.1930 (.0801)	.6964 (.2161)	.6677 (.1824)	.2615 (.0843)
$\alpha$	–.0155 (.0164)	–.0230 (.0182)	–.0209 (.3784)	–.0352 (.0215)	–.0398 (.0230)	.0016 (.2161)
$J(12)$	27.46 [.007]	36.14 [.000]	7.64 [.813]	27.02 [.008]	28.38 [.005]	14.22 [.287]
$J(13) - J(12)$	196.52 [.000]	198.80 [.000]	10.50 [.001]	164.02 [.000]	294.40 [.000]	3.82 [.051]
GH(15)				27.16 [.027]	29.20 [.015]	32.60 [.005]

NOTE.—See table 2.

we can measure that are also in agents' information sets. Estimation is based on finding parameter values that make sample analogues of these population orthogonality restrictions close to zero. Choosing instruments, therefore, embodies assumptions regarding agents' information. Moreover, the econometric issues of identification and efficiency depend on the set of instruments used (see Hansen [1985] and Hansen, Heaton, and Ogaki [1988] for details). The results in tables 2–5 are based on estimation that exploits 15 orthogonality restrictions. That is, we use three instruments for each of the five Euler equations. Estimating three parameters from 15 orthogonality restrictions leaves 12 overidentifying restrictions that can be used as a fur-

TABLE 5

GMM RESULTS FOR BONDS, STOCKS, AND NONDURABLES SERVICES

	1959:4–1986:12			1959:4–1978:12		
	INST1	INST2	INST3	INST1	INST2	INST3
Nondurables						
$\delta$	-.0041 (.0018)	-.0029 (.0018)	-.0020 (.0055)	-.0050 (.0031)	-.0068 (.0031)	-.0074 (.0061)
$\gamma$	-.0002 (.0528)	-.0083 (.0546)	-.4120 (.4894)	-.0027 (.0613)	.1408 (.0567)	-.0087 (.2207)
$\sigma$	.2650 (.0764)	.2814 (.0710)	.4103 (.3184)	.2851 (.1042)	.2606 (.0797)	.2064 (.1334)
$\alpha$	.0005 (.1492)	.0211 (.1455)	.5922 (1.070)	.0069 (.1564)	-.3994 (.1533)	.0333 (.8569)
$J(12)$	30.37 [.002]	34.96 [.000]	5.37 [.944]	27.61 [.006]	26.86 [.008]	11.91 [.453]
$J(13) - J(12)$	246.43 [.000]	224.20 [.000]	26.80 [.000]	249.27 [.000]	158.38 [.000]	8.07 [.005]
GH(15)				22.55 [.094]	27.08 [.028]	17.90 [.268]
Net Nondurables and Services						
$\delta$	-.0036 (.0018)	-.0032 (.0018)	-.0024 (.0036)	-.0049 (.0031)	-.0048 (.0031)	-.0072 (.0046)
$\gamma$	.0013 (.0564)	.0090 (.0600)	-.1461 (.5385)	-.0067 (.0659)	-.0014 (.0629)	-.0087 (.1748)
$\sigma$	.2477 (.0728)	.2570 (.0728)	.3081 (.2056)	.2637 (.0950)	.2652 (.0812)	.1918 (.0920)
$\alpha$	-.0041 (.1728)	-.0260 (1.203)	.3281 (3.038)	.0188 (1.493)	.0039 (1.266)	.0365 (2.764)
$J(12)$	28.20 [.005]	35.89 [.000]	5.51 [.939]	24.61 [.017]	23.32 [.025]	12.14 [.435]
$J(13) - J(12)$	189.28 [.000]	173.25 [.000]	16.55 [.000]	181.61 [.000]	179.86 [.000]	4.36 [.037]
GH(15)				26.60 [.032]	29.41 [.014]	19.63 [.187]

NOTE.—See table 2.

ther test (in addition to the expected utility restriction) of the implications of the model.

