Price Theory
Proposed Solution to Problem Set 2, Question 1

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Many goods (e.g. meals) can be produced at home or purchased in the market. Other goods (e.g. cars) are difficult to produce in the household due to economies of scale etc. In this problem we will explore some of the trade-offs between market and household production. To keep things simple we will consider two goods (meals and cars). Each consumer is endowed with one unit of time that can be allocated between the production of meals and cars. Meal production is equally efficient in the household and in the market while cars can only be produced in the market. Meals produced at home can only be consumed by the individual that produced the meal while meals produced in the market can be sold to anyone.

1 Setting

There are two goods, *meals* and *cars* and three prices: the wage w, the price of meals (in the market) P_{meals} and the price of cars, P_{cars}. It will make sense to take *meals* as the numeraire, so that $P_{meals} = 1$ and all prices are measured in units of *meals*.

All individuals are identical, and we will assume that the marginal utility of each good increases without bounds as its consumption levels nears zero. Thus, the individual will want to consume a positive amount of each good no matter the price and her income.

a.

Assume that the output of meals is m per unit time spent producing meals (either at home or in the market) and that the output of cars is c per unit of time spent producing cars. What will determine the allocation of time between producing meals and cars? How will time usage vary with m and c? Why?

We can build our target equilibrium (P^*_{cars}, w^*) by examining the behavior of each of our agents: the representative consumer, the firm that produces *cars* in the market, and the firm that produces *meals* in the market.

Consider first the firm that produces *meals* in the market. Having a wage which is **lower** than the (value of) marginal product of labor for meals would allow it to reach unbounded levels of profits, so that its labor demand at such a wage would also be unbounded. But our individual only has 1 unit of time. Second, a wage which is **higher** than the marginal product of labor is equally problematic: the firm does not wish to hire any workers, but the individual will abandon any home production to sell his labor in the market, where it is worth more. And after working, she will have a positive demand for market meals.[2] It then follows that any equilibrium must have: $w = m$.

Similarly, the value of the marginal product of labor for *cars* must equal the wage: $w = P_{cars}c$.[3]

Therefore, the relative price of *cars* and *meals* is pinned down by the technology that prevails in the economy:

$$P_{cars} = \frac{m}{c}.$$

Notice now that the individual's time can produce the same amount of *meals* in the home and in the market sector. However, she is not indifferent between the two sectors when choosing where to allocate time: *meals* produced at home cannot be exchanged for *cars*, and she wants a positive amount of *cars*.

Figure 1 illustrates this. If our individual were to use her whole unit of labor in the market, she would face the budget constraint colored in black. On the other hand, a small amount of time spent in household production results in a budget constraint colored in gray. The light gray budget constraint is one where the individual spends even more time in home production.

[2] This is a consequence of our assumption on marginal utility.

[3] If it were higher than the wage, there would be an unbounded demand for labor. If it were lower, no car would be produced but a positive amount of cars would be demanded.

