迴歸分析　期末報告

世界幸福探索性數據分析

World Happiness Exploratory Data Analysis

第六組

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一、 關於世界幸福指數

世界幸福報告（英語：World Happiness Report）為聯合國為衡量幸福的可持續發展方案，於網路出版的國際調查報告。

1. 計算方式：

《世界幸福報告》的排名是使用蓋洛普世界民意調查的數據，這是由民意調查中提出的生活評估問題的回答，它要求受訪者想出一個階梯，對他們來說最好的是 10，最壞的是 0，這被稱為 Cantril 階梯。

評分的細項共有 10 點：

(1) 人均 GDP

(2) 預期的健康壽命(Healthy Life Expectancy)

(3) 社會支援(Social support)

(4) 人生抉擇的自由(Freedom to make life choices)

(5) 慷慨(Generosity)

(6) 正面影響(Positive affect)

(7) 負面影響(Negative affect)

(8) 家庭收入報告的基尼係數(GINI of household income reported)

(9) 世界銀行提供的基尼指數(GINI index from the World Bank)

(10) 對於政府機關的信任程度(Institutional trust)

這十點細項將表現出以下六大因素，這六大因素（GDP 水平、預期壽命、慷慨、社會支持、自由和腐敗）中的每一個皆有助於評估每個國家。

2. 殘差：

除了上面這些指標外，因為統計方法的緣故，分數裡還加了一項「 Dystopia + 殘值 」這個數字。在這裡， Dystopia 是個「作為標準的虛擬國家」，擁有上面六項指標的最低分數；「殘值」則是代表實際值和這個統計模型預估值的差異：以迴歸分析算出來的值跟實際值的差異（高為正數、低為負數），再加上 dystopia 的實際值，簡單來說，可以視為這個統計模型中「無法解釋的部分」。

3. 樣本來源 ：

每年每個國家的典型樣本為 1,000 人，聯合國使用最近三年的回復來提供最新的生活評估，所以如果一個典型的國家每年進行調查，樣本量將是 3,000，其樣本數夠多，可以減少隨機抽樣誤差；然而，目前還有許多國家沒有進行年度調查。

4. 數據“浪潮”：

蓋洛普將每個日曆年收集的調查作為當年調查浪潮的一部分。在絕大多數情況下，浪潮對應於日曆年，但也有一些例外。一些在 2022 年初完成的調查被認為是 2021 年浪潮的一部分。

並非每個國家每年都接受調查。因此，調查波的規模每年也不同。

二、 執行過程

Step 0. 資料匯入及預處理

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.api as sm
from collections import Counter
from pandas import DataFrame

import sklearn
import statsmodels.api as sm
from sklearn.linear_model import LinearRegression
from statsmodels.sandbox.regression.predstd import wls_prediction_std
from sklearn.model_selection import train_test_split
from patsy import dmatrices
from statsmodels.stats.outliers_influence import variance_inflation_factor

import scipy.stats as stats
from sklearn.metrics import mean_squared_error

import seaborn as sns
import matplotlib.mlab as mlab

sns.set_style("whitegrid")
sns.set_context("paper")
df = pd.read_csv("2021a.csv")
```

```
df.info()
print('\n')
print('len = ' ,len(df))
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 149 entries, 0 to 148
Data columns (total 20 columns):
 #   Column                              Non-Null Count  Dtype
---  ------                              --------------  -----
 0   CountryName                         149 non-null    object
 1   RegionalIndicator                   149 non-null    object
 2   LadderScore                         149 non-null    float64
 3   StandardErrorOfLadderScore          149 non-null    float64
 4   upperwhisker                        149 non-null    float64
 5   lowerwhisker                        149 non-null    float64
 6   LoggedGDPPerCapita                  149 non-null    float64
 7   SocialSupport                       149 non-null    float64
 8   HealthyLifeExpectancy               149 non-null    float64
 9   FreedomToMakeLifeChoices            149 non-null    float64
 10  Generosity                          149 non-null    float64
 11  PerceptionsOfCorruption             149 non-null    float64
 12  LadderScoreInDystopia               149 non-null    float64
 13  ExplainedbyLogGDPpercapita          149 non-null    float64
 14  ExplainedbySocialsupport            149 non-null    float64
 15  ExplainedbyHealthylifeexpectancy    149 non-null    float64
 16  ExplainedbyFreedomtomakelifechoices 149 non-null    float64
 17  ExplainedbyGenerosity               149 non-null    float64
 18  ExplainedbyPerceptionsofcorruption  149 non-null    float64
 19  Dystopiaresidual                    149 non-null    float64
dtypes: float64(18), object(2)
memory usage: 23.4+ KB


len =  149
```

a. 檢查資料是否有缺失值

```
df.isnull().sum(axis=0)

