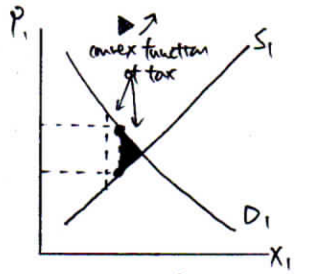
Theory of Income II

Model 6 : Fiscal Policy

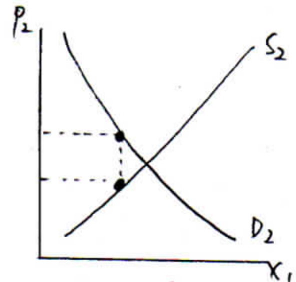
[2/22] Fiscal Policy → expenditure side / → financing.

1. timing question   Barro . Lucas-Stokey   … exogenous path of spending
2. tax on capital   … attractive in short run (inelastic)   keep the total distortion (DWL) to be minimal

   but discourage investment in long run.



Good 1 / Date 1



Good 2 / Date 2

discount on future distortion

"smooth" the distortions.

<u>Government budget constraint</u>. $\sum_{t=0}^{\infty} P_t \left[g_t + {}_0b_t - R_t \right] \le 0$

where $P_0 = 1$, $P_t = \prod_{s=1}^{t} \frac{1}{1+r_s}$ $t = 1, 2, 3, \ldots$

$\{ g_t , R_t \}_{t=0}^{\infty}$ are government spending and revenue.

$\{ {}_0b_t \}$ is debt (real) outstanding at date 0.

<u>Walras' Law</u>: If (the representative H.H. / all households) satisfy their budget constraint, <u>and</u> markets clear at every date, then the government BC also holds.

<u>Ricardian Equivalence</u>: HH's BC is $\sum_{t=0}^{\infty} P_t \left[C_t + R_t - {}_0b_t - Y_t \right] \le 0$.

R.E. says, if the taxes are lump-sum, only $\underline{\bar{R} = \sum_{t=0}^{\infty} P_t R_t}$ matters

Ramsey : fixed tax instrument (not lump-sum)   distortion by tax.

… benevolent government wants to maximize the utility

<u>Barro's Model</u>

ignore any feedback from tax rates.

$\{ G_t \}_{t=0}^{\infty}$: exogenous government spending. $\{ Y_t \}_{t=0}^{\infty}$: (exogenous) income.

b_0 : government debt (short-term) outstanding at $t=0$.

r : constant (exogenous) interest rate. (small open economy).

Government BC. (BC)

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t G_t + b_0 \le \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t R_t$$

— choose the sequence of revenue (given the mix of tax instruments)

where $\{R_t\}_{t=0}^{\infty}$ is revenue at t.

Excess burden (DWL) of collecting R_t is: — benevolent government.

$$L_t = Y_t f\left(\frac{R_t}{Y_t}\right)$$ where Y_t is income at t.

where $f(0)=0$, $f'>0$, $\underline{\underline{f''>0}}$ CRS. double revenue → double DWL.

↳ triangle grows square of tax rates.

<u>Accounting</u> $b_{t+1} = (1+r)(G_t + b_t - R_t)$ or $b_{t+1} - b_t = (1+r)(G_t - R_t) + rb_t$, $\forall t$.

debt growing/declining. deficit + interest on b_t.

Government's problem choose $\{R_t\}_{t=0}^{\infty}$ to

$$\min \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t Y_t f\left(\frac{R_t}{Y_t}\right) \quad \text{s.t. (BC)}.$$

→ solution: $f'\left(\frac{R_t}{Y_t}\right) = \lambda \quad \forall t.$

$\Rightarrow \frac{R_t}{Y_t} = \text{constant} \equiv \theta$, where θ is chosen to satisfy the BC.

<u>Example 1.</u> Suppose $G_t = \bar{G}$, $Y_t = \bar{Y}$ $\forall t$, and $b_0 \ge 0$ given.

$\Rightarrow R_t = \bar{R}$ s.t. $\bar{R} = \bar{G} + \frac{r}{1+r} b_0$. (current spending & debt).

The optimal plan repays the interest on the debt but rolls over the principle.