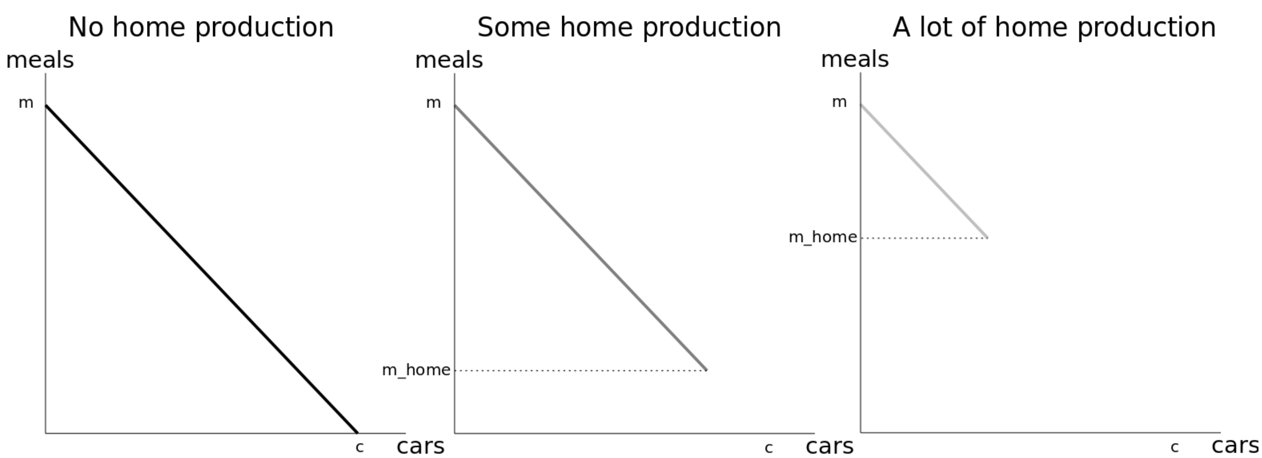

Figure 1: Budget constraints for different time market/household time allocations, evaluated at equilibrium prices.

How much time to allocate to home production then? If home-produced goods were exchangeable in the marketplace (or, in this case, if the individual worked in the market full time), the optimal consumption bundle would be the one given in Figure 2.

Here, $cars^*$ is the optimal amount of cars chosen given P^{cars}, w. Now, if the individual were to spend too much time in home produc-

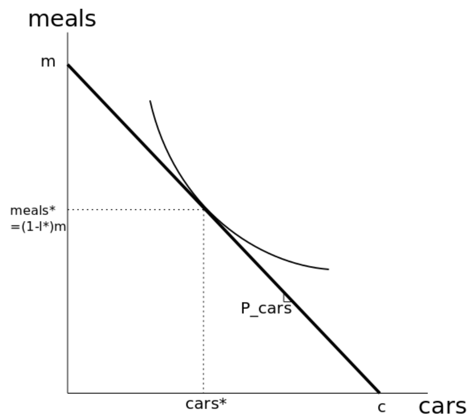

Figure 2: Optimal consumption bundle.

tion, she would not be able to afford *cars**. Thus, there is a minimum amount of labor l^* that must be spent in the market, given by

$$l^* = \frac{P_{cars} cars^*}{w} = \frac{cars^*}{c}.$$

Since any amount of market labor $l \in [l^*, 1]$ allow the individual to consume $(cars^*, meals^*)$, we have multiple equilibria.

Consider now an increase in *meals* productivity leading to $m' > m$. This drives up the equilibrium price of *cars* but also raises income, since the equilibrium wage increases. Assuming both *meals* and *cars* are normal goods, consumption of *meals* increases and consumption of *cars* could increase or decrease, leading the minimum amount of labor on the market to increase or decrease correspondingly.

On the other hand, consider an increase in *cars* productivity. This will increase l^* iff the price elasticity of *cars* exceeds one.[4]

[4] Recall that

$$l^* = \frac{cars^*(m/c, m)}{c}$$

and note that a one percent increase in c is roughly equal to a one percent decrease in $1/c$.

b.

Will it matter if meals are produced in the market or in the household? Why or why not?

It matters whether meals are produced in the market or in the household, even though home and market production are equally productive. This is because household meals are not exchangeable in the market.

c.

Now assume that all earnings in the market (from producing either cars or meals for sale to others) are subject to a 10% tax and that the proceeds of the tax are rebated to consumers on a per capita basis. What will the equilibrium *look like now? Why? What will happen to the consumption of cars and food relative to part A? Why? What can you say about the wage, the price of cars and the price of meals?*

Note that some of the previous logic continues to hold: under the income tax, in equilibrium, the wage paid by the firms, w^{firm} is still m, and the price of *cars* received by the firms, P^{firm}_{cars} is still $\frac{m}{c}$. The fact that the tax falls on the worker is a consequence of the perfectly elastic supplies of *cars* and *meals* given by the constant returns to scale technology.

In contrast to the previous questions, time allocated to home production yields more *meals* for the worker than time allocated to the market. A consequence of this is that no meals will be produced in the market: an individual that purchases market *meals* could use the time allocated to the market, which finances these *meals*, in home production. This reallocation of time would result in a larger consumption of *meals* and hold consumption of *cars* constant. Thus, for any consumption bundle that features a positive amount of market meals, there is another, better, consumption bundle that is feasible and only requires a reallocation of market time towards household time.

