Economics 33100 Reading Summaries

Winter 2010

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Contents

1 Labor market search

1.1 Ljungqvist, Lars, and Thomas J. Sargent. 1998. The European unemployment dilemma, Journal of Political Economy 106: 514-550. (Elisa Olivieri)

Figure 1: Abstract (in case you're too lazy to even read the summary)

> Post–World War II European welfare states experienced several decades of relatively low unemployment, followed by a plague of persistently high unemployment since the 1980s. We impute the higher unemployment to welfare states' diminished ability to cope with more turbulent economic times, such as the ongoing restructuring from manufacturing to the service industry, adoption of new information technologies, and a rapidly changing international economy. We use a general equilibrium search model in which workers accumulate skills on the job and lose skills during unemployment.

Note: this model is the father of the search model we did in class, and the Steve the Stevedor problem we did in the homework. Please refer to those for the basic mechanics of dynamic employment models.

The motivation and thesis: European welfare states have seen high and persistent (long duration) unemployment since the 1980s, while the US hasn't. Why? The authors claim that high taxes and generous welfare benefits distort worker's labor supply decisions, and the focus of the paper is on how **welfare states 'adversely affect dynamic responses to economics shocks and to increasing turbulence in the economic environment". Thesis: "our thesis is that changed economics conditions from the mid 1970s onward can explain the high (long term) unemployment in welfare states."**

Changing environment: Since 1980s, both permanent and transitory components of individual earnings have spread out—ie. more turbulence/ variance in wages. Oil shocks were frequent. New technologies and globalization have dramatically altered work and the economics environment.

The Model: the authors create a general equilibrium search model like the one presented in class, in which worker's skill's depreciate during unemployment spells, and in which unemployment benefits are determined by worker's past earnings. (The ideas are very similar to the model we had to deal with for Steve the Dockworker, so go look at the answer key!)

The crucial thing to look at is the amount of skills lost at layoff. Long-time displaced workers suffer huge and enduring human capital depreciation, though they rapidly regain some of the lost skills if they go back to work. This is very important because it means that incentives to stay out of work, like high unemployment benefits, might cause debilitating depreciation of skills. The authors model this, and their findings suggest that workers facing European incentives might often wind up in the ranks of the long-term unemployed. Below is the worker's bellman, with F(w) being the wage distribution, pi the probability of getting a wage offer, gamma the probability of being fired, s being the intensity of search for work, h being today's skill and h' tomorrow's skill. Skills are accumulated/deteriorate stochastically, with some transition matrix μe for employed people, μl for the period they get laid off, and µu for unemployed people. The workers sees his skill, then decides whether to accept his wage offer, choose a search intensity, or quit his job. Unemployed workers face unemployment b(I), where I is their last earnings. I(I) is a government threshold; unemployed workers who take a job that earns below I(I) may still receive government benefits; those who make more than that cannot. Tao is a (flat) tax parameter.

Figure 2: The Model

$$V(w, h) = \max_{\text{accept, reject}} \Big\{ (1-\tau)wh + (1-\alpha)\beta\Big[(1-\lambda)\sum_{h'}\mu_e(h,h')V(w,h') + \lambda\sum_{h'}\mu_l(h,h')V_b(wh,h')\Big], V_o(h)\Big\},$$

$$\begin{aligned} V_b(I, h) = \max_s \Big\{ &-c(s) + (1-\tau)b(I) + (1-\alpha)\beta\sum_{h'}\mu_u(h,h') \\ &\times\Big[[1-\pi(s)]V_b(I,h') + \pi(s)\Big(\int_{w\geq I_g(I)/h'} V(w,h')\,dF(w) \\ &+ \int_{w<I_g(I)/h'} \max_{\text{accept, reject}}\Big\{(1-\tau)wh' \\ &+ (1-\alpha)\beta\Big[(1-\lambda)\sum_{h''}\mu_e(h',h'')V(w,h'') \\ &+ \lambda\sum_{h''}\mu_l(h',h'')V_b(wh',h'')\Big], V_b(I,h')\Big\}dF(w)\Big)\Big]\Big\}, \end{aligned} \tag{2}$$

and

$$\begin{aligned} V_o(h) = \max_s \Big\{ &-c(s) + (1-\alpha)\beta\sum_{h'}\mu_u(h,h') \\ &\times\{[1-\pi(s)]V_o(h') + \pi(s)\textstyle\int V(w,h')\,dF(w)\}\Big\}. \end{aligned} \tag{3}$$

The optimal decision rules are: a reservation wage and a search intensity for an unemployed worker eligible for benefits, and a reservation wage and search intensity for an unemployed worker who is not eligible for benefits.

The authors calibrate the hell out of this model, trying all sorts of combinations and introducing economic shocks to see how reservation wages and search intensities vary in the face of different parameters (welfare state vs non-welfare state) and different types of turbulence.

The results are summed up as follows:

Figure 3: Results Summary

Our analysis suggests that the smooth performance of the welfare states in the 1950s and 1960s concealed an inherent instability in these economies. In our model, a welfare state with a very generous entitlement program is a virtual "time bomb" waiting to explode. As long as the economy is not subject to any major economic shocks, the welfare state can function well. Workers who get laid off with generous unemployment compensation can without too much trou-

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ble get back into employment at working conditions similar to those of their previous jobs. That is, the availability of "good jobs" for unemployed workers counteracts the adverse effects of generous unemployment compensation. However, at the time of a large shock, generous unemployment compensation hinders the process of restructuring the economy. Laid-off workers then lack the incentives to quickly accept the transition to new jobs in which skills will once again have to be accumulated. Consequently, there can be a lengthy transition phase with long-term unemployment largely attributable to the existence of welfare programs. This causality is hard to detect from time-series data because there need not have been any changes in the welfare programs at the time of the shock.

There are interesting implications for who is unemployed longest:

long-term unemployment during their working lives. The workers who end up being unemployed for long terms are ones who have lost considerable amounts of skills at the time of their layoffs or during their unemployment spells. The fact that welfare benefits are based on past earnings causes these workers with depreciated skills literally to "bail out" from the active labor force by choosing low search intensities and high reservation wages. In other words, our model predicts that workers who have accumulated significant amounts of skills and subsequently lose these skills are more prone to end up as long-term unemployed. This view is consistent with the

2 Technical change, real business cycles

2.1 Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell. 1997. Long-run implications of investment-specific technical change. American Economic Review, 87: 342–362. (Daisuke Fujii)

The purpose of this paper is **to investigate the role of investment-specific technological change in pose-war US growth.** Decompose investment (and capital) into equipment (like computers, telecommunication and transportation) and structures. There are two striking features in data.

1. The price of equipment has declined constantly and the equipment-GNP raio has increased whereas the price and quantity of structures have been stable (Long run).

2. The short-run data display a negative correlation between the price of equipment and the investment in equipment.

   These results suggest that the investment-specific technological change may be an engine of growth and a source of economic fluctuations. What is the quantitative role of investment-specifit technological change as an engine of growth? By building an appropiate model, the calibration of the role is 60% of the growth in output. Residual, neutral productivity change then accounts for the remaining 40 percent.

The Model

Setup

$$\begin{aligned} \text{Preferences} &: E\left[\sum_{t=0}^{\infty}\beta^t U\left(c_t,l_t\right)\right] \text{ with } U\left(c_t,l_t\right)=\theta\ln c+(1-\theta)\ln(1-l) \\ \text{Production} &: y=zF\left(k_e,k_s,l\right)=zk_e^{\alpha_e}k_s^{\alpha_s}l^{((1-a_e-\alpha_s)} \\ \text{Constraint} &: y=c+i_e+i_s \\ \text{Law of motion (s)} &: k_s'=\left(1-\delta_s\right)k_s+i_s \\ \text{Law of motion (e)} &: k_e'=\left(1-\delta_e\right)k_e+i_e q \text{ ($\leftarrow$this q is critical!)} \\ \text{Govt. budget constraint} &: \tau=\tau_k\left(r_e k_e+r_s k_s\right)+\tau_l wl \end{aligned}$$

Balanced Growth Path

Let γ_z,γ_q be the average growth rate of z and q. On BGP, output, consumption, investment and the stock of structures all have to grow at the same rate g. **Equipment grows faster because of q.** We obtain

$$\begin{aligned} g &= \gamma_z^{\frac{1}{(1-\alpha_e-\alpha_s)}}\gamma_q^{\frac{\alpha_e}{(1-\alpha_e-\alpha_s)}} \\ g_e &= \gamma_z^{\frac{1}{(1-\alpha_e-\alpha_s)}}\gamma_q^{\frac{1-\alpha_s}{(1-\alpha_e-\alpha_s)}} \end{aligned}$$

The rental price of a unit of equipment must be falling over time on BGP. How then can the real interst rate remain constant? The answer is that the cost of a unit of equipment in terms of consumpiton or $\frac{1}{q}$ is also declining.

Matching the Data

We have data for some of the parameters in the model. The esimate for other parameters can be constructed from calculatoins. The average annual growth rage in neutral productivity (z) is 0.39 percent whereas that of investment-specific productivity (q) is 3.21 percent. There is a dramatic downturn in total factor productivity which began in the seventies until now→two explanations. First investment-specific technological change was high when total productivity growth was low, that is, growth in q accelerated at the same time as there was a slowdown in z.

Conclusions

The model has the prperty that equipment-GNP raito increases over time as the relative price of new capital goods declines. Otherwise, the standard results of neoclassical model apply. The BGP of the model was calibrated in the long-run US data. It was found that 60% of postwar productivity growth can be attributed to investment-specific technological change. Also, a more striking picture emerges of the much-discussed productivitiy slowdown that started in 1970's. Once the recent rapid technological improvement in the pdocution of new capital goods is taken into account, the decline in the productiviity of other factors is dramatic. These findings point to a gery specific and potnetially iportant source of economic growth. For future reserach, we can try to endogenize the investment-specific technological change allowing human capital formulation, endogenous R&D or monopolistic competition.

2.2 Krusell, Per, Lee E. Ohanian, Jose-Victor Rios-Rull and Giovanni L. Violante. 2000. Capital-skill complementarity and inequality: a macroeconomic analysis. Econometrica, 68(4): 1029-53. (Matt Schiffman)

- Looks at market for skilled (college educated) and unskilled labor.
  - Relative quantity of skilled labor increased postwar period
  - Wage of skilled labor increased relatively as well, but only since 1980
- Paper tries to formulate an explicit model of skill-biased technical change
  - Model compares the elasticity of substitution's for capital/unskilled labor and capital/skilled labor
    - Higher for capital/unskilled labor
  - If this is correct then as capital stocks increase, this elasticity works against unskilled labor as its substituted away.

