Price Theory I
Problem Set 1, Question 2

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Utility is defined over interior house temperature H and consumption good X. Households purchase the consumption good directly in a national market but 'produce' house temperature using insulation and energy. The key to this question is to identify another relevant decision: there is mobility, so households decide whether to live in the North or the South; that is, they solve the problem:

$$\max\{V_N, V_S\}$$

where V_i is the value function achieved by living in region $i \in \{N, S\}$. Conditional on deciding where to live, a typical household of region i solves the following problem:

$$\begin{aligned} V_i = \max_{X_i, I_i, K_i} \; & u(X_i, H_i) \qquad (1) \\ \text{s.t. } & X_i + C(I_i)K_i + P_i I_i = W_i \\ & H_i = E_i + K_i \end{aligned}$$

First, note that I normalized the price of the consumption good (which is equal in the North and the South since X is sold in a National market) to be 1, which means that the other prices are expressed in units of X.[1] Second, note that 'energy cost' denotes 'energy expenditure', $C(I_i)K_i$, so the 'price' of interior house temperature is $C(I_i)$. P_i denotes the price of insulation, W_i denotes

*These answers largely benefited from discussion with Kevin Murphy. All errors are my own.

[1]Normalizing is quite common, done for simplicity and without loss of generality. Recall that Marshallian demand is homogeneous of degree zero on income and prices; that is, the consumer's problem is unaltered if we scale all prices and income proportionally by some positive constant λ. We can set $\lambda = 1/p_j$ to normalize by the price of good j.

wage income (WLOG, households supply inelastically one unit of labor) and E_i denotes exterior house temperatures (exogenous). Additionally, assume $E_N < E_S$ WLOG. Finally, we ignore the nonnegativity constraints (in particular, $K_i \geq 0$) to avoid unnecessary complications.

a.

We're told that insulation is fixed in the North and South (WLOG, assume it's equal to 1), $I_N = I_S = 1$. Immediately we know the price of interior house temperature is the same in both regions ($C(1)$). We could make two assumptions about how insulation is being fixed. First, we could assume that there is a market for insulation in both regions and the Government controls the supply or imposes a quota at $I = 1$. In this case, prices would have to adjust to clear the market, so we would expect $P_N > P_S$. This case, however, is more complicated and does not add much to the intuition in part a) and b), so let's assume for these parts that 'amount of insulation in each house is fixed' means that households are not able to choose the level of insulation.

Now, here is where mobility of consumers comes into play. Mobility requires the marginal household to be indifferent between living in the North and in the South; otherwise, there will be migration toward the most attractive option until this condition is satisfied. Since we assumed that households have the same preferences,[2] this means that utility *level* has to be the same in both regions.

If the two regions have the same utility levels and face the same prices, they will consume the same X and H. To see why, rewrite problem (1) as:

$$\max_{X_i, H_i} u(X_i, H_i) \tag{2}$$
$$\text{s.t. } X_i + C(1)H_i = \underbrace{W_i - P + C(1)E_i}_{\equiv \widetilde{W}_i}$$

which allows us to interpret the problem as a simple two-good consumer problem. We know from class that the Hicksian demands (solution to the dual problem) will be: $X_i(1, C(1), \overline{U}_i)$ and $H_i(1, C(1), \overline{U}_i)$ for $i \in \{S, N\}$. Now, if both regions have the same prices, $1, C(1)$, and the same utilities, $\overline{U}_S = \overline{U}_N$, then their demands will be the same.[3]

[2] There is really no gain in complicating the model assuming heterogeneity in utility, since the interesting variation comes from regional temperature differences.

[3] Of course, this depends on assuming regularity conditions to guarantee uniqueness of the

If H is the same in both regions, then K has to be higher in the region with the lowest external temperature (e.g., the North). From the budget constraint it is then straightforward to see that wages must be higher in the region with the cooler temperatures (since heating costs are higher there), such that incomes net of heating costs are the same in each location:

$$W_S - C(1)K_S = W_N - C(1)K_N \tag{3}$$

b.

Insulation now increases to $I > 1$, so the price of interior house temperature is lower ($C'(I) < 0$). Note that utility must still be the same between both regions (although it need not be at the same level as in the previous part), so the consumption good and interior house temperatures are still the same between both regions. The wage difference between both regions—using Equation (3), equality $H = E + K$, and the fact that $H_S = H_N$—can be written as:

$$W_N - W_S = C(I)(E_S - E_N)$$

Note that exterior temperatures remain constant (with $E_S > E_N$), so the heating cost difference, $C(I)(E_S - E_N)$ decreases. As a result, the wage difference $W_N - W_S$ will decrease as well, which requires people to migrate from warmer regions to colder regions, raising wages in the warmer regions and reducing them in the colder regions (recall wages depend negatively on the number of people).

To see what happens with house temperatures, observe how the budget constraint changes:

$$X_i + C(I)K_i = W_i$$

The price of interior house temperatures is falling and income is rising for a house in the South; hence, interior house temperatures increase (in both the South and the North).

c.