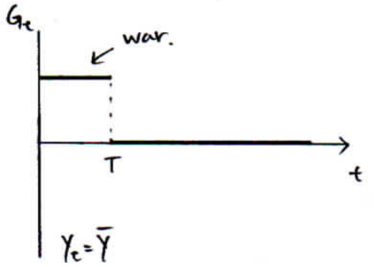
<u>Example 2.</u> $G_t = \bar{G}(1+\gamma)^t$, $Y_t = \bar{Y}(1+\gamma)^t$ where $0 \le \gamma < r$ and $b_0 \ge 0$ given

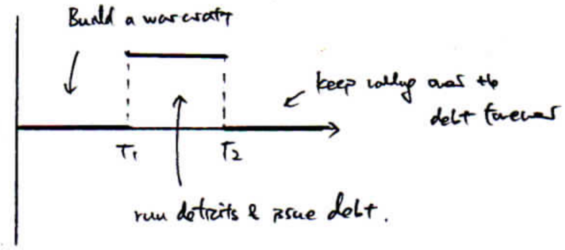
↑ r greater than the growth rate.

$\Rightarrow R_t = \bar{R}(1+\gamma)^t$ (keep the debt to GDP ratio constant).

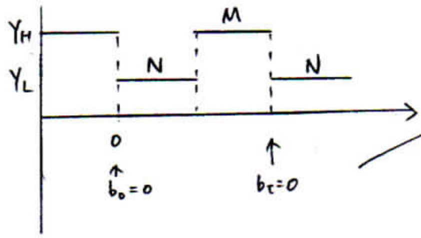
$\bar{R}$ is chosen s.t. $b_t = b_0(1+\gamma)^t$ $\forall t$.

and $\underline{\frac{b_t}{Y_t} \text{ is constant}}$





perfectly foreseen cycles.



$G_t = \bar{G}$ $\quad b_0 = 0.$

balance the budget over one cycle.

(not every period)

save in good time, borrow/dissave in bad time.

- add uncertainty ... smooth across shocks & time.

Problem
- exogenous r ... problematic for the US
- exogenous $\{Y_t\}$ ... tax may change income
- where does $f\left(\frac{B_t}{Y_t}\right)$ come from?.

Lucas - Stokey

Add a RH with $\boxed{\text{preference}}$ $\sum_{t=0}^{\infty} \beta^t u(c_t, \overbrace{1-n_t}^{l_t})$

and $\boxed{\text{technology}}$ $c_t + g_t = 1 - l_t \rightarrow c_t + g_t + l_t = 1.$

no capital, goods cannot be moved across time.

interest rates are endogenous.

Government takes $\{g_t\}_{t=0}^{\infty}$ as exogenous. Choose $\{\tau_t\}_{t=0}^{\infty}$

↳ benevolent

flat-rate tax on labor income.

... maximize the welfare of R.H.

time consistency problem & maturity structure of the debt.

... does the government want to reoptimize at $t+1$?.

[2/27]

① Solve the Ramsey problem of the government.
- analyze the HH's problem. given tax and prices
- formulate the implementability constraint
- analyze the government's problem ... solution: purchases $\{c_t, x_t\}_{t=0}^{\infty}$ & prices $\{p_t\}$

② Write BC involving debt $\{{}_0b_t\}_{t=1}^{\infty}$

③ Ask about time consistency.

Write the FOC's for the government that would have to hold at $t=1$.

These involve $\{{}_1b_t\}_{t=1}^{\infty}$ Ask if there exists a debt policy that satisfies the new FOC's and the BC.

<u>implementability</u>: use FOC to replace (tax adjusted) prices with quantity.

<u>Problem</u>: $$U = \sum_{t=0}^{\infty} \beta^t u(c_t, x_t)$$ (both goods are normal).

<u>Technology</u>: $$c_t + g_t + x_t = 1 \quad \forall t \quad (RC)$$

Given $\underbrace{\{g_t\}_{t=0}^{\infty} \ \& \ \{{}_0b_t\}_{t=0}^{\infty}}$ ... debt as of date 0 & when the debt is due.

↳ small enough so that the government can finance.