- Data is stock of capital in united states divided into "structures" (less relevant) and "equipment" (more relevant)
- Creates a 4 factor production model using structures, equipment, skilled, and unskilled labor.
  - Production function CRS, Cobb-Douglas over structures and CES over everything else
  - Dynamic model with stochastic shocks
    - Shocks to relative prices of the types of capital
    - Shocks to efficiency of types of labor
- Paper concludes that elasticity of substitution between unskilled labor and equipment is 1.67 (which is inline with literature)

2.3 Cooley, Thomas F. and Edward C. Prescott. 1995. Economic growth and business cycles, in Frontiers of Business Cycle Research, ed. by Thomas F Cooley, Princeton: Princeton University Press, pp. 1-38. (Philipp Tillman)

Stylized Growth Facts

1. Real output grows at constant rate
2. Stock of real capital grows at constant rate greater than the labor input growth rate
3. Growth rate of real output = growth rate of capital
4. Rate of profit on capital has horizontal trend
5. Growth rate of output per-capital varies greatly between countries
6. Economies with high share of profits in income have high ratio of investment to output

(1)-(4): balanced growth path. Neoclassical growth model captures many of the stylized facts.

Real business cycle theory: maintain neoclassical growth model and study fluctuations around trend due to various (real) shocks (as opposed to using separate models for growth and business cycles and explaining fluctuations with nominal shocks such as herd behavior, money illusion, etc.).

Table 1: Sources of Growth

Changes in output per worker	Secular (long-term) growth	Business Cycle
Due to changes in capital	1/3	0
Due to changes in labor	0	2/3
Due to changes in TFP	2/3	1/3

Calibration

- *Step 1*: Restrict processes (output, investment,etc.) to a parametric class (use theory) (e.g. choose production and utility function that is consistent with data and that can be parameterized)
- *Step 2*: construct a set of measurements that are consistent with the parametric class of models (manipulate the data such that they match our model's parameters and variables)
- *Step 3*: Assign values to the parameters of the model (so that the behaviour of the model matches features of the measured data) (e.g. set labor and capital share in Cobb-Douglas function according to data)

Business Cycle Facts

- magnitude of fluctuationsin output and aggregate hours of work equal
- most fluctuations in total hours represents movements into and out of labor force rather than adjustments in average hours of work
- consumption of nondurables pretty smooth
- investment in durables more volatile than output
- capital stock fluctuates much less than output and is uncorrelated with output
- productivity slightly procyclical but varies much less than output
- wages vary less than productivity
- no correlation between average wage and output
- government expenditures uncorrelated with output
- imports are more strongly procyclical than exports

2.4 Hansen, Gary D. and Edward C. Prescott. 1995. Recursive methods for computing equilibria of business cycle models, in FBCR, pp. 39-64. (Dimitris Batzilis)

Introduction

The problem that concerns the authors is that generally it's not possible to compute equilibria of business cycle models analytically. However, this is possible for some structures, like the one with a quadratic objective function, linear constraints and exogenous disturbances that are generated by a first order linear vector-autoregressive process. The quadratic objective that is chosen in the algorithms presented in the paper is the second-order Taylor series expansion of the return function for the deterministic version of the model evaluated at the steady state.

Social Planning Problems

This section is about economies where the second welfare theorem applies, that is CE are PO. The dynamic programming problem of the social-planning problem has the following form

$$v(z,s) = \max\left\{\left\{r(z,s,d) + \beta E\left[v(z',s')\,|z,s\right]\right\}\right\} \tag{1}$$

subject to the following two constraints- laws of motion:

$$z' = A(z) + \epsilon' \tag{2}$$

$$s' = B(z,s,d) \tag{3}$$

Notation:

- $r(z,s,d)$ is the return function
- $v(z,s)$ is the optimal value function for the problem
- z is a vector of exogenous state variables
- ϵ is a vector of random variables distributed independently over time with mean zero and finite variance
- d is a vector of decision variables.
- A and B are linear functions

The Basic Model

The goal here is to see how a typical stochastic growth model can be reformulated so that it looks just like the model above.

A representative agent maximizes the utility function:

$$E\sum_{t=0}^{\infty}\left\{\beta^t U(c_t,l_t)\right\},\ 0<\beta<1 \tag{4}$$

With z_0 and k_0 given, and subject to the following constraints:

$$y_t = z_t F(k_t,h_t) \tag{5}$$

$$z_{t+1} = A(z_t) + \epsilon_{t+1} \tag{6}$$

$$y_t = c_t + i_t \tag{7}$$

$$k_{t+1} = (i-\delta)k_t + i_t, 0<\delta<1 \tag{8}$$

Notation and Properties:

- U is a concave, strictly increasing, and twice continuously differentiable (in both arguments) utility function.
- c, l and h are consumption, leisure, and work-hours respectively.
- y and k stand for output and capital respectively.
- F is a concave, CRS, twice differentiable, increasing in both arguments production function
- z is the technology shock, which is observed at the beginning of the period.
- ϵ's are random iid variables with mean zero and finite variance
- A is linear

Now, we:

- combine constraints 5-7 to eliminate y_t
- solve the resulting equation for c_t
- substitute this in the utility function

Note that: $s = k, d, = (h, i)$. The new return function is

$$r(z, k, h, i) = U[zF(k, h) - i, 1 - h] \tag{9}$$

and the law of motion for the endogenous state variable s is

$$B(z, k, h, i) = (i - \delta)k + i \tag{10}$$

In the rest of section 2, the authors expand this model to add a) labor-augmenting technological growth b) the requirement that multiple periods are required to build productive capital c) the assumption of indivisible labor.

Solving a Social Planning Problem

In this section, the authors describe how to solve the problem when the return function is quadratic. Because I cannot make a summary of the math without just copying the paper, I will briefly describe the general ideas.

First, when the return function is not quadratic, we need to form its quadratic approximation. The quadratic approximation corresponds to the first three terms of a Taylor series expansion of the r.f. at the steady state values $(r(\bar{z}, \bar{s}, \bar{d}))$.

The rest of the section descibes the steps to solve the dynamic program by successive approximations. This is meant to be more like an algorithm for constructing computer programs that will do the computations.

2.4 Hansen, Gary D. and Edward C. Prescott. 1995. Recursive methods for computing equilibria of business cycle models, in FBCR, pp. 39-64. (Dimitris Batzilis)

Recursive Competitive Equilibrium for Homogeneous-Agent Economies

This section is about the economies where the second welfare theorem doesn't hold. For example this happens in economies with distorting taxes, or with money that is introduced by imposing a cash-in-advance constraint.

General dynamic programming problem

$$v(z, S, s) = max\,\{r(z, S, s, D, d) + \beta E[v(z', S', s')|z]\} \tag{11}$$

subject to the following constraints

$$z' = A(z) + \epsilon' \tag{12}$$

$$s' = B(z, S, s, D, d) \tag{13}$$

$$s' = B(z, S, S, D, D) \tag{14}$$

$$D = D(z, S) \tag{15}$$

Notation:

- z, A and ϵ are defined the same way as above
- s is a vector of endogenous household-specidic state variables
- S is a vector containing their economy-wide per capita values
- d is a vector of household decision variables
- D is the vector of per capita values of these same variables

Note that the third constraint is obtained from the second by aggregating over all households. The function D in the last constraint expresses the relationship between the per capita values of the decision variables, D, and the state variables, z and S. This function is determined as part of the equlibrium

The goal is to find a recursive competitive equilibrium, that consists of

- household decision rules: d=d(z,S,s)
- a rule determining the per capita values of these variables: D=D(z,S)
- a valu function v(z,S,s), such that
  - given the aggregate decision rules (D), the value function satisfies the objective function, with d being the associated decision rules
  - the function D satisfies the relationship D(z,S)=d(z,S,S)

Solving for a Recursive Competitive Equilibrium

This is simply the equivalent to section 3, but for the type of problem described in 4.

Extensions to Heterogeneous-Agent Economies

This section extends the methods described above to the case where the agents are not ex ante identical.

2.5 Prescott, Edward C. 1986. Theory ahead of business cycle measurement, Federal Reserve Bank of Minneapolis Quarterly Review 10,4. (Avihai Rapaport)

Economists have long been puzzled by the observations that during peacetime industrial market economies display recurrent, large fluctuations in output and employment over relatively short time periods. These observations are considered puzzling because the associated movements in labor's marginal product are small. Economic theory implies that given the nature of the shocks to technology and people's willingness to substitute inter- and intra-temporally, the economy will display fluctuations from trend. Theory predicts very well that consumption would be about half as volatile as output and investment about three times or more volatile that output. The point of this short article is to show that theory has done a good job in explaining the data but suggest that changing the measurement methods of the data to be more aligned with theory would improve the fit between theoretical prediction and the data. Hence the name: "Theory ahead of business cycle measurement"

The Business Cycle Phenomena

Here they say they use the smoothing filter mentioned in class to distinguish between trend and deviations from trend to define business cycles. "our curve-fitting method is to take the logarithms of variables and then select the trend path which minimizes the sum of the squared deviations from a given series subject to the constraint that the sum of the squared second differences not be too large. They choose the Lagrange multiplier of the constraint to be 1600.

The Growth Model

Here they introduce the basic model described in class with Cobb-Douglass production fct with an exogenous productivity shock Z. Only difference is that Z is now serially correlated (vs. iid in class) which is assumed in order to explain the serial correlation in the data for the economic variables. They maximize expected life time utility over consumption and leisure of a representative agent and say it's the same results as the competitive equilibrium outcome.

Empirical Labor Elasticity

One key deviation of the data from the model is that the empirical labor elasticity of output (the exponent of labor in the CobbDouglas prod. fct.) is less than predicted by theory. Reasons why this is so:

- Cyclical measurement errors in output. A sizable part of the investment component of output is hard to measure and therefore not included in the US National Product Accounts measure of output, the gross national product (GNP). In particular, a firm's major maintenance expenditures, research and development expenditures, and investment s in human capital are not included in GNP. In good time – namely, when output is above trend – firms may be more likely to undertake major repairs of a not fully depreciated asset, such as replacing the roof of a 30-year-old building which has a tax life of 35 years. Such expenditure is counted as maintenance and therefore not included in GNP even though the new roof will provide productive services for many years. The incentive for firms to do this is tax savings. If this and other types of unmeasured investment fluctuate in % terms more than output, as do all the measured investment components, the volatility of GNP is larger than measured which would lead to the data understating the elasticity of output wrt labor (the exponent of the Cobb-Douglas prod. fct.)

- Another reason to expect the American economy's' labor elasticity to be less than the model's is that the model shocks are perfectly neutral with respect to the consumption and investment good transformation. Persistent shocks which alter the product transformation frontier between these goods would cause variation in output and employment but not in the productivity parameters. For fluctuations so induced, the empirical labor elasticity of output would be the true elasticity. Similarly, relatively permanent changes in the taxing of capital – such as altering depreciation rates, the corporate tax rate, or the investment tax credit rate would all result in fluctuations in output and employment but not in the productivity parameters.
- A final reason for actual labor elasticity to be less than the model's is the way imports are measured. An increase in the price of imported oil, that is, an increase in the quantity of output that must be sacrificed for a given unit of that input, has no effect on measured productivity. From the point of view of the growth model, however, an oil price increase is a negative technology shock because it results in less output, net of the exports used to pay for the imported oil, available for domestic consumption and investment. Theory predicts that such shocks will induce variations in employment and output, even though they have no effect on the aggregate prod. fct. Therefore, insofar as they are important, they reduce the empirical labor elasticity of output.