CountryName                            0
RegionalIndicator                      0
LadderScore                            0
StandardErrorOfLadderScore             0
upperwhisker                           0
lowerwhisker                           0
LoggedGDPPerCapita                     0
SocialSupport                          0
HealthyLifeExpectancy                  0
FreedomToMakeLifeChoices               0
Generosity                             0
PerceptionsOfCorruption                0
LadderScoreInDystopia                  0
ExplainedbyLogGDPpercapita             0
ExplainedbySocialsupport               0
ExplainedbyHealthylifeexpectancy       0
ExplainedbyFreedomtomakelifechoices    0
ExplainedbyGenerosity                  0
ExplainedbyPerceptionsofcorruption     0
Dystopiaresidual                       0
dtype: int64
```

b. One hot encoding :

可以處理數字但不能直接處理字串值， 需先將字串對應成數值。

```
data_one_hot = pd.get_dummies(df)
data_one_hot.head()
```

c. Drop 無關值

```
df = df.drop(['CountryName','RegionalIndicator','upperwhisker','lowerwhisker','ExplainedbyLogGDPpercapita',
              'ExplainedbySocialsupport','ExplainedbyHealthylifeexpectancy','ExplainedbyFreedomtomakelifechoices',
              'ExplainedbyGenerosity','ExplainedbyPerceptionsofcorruption','Dystopiaresidual'], axis = 1)
df.head()
```

Step 1. 複迴歸模型 ：

score 會透過R^2來判定我們模型的精準程度；如果訓練集的分數很高，但測試集的分數卻很低，那就是過度擬和

```
model = LinearRegression()
X, y = df[['LoggedGDPPerCapita','SocialSupport','HealthyLifeExpectancy','FreedomToMakeLifeChoices',
           'Generosity','PerceptionsOfCorruption','LadderScoreInDystopia']], df['LadderScore']

model.fit(X, y)
print('score = ',model.score(X, y))
```

```
score =  0.7558471374226855
```

a. 從理論公式推導

```
yhat = model.predict(X)
```

```
SS_Residual = sum((y-yhat)**2)
SS_Total = sum((y-np.mean(y))**2)
r_squared = 1 - (float(SS_Residual))/SS_Total
adjusted_r_squared = 1 - (1-r_squared)*(len(y)-1)/(len(y)-X.shape[1]-1)
print(r_squared, adjusted_r_squared)
```

```
0.7558471374226854 0.7437260733231024
```

b. 使用 sklearn linear_model 計算

雖然無法直接從文檔中找到任何計算 adjusted R^2方式的函數。

```
print(model.score(X, y), 1 - (1-model.score(X, y))*(len(y)-1)/(len(y)-X.shape[1]-1))
```

```
0.7558471374226855 0.7437260733231026
```

c. 使用 statsmodels 計算，手動添加截距

```python
# 通過手動添加截距後使用 statsmodels 計算
import statsmodels.api as sm
X1 = sm.add_constant(X)
result = sm.OLS(y, X1).fit()
#print dir(result)
print(result.rsquared, result.rsquared_adj)
```

```
0.7558471374226855 0.7455308192856158
```

d. 使用 statsmodels 計算，使用公式的另一種方法

```python
# 使用 statsmodels的另一種公式計算
import statsmodels.formula.api as sm
#result = sm.ols(formula="Ladder score ~ NumberofEmployees + ValueofContract", data=df).fit()
#print result.summary()
print(result.rsquared, result.rsquared_adj)
```

```
0.7558471374226855 0.7455308192856158
```

```python
print('   R^2                adj R^2')
print('a:', r_squared1, adjusted_r_squared1)
print('b:', result2_rsquared, result2_rsquared_adj)
print('c:', result3.rsquared, result3.rsquared_adj)
print('d:', result4.rsquared, result4.rsquared_adj)
```