<u>Efficient solution</u>: $U_c(c_t, x_t) = U_x(c_t, x_t) \quad \forall t$ (lump-sum taxes)

<u>Household's problem</u>: given $\{P_s, \tau_s\}_{s=0}^{\infty}$, max U

$$\text{s.t.} \quad \sum_{s=0}^{\infty} P_s \left[c_s - {}_0b_s - (1-\tau_s)(1-x_s) \right] \le 0.$$

FOCs. $$\frac{U_x(c_t, x_t)}{U_c(c_t, x_t)} = 1 - \tau_t \quad \forall t. \qquad \beta^t \frac{U_c(c_t, x_t)}{U_c(c_0, x_0)} = \frac{P_t}{P_0} \quad \forall t.$$

<u>Implementability constraint</u>: use FOC's to write the HH budget constraint.

$$\sum_{s=0}^{\infty} \beta^s \left\{ U_c(c_s, x_s)\left[c_s - {}_0b_t\right] - U_x(c_s, x_s)\left[1 - x_s\right] \right\} \underset{\uparrow}{\ge} 0 \quad (IC)$$

for the sign of Lagrange multiplier.

<u>Lemma 1</u> Given $\{g_t, {}_0b_t\}$, an allocation $\{c_t, x_t\}$ is <u>implementable</u> (with some tax policy $\{\tau_t\}_{t=0}^{\infty}$) if and only if it satisfies (IC) and (RC) (characterization of what is feasible for the government)

<u>Ramsey government problem</u>

Choose $\{c_t, x_t\}_{t=0}^{\infty}$ to maximize U s.t. (RC) and (IC)

↳ back out the associated $\{\tau_t, P_t\}$ from HH's FOCs.

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t, x_t) + \lambda_0 \sum_{t=0}^{\infty} \beta^t \left[(c_t - ob_t) U_c(c_t, x_t) - (1-x_t) U_x(c_t, x_t) \right]$$
$$- \sum_{t=0}^{\infty} \underbrace{\beta^t \mu^t}_{\hat{\mu}_t} \left[c_t + g_t + x_t - 1 \right]$$

FOCs.
$$(1+\lambda_0) U_c + \lambda_0 \left[(c_t - ob_t) U_{cc} - (1-x_t) U_{cx} \right] - \mu_t = 0, \quad \forall t.$$
$$(1+\lambda_0) U_x + \lambda_0 \left[(c_t - ob_t) U_{cx} - (1-x_t) U_{xx} \right] - \mu_t = 0, \quad \forall t.$$

or
$$\underbrace{(U_c - U_x)}_{=0 \text{ when undistorted}} + \theta \underbrace{\left[(c_t - ob_t)(U_{cc} - U_{cx}) + (1-x_t)(U_{xx} - U_{cx}) \right]}_{\text{distortion from the tax}} = 0, \quad \forall t.$$

where $\theta = \frac{\lambda_0}{1+\lambda_0}$

necessary condition for the optimal policy (may not exist. e.g. g_t too high).

If $\theta_0 = 0$, then the efficient solution satisfies the FOC & RC $\forall t$.

Suppose $g_t + ob_t = 0 \ \forall t$ ... asset is sufficient to cover the spending.

↳ same $g_t, g_{t'}, ob_t, ob_{t'}$ for t & t' → symmetric solution for consumption & leisure.

Compared with economy with lump sum taxes,

the Ramsey solution distorts $c_t \downarrow$ & $x_t \uparrow$?

... efficient solution: if $g_t > 0$, $c_t \downarrow$ & $x_t \downarrow$ (both goods are normal).

If $g_t = \bar{g} \ \forall t$ & $ob_t = \bar{b} \ \forall t$, then $\tau_t = \tilde{\tau} \ \forall t$.

[3/1]. <u>Recap.</u> Given $\{g_t, ob_t\}$, the Ramsey solution $\{c_t, x_t\}$ & θ_0 satisfy.

(1) $c_t + g_t + x_t = 1 \quad \forall t$

(2) $\sum_{t=0}^{\infty} \beta^t \left[(c_t - ob_t) U_c(c_t, x_t) - (1-x_t) U_x(c_t, x_t) \right] = 0$ ... ≥ 0 from government's perspective.