2.6 Long, John B., Jr. and Charles I. Plosser. 1983. Real business cycles, Journal of Political Economy, 91(1): 39-69. (Dave Toniatti)

Overview

The authors present a simple model with technology shocks that induces market fluctuations that resemble business cycles.

Abstract

In this paper we demonstrate how certain very ordinary economic principles lead maximizing individuals to choose consumption-production plans that display many of the characteristics commonly associated with business cycles. Our explanation is entirely consistent with (i) rational expecatations, (ii) complete current information, (iii) stable preferences, (iv) no technilogical change, (v) no long-lived commodities, (vi) no frictions or adjustment costs, (vii) no government, (viii) no money, and (ix) no serial dependence in the stochastic elements of the environment. We also provide a completely worked out example of the type of artificial economy we have in mind. The time-series properties of the example exhibit some major features o the observed business cycles. Although this type of model may not be capable of explaining all of the regularities in the actual business cycles,

we believe that it provides a useful, well-defined benchmark for assessing the relative importance of factors (e.g. monetary disturbances) that we have deliberates ignored.

The Model

The paper presents a Robinson Crusoe model with N commodities. The agent maximizes consumption over infinite periods, and has a log utility function over all commodities and leisure. The agent has a Cobb-Douglas production function that uses all N commodities and labor in the production of N commodities for the next period. There is no joint production, and the production of each commodity has serially uncorrelated and iid technology shocks. Depreciation rate for commodities is 100%. Mathematically,

$$u(C_t, Z_t) = \theta_0 \ln Z_t + \sum_{i=1}^{N} \theta_i \ln C_{it}$$

where $\theta_i \geq 0\ \forall i$, and Z_t is leisure.

$$Y_{i,t+1} = \lambda_{i,t} L_{it}^{b_i} \prod_{j=1}^{N} X_{ijt}^{a_{ij}},\ i = 1, 2, \ldots, N$$

where $b_i + \sum_{i=1}^{N} a_{ij} = 1,\ i = 1, 2, \ldots, N$, $L_{it}^{b_i}$ is the labor input for commodity i with input share b_i, $X_{ijt}^{a_{ij}}$ is the commodity input j in period t for production of commodity i with capital input share a_{ij}. Technology shocks $\{\lambda_{i,t}\}$ are iid and serially uncorrelated.

We create a stochastic process by taking logs of the production function.

$$y_{t+1} = Ay_t + k + \ln \lambda_{t+1}$$

The transition matrix A uses input shares $\{a_{ij}\}$ as elements and the constant k incorporates utility-specfic paramters θ and β.

Key Results

Since the matrix A is stable (capital shares sum to less than unity) and the agent smooths consumption, technology shocks generate damped oscillations in the expected output path. In other words, consumers absorb output fluctuations resulting in both persistance as well as comovement among consumption, inputs, and output. Since log likelihood utility is additively separable and there is no joint production, shocks do not change in the marginal utility of labor which determines wages.

The paper confirms this cyclical behavior with 1967 US data from six industries. The empirical exercise demonstrates that the largest response is from the shocked industry, all sectors show a lagged response to one industry-specific shock, and the size of response in a non-shocked industry is a function of size of the shocked industry's input share. Since this model is relatively abstract and stimulates the economy with uncorrelated productivity shocks to different industries, any comparison with historical trends is uninformative.

3 Asset Pricing

3.1 Lucas, Robert E. 1978. Asset prices in an exchange economy, Econometrica 46: 1462-1445. (Ryan Hughes)

A look at stochastic behavior of equilibrium asset prices in a one-good pure exchange economy with representative consumers.

Setup

We treat the economy as a single representative consumer who wants to maximize consumption over time:

$$\sum_{t=0}^{\infty} U(c_t)$$

There are n distinct productive units, and output cannot be stored, so feasible consumption is given by, for all t,

$$0 \leq c_t \leq \sum_{j=1}^{n} y_{jt}$$

(Informal note: We can think of these productive units as trees or firms or robots or whatever, but I will refer to them as firms more often than not)

The vector of aggregate potential consumption goods y is stochastic and Markov, so its behavior is completely described by the conditional cdf:

$$F(y', y) = p(y_{t+1} = y' | y_t = y)$$

Each productive unit/firm (or component of the y vector, if you will) has one perfectly divisible equity share, and a person owning a portion of such a share at period t is entitled to an equal portion of the firm's output in that period. After dividends (claims on consumption) are distributed, shares are traded in a competitive market with price vector p. So let $p = (p_1, \ldots, p_n)$ be the vector of prices, $z = (z_1, \ldots, z_n)$ be the vector of share holdings by the representative consumer at the beginning of period t. Then it is easy to determine our

Equilibrium Conditions:

There is exactly one equilibrium price function for this economy so that:

All output is consumed, so $c_t = \sum_{j=1}^{n} y_{jt}$, all t.

All shares are held: $z_t = \vec{1}$, for all t.

Consumers have *rational expectations*: given what they think is going to happen, they behave optimally in such a way as to make the economy realize the outcome they originally predicted.

Using standard Bellman equation/FOC/EE tricks, we find that the equilibrium price vector needs to satisfy

$$U'\left(\sum_j y_j\right) p_i(y) = \beta \int U'\left(\sum_j y'_j\right)(y'_i + p_i(y'))\, dF(y', y) \tag{16}$$

to make sense of (16) define

$$g_i\left(y\right) = \beta \int U'\left(\sum_j y'_j\right) y'_i dF\left(y', y\right)$$

then for n independent function equations

$$f\left(y\right) = g_i\left(y\right) + \beta \int f\left(y'\right) dF\left(y', y\right)$$

have n solutions $f_1(y), f_2(y), \ldots, f_n(y)$ then

$$p_i\left(y\right) = \frac{f_i\left(y\right)}{Y'\left(\sum_j y_j\right)},\ i = 1, \ldots, n$$

are the components of our equilibrium price vector.

Examples

1. Linear Utility: here we have
$$p_i(y) = \sum_{s=1}^{\infty} \beta^2 \mathbb{E}\left[y_{i,t+s} | y_t\right]$$

2. One asset: In this case
$$\frac{yp'(y)}{p(y)} = -\frac{yU''(y)}{U'(y)}$$

Concluding notes

A series that has the martingale property:

$$w_{i,t+1} - w_{i,t} = \beta U'\left(\sum_j y_{j,t+1}\right)\left(y_{i,t+1} + p_{i,t+1}\right) - U'\left(\sum_j y_{jt}\right) p_{it}$$

"The presence of a diminishing MRS of future for current consumption is inconsistent with this " [Martingale property of asset price sequences $\{w_{i,t}\}$]. (Not much else, just talking about future research)

3.2 Mehra, Rajnish and Edward C. Prescott. 1985. The equity premium: a puzzle, Journal of Monetary Economics 15: 145-162. (Brian Weller)

Overview

Over the 90-year period, 1889-1978, the average real return on the S&P 500 was 7%, while the average yield on short-term debt (60-90 day commercial paper and, later, 90-day Treasury Bills) was <1%. The authors find that the resultant 6% spread cannot be explained in models without transactions costs or other market frictions. Hence, the celebrated **equity premium puzzle**.

"Intuitively, the reason why the low average real return and high average return on equity cannot simultaneously be rationalized in a perfect market framework is as follows: With real per capita consumption growing at nearly two percent per year on average, **the elasticities of substitution between the year t and year t + 1 consumption good that are sufficiently small to yield the six percent average equity premium also yield real rates of return far in excess of those observed**. In the case of a growing economy, agents with high risk aversion effectively discount the future to a greater extent than agents with low risk aversion (relative to a non-growing economy). Due to growth, future consumption will probably exceed present consumption and since the marginal utility of future consumption is less than that of present consumption, real interest rates will be higher on average." (146).

In sum, the **equity premium puzzle** and the **risk free rate puzzle** are intimately linked and cannot be simultaneously resolved in standard power utility models. Later work (e.g., Kocherlakota 1996) disentangles the puzzles by introducing alternative utility specifications.

The Model

- The growth rate of the endowment, rather than the endowment level, is assumed to follow a Markov process
- The agent's lifetime utility is

$$\begin{aligned} E_0\left[\sum_{t=0}^{\infty}\beta^t U\left(c_t\right)\right],\ 0 &< \beta<1 \\ U\left(c,\alpha\right) &= \frac{c^{1-\alpha}-1}{1-\alpha} \end{aligned}$$

- The asset pricing equations become

$$\begin{aligned} P_t &= E_t\left[\sum_{s=t+1}^{\infty}\frac{\beta^{s-t}U'\left(y_s\right)}{U'\left(y_t\right)}d_s\right] \\ &= E_t\left[\sum_{s=t+1}^{\infty}\frac{\beta^{s-t}y_t^{\alpha}}{y_s^{\alpha}}d_s\right] \end{aligned}$$

- The expected returns on the equity and risk free security are

$$R^e=\sum_{i=1}^{n}\pi_i R_i^e \text{ and } R^f=\sum_{i=1}^{n}\pi_i R_i^f$$

  where π is the vector of probabilities associated with the stationary distribution of the Markov chain.

- Mehra and Prescott specialize this model to a two-state Markov chain that is calibrated to match the

- Average growth rate of per capita consumption
- Standard deviation of the growth rate of per capita consumption
- First-order serial correlation of this growth rate
- all with respect to the model's stationary distribution

Results

Mehra and Prescott find the average equity premium to be 6.18 (standard error 1.76%). This result is robust to model misspecification in the average and standard deviation of the growth rate of consumption and in the persistence of the state variable. They also add additional states to their two-state model to increase higher moments without altering the first and second moments—think a mean-preserving spread—and find an increase in the maximal equity premium from 0.04 to 0.39, still a far cry from 6.18. The authors also allow for non-stationarities in the growth rate to no effect. They also briefly examine the effect of leverage (<10% change) and of introducing production (no change).

3.3 Kocherlakota, Narayana R. 1996. The equity premium: it's still a puzzle, Journal of Economic Literature, 34: 42-71. (Brian Weller)

Summary

Mehra and Prescott (1985) present the **equity premium puzzle**—equities do not covary sufficiently with consumption to justify the spread in returns. Weil (1989) expound on the related **risk free rate puzzle**—"although individuals like consumption to be very smooth and the risk free rate is very low, they still save enough that per capita consumption grows rapidly" (43). We've seen this feature of asset returns before as a motivation for our study of the Aiyagari (1994) model.

Kocherlakota examines the robustness of these puzzles and avenues of attack for resolving them. The puzzles rely on three basic assumptions

1. Individuals are expected utility maximizers
2. Asset markets are complete, i.e., contingent claims exist for all states of the world
3. Asset trading is costless

Although the risk free rate puzzle can be explained by relaxing one or more of these assumptions, the equity premium puzzle is more robust. The 1991 state of research suggested that the only way to explain the equity premium puzzle was to have very large trading cost spreads between stocks and bonds or to let consumers have (unrealistically) high coefficients of relative risk aversion. The equity premium puzzle's universality induces Kocherlakota to suggest these puzzles carry broad implications for macroeconomics

1. Why do people save when returns are low? Our model of aggregate savings is lacking.
2. Why are individuals so averse to procyclical (equity) risk? How can we evaluate the costliness of business cycles absent this knowledge?