Since we have very little guidance in picking instruments, we replicate the estimation using three different sets of instruments. The first set has a constant, consumption growth lagged, and consumption growth lagged twice. The second set has a constant, consumption growth lagged, and the market return lagged. This set of instruments was used by Hansen and Singleton (1982). Finally, we use as our third set the second set of variables lagged an additional period. This requires weaker assumptions on the information structure of the problem than the other two instrument sets do. Hall (1988) argues

that the additional lag helps in reducing the effects of time aggregation and the mismatching of measurement time periods with planning time periods. Ogaki (1988) shows that the additional lag is consistent with the information structure of a monetary economy with cash-in-advance constraints.<sup>11</sup>

Before examining the actual results presented in tables 2–5, we briefly review how these numbers were obtained. Numerical minimization was accomplished with the DFP algorithm of the GQOPT numerical optimization package. Analytical derivatives were used in this optimization as well as in the construction of variance-covariance estimates. Convergence of this algorithm occurred when either the norm of the gradient or the norm of the change in the parameter vector was less than  $10^{-7}$ . A first round of consistent but inefficient estimates was obtained in each case using a nonlinear two-stage least-squares estimator, that is, using a block-diagonal weighting matrix with diagonal blocks equal to the inverse of the moment matrix of the instrument vector. At this stage a variety of starting values were attempted in each case. If the algorithm found more than one local optimum, the one with the smallest function value was taken as the global optimum. These consistent estimates were then used to construct the efficient weighting matrix for the relevant instrument vector and were also used as starting values for another round of estimation based on this efficient weighting matrix.<sup>12</sup> These estimates, reported in the tables, are consistent, relatively efficient, and asymptotically normally distributed.

The models of Section II allow some freedom in choosing a parameterization; for example, we could estimate  $\alpha$  directly or indirectly by estimating  $\gamma$ . We found our numerical methods to be most reliable when we estimated the parameters  $\delta$  (the rate of time preference),  $\gamma$  (the ratio of the risk parameter to the substitution parameter), and  $\sigma$  (the elasticity of intertemporal substitution in consumption). An estimate of  $\alpha$  and its asymptotic standard error can be derived from the results for the other parameters. The test of the expected utility hypothesis under this parameterization is a test of  $\gamma$  equal to one. In finite samples, a  $t$ -test of this hypothesis will not be invariant to nonlinear transformations or the hypothesis, that is, alternative nonlinear parameterizations. We therefore also estimate the model using the

<sup>11</sup> We assume that the additional lag in this instrument set implies that the forecast error has a moving average component of order one. In this case, we use an optimal GMM weighting matrix that takes account of this moving average process.

<sup>12</sup> To investigate the stability of these numerical procedures, a third round of estimates was computed using the second-round estimates as starting values and to compute the weighting matrix. In all cases, there was little movement in either the parameter estimates or their estimated standard errors.

efficient weighting matrix imposing the expected utility constraint,  $\gamma = 1$ . The difference in the values of the objective functions forms a test analogous to the likelihood ratio statistic, which is invariant to the parameterization and has a  $\chi^2(1)$  distribution in large samples.<sup>13</sup>

With this description of the details of the estimation in mind, we can now turn to the actual results.

### *C. Empirical Results*

The empirical results presented in tables 2–5 show some broad patterns that appear to hold over time periods, consumption measures, asset returns, and instrument sets. The elasticity of substitution is typically small (i.e., always less than one), corroborating the empirical work of Hall (1988). Risk preferences do not differ statistically from the logarithmic specification. This is consistent with intuition in Arrow (1965) and also the cross-sectional evidence of Mankiw and Shapiro (1986). With logarithmic risk preferences, the asset return equations reduce to those tested by Hansen, Richard, and Singleton (1981) and Brown and Gibbons (1985). Our model differs in that the elasticity of substitution is still a free parameter and equation (19) contains independent information. A troubling pattern that emerges is that the rate of time preference,  $\delta$ , is often significantly less than zero; that is, the discount factor is bigger than one. This result indicates a problem that this model shares with the expected utility model in fitting the levels of asset returns.<sup>14</sup> The point estimates for  $\gamma$  typically imply a preference for the late resolution of uncertainty.

Many of the results are sensitive to the consumption measure adopted and to the choice of instrumental variables (which determines the moment restrictions employed in the estimation and tests). The inclusion of services in the consumption measure can substantially alter both the point estimates and the economic interpretation of the model. The exclusion of clothing, shoes, and medical services has a much smaller impact on the results. To see this, consider first the results relating to the entire sample and the first two instrument sets.<sup>15</sup> For both sets of moment restrictions and for both sets of re-

<sup>13</sup> See Newey and West (1987) and Eichenbaum, Hansen, and Singleton (1988) for a complete discussion of this test statistic.