(3). $U_c - U_x + \theta_0 \left[(c_t - ob_t)(U_{cc} - U_{cx}) + (1-x_t)(U_{xx} - U_{cx}) \right] = 0 \quad \forall t.$

The associated prices and tax rates are:

$$p_t = \beta^t \frac{U_c(c_t, x_t)}{U_c(c_0, x_0)} \qquad 1 - \tau_t = \frac{U_x(c_t, x_t)}{U_c(c_t, x_t)} \qquad \forall t.$$

see if the solution is time consistent

... suppose we have sequential markets.

Sequential markets — Suppose the government BC holds in period t.

$$\sum_{s=t}^{\infty} p_s \left[g_s + {}_t b_s - \tau_s (1 - x_s) \right] = 0$$

spending — debt outstanding at time t — revenue

present value $= 0$.

period / flow BC. ⊛

Then, the government chooses $\{{}_{t+1}b_s\}$ satisfying.

$$p_t \left[g_t + {}_t b_t - \tau_t (1 - x_t) \right] = \sum_{s=t+1}^{\infty} p_s \left[{}_{t+1}b_s - {}_t b_s \right]$$

present value of additional debts (some positive, some negative) covers the current deficit.

Subtract to get: $\displaystyle\sum_{s=t+1}^{\infty} p_s \left[g_s + {}_{t+1}b_s - \tau_s (1 - x_s) \right] = 0$ … BC holds at $t+1$.

Time consistency

We want to find $\{{}_1 b_s\}_{s=1}^{\infty}$ and θ_1 s.t. $\displaystyle\sum_{s=1}^{\infty} \beta^s \left[(c_s - {}_1 b_s) U_c - (1 - x_s) U_x \right] = 0$

and $(U_c - U_x) + \theta_1 \left[(c_s - {}_1 b_s)(U_{cc} - U_{cx}) + (1 - x_s)(U_{xx} - U_{cx}) \right] = 0 \quad \forall s \geq 1$

where $\{{}_1 b_s\}$ satisfies ⊛ (θ_1 not the same as θ_0).

$$(3) \Rightarrow \theta_0 \left[(c_s - {}_0 b_s) + (1 - x_s) \frac{U_{xx} - U_{cx}}{U_{cc} - U_{cx}} \right] = \theta_1 \left[(c_s - {}_1 b_s) + (1 - x_s) \frac{U_{xx} - U_{cx}}{U_{cc} - U_{cx}} \right] \quad \forall s$$

↳ so $\{{}_1 b_s\}$ must satisfy $\underline{\underline{\theta_1 [\hat{a}_s - {}_1 b_s] = \theta_0 [\hat{a}_s - {}_0 b_s]}}$ $\forall s$.

where $\underline{\underline{\hat{a}_s = c_s + (1 - x_s) \dfrac{U_{xx} - U_{cx}}{U_{cc} - U_{cx}}}}$ $\forall s$.

If $\theta_0 = 0$, then $\theta_1 = 0$ and the new debt sequence is indeterminate. … impose no distortion at time 0 ($\theta_0 = 0$) → $\theta_1 = 0$ Ricardian equivalence.

Otherwise, $\quad {}_1 b_s = {}_0 b_s + \left(1 - \dfrac{\theta_0}{\theta_1} \right) (\hat{a}_s - {}_0 b_s) \quad \forall s.$

… Solve for θ_1 from the PV BC.

${}_0 b_s = 0$ (no debt outstanding at $t=0$) $\Rightarrow$ ${}_1 b_s = \left(1 - \dfrac{\theta_0}{\theta_1} \right) \hat{a}_s$.

positive or negative depending on whether the government BC gets looser or tighter.

${}_2 b_s = {}_1 b_s + \left(1 - \dfrac{\theta_0}{\theta_1} \right) (\hat{a}_s - {}_1 b_s)$. … still scaled version of $\hat{a}_s$

maturity structure & interest rate. the government can affect interest rate through its policy.

If $\{g_t\}$ is stochastic, the same principles apply.

Note: complete market → contingent claims for all dates & states.

smoothing over time & smoothing over the states of the world.