```
   R^2                adj R^2
a: 0.7558471374226854 0.7437260733231024
b: 0.7558471374226855 0.7437260733231026
c: 0.7558471374226855 0.7455308192856158
d: 0.7558471374226855 0.7455308192856158
```

=> 由上述 4 種方式得知調整後的 R^2 最好的是 c、d 的模型

Step 2. OLS 線性關係判斷

線性回歸模型，顧名思義，首先要保證自變量與因變量之間存在線性關係。關於線性關係的判斷，我們可以通過圖形或 Pearson 相關係數來識別

一般情況下的評判標準：

當相關係數 低於 0.4 ，則表明變量之間存在弱相關關係；

當相關係數在 0.4~0.6 之間 ，則說明變量之間存在中度相關關係；

當相關係數在 0.6 以上時 ，則反映變量之間存在強相關關係。

a. 創建線性迴歸最小平方方法模型

```
import statsmodels.formula.api as smf
import statsmodels.api as sm

model = sm.OLS(y,X)

results = model.fit()

results.summary()
```

OLS Regression Results

Dep. Variable:	LadderScore	R-squared:	0.756
Model:	OLS	Adj. R-squared:	0.746
Method:	Least Squares	F-statistic:	73.27
Date:	Mon, 13 Jun 2022	Prob (F-statistic):	5.06e-41
Time:	17:09:58	Log-Likelihood:	-116.50
No. Observations:	149	AIC:	247.0
Df Residuals:	142	BIC:	268.0
Df Model:	6		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
LoggedGDPPerCapita	0.2795	0.087	3.219	0.002	0.108	0.451
SocialSupport	2.4762	0.668	3.706	0.000	1.155	3.797
HealthyLifeExpectancy	0.0303	0.013	2.274	0.024	0.004	0.057
FreedomToMakeLifeChoices	2.0105	0.495	4.063	0.000	1.032	2.989
Generosity	0.3644	0.321	1.134	0.259	-0.271	0.999
PerceptionsOfCorruption	-0.6051	0.291	-2.083	0.039	-1.179	-0.031
LadderScoreInDystopia	-0.9207	0.259	-3.548	0.001	-1.434	-0.408

Omnibus:	12.908	Durbin-Watson:	1.614
Prob(Omnibus):	0.002	Jarque-Bera (JB):	13.688
Skew:	-0.667	Prob(JB):	0.00107
Kurtosis:	3.650	Cond. No.	1.05e+03

[1] 標準誤差假設正確指定了誤差的共變異數矩陣。

[2] 條件數很大，1.05e+03。 這可能表明存在強多重共線性或其他數值問題。

b. LadderScore 與自變量之間的相關係數

```
df.corrwith(df['LadderScore'])
```

```
LadderScore                    1.000000
StandardErrorOfLadderScore    -0.470787
LoggedGDPPerCapita             0.789760
SocialSupport                  0.756888
HealthyLifeExpectancy          0.768099
FreedomToMakeLifeChoices       0.607753
Generosity                    -0.017799
PerceptionsOfCorruption       -0.421140
LadderScoreInDystopia               NaN
dtype: float64
```

經過對比發現，LadderScore 與 Generosity 之間的為弱相關關係，可以不考慮將該變量納入模型。當然，變量之間不存在線性關係並不代表不存在任何關係，可能是二次函數關係、對數關係等，所以一般還需要進行檢驗和變量

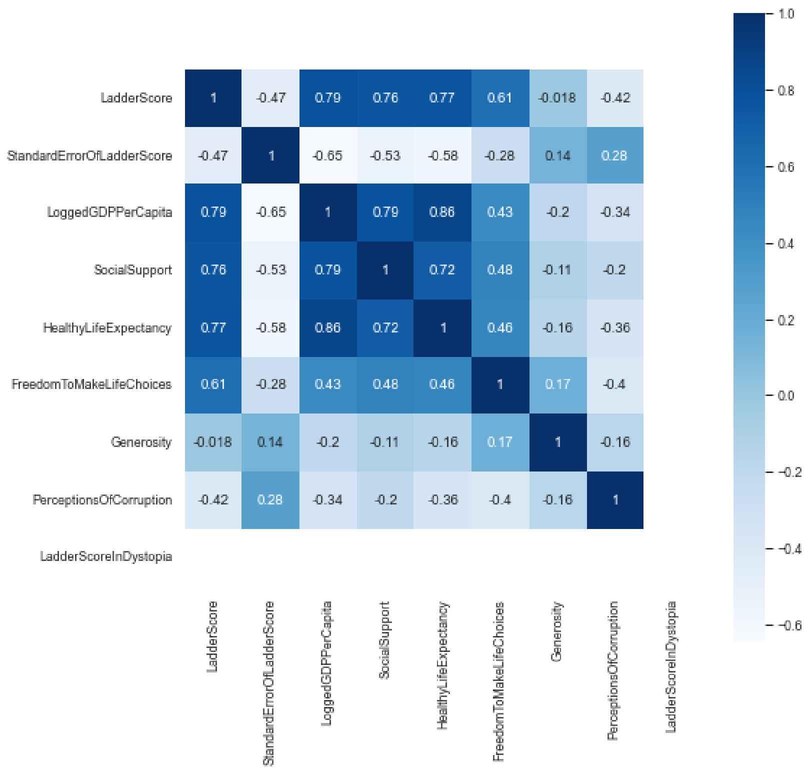
	轉換。

相關係數較大的是 Logged GDP per capita、Healthy life expectancy、Social support、Freedom to make life choices。

Step 3. 多重共線性判斷：

a. 相關性(使用 heatmap)