3.3 Kocherlakota, Narayana R. 1996. The equity premium: it's still a puzzle, Journal of Economic Literature, 34: 42-71. (Brian Weller)

Restatement of Mehra and Prescott

The notation and assumptions of Mehra and Prescott (1985) are quite antiquated, so this brief update may prove useful. The pertinent asset pricing equations can be represented as

$$\begin{aligned} E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\left(R^e_{t+1} - R^b_{t+1}\right)\right] &= 0 \text{ (properly discounted excess returns are zero)} \\ \beta E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha} R^b_{t+1}\right] &= 1 \text{ (standard Euler Equation condition for the riskless asset)} \end{aligned}$$

With complete markets, the Mehra and Prescott representative agent assumption is easier to justify: "individuals become *marginally* homogeneous even though they are initially heterogeneous" (48). Kocherlakota presents two GMM-born tables that nicely summarize the two puzzles (depicted below). Note that high coefficients of relative risk aversion are required to address the equity premium puzzle, but that low coefficients of risk aversion are required to address the risk free rate puzzle.

Figure 4: The Equity Premium and Risk Free Rate Puzzles

(a) The Equity Premium Puzzle

TABLE 2
THE EQUITY PREMIUM PUZZLE

a	$\bar{e}$	t-stat
0.0	0.0594	3.345
0.5	0.0577	3.260
1.0	0.0560	3.173
1.5	0.0544	3.082
2.0	0.0528	2.987
2.5	0.0512	2.890
3.0	0.0496	2.790
3.5	0.0480	2.688
4.0	0.0464	2.584
4.5	0.0449	2.478
5.0	0.0433	2.370
5.5	0.0418	2.262
6.0	0.0403	2.153
6.5	0.0390	2.044
7.0	0.0372	1.934
7.5	0.0357	1.824
8.0	0.0341	1.715
8.5	0.0326	1.607
9.0	0.0310	1.501
9.5	0.0295	1.395
10.0	0.0279	1.291

In this table, $\bar{e}$ is the sample mean of $e_t = (C_{t+1}/C_t)^{-\alpha}(R^s_{t+1} - R^b_{t+1})$ and α is the coefficient of relative risk aversion. Standard errors are calculated using the implication of the theory that e_t is uncorrelated with e_{t-k} for all k; however, they are little changed by allowing e_t to be $MA(1)$ instead. This latter approach to calculating standard errors allows for the possibility of time aggregation (see Hansen, Heaton, and Yaron 1994).

(b) The Risk Free Rate Puzzle

TABLE 3
THE RISK FREE RATE PUZZLE

a	$\bar{e}$	t-stat
0.0	0.0033	0.0567
0.5	−0.0081	−1.296
1.0	−0.0162	−2.233
1.5	−0.0239	−2.790
2.0	−0.0313	−3.100
2.5	−0.0382	−3.263
3.0	−0.0448	−3.339
3.5	−0.0510	−3.360
4.0	−0.0569	−3.348
4.5	−0.0624	−3.312
5.0	−0.0675	−3.259
5.5	−0.0723	−3.195
6.0	−0.0768	−3.123
6.5	−0.0808	−3.043
7.0	−0.0846	−2.959
7.5	−0.0880	−2.871
8.0	−0.0910	−2.779
8.5	−0.0937	−2.685
9.0	−0.0960	−2.590
9.5	−0.0980	−2.492
10.0	−0.0997	−2.394

In this table, $\bar{e}$ is the sample mean of $e_t = \beta(c_{t+1}/c_t)^{-\alpha}(R^b_{t+1} - 1)$ and α is the coefficient of relative risk aversion. The discount factor β is set equal to 0.99. The standard errors are calculated using the implication of the theory that e_t is uncorrelated with e_{t-k} for all k; however, they are little changed by allowing e_t to be $MA(1)$ instead. This latter approach to calculating standard errors allows for the possibility of time aggregation (see Hansen, Heaton, and Yaron 1994).

These puzzles can be solved simultaneously if β is allowed to be larger than 1: for $\beta = 1.08$, $\alpha = 17.95$ satisfies both puzzles—$\beta > 1$ and such high risk aversion run contrary to all experimental evidence.

Relaxing Assumption 1 OR The Equity Premium Puzzle is Robust

Epstein-Zin Preferences Epstein-Zin preferences offer a recursive generalization of the standard power utility preferences

$$U_t = \left\{ c_t^{1-\rho} + \beta \left\{ E_t U_{t+1}^{1-\alpha} \right\}^{(1-\rho)/(1-\alpha)} \right\}^{1/(1-\rho)}$$

where α is the coefficient of relative risk aversion and ρ is the reciprocal of the intertemporal elasticity of substitution. Decoupling smoothing across states (α) from smoothing across time (ρ) gives the flexibility to solve the risk free rate puzzle by allowing α high and ρ low.

Habit Formation Habit formation is as it sounds. Internal habit formation has the effect of enabling a simultaneous satisfying of both puzzles with $\beta < 1$, but α is still high (estimated to be 15.384). External habit formation, "keeping up with the Joneses," enables a lower α to explain the equity premium puzzle, but it fails to resolve the risk free rate puzzle.

Relaxing Assumptions 2 and 3 OR

Incomplete Markets Constantinides and Duffie (1995) show that the *absence of labor income insurance combined with permanent labor income shocks* can generate a risk free rate much lower than that which would exist in complete markets. Unfortunately, even persistences of 0.8 are insufficient to generate much spread between incomplete and complete markets' risk free interest rates. So long as agents can accumulate a stock of assets and trade costlessly, they can self-insure and guard against idiosyncratic risk, thereby reducing the power of this explanation substantially.

Trading Costs To support an explanation of the equity premium, there must be a sizable difference in trading costs between stocks and bonds, a fact for which there is minimal empirical evidence. Market segmentation into stockholders and non-stockholders partly alleviates the equity premium puzzle by enabling a reduction in the necessary coefficient of relative risk aversion, but it doesn't go nearly far enough.

4 Equilibrium Search

4.1 Lucas, Robert E. Jr., and Edward C. Prescott. 1974. Equilibrium search and unemployment, Journal of Economic Theory 7(2): 188-209. (Philipp Tillman)

Why is it that workers choose (under some conditions) to be unemployed rather than to take employment at lower wage rates?

A particularly interesting class of models arises when the alternate activity is taken to be job search: The worker is faced with a wage offer which he views as a drawing from a probability distribution; his choices are to accept the offer or to take another drawing.

Structure of the Model

- We think of an economy in which production and sale of goods occur in a large number of spatially distinct markets.
- Product demand in each market shifts stochastically, driven by shocks which are independent over markets (so that aggregate demand is constant) but autocorrelated within a single market.
- Output to satisfy current period demand is produced in the current period, with labor as the only input.
- Each worker may either work at this wage rate, in which case he will remain in this market into the next period, or leave.

- If he leaves, he earns nothing this period but enters a "pool" of unemployed workers which are distributed in some way over markets for the next period.
- In this way, a new workforce distribution is determined, new demands are "drawn," and the process continues.
- Workers are assumed to be aware of the values of the variables affecting the market where they currently are (i.e., demand and workforce) and of the true probability distributions governing the future state of this market and the present and future states of all others. That is, expectations are taken to be rational.
- The economic interpretation of this assumption of rational expectations is that agents have operated for some time in a situation like the current one and have therefore built up experience about the probability distributions which affect them.
- For this to have meaning, these distributions must remain stable through time.
- Mathematically, this means that we will be concerned only with stationary distributions of demand and workforce and with behavior rules under these stationary distributions.

Equilibrium in a Single Market

In this section and the next, we study wage and employment determination in a single market, representing the impact of the rest of the economy on this market by certain given parameters. This impact takes three forms:

1. Product demand functions shift in an exogenously determined, stochastic manner
2. The outside economy offers alternative employment to workers
3. New workers arrive from the rest of the economy, augmenting the local work force

The labor supply curve will not be flat, as would be the case if workers held a fixed "reservation wage" above which they accept employment and below which they do not. The reason this does not occur lies in the fact that as demand varies, wage and price changes convey information about future wage prospects as well as current earnings. Thus, as demand shifts to the left and employment declines, future prospects are affected in two ways: first, lower demand this period increases the probability of a low demand next period as well; second, lower employment this period implies a lower workforce next period.

The Equilibrium Distribution of the Workforce

Lots of formulas. No intuition. Sorry, don't know how to summarize this without copying all formulas.

Economywide Equilibrium

The preceding propositions (not covered in this summary) describe the determination of the stationary distributions of employment, workforce, and wages in a representative market, with the expected return from search, λ, treated as a given parameter. From an economy-wide viewpoint, however, it is the size of the workforce which is fixed and the "price" λ which adjusts to clear the market.

It follows the derivation of a proposition that says that for all values of workforce-per-market μ, there is a unique positive equilibrium value of λ.

Conclusion

Although there are (by assumption) no aggregate dynamics in the model developed above, it should be obvious that the mechanism we have described is consistent with the now familiar account of the observed Phillips curve in terms of expectations. Thus, an unanticipated change in aggregate demand will move unemployment and wage changes in opposite directions. Of course, if aggregate demand changes were a recurrent event, as they are in reality, this fact would become incorporated rnto the maximum problem facing workers and would result in different equilibrium wages and employment.

Last comment which tells you why there wasn't much to summarize:

Our intention in this paper has been to indicate the general kind of framework within which such discussions can be conducted and to begin to develop suitable analytical methods.

which means that the model is very abstract and doesn't give us much intuition

5 Consumption and Saving

5.1 Aiyagari, S. Rao. 1994. Uninsured idiosyncratic risk and aggregate saving. Quarterly Journal of Economics 109: 659-684. (Avihai Rapaport)

The exposition is built around the standard growth model modified to include a role for uninsured idiosyncratic risk and borrowing constraints. This is done by having a large number of agents who receive idiosyncratic labor endowment shocks that are uninsured. As a result of this market incompleteness in combination with the possibility of being borrowing constrained in future periods, agents accumulate excess assets in order to smooth consumption in the face of uncertain individual labor incomes.

Motivation for borrowing constraints and idiosyncratic uninsured risk (labor endowment shock that cannot be hedged against or insured for)

Casual empiricism indicates that individual consumptions are much more variable than aggregate consumption. Further, individual consumptions are not very highly correlated either with each other or with aggregate consumption as would be the case with complete frictionless Arrow-Debreu markets. This suggests that heterogeneity due to incomplete markets may be important. Heterogeneity is clearly necessary for studying the importance of borrowing constraints. Also, individual wealth holdings appear to be highly volatile.

Notes on model

Borrowing limit is introduced to prevent a Ponzi scheme where an individual can borrow more than his total present value. An important point is that if r equals or exceeds the rate of time preference, then the individual will accumulate an infinitely large amount of assets. Intuitively, if r exceeds ρ, then the individual wants to postpone consumption to the future. The consumption profile will be upward sloping, and the agent will accumulate an infinitely large amount of assets to finance an infinitely large amount of consumption in the distant future. This conclusion carries over to the borderline case of r equal to ρ. In this case, the consumer attempts to maintain a smooth marginal utility of consumption profile. At the margin it is costless for the consumer to acquire an additional unit of the asset. However, since there is a positive probability of getting a sufficiently long string of bad draws of labor shocks, maintaining a smooth marginal utility of consumption profile is only possible if the consumer has an arbitrarily large amount of assets to buffer the shocks.

