2015 Final Exam Q2

Two risk neutral partners seek to dissolve their partnership. Together they own and indivisible asset, and they wish to assign the asset to the one of them who values it most with the other being sufficiently compensated. Both partners own an equal share of the asset. The partners' private values for the asset are independent and drawn from a uniform distribution on $[0, 1]$. Each partner knows this and knows his own value, but does not know the values of the other partners. Assume throughout that if partner i's value is v_i, then, because he owns $1/2$ of the asset, he must obtain interim expected utility at least $v_i/2$ from the mechanism. Otherwise he will not participate and the deal will not go through. Show that there exists an expost efficient, incentive compatible, individually rational, expost budget balanced direct mechanism with which the two partners can dissolve their partnership. Does this contradict the Myerson-Satterthwaite theorem? Briefly explain.

JR 9.11

Suppose all bidders' values are uniform on $[0, 1]$. Construct a revenue maximising auction. What is the reserve price?

9.12

Consider again the case of uniformly distributed values on $[0, 1]$. Is a first-price auction with the same reserve price as in the preceding question optimal for the seller? Prove your claim using the revenue equivalence theorem.

9.13

Suppose the bidders' values are iid, each according to a uniform distribution on $[1, 2]$. Construct a revenue-maximising auction for the seller.

9.15

A drawback of the direct mechanism approach is that the seller must know the distribution of the bidders' values to compute the optimal auction. The following exercise provides an optimal auction that is distribution-free for the case of two asymmetric bidders, 1 and 2, with independent private values. Bidder i's strictly positive and continuous density of values on $[0, 1]$ is f_i with distribution F_i. Assume throughout that

$$v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}$$

is strictly increasing for $i = 1, 2$.

The auction is as follows. In the first stage, the bidders each simultaneously submit a sealed bid. Before the second stage begins, the bids are publicly revealed. In the second stage, the bidders must simultaneously declare whether they are willing to purchase the object at the other bidder's revealed sealed bid. If one of them says "yes" and the other "no", then the the "yes" transaction is carried out. If they both say "yes" both say "no" then the seller keeps the object and no payments are made.

Note that the seller can run this auction without knowing the bidder's value distributions.

(a)

Consider the following strategies for the bidders. in the first stage, when her value is v_i, bidder $i \neq j$ submits the sealed bid $b_i^* (v_i) = b_i$, where b_i solves

$$b_i - \frac{1 - F_j (b_i)}{f_j (b_i)} = \max \left(0, v_i - \frac{1 - F_i (v_i)}{f_i (v_i)} \right).$$

(While such a b_i need not always exist, it will always exist if the functions $v_1 - (1 - F_1 (v_1)) / f_1 (v_1)$ and $v_2 - (1 - F_2 (v_2)) / f_2 (v_2)$ have the same range. So, assume this is the case.)

In the second stage, each bidder says "yes" if and only if her value is above the other bidder's first-stage bid.

Show that these strategies constitute an equilibrium of this auction.

(Also, note that while the seller need not know the distribution of values, each bidder needs to know the distribution of the other bidder's values in order to carry out her strategy. Hence, this auction shifts the informational burden from the seller to the bidders.)

(b)

(i) Show that in this equilibrium the seller's expected revenues are maximized. (ii) Is the outcome always efficient?

(c)

(i) Show that it is also an equilibrium for each bidder to bid his value and to then say "yes" if and only if his value is above the other's bid. (ii) Is the outcome always efficient in this equilibrium? (iii) Show that the seller's revenues are not maximal in this equilibrium.

9.31

Consider Example 9.3. Add the social state, "Don't Build" (D) to the set of social states so that $X = \{D, S, B\}$. Suppose that for each individual i,

$$v_i (D, t_i) = k_i$$

is independent of t_i.

(a)

Argue that one interpretation of k_i is the value of the leisure time individual i must give up towards the building of either the pool or the bridge. (For example, all the k_i might be zero except $k_1 > 0$, where individual 1 is the town's only engineer).

(b)

What are the interim individual rationality constraint if individuals have property rights over their leisure time?

(c)

When is it efficient to build the pool? The bridge?

(d)

Give sufficient conditions for the existence of ex post efficient mechanism both when individuals have property rights over their leisure time and when they do not. Describe the mechanism in both cases and show that the presence of property rights makes it more difficult to achieve ex post efficiency.

Core 2017 IV

Consider the standard independent private value auction model with n bidders. Assume that the distribution of valuation of each bidder is given by the CDF F supported on the interval $[1, 2]$. (There is a pdf f on the interval $[1, 2)$ corresponding to the CDF F.) In addition, the virtual valuation of each bidder is a weakly increasing function.

Consider the following allocation:

(i) if the valuation of each bidder is strictly smaller than two, the seller keeps the objective with probability one half and, with the remaining probability, the object is randomly assigned to a bidder with equal probabilities.

(ii) if there is at least one bidder whose valuation is two then the object is randomly assigned to one of those bidders whose valuation is two with equal probabilities.

MWG 23.E.7

Consider a bilateral trade setting in which the buyer's and seller's valuations are drawn independently from the uniform distribution on $[0, 1]$.

(a)

Show that if f is a Bayesian IC and interim IR social choice function that is expost efficient then the sum of buyer's and seller's expected utility is no less than $5/6$.

(b)

Show that there is no social choice function in which the sum of the utility is more than $2/3$.