This argument can also be seen graphically. Panel A in Figure 3 depicts the budget constraints faced by an individual that (i) produces all *meals* in the household, and (ii) works full time in the market. Thus, anyone who likes more *meals* and *cars* as opposed to less will prefer to face the budget constraint in (i). An intermediate case is presented in Panel B. Here, as more *cars* are sacrificed for *meals*, the individual decides to no longer spend the additional time in the household, at the level l^{house} of household labor.

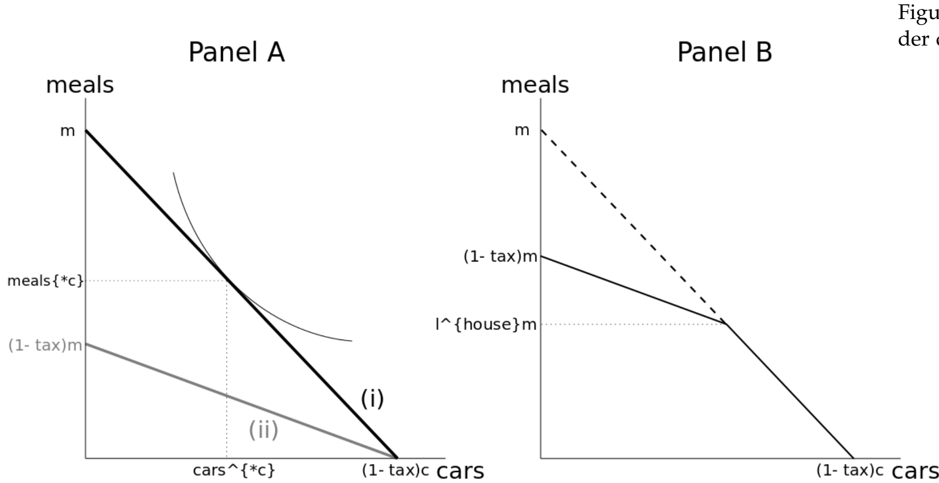

Figure 3: Budget Constraints faced under different allocations of time.

An equilibrium will therefore be one where everyone faces the budget constraint (i), and quantities consumed are as in Panel A in Figure 3: *cars*^{*c} and *meals*^{*c}.[5]

[5] Superscript c as in "equilibrium in question c." Note that there are actually multiple equilibria if we think of different wages across industries. If the *meals* firms paid a wage slightly larger than m, there would be zero demand for labor in this industry, and supply is also zero, since home production is still a superior way of procuring *meals*.

It can be seen now that the tax on market earnings is in fact equivalent an increase in the price of *cars*. Since the revenues of the tax are rebated to the household, there is no income effect, only a substitution effect. Thus, consumption of *cars* decreases and that of *meals* increases. This exercise is summarized in Figure 4.

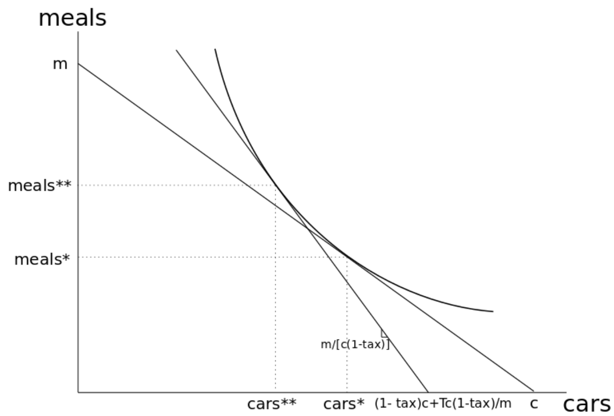

Figure 4: The effect of taxes on market earnings in equilibrium.

d., e.

Now assume in addition that each individual producing cars must spend some fixed amount of time, k, learning how to produce his productivity is c per unit time. For simplicity consider the case where cars and meals are consumed in fixed proportions. If k is small what will the equilibrium look like? Will everyone work in the market? Will meals be produced and sold in the market?

What can you say about the equilibrium wage rate and prices for meals and cars?

We will proceed without making special assumptions on preferences, for now. The starting point in this case is to realize that wages in both industries cannot be the same. If they were the same, why would anyone choose to spend time in the *cars* industry?[6] For there to be a positive supply of *cars*, we must have $w^{cars} > w^{meals}$.