Simulation is used to come up with the following results

- One main point is that for high values of (1) relative risk aversion and (2) variation in endowments the difference between the savings rate with and without insurance (uncertainty vs certainty) is high (7-14 percentage points).
- Consumption varies about 50-70 percent as much as income while savings vary about three times as much as income and assets vary about twice as much.
- Consumption variability rises with persistence in labor endowment shocks and falls with risk aversion.
- There is much less dispersion across households in consumption compared with income and much greater dispersion in wealth compared with income. All of the cross-section distributions are positively skewed (median<mean). However, the degree of skewedness is somewhat less than in the data.

5.2 Deaton, Angus. 1991. Saving and liquidity constraints. Econometrica, 59: 1221-1248. (Daisuke Fujii)

Summary

The paper provides a theory of saving (when consumers are not permitted to borrow) which explains some of the stylized facts of saving behavior. As we assumed in class, consumers are impatient in the sense that their discout factor is larger than the depreciation and interest rate. In the model,

- If consumers are impatient and labor income shock is iid$\Longrightarrow$assets act like a buffer stock (protecting consumption against bad news)
- If the income shock process is autocorrelated (but stationary)$\Longrightarrow$ assets still act like a buffer stock but less effectively (greater cost of consumption is needed)
- If the income shock proceess is random walk (exterme case of autocorrelation)$\Longrightarrow$consumers simply consume the current income

For the liquidity constrained representative agent, if the agent recives aggregate labor income, the observed aggregate US saving behavior cannot be generated because there should be no saving (random walk case) or the saving is countercyclical (positive autocorrelation). However, microeconomic income processes do not resemble their average, and thus, it is possible to build a microeconomic model for saving behaviour which generates the observed stylized facts when it's aggregated.

Stylized Facts of Saving Behavior in the U.S.

- In macro level, changes in consumption are positively related to predictable changes in income.
- Microeconomic evidence is more mixed.
- Consumption tracks household income quite closely over the life-cycle.
- Contrary to the life-cycle saving model, most households in the U.S. hold very few assets (median is around $1000).
- Individual incomes are negatively autocorrelated whereas the aggregate income is positively autocorrelated.

Model

The restriction is not symmetric, so consumers can save as much as they want to save, but cannot borrow (or their is a bound for borrowing). By changing the income shock processes, the model can generates some interesting saving behaviors. The approach uses partial equilibrium, so the real interest rate is given.

$$u = E_t \left\{ \sum_{t=\tau}^{\infty} (1+\delta)^{t-\tau} v(c_\tau) \right\}$$

where δ is the rate of time preference and $v(c_t)$ is the instantaneous (increasing, concave) utility function. The evolution of assets is

$$A_{t+1} = (1+r)(A_t + y_t - c_t)$$

where y_t is the stochastic labor income with support $[y_0, y_1]$. The restriction on A is

$$A_t \geq 0$$

or

$$A_t \geq B$$

for some negative B. There are bunch of optimization equations in the paper, but I don't think we have to know all those.

5.3 Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante. 2008. Insurance and opportunities: a welfare analysis of labor market risk, Journal of Monetary Economics, 55(3): 501-525. (Daisuke Fujii)

Summary

The paper analyzes three types of welfare effects; (1) the welfare effect of a rise in wage dispersion, (2) the welfare gain from completing markets and (3) the welfare effect from eliminating risk.

Cross-sectional wage dispersion and individual wage volatility are large (the variance of the growht rate of individual wages in the U.S. is over 100 times larger than the variance of average

wage growth rate). Also, there has been a sharp increase in wage dispersion in the U.S. over the past 30 years.

When labor supply is flexible, increased wage inequality impacts not just consumption inequality, but also leisure inequality and the average values for consumption and leisure. More precisely, welfare effects are driven by two offsetting forces: an increase in idiosyncratic wage risk increases the need for insurance, but also presents opportunities to increase the level of aggregate productivity, measured as output per hour worked. There is an important difference between insuring risk and eliminating risk when labor supply is flexible. In fact, eliminating risk will always lead to smaller welfare gains than insuring risk, because removing risk also takes away opportunities to increase average labor productivity.

Model

The model is characterized by constant relative risk aversion (CARA) preferences, endogenous labor supply and partial insurance against wage shocks. Agents may use hours worked to mitigate fluctuations in earnings.

Results

1. Welfare effects of rising wage dispersion

   The welfare losses associated with the observed rise in wage dispersion are quite similar across the two alternative preference specifications, between 2.5% and 3% of lifetime consumption.

2. Welfare gains from completing markets

   A household in the partialinsurance economy values the availability of a complete set of insurance markets against the permanent component of wages at 39% of her lifetime consumption. The striking feature of these results is that the gains associated with better productive opportunities in complete markets are twice as large as the gains from reduced dispersion.

3. Welfare gains from eliminating risk

   Eliminating risk implies a welfare gain less than half that from completing markets. The fact that eliminating the insurable component of wage risk is welfare-reducing leaves open the theoretical possibility that the welfare effect from eliminating all idiosyncratic wage risk through some redistributive policy might be negative

Conclusion

From their analysis, eliminating idiosyncratic wage risk implies a welfare gain that is at least two orders of magnitude larger than most estimates of the welfare gains from eliminating business cycle risk. Thus, according to our model, the potential gain for a society from applying progressive taxes and wage compression is much larger than the potential gain from aggregate stabilization. However, the welfare gains from eliminating wage risk (through policies that compress after-tax wages) are only around half as large as the gains that would accrue from perfectly insuring wage risk. From a policy perspective, an important implication is that the government should develop the legal and institutional frameworks that will allow new insurance markets to develop.

6 Sticky Prices, Menu Cost Models

6.1 Blanchard, Olivier and Nobu Kiyotaki. 1987. Monopolistic competition and the effects of aggregate demand, American Economic Review 77(4): 647-666. (Avihai Rapaport)

The mathematical model is the essence of this paper and in order to learn it you are best off going through the solutions to PS8 Q.1. I present here the main points without going into technical detail.

Effect of Monopolistic Competition

Compared with the competitive case:

- Real money balances are lower in the monopolistic equilibrium.
- Price level is higher in the monopolistic equilibrium.
- Employment and output are lower in the monopolistic equilibrium.

Under monopolistic competition, output it too low. The intuition for this is that in the monopolistically competitive equilibrium, each price (wage) setter has, given other prices, no incentive to decrease its own price (wage) and increase its output (labor). Suppose however that all price setters decrease their prices simultaneously; this increases real money balances and aggregate demand (weighted sum of consumer demands). The increase in output reduces the initial distortion of underproduction and underemployment and increases social welfare.

Menu Costs (Small costs to changing prices)

Consider the effect of a small change in nominal money, dM. The intuitive argument is:

At the initial nominal prices and wages, the change in nominal money leads to a change in aggregate demand, thus to a change in the demand facing each firm. If demand is satisfied, the change in output implies in turn a change in the derived demand for labor, thus a change in the demand facing each worker. Unless firms operate under constant returns, each firm wants to change its relative price. Unless workers have constant marginal utility of leisure, each worker wants to change his relative wage. The loss in value to a firm which doest not adjust its relative price is, however, of second order. Thus second-order menu costs may prevent firms from adjusting relative prices. The implication is that nominal prices and nominal wages do not adjust to the change in nominal money.

Strategic Complementarities and Multiple Equilibria

Strategic complementarity corresponds to the case where an increase in the price level leads a firm, absent menu costs, to increase its own nominal price. Strategic substitutability corresponds to the case where an increase in the price level leads the firm instead to decrease its nominal price. If we have strategic complementarity then the equilibrium with menu costs may not be unique. Consider what a particular firm should do after an increase in nominal money. If other firms do adjust their prices, and if prices are strategic complements, the firm's optimal price increases because of both the increase in M and the increase in the general price level. If, instead, other firms do not adjust their prices, so that the price level doesn't change, the optimal price Pi increases but by less. The larger the proportion of firms which do not adjust their prices, the lower the opportunity cost to a firm of not adjusting its price. Thus, for some values of the menu costs, there will be two equilibria, one in which all firms adjust prices and one in which no firm adjusts prices.

Conclusions

Under monopolistic competition output is too low because of an aggregate demand externality. This externality, together with small menu costs, implies that movements in demand can affect output and welfare. In particular, increases in nominal money can increase both output and welfare.

6.2 Caplin, Andrew S., and Daniel F. Spulber. 1987. Menu costs and the neutrality of money. Quarterly Journal of Economics, 102: 703-725. (Matt Schiffman)

This paper tries to make the money supply matter using sticky prices. Time is indexed by t while consumers are indexed by $i \in [0,1]$. We have the following variables
$M(t)$ - log of the money supply at time t with $M(0) = 0$ and continously/montonically increasing.
$q_i(t)$ - nominal price charged by firm i
$p_i(t)$ - log of nominal price
$r_i(t)$ log of real price
$Q(t)$, $P(t)$ are indices of aggregate prices (details of which aren't specified)
$\Gamma_i(t)$ consumer demand for good i
$B_i(t)$ flow of profits to firm i
β - Cost to change prices for a firm
$X(t)$ - Aggregate Real Output

Firms have a (s, S) pricing policy. What this means is that whenever a firms real price $r_i(t)$ is reduced to s, the firm will pay the menu cost to increase their price to S. This requires an *Assumption*: $M(t) - P(t) = -P(0)$ (this gives stationary demand). We also need to specify the initial distribution of prices $p_i(0)$. This requires another *assumption*: Let $F_0(p)$ be the portion of firms with initial price at most p. Then

$$F(p) = \begin{cases} 0 & p \leq s \\ \frac{b}{S-s} & p \in (s, S) \\ 1 & p \geq S \end{cases}$$

With these assumptions we get neutraility of money. Formally - real aggregate output is invariant to money shocks so $X(t) = X(0)$ for all t.

Next the paper talks about the levels of inflation. Some more definitions: $\Pi^\tau(t) = P[\tau(t+1)] - P[\tau t] - M[\tau(t+1)] - M[\tau t]$ - which in other words is a function which takes discrete snapshots of the inflation at periods of time length τ.
$\Pi_i^\tau(t)$ - measure of change in firm prices at intervals of length τ.
$b^\tau(t) = \Pi^\tau(t)$ Modulo $S - s$ - residual inflation process
$V^\tau(t)$ - Relative price variability - a measure of the dispersion between Π_i and Π (so how much individual prices are spread about). $\Pi^\tau(t)$ is assumed to be a stationary process with continous density function $\phi^\tau(\Pi)$. This gives us that expected price volatility is related to $S - s$ and the

residual inflation process. This is not too surprising - its just saying the dispersion of prices is related to the minimum and maximum prices charged and the level of inflation. Finally, we get that in the limit $E[V]/E[\Pi] = S - s$. So the only thing that matters is the minimum and maximum values of prices when looking at inflation (not money supply!).

