Problem Set 1 Q6

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1 Question 2

Consider a simple monopoly setting in which a firm sells to a consumer whose unit valuation is distributed uniformly on $[0, 1]$. The firm's unit costs are zero.

1.1 (a).

What is the optimal price for the monopolist? What is the expected consumer surplus?

Solution: The optimal price maximizes the profits of the monopolist seller in expectation. Since consumer's valuation θ is distributed uniformly on $[0, 1]$, $\mathbb{E}[\theta] = \frac{1}{2}$. Suppose the optimal price $p^\star < \mathbb{E}[\theta] = \frac{1}{2}$, then there always exists $\varepsilon \in \left(0, \frac{1}{2} - p^\star\right)$ such that $p^\star + \varepsilon$ generates higher profits for the firm. Suppose $p^\star > \mathbb{E}[\theta] = \frac{1}{2}$, then the consumer won't buy the unit and it cannot be optimal for the firm. Therefore, the optimal price is $p^\star = \mathbb{E}[\theta] = \frac{1}{2}$, and the expected consumer surplus is zero.

For the remainder of this problem, suppose that before the monopolist sets the price, the consumer can credibly reveal any closed subset, $M \subseteq [0, 1]$ which contains their valuation. E.g., a consumer of type $\theta = 0.6$ can reveal the simple message $\{0.6\}$ to the monopolist, or she could reveal a coarser signal such as "my value is in$[0.5, 0.7] \cup [0.8, 0.9]$", but she cannot lie and say her type is in a set when it is not. A consumer can also send the message $M = [0, 1]$ which conveys no information. Revealing credible information does not have any direct cost to the consumer.

1.2 (b).

Show that the original monopoly outcome is an equilibrium outcome in this certified disclosure game. Indicate the equilibrium messages and monopolist's beliefs (including out-of-equilibrium beliefs) which sustain the equilibrium.

Solution: The original monopoly outcome is the babbling equilibrium.

- The equilibrium message is $M = [0, 1]$, which conveys no meaningful information

- Monopolist's beliefs are: for $M = [0,1]$, $\mu(\theta \mid M) = 1$ for $\theta \in [0,1]$. For $M \neq [0,1]$

$$\mu(\theta \mid M) = \begin{cases} 0 & \theta \neq 1 \\ 1 & \theta = 1 \end{cases}$$

It is an equilibrium because the firm knows that the consumer is babbling, so firm pays no attention to the consumer. The consumer knows that the firm pays no attention, so the consumer is babbling. Given the belief of the other side, each side has no incentive to deviate.

1.3 (c).

Show that full revelation with simple messages, followed by prices set to equal valuations, is also an equilibrium and in this equilibrium the monopolist extracts all of the consumer's surplus. Again, indicate the monopolist's beliefs that you are using to sustain this equilibrium.

Solution: Full revelation with simple messages is an equilibrium outcome with the following beliefs and messages:

- The equilibrium message is $M = \{\theta\}$, which conveys the exact type of the consumer
- Monopolist's beliefs are: for singleton message $M = \left\{\tilde{\theta}\right\}$

$$\mu(\theta \mid M) = \begin{cases} 1 & \text{for } \theta = \tilde{\theta} \\ 0 & \text{for } \theta \neq \tilde{\theta} \end{cases}$$

For non-singleton message M,

$$\mu(\theta \mid M) = \begin{cases} 0 & \theta \neq 1 \\ 1 & \theta = 1 \end{cases}$$

It is an equilibrium because the firm knows that the consumer is conveying exact information, so the monopolistic firm sets price equal to valuation to maximize profit and firm has no incentive to deviate. Consumer knows that the firm completely believes what he conveys, and knows the system of beliefs of firm. Given that the consumer cannot lie, the only possible deviation is sending a message that is not a singleton, which will result in the price $p = 1$ and lead to zero surplus of the consumer, so consumer has no incentive to deviate.

In this equilibrium, consumer always accurately disclose his type and firm sets price equal to the valuation, which results in zero surplus of the consumer.

1.4 (d).

Construct an equilibrium in which the consumer does better (from an ex ante perspective) than the standard monopoly outcome in (a). [Hint: look for a simple 2-step partition, $M_1 = [0, a)$ and $M_2 = [a, 1]$.]

Solution: In this two step equilibrium,

- Monopolist's beliefs are: for message M_1

$$\mu(\theta \mid M_1) = \begin{cases} \frac{1}{a} & \text{for } \theta \in M_1 \\ 0 & \text{for } \theta \notin M_1 \end{cases}$$

for message M_2

$$\mu(\theta \mid M_2) = \begin{cases} \frac{1}{1-a} & \text{for } \theta \in M_2 \\ 0 & \text{for } \theta \notin M_2 \end{cases}$$

for message $M \neq M_1$ and $M \neq M_2$,

$$\mu(\theta \mid M) = \begin{cases} 0 & \theta \neq 1 \\ 1 & \theta = 1 \end{cases}$$

- The equilibrium message is $M \in M_1$ if $\theta \in M_1$, and $M \in M_2$ if $\theta \in M_2$
- The equilibrium price is $p = \frac{a}{2}$ if $M \in M_1$, and $p = \frac{1+a}{2}$ if $M \in M_2$
- Given firm's beliefs and actions, the consumer would like to do this:
  - If the consumer send an message $M \neq M_1$ and $M \neq M_2$, the payoff is definitely 0, so the consumer would not have incentive to do that
  - The consumer cannot lie, so he has to send message according to the equilibrium condition.
- Given the consumer's message sending strategy, the firm's optimal pricing is by setting the pricing equal to the expected valuation, which is the equilibrium pricing strategy, so the firm has no incentive to deviate.

To make the consumer do better than the standard monopoly outcome in (a), we need his expected surplus ex ante strictly postive. So,

$$\int_0^a \left(\theta - \frac{a}{2}\right) d\theta + \int_a^1 \left(\theta - \frac{1+a}{2}\right) d\theta = 0$$