Consumers will now use more insulation in the colder regions (where they do more heating and hence save more on energy by insulating). The higher insula-

solutions to the consumer's problem (which will be covered in detail in Price Theory II). For now, there is really no gain in assuming otherwise.

tion will lower the marginal cost of interior temperature and cause them to have higher interior temperature than in the warmer regions. Since interior temperatures are higher in the colder regions, the consumption of X will be lower there (utilities still have to be equal in both regions). Wages will be higher in the colder regions to compensate for the higher heating and insulation costs there.

In the TA session I mentioned an intuitive explanation for why this is the case, but here is a more formal argument. Consider the cost minimization problem, which is the dual of Problem (1):

$$\begin{aligned} \min_{X_i, H_i, I_I} \; & X_i + C(I_i)(H_i - E_i) + PI_i \\ \text{s.t. } & U(X_i, H_i) = U_0 \end{aligned} \tag{4}$$

Note we set U_0 at the same level for both, the North and the South, due to the mobility argument. Add and subtract E_S to rewrite the cost function in the North as:

$$\underbrace{X_N + C(I_N)(H_N - E_S) + PI_N}_{\text{Same function as in the South}} + \underbrace{C(I_N)(E_S - E_N)}_{>0}$$

and note that, for the same (X, I, H), the cost function will always be higher in the North than in the South (which is mechanical, since in the North they require higher energy expenditures to get the same bundle). This immediately tells us that wages have to be higher in the North (recall, wages equal minimized cost function, which is higher in the North).

To see what happens to insulation, rewrite Problem (4) as:

$$\begin{aligned} \min_{X_i, H_i} \; & \left\{ \min_{I_i} \; X_i + C(I_i)(H_i - E_i) + PI_i \right\} \\ \text{s.t. } & U(X_i, H_i) = U_0 \end{aligned}$$

This way we break the minimization problem in parts, solving an inner minimization problem over I_i first (which gives as a solution a policy function $I(X, H)$).[4] Take first order conditions to get:

$$-C'(I_i)(H_i - E_i) = P \tag{5}$$

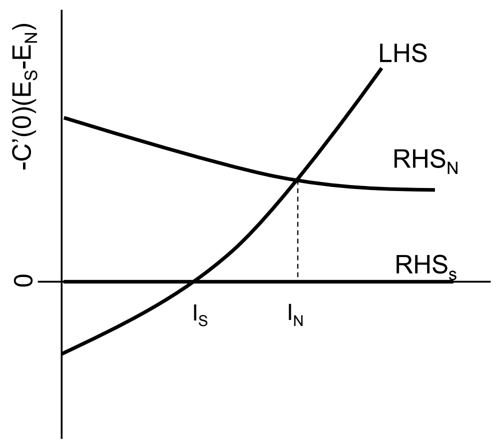
[4]This 'trick' will be useful in other classes, like Theory of Income.

add and subtract E_S as needed in the equation in the North to get:

$$\underbrace{C'(I_N)(H-E_S)+P}_{\text{LHS}} = \underbrace{-C'(I_N)(E_S-E_N)}_{>0}$$

Note that the LHS is the same function in the North and in the South, but the RHS is higher (positive) in the North than in the South (zero). It is possible to show that the LHS is increasing in I,[5] which means that $I^N(X,H) > I^S(X,H)$ as in Figure 1. Again, this is not surprising. The marginal benefit of an extra unit of insulation is higher in the North than in the South (that's the RHS term) and the marginal cost (P) is the same in both regions; hence, the North will prefer higher insulation.

Figure 1: **Insulation in the North and in the South**



Take FOC from the outer minimization problem then to get:[6] $1 = \lambda \frac{\partial U}{\partial X}$ and $C(I) = \lambda \frac{\partial U}{\partial H}$, or $\frac{\partial U/\partial H}{\partial U/\partial X} = C(I)$. Since $I_N > I_S$, the opportunity cost of inner room temperature is lower in the North, so $H_N > H_S$ and $X_S > X_N$.

[5] To see why, rewrite the FOC with respect to I as: $-C'(I)K = P$. Differentiate to get: $-C''(I)KdI - C'(I)\frac{\partial K}{\partial P}dP = dP$. Rearrange to get the demand for insulation: $\frac{dP}{dI} = -C''(I)K/[1 + C'(I)\frac{\partial K}{\partial P}]$. If we want the demand for insulation to be downward sloping ($dP/dI < 0$) and if I and K are complements ($\frac{\partial K}{\partial P} < 0$) then we need C to be convex; $C''(I) > 0$.

[6] If you're wondering what happened to the terms $\partial I(X,H)/\partial X$ and $\partial I(X,H)/\partial H$ in the FOC, observe that these terms appear multiplying $[P + C'(I)(H-E)]$ which is equal to zero, from Equation (5).