<sup>14</sup> This property is predictable given the general equilibrium simulations that have been done with these models. See Kocherlakota (1988) for an expected utility model with this feature that can match levels, and Weil (1989) for a non-expected utility model without this feature that cannot match levels of returns.

<sup>15</sup> Recall that the first instrument set restricts the unconditional means of the Euler equation errors and their correlations with consumption growth at 1- and 2-month lags. The second instrument set restricts the unconditional means of the errors and their correlations with consumption growth and the market return at a 1-month lag.

turns, we find that (i) the risk aversion parameter is larger and less precisely estimated for nondurables and services than for just nondurables; (ii) the elasticity of substitution has the opposite pattern, being smaller and more precisely estimated for nondurables and services than for just nondurables (e.g., neither  $\sigma = 0$  nor  $\sigma = 1$  can be rejected with nondurables, whereas both hypotheses can be rejected with nondurables and services); (iii) the rate of time preference is positive for nondurables and negative for nondurables and services; and (iv) the overidentifying restrictions tests provide more evidence against the model for nondurables and services than for just nondurables. (In addition, this test provides slightly more evidence against the model when the bond return is included than when only stocks are used.)

For the first two instrument sets, rejection of the expected utility model by either the Wald test of  $\gamma = 1$  or the likelihood ratio-type test is not sensitive to the consumption measure. Furthermore, all these patterns carry over to the pre-1979 subsample (with the possible exception of the risk aversion estimates, which can be extremely imprecise). This stability is reflected in the Ghysels-Hall (1990) test: parameter stability cannot be rejected at the 1 percent level.

The third instrument set does not impose any restrictions on the correlations of the Euler equation errors and the information measurable one period earlier. It does, however, still restrict the unconditional means and the two-period lag correlations of the error with consumption growth and the market return.<sup>16</sup> The results for this instrument choice for the vector of returns that includes the Treasury bill, when the entire sample and only nondurables are used, provide perhaps the most favorable evidence for our model. The overidentifying restrictions test does not provide evidence against the model, whereas both tests of the expected utility model clearly reject. The elasticity of substitution estimate is small and quite precise. The rate of time preference, however, is still less than zero.

These favorable results are tempered by the evidence across consumption measures and time periods. For example, for the stock returns, even though the model is not rejected, the expected utility model is not rejected by either test when the nondurables and services consumption measure is used. Further, the point estimate for  $\alpha$ , though still small, has an extremely large standard error when nondurables and services are used. Related to this, we find that for the pre-1979 subsample, it is not uncommon for the two tests of the expected utility restriction to disagree. Therefore, even though these

<sup>16</sup> Finn, Hoffman, and Schlagenhauf (1989) have investigated this lag structure for the expected utility model. Their conclusions are consistent with our findings.

instruments provide the most favorable evidence for our model, they are to some degree *poor* instruments since they can lead to fragile conclusions.

#### IV. Conclusion

This paper continues the work that was begun in the precursor to this paper (Epstein and Zin 1989). Models that generalize the conventional, time-additive, expected utility specifications and that retain empirical tractability are used to analyze the *stylized facts* of consumption and asset market data and to estimate and test the model using formal econometric procedures. Generalized method of moments estimates of the parameters of preference for a variety of monthly U.S. data sets generally lead to rejections of the expected utility hypothesis. The performance of the non-expected utility model is sensitive to the choice of consumption measure and instrumental variables. The dating of information is crucial: period  $t + 1$  "forecast errors" appear to be correlated with variables measured in period  $t$  but not with variables measured in period  $t - 1$ . This fact may be an indication of time aggregation problems or institutional constraints such as cash-in-advance restrictions. In general, we find that the elasticity of intertemporal substitution is less than one, relative risk aversion is close to one, and consumers prefer the late resolution of uncertainty.

Future research will investigate the sensitivity of these results to the functional forms adopted in this paper. In particular, we shall examine alternative characterizations of risk preferences that relax the independence axiom (as in Epstein and Zin [1989]). Another direction that merits investigation is the integration of stochastic labor income into the model of consumption and portfolio selection.

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