6.3 Bils, Mark and Peter J. Klenow. 2004. Some evidence on the importance of sticky prices. Journal of Political Economy 112: 947-985. (Elisa Olivieri)

The question: How sticky are prices, really?

The Method

"We examine the frequency of price changes for 350 categories of goods and services covering about 70 percent of consumer spending, on the basis of unpublished data from the Bureau of Labor Statistics for 1995– 97."

The Results

"In comparison with previous studies, we find much more frequent price changes, with half of prices lasting less than 4.3 months. Even excluding temporary price cuts (sales), we find that half of prices last 5.5 months or less."

On Price Changes

- goods with little value added in final production, that is, energy-related goods and fresh foods, display much more frequent price change
  - But excluding these goods, they still find much more frequent price changes than reported in prior work.
- prices seldom change for some goods: newspapers, men's haircuts, and taxi fares change less than 5 percent of months
- some prices change very frequently, with prices of gasoline, tomatoes, and airfares changing more than 70 percent of months.
- **Notably, durable goods actually show more frequent price changes than the overall consumer bundle.**

The Implication

Prices are less sticky, and more volatile, than a lot of calibrations used in macro models. The authors site a number of very recent papers that use price-setting timeframes that are much too long to resemble reality—eg. prices being set once a year.

On Inflation

The authors examine time-series data across 123 categories of goods to test whether goods' inflation rates behave as suggested by time- dependent pricing models. The most popular sticky-price models fail.

"We found that, for nearly all consumer goods, these models predict inflation rates that are much more persistent and much less volatile than we observe. The models particularly overpredict persistence and underpredict volatility for goods with less frequent price changes."

If you get really bored, check out the ridiculous tables at the end, where you can find detailed price info for important goods like "fresh whole chicken", "frankfurters", "other beef", and "Cemetery lots and crypts"

6.4 Golosov, Mikhail and Robert E. Lucas, Jr. 2007. Menu costs and Phillips curves. Journal of Political Economy 115: 171-199. (David Toniatti)

Abstract

This paper develops a model of a monetary economy in which the individaul firms are subject to indiosyncratic productivity shocks as well as general inflation. Sellers can change the price only by incurring a real "menu cost." We calibrate this cost and the variance and autocorrelation of the idiosyncratic shock using a new U.S. data set of indivudal prices due to Klenow and Kryvstov. The prediction of the calibrated model for the ffects of high inflation of hte frequency of price changes accords well with the international evidence from various studies. The model is also used to conduct numerical experiments on the economy's response to various shocks. In none of the simulations we conducted did the monetary shocks induce large or or persistent real responses.

Place in the Literature

This paper is chiefly motivated by "New Keynesian" models–namely Calvo (1983)–that use random firm repricing to show large, persistent changes in output in response to monetary shocks. This paper expands on an endogenous repricing model by Caplin and Spulber (1987), but it also adds a stochastic productivity component, and calibrates the model with BLS data.

The Model

Law of motion of money supply

$$d\log(m_t) = \mu dt + \sigma_m Z_m$$

where $Z_m \sim N(0,1)$. Absent menu costs, money supply has no effect on resource allocation.

Firm-specific productivity shocks ν_t

$$d\log(\nu_t) = -\eta\log(\nu_t)dt + \sigma_\nu dZ_\nu$$

where $\eta > 0$ and $Z_\nu \sim N(0,1)$ and independent of Z_m.

Consumers maximize over infinite-period consumption $\{c_t\}$, leisure $\{l_t\}$, and money-holding $\{\hat{m}_t\}$.

$$E\left[\int_0^\infty e^{-\rho t}\left[\frac{1}{1-\gamma}c_t^{1-\gamma}-\alpha l_t+\log\left(\frac{\hat{m}_t}{P_t}\right)\right]dt\right]$$

subject to a budget constraint. There are key assumptions in this utility function–(1) utility is separable, (2) disutility of labor is linear, (3) utility of money is logarithmic–without which the model becomes technically very complicated.

Firms maximize profits subject to consumer demand, nominal wage rate, and productivity shock. By choosing a different price than the previous period, the firm incurs cost kw_t. The idea is that it requires a certain number of labor hours to change the price. The firm's Bellman equation is

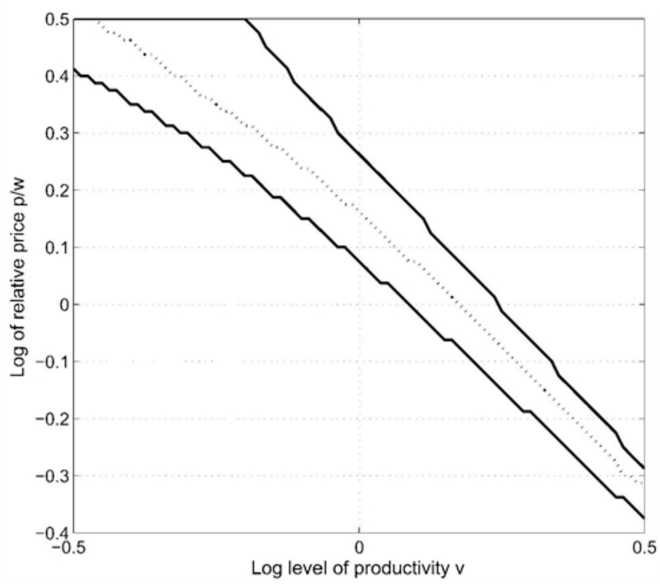
$$\varphi(p,\nu,w,\phi_t)=\max_T E_t\left[\int_t^{t+T} Q_sC_s(p)\left(p-\frac{w_s}{\nu_s}\right)ds+Q_T\max_q\left[\varphi(q,\nu_{t+T},w_{t+T},\phi_{t+T})-kw_{t+T}\right]\right]$$

where ϕ is the joint distribution of a firm's previous price and its productivity shock, Q represents the prices in a capital market in which money units are traded, $C(p)$ is consumer demand, and $\left(p-\frac{w_s}{\nu_s}\right)$ is the firm's profit margin. Notice that the firm's choose a stopping time T and the prices q simultaneously. In other words, a firm's "pricing strategy" determines how far to look into the future and what prices to set.

Main Results

CASE I: Set $\sigma_m = 0$ so inflation is a deterministic process.

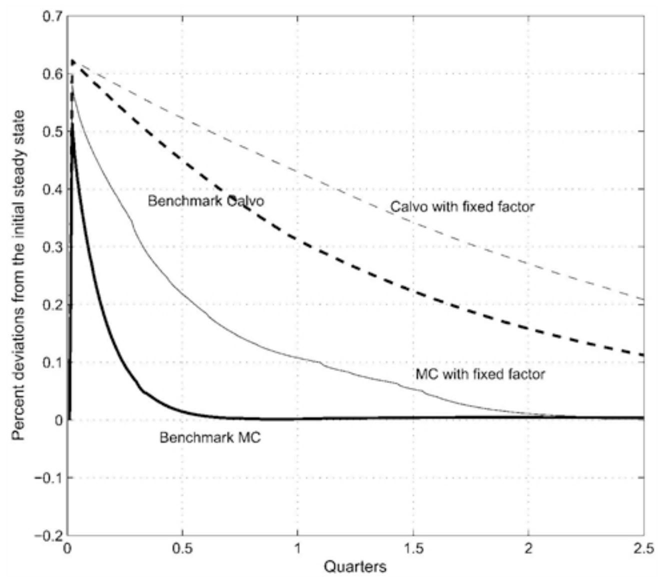
Figure 5: Relative Price v. Productivity



This figure shows optimal price levels and upper and lower bounds Notice that interval is wider for low productivity shocks (i.e. setting prices is more important when more goods are sold).

The data is calibrated usin 80,000 Bureau of Labor Studies price quotes from 1988-97. (In the first model, σ_m is fixed to 0.) Sales are removed from data. One key calibration: Labor required to adjust prices is 0.5% of overall employment.

Figure 6: Deivations from the Initial Steady State v. Time



This impulse response function graph shows the change in output over time from this paper versus Calvo. As summarized in the abstract, the change in output is smaller and less persistant.

CASE II: Allow for stochastic inflation

Using calibrated $\sigma_m = 0.0062$, standard deviation of output is 0.0006 and actual US data is estimated 0.015 for same period. The authors conclude that montary flunctuations account for lesss than 10 percent of observed fluctuations in output. Increasing nominal wages rates 1% leads to increase of 0.049% (SE: 0.008%) of real output.

7 Monetary Policy

7.1 Svensson, Lars E. O. 1985. Money and asset prices in a cash-in-advance economy, Journal of Political Economy 93: 919-944. (David Toniatti)

Abstract

An asset-pricing model with money intrdoduced via a cash-in-advance constraint is presented. The monetary velocity is variable; hence, money demand does not obey the trivial quantity equation. The effects of disturbances in output and money growth on real balances, the price level, and interest rates are examined. Monetary policy has effects on real asset prices. The Fisher relation and the premium on nominal bonds are discussed. The precise role of the timing of information and transactions for properties of price levels and interest rates are clarified.

Place in the Literature

First, previous work included exogenous stochastic processes for prices and interest rates. This paper uses Lucas' GE asset pricing model to include prices and interest rates as endogenous functions of money supply and output. Second, previous work introduced real balances directy into the utility function. This paper uses a cash-in-advance model. This paper also presents an important addition to the cash-in-advance literature by stipulating a reasonable condition: consumers decide on how much cash to hold in the next period before they know their current state (ie. shocks). Notably, agents in Lucas (1982) observe shocks before the asset market so this model only consider equilibria in which the liquidty constraint is binding. Svensson's shock comes after the asset market which allows for nonpositive nominal interest rates and variable velocity of money.

The Model

This model is very similar to the CIA model Professor Stokey presented in class (with the exception of the timing consideration mentioned above). Consumers maximize preferences

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau), \ \beta \in (0,1)$$

subject to the budget constraint and liquidity constraints:

$$\begin{aligned} c + \pi M' + qz' &\leq \pi M + (q+y)z + \pi(\omega - 1)\bar{M} \equiv w \\ c &\leq \pi M \end{aligned}$$

where

c	consumption
π	$1/P_t,$ real price of money
M, M'	consumer's supply of money in current/next period
q	real price of shares
y	stochastic output
z	holdings of shares
ω	stochastic growth rate of money
$\bar{M}$	money stock (average over entire economy)
w	wealth

In equilibrium, goods, money, and share markets clear.

$$\begin{aligned} c &= y \\ M &= \bar{M} \\ M &= \bar{M}' = \omega\bar{M} \\ z &= 1 \end{aligned}$$

The Bellman equation is not particularly informative so I omit it. Second, the paper is notationally dense, so I will summarize the main results of the model without their notation. The key constraint in this model is the liquidity constraint. A few observations on how the liquidity constraint is important:

When the Liquidity Constraint IS NOT Binding:

- Marginal Utility of Wealth = Marginal Utility of Consumption
- Velocity of Money = 1
- Inflation is independent of monetary expansion
- Nominal interest rate is independent of monetary expansion
- Real interest rate is independent of monetary expansion

When the Liquidity Constraint IS Binding:

- Marginal Utility of Wealth < Marginal Utility of Consumption
- Velocity of Money < 1
- Inflation is increasing in both monetary expansion and output
- Nominal interest rate is independent of monetary expansion[1]

[1] Under a monetary expansion, the value of money is lower in all states, this change does not affect the relative attractiveness between nominal bonds and money.