```
plt.figure(figsize=(8,8))
sns.heatmap(df.corr(), annot=True, vmax=1, square=True, cmap='Blues')
plt.show()
```



b. 擬合模型-模型顯著性和參數顯著性判斷

```
Train,Test = train_test_split(df, train_size = 0.8, random_state=0)
fit = smf.ols('LadderScore~ LoggedGDPPerCapita + SocialSupport + \
            HealthyLifeExpectancy + FreedomToMakeLifeChoices + Generosity + \
            PerceptionsOfCorruption + LadderScoreInDystopia'
            , data = Train).fit()

fit.summary()
```

OLS Regression Results

Dep. Variable:	LadderScore	**R-squared:**	0.752
Model:	OLS	**Adj. R-squared:**	0.739
Method:	Least Squares	**F-statistic:**	56.63
Date:	Mon, 13 Jun 2022	**Prob (F-statistic):**	1.14e-31
Time:	17:09:59	**Log-Likelihood:**	-97.408
No. Observations:	119	**AIC:**	208.8
Df Residuals:	112	**BIC:**	228.3
Df Model:	6		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-0.3658	0.108	-3.389	0.001	-0.580	-0.152
LoggedGDPPerCapita	0.2185	0.111	1.968	0.052	-0.002	0.438
SocialSupport	2.8474	0.799	3.565	0.001	1.265	4.430
HealthyLifeExpectancy	0.0424	0.016	2.689	0.008	0.011	0.074
FreedomToMakeLifeChoices	1.6991	0.568	2.990	0.003	0.573	2.825
Generosity	0.3891	0.358	1.088	0.279	-0.320	1.098
PerceptionsOfCorruption	-0.5927	0.333	-1.779	0.078	-1.253	0.067
LadderScoreInDystopia	-0.8889	0.262	-3.389	0.001	-1.409	-0.369

Omnibus:	7.976	**Durbin-Watson:**	2.318
Prob(Omnibus):	0.019	**Jarque-Bera (JB):**	7.660
Skew:	-0.592	**Prob(JB):**	0.0217
Kurtosis:	3.375	**Cond. No.**	7.23e+17

[1] 標準誤差假設正確指定了誤差的共變異矩陣。

[2] 最小特徵值為 9.97e-31。 這可能表明存在強多重共線性問題或設計矩陣是奇特的。

=> 通過上面結果我們清楚看到：

7 個回歸係數的 t 統計量 p 值除了 Generosity、PerceptionsOfCorruption、LoggedGDPPerCapita 其餘的都<0.05，說明剩下的迴歸係數較顯著，因此需要 Drop 掉 P 值最大的 Generosity 重新建模

c. 第一次重新建模

```python
fit2 = smf.ols('LadderScore~ LoggedGDPPerCapita \
                + SocialSupport + HealthyLifeExpectancy + \
                FreedomToMakeLifeChoices + PerceptionsOfCorruption + \
                LadderScoreInDystopia'
                ,data = Train.drop('Generosity', axis = 1)).fit()