[6] Think of the budget constraint faced by someone who works full time in the *meals* market sector compared to someone who works full time in the *cars* sector when wages offered in both sectors are equal.

Now consider a worker who has decided to pay the time cost. Is it advantageous for her to spend her time left anywhere other than the *cars* sector? No: doing so would have a negative impact on her budget constraint, since an hour of work there is worth more *meals* than an hour of work in the *meals* sector or in the household sector.

Given that $w^{cars} > w^{meals}$, how large should w^{cars} be? It cannot be too large as to drive everyone to the *cars* industry, and it cannot be too small as to not bring in any labor. In equilibrium, workers will be indifferent between paying the fixed cost and joining the *cars* sector or not doing so and produce potentially both market and household *meals*. Moreover, people are indifferent if the income earned as a full-time *cars* worker equals that earned as a full-time market *meals* worker:

$$(1-k)w^{cars} = w^{meals} \implies w^{cars} = \frac{m}{1-k}.$$

where we substituted in the equilibrium wage paid in the *meals* sector.

Using the same logic that follows from the constant returns to scale technology in question a., we must also have: $P^{cars}c = w^{cars}$, so that

$$P^{cars} = \frac{1}{1-k}\frac{m}{c}.$$

This answers question *e*.

We are now left to specify our agents' equilibrium occupational choices, that is, the fraction of workers in the *cars* industry – ϕ – and the fraction of time spent in the market by the *meals* workers. ϕ is simply pinned down with the market clearing condition of the *cars* market:

$$\begin{aligned} D^{cars}(1, P^{cars}, m) &= \phi(1-k)c \\ \iff \phi &= \frac{D^{cars}(1, P^{cars}, m)}{(1-k)c}. \end{aligned}$$

where D^{cars} is the Marshallian demand for *cars*.

Finally, notice there exists an equilibrium where all production takes place in the market: *meals* workers may as well spend all their time in the market since their budget constraint is as in Figure 1. On the other hand, the *meals* market clears.[7]

To recap, the equilibrium we have found is the following:

$$\begin{aligned} P^{meals} &= 1 \\ w^{meals} &= m \\ w^{cars} &= \frac{m}{1-k} \\ P^{cars} &= \frac{m}{(1-k)c} \\ \phi &= \frac{D^{cars}\left(1, \frac{m}{(1-k)c}, m\right)}{(1-k)c} \end{aligned}$$

Meals workers work full-time in the market.

It will exist as long as

$$\phi < 1 \iff D^{cars}\left(1, \frac{m}{(1-k)c}, m\right) < (1-k)c.$$

[7] To see why this is the case, apply Walras' Law: start from the market clearing condition in the *meals* market and use the budget constraint to recover the market clearing condition in the *cars* market, which is guaranteed to hold for the equilibrium fraction of workers in the *cars* market, ϕ.

Consider now the case where *meals* and *cars* are perfect complements, so that

$$U(meals, cars) = \min\{meals,\ \alpha\, cars\}, \qquad \alpha > 0.$$

In this case, people will want to consume α units of *cars* for every unit of *meals*: otherwise one would be consuming stuff that does not enhance utility and has a positive price. Together with the budget constraint, this condition pins down the demand curve for *cars* (and for *meals* as well):[8]

$$D^{cars}(1, P^{cars}, w) = \frac{w}{\alpha + P^{cars}}.$$

[8] To get this, simply use the budget constraint and the optimal consumption rule we just described.

Therefore, substituting for equilibrium prices we get that

$$\phi < 1 \iff \frac{m}{\alpha + \frac{m}{(1-k)c}} < (1-k)c \iff (1-k)c\alpha > 0.$$

This is always the case, so that the equilibrium we have found exists.

This equilibrium is such that everyone works in the market, and hence all meals are produced and sold in the market.

f.

If k is large what will the equilibrium look like? Will everyone work in the market? Will meals be produced and sold in the market? What can you say about the equilibrium wage rate and prices for meals and cars? Will everyone work the same amount in the market?

Notice that the equilibrium we have found exists for any $k \in [0, 1)$. If k were to equal one, no *cars* production would be possible. We would end up in a world where only *meals* can be produced but bring no utility whatsoever.