- Real interest rates are decreasing in monetary expansion and output

Note: The authors also present an asset pricing model and prove that the liquidity constraint affects the real assset of prices.

Final Note

The paper also discusses how this model relates to the Fisher relation. [2]

Generally, this result does not hold under uncertainty. Second, the paper asks, is there a premium on nominal bounds[3]In general, large variance in income creates a premium on nomimal bonds. This paper's contributions to these questions are subtle and beyond my understanding after multiple readings. If Stokey asks this on the exam, I will eat my hat and apolgize profusely.

7.2 Lucas, Robert E., Jr. 2000. Inflation and welfare. Econometrica 68: 247-274. (Dimitris Batzilis)

Introduction

Every agent in the economy wants to avoid holding non-interest-bearing cash and reserves, and spends a lot of time and money for that reason. However, someone has to hold it all, so the efforts cancel out.

The opportunity cost of holding money is the nominal rate of interest. So the higher the rate is, the more time people spend trying to economize on cash holdings. Big macro question: estimate the gains from reducing inflation and interest rates.

Money Demand and Consumer Surplus

There are three main graphs that summarize the evidence.

[2] The Fisher relation states that the real interest rate equals the nominal interest rate minus the expected rate of inflation.

[3] A premium on nominal bonds exists if the real rate of return of the bonds are higher than the real interest rate

Figure 7: U.S. Money, Income, and Interest Rates

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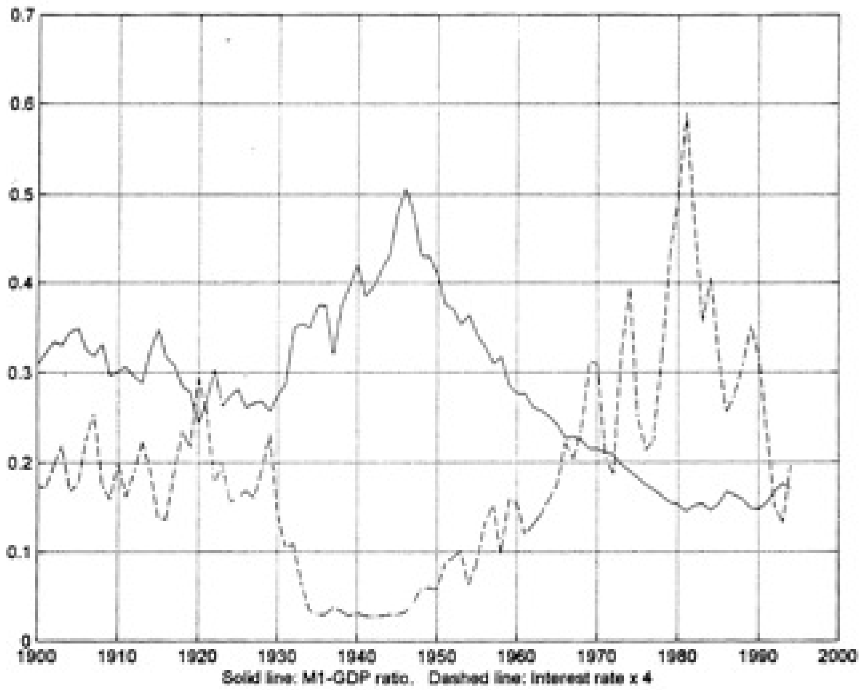

FIGURE 1.—U.S. money, income, and interest rates.

Conclusions: the money-income ratio is trendless over the century but has a downward trend the last 50 years. Improvements in transactions services would produce such a downward trend. An income elasticity of money demand that is bigger than 1 would work the other way. Lucas says that neither trend appears in the data, which might mean that they are both there and offset each other.

Then he presents the time series as points in the money demand function.

Figure 8: U.S. Money Demand (log-log)

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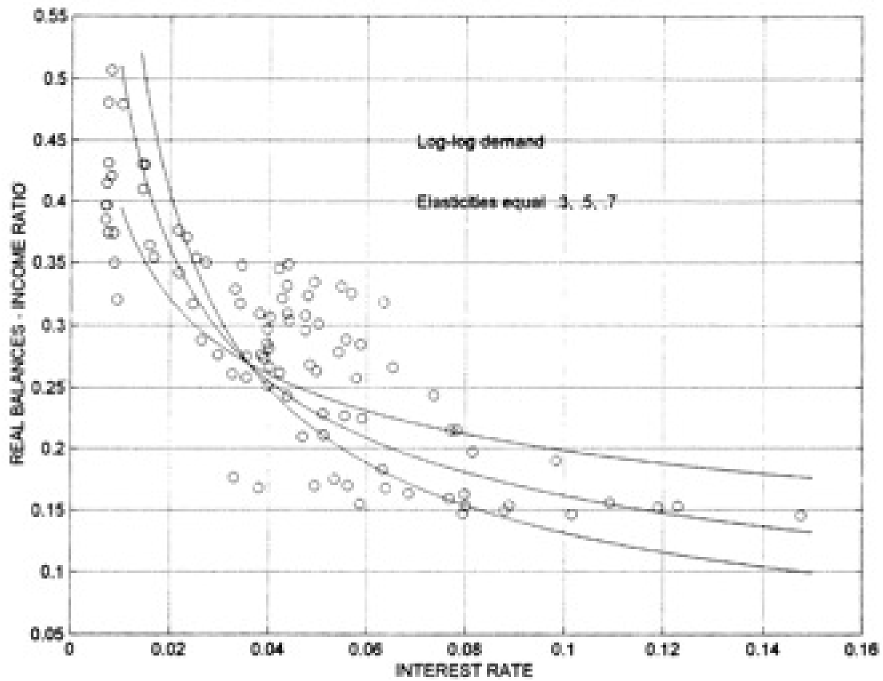

FIGURE 2.—U.S. money demand, 1900–1994.

Figure 9: U.S. Money Demand (semi-log)

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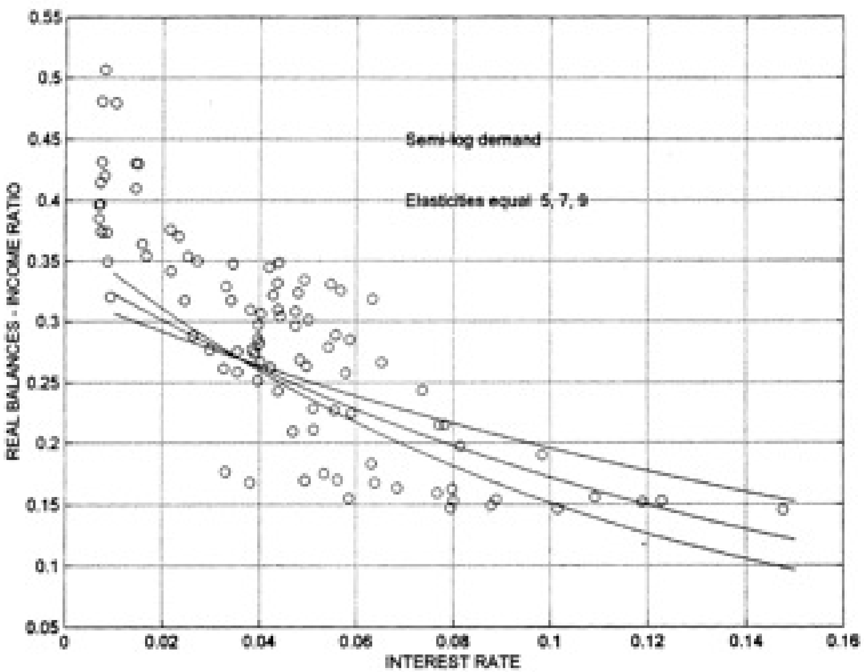


FIGURE 3.—U.S. money demand, 1900–1994.

Note that the first is a log-log demand while the second is a semi-log. The first one fits the data better. The demand curves are constructed for different elasticities, in a way that they pass through the geometric mean of the data. According to Lucas, the curve with elasticity equal to .5 in the first graph fits the data better.

In the fourth figure, Lucas uses the log-log demand curve with elasticity .5 to construct the predicted real balances, and compares them to the actual ones.

Figure 10: Actual and Predicted Real Balances

FIGURE 4.—Actual and predicted real balances, 1900–1994.

Lucas calculates the welfare cost by estimating the area under the inverse demand function (the consumer surplus).

The Sidrauski Framework

Here he uses the Sidrauski model to provide a general equilibrium rationale. Assumption: the monetary policy can be carried out with lump sum fiscal transfers

Fiscal Considerations

Again: the same model, for the same purpose, but he drops the above assumption. New assumption: Only flat rate income taxes can be used. Government is financed either with inflation taxes or income taxes

The McCallum-Goodfriend Framework

Uses the McCallum and Goodfriend model of transaction technology to provide another general equilibrium rationale.

Conclusion

Here he discusses some related literature. Interesting, but not important, so I will leave it out.

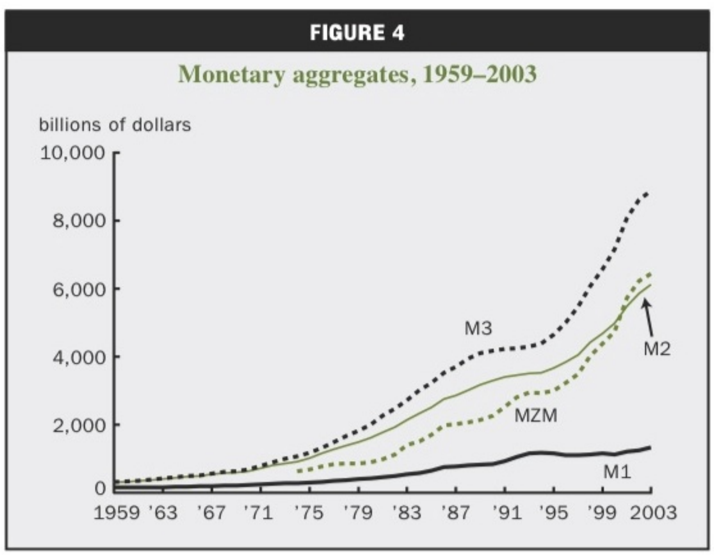
7.3 Teles, Pedro and Ruilin Zhou. 2005. A stable money demand: looking for the right monetary aggregate, Economic Perspectives, Federal Reserve Bank of Chicago, issue Q I, 50-63. (Matt Schiffman)

So this paper basically takes a bunch of other papers, reproduces graphs from their models, but looks at different measures of the money supply. They mention four types:

M1: Currency held by the public + Travelers checks + Demand deposits + Other checkable deposits, including NOWs (negotiable orders of withdrawal accounts), ATS (automatic transfer services), and share draft account balances.
M2: M1 + Savings deposits, including MMDAs (money market deposit accounts) + Small-denomination time deposits + Retail money market mutual funds
M3: M2 + Institutional MMMFs (money market mutual funds) + Large-denomination time deposits + Repurchase agreements + Eurodollars
MZM (Money zero maturity): M2 − Small-denomination time deposits + Institutional MMMFs

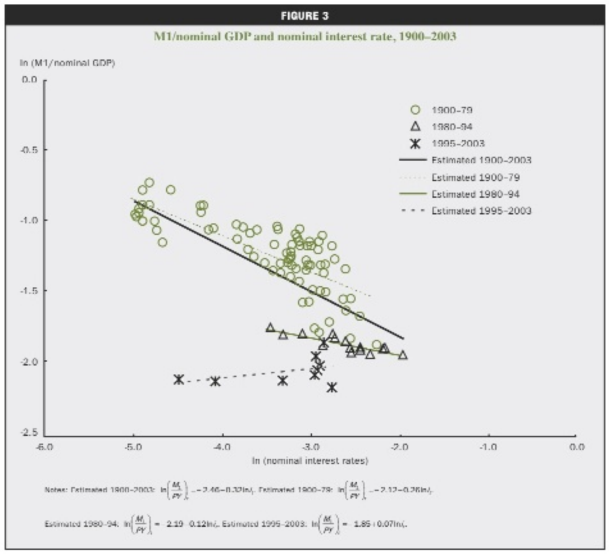
The level of these look like this:

Figure 11: Monetary Aggregates



The paper basically says that Lucas 2000 uses M1 as the measure of the money supply, but should have instead used MZM as the measure for the period following the banking deregulation of the 1980s. We can compare the following options in this graph (see writeup of Lucas's paper for comaprable one)

Figure 12: M1/Nominal GDP v. Nominal Interest Rate



Why is this better? If the interest elasticity is what Lucas 2000 predicted then we would expect M1 to have increased substantitally (which it did not). Why MZM? The article doesn't really say...

7.4 McCandless, George T., Jr. and Warren E. Weber. 1995. Some monetary facts, Federal Reserve Bank of Minneapolis Quarterly Review 19,3: 1-11. (Elisa Olivieri)

"This article describes three long-run monetary facts derived by examining data for 110 countries over a 30-year period (1960-1990), using three definitions of a country's money supply and two subsamples of countries."

The article is written by Fed economists, so the underlying motivation is to understand the strengths and limitations of monetary policy (ie. changes in money supply) as a means for achieving economic growth/stability. The goal is to help establish the links between money and other economics variables, mainly output, price levels, and inflation.

The three long-run facts are:

1. Growth rates of the money supply and the general price level are highly correlated for all three money definitions, for the full sample of countries, and for both subsamples. The correlation is near unity!

   (a) The three money definitions are M0, M1, M2 (for those of you who weren't in love with the fed when they were in high school: M0=cash/coins +bank reserves, M1=cash+coin+demand deposits+travelers checks, M2=all that stuff+ money easily converted into use for transactions.)

   (b) A particularly cool thing about coefficient of 1 is that it means that the usual equation:

$$M \times V = P \times Y, \text{ or in growth rates: } m + v = p + y$$

captures reality quite nicely, when we treat v and y as constants.

2. The growth rates of money and real output are not correlated, except for a subsample of countries in the OECD, where these growth rates are positively correlated.

    (a) in addition, looking at the OECD sample only, the authors find that "increases in money growth are associated with increases in real output growth about one-tenth as large."

    (b) So, should we just go and print lots of money then?? No."the positive correlations in the data may reflect not a causal relationship, but rather a similarity in central bank policy; the central banks of the OECD countries may all be following similar feedback rules from real output growth to money growth, increasing or decreasing money growth as real output growth increases or decreases."

3. The rate of inflation and the growth rate of real output are essentially uncorrelated. This holds for both subsamples.

    (a) **Authors' comment**: "the fact that the growth rates of money and real output are not correlated suggests that monetary policy has no long-run effects on real output. Of course, this does not rule out the possibility that it might have short- run effects."

The authors are cautious about taking the results too causally, but feel their results are extremely supportive of the stylized facts listed above. One obvious critique is that policy rules themselves influence the correlations we find in the data; but the hope is that by looking across more than 100 countries, all with their own policy rules, this won't be a problem.

7.5 Lucas, Robert E., Jr. 1996. Monetary neutrality. Journal of Political Economy 104:661-682. (Dimitris Batzilis)

Introduction

Lucas discusses Hume's view on monetary neutrality.

Evidence

Figure 13: Inflation Rate v. Money Growth

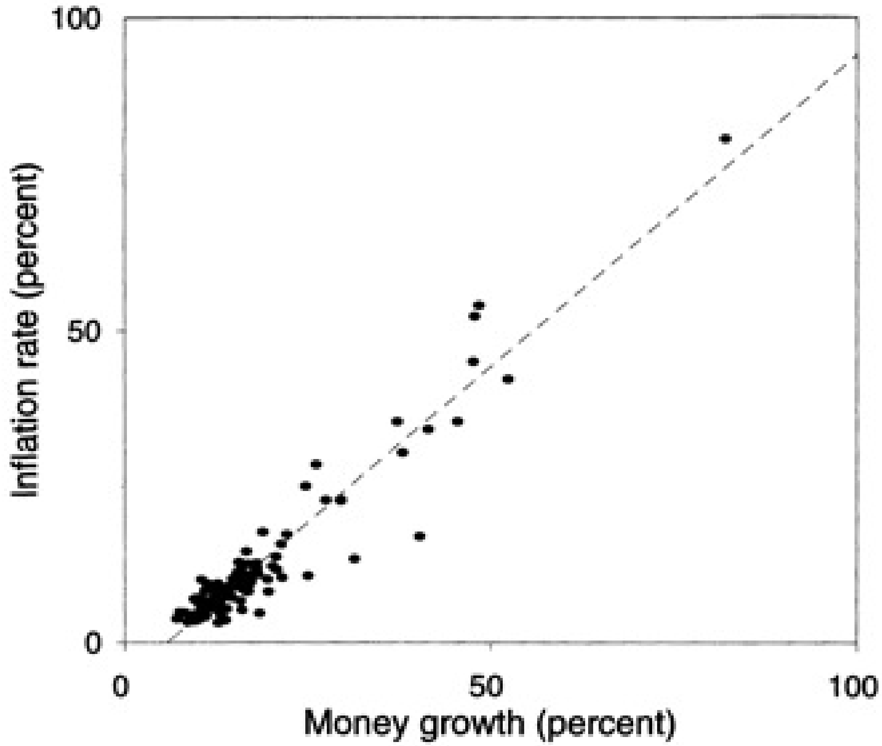


Fig. 1

Plots 30-year (1960-90) average annual inflation rates against average annual growth rates of M2 in 110 countries. The points are approximately on the 45 degree line which is in accordance with the monetary neutrality theory. Monetary neutrality holds in the long run.

The 2nd plot and the 3.f) plot support the theory too.

7.5 *Lucas, Robert E., Jr. 1996. Monetary neutrality. Journal of Political Economy 104:661-682. (Dimitris Batzilis)*

Figure 14: Real Output Growth v. Money Growth

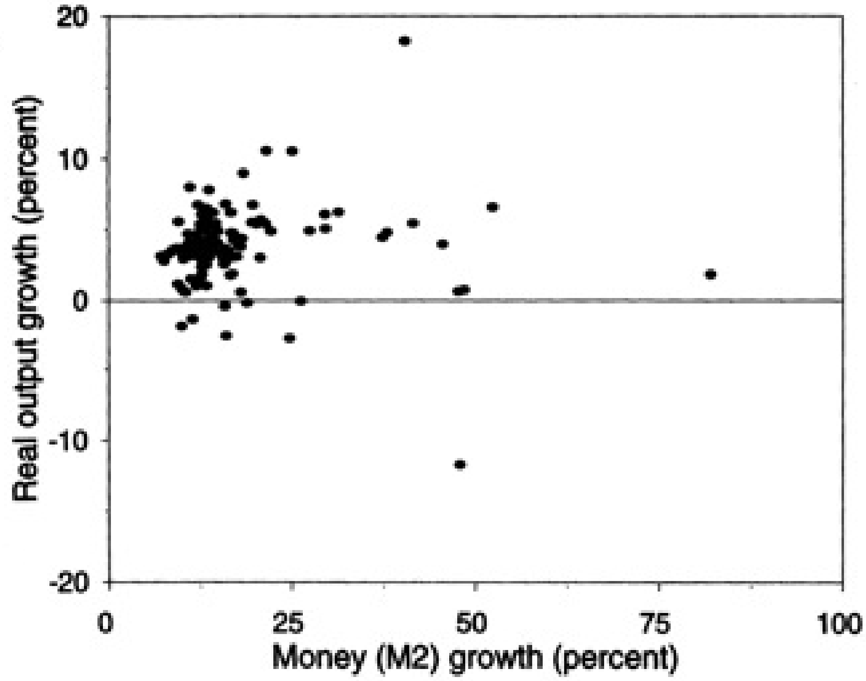


FIG. 2

Figure 15: Inflation Rate v. Unemployment Rate

Theoretical Responses

There is not much to say about this section. It's more about the history of economic thought on this issue.

General Equilibrium Macroeconomics

Lucas used Samuelson's general equilibrium model to reconcile monetary neutrality with the fact that a monetary expansion can provide short-term stimulus.

Conclusion

There is an important difference between anticipated and unanticipated changes in money growth. Anticipated expansions have inflation tax effects, but they do not stimulate employment and production. Unanticipated monetary expansions though can stimulate the economy.

7.6 Atkeson, Andrew and Lee E. Ohanian. 2001. Are Phillips curves useful for forecasting inflation? Quarterly Review, Federal Reserve Bank of Minneapolis, Winter, pp 2-11. (Philipp Tillman)

Abstract This study evaluates the conventional wisdom that modern Phillips curve-based models are useful tools for forecasting inflation. These models are based on the non-accelerating inflation rate of unemployment (the NAIRU). The study compares the accuracy, over the last 15 years, of

three sets of inflation forecasts from NAIRU models to the naive forecast that at any date inflation will be the same over the next year as it has been over the last year. The conventional wisdom is wrong; none of the NAIRU forecasts is more accurate than the naive forecast. The likelihood of accurately predicting a change in the inflation rate from these three forecasts is no better than the likelihood of accurately predicting a change based on a coin flip. The forecasts include those from a textbook NAIRU model, those from two models similar to Stock and Watson's, and those produced by the Federal Reserve Board.

- A Phillips curve is an equation that relates the unemployment rate, or some other measure of aggregate economic activity, to a measure of the inflation rate
- Modern specifications of Phillips curve equations relate the current rate of unemployment to future changes in the rate of inflation
- These specifications are based on the idea that there is a baseline rate of unemployment at which inflation tends to remain constant
- The idea is that when unemployment is below this baseline rate, inflation tends to rise over time, and when unemployment is above this rate, inflation tends to fall
- The baseline unemployment rate is known as the non-accelerating inflation rate of unemployment (the NAIRU), and modern specifications based on it are known as NAIRU Phillips curves.