fit2.summary()
```

OLS Regression Results

Dep. Variable:	LadderScore	**R-squared:**	0.749
Model:	OLS	**Adj. R-squared:**	0.738
Method:	Least Squares	**F-statistic:**	67.61
Date:	Mon, 13 Jun 2022	**Prob (F-statistic):**	2.35e-32
Time:	17:09:59	**Log-Likelihood:**	-98.033
No. Observations:	119	**AIC:**	208.1
Df Residuals:	113	**BIC:**	224.7
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-0.3448	0.106	-3.244	0.002	-0.555	-0.134
LoggedGDPPerCapita	0.1996	0.110	1.818	0.072	-0.018	0.417
SocialSupport	2.9098	0.797	3.649	0.000	1.330	4.490
HealthyLifeExpectancy	0.0413	0.016	2.623	0.010	0.010	0.072
FreedomToMakeLifeChoices	1.8121	0.559	3.241	0.002	0.704	2.920
PerceptionsOfCorruption	-0.6498	0.329	-1.973	0.051	-1.302	0.003
LadderScoreInDystopia	-0.8379	0.258	-3.244	0.002	-1.350	-0.326

Omnibus:	8.416	**Durbin-Watson:**	2.282
Prob(Omnibus):	0.015	**Jarque-Bera (JB):**	8.142
Skew:	-0.606	**Prob(JB):**	0.0171
Kurtosis:	3.419	**Cond. No.**	7.22e+17

[1] 標準誤差假設正確指定了誤差的共變異矩陣。

[2] 最小特徵值為 9.98e-31。 這可能表明存在強多重共線性問題或設計矩陣是奇特的。

=> 通過上面結果我們清楚看到：

剩下 6 個回歸係數的 t 統計量 p 值除了 LoggedGDPPerCapita、PerceptionsOfCorruption、其餘的都<0.05，說明剩下的迴歸係數較顯著，因此需要 Drop 掉 P 值最大的 LoggedGDPPerCapita 繼續重新建模。

d. 重新建模(第二次)

```
fit3 = smf.ols('LadderScore~ SocialSupport + HealthyLifeExpectancy + \
               FreedomToMakeLifeChoices + PerceptionsOfCorruption + \
               LadderScoreInDystopia'
               ,data = Train.drop('LoggedGDPPerCapita', axis = 1)).fit()

fit3.summary()
```

OLS Regression Results

Dep. Variable:	LadderScore	**R-squared:**	0.742
Model:	OLS	**Adj. R-squared:**	0.733
Method:	Least Squares	**F-statistic:**	82.02
Date:	Mon, 13 Jun 2022	**Prob (F-statistic):**	1.22e-32
Time:	17:09:59	**Log-Likelihood:**	-99.749
No. Observations:	119	**AIC:**	209.5
Df Residuals:	114	**BIC:**	223.4
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-0.3122	0.106	-2.951	0.004	-0.522	-0.103
SocialSupport	3.7287	0.665	5.610	0.000	2.412	5.045
HealthyLifeExpectancy	0.0607	0.012	5.195	0.000	0.038	0.084
FreedomToMakeLifeChoices	1.5791	0.550	2.872	0.005	0.490	2.668
PerceptionsOfCorruption	-0.7708	0.326	-2.366	0.020	-1.416	-0.125
LadderScoreInDystopia	-0.7586	0.257	-2.951	0.004	-1.268	-0.249

Omnibus:	6.199	**Durbin-Watson:**	2.225
Prob(Omnibus):	0.045	**Jarque-Bera (JB):**	5.773
Skew:	-0.524	**Prob(JB):**	0.0558
Kurtosis:	3.257	**Cond. No.**	7.15e+17

[1] 標準誤差假設正確指定了誤差的共變異矩陣。

[2] 最小特徵值為 9.98e-31。 這可能表明存在強多重共線性問題或設計矩陣是奇特的。

=> 通過模型反饋的結果我們可知，模型是通過顯著性檢驗的，即剩下 6 個迴歸係數的 t 統計量 p 值遠遠小於 0.05 這個閾值的，說明需要拒絕原假設(即認為模型的所有回歸係數都不全為 0)。

=>模型的顯著性通過檢驗的話，並不代表每一個自變量都對因變量是重要的，

所以還需要進行偏回歸係數的顯著性檢驗。通過上圖的檢驗結果顯示，除變量 newspaper 對應的 P 值超過 0.05，其餘變量都低於這個閾值，說明 newspaper 這個廣告渠道並沒有影響到銷售量的變動，故需要將其從模型中剔除。

e. RMSE 比較

<pre>pred = fit.predict(exog = Test)
pred2 = fit2.predict(exog = Test.drop('Generosity', axis = 1))
pred3 = fit3.predict(exog = Test.drop('LoggedGDPPerCapita', axis = 1))
 	
RMSE = np.sqrt(mean_squared_error(Test.LadderScore, pred))
RMSE2 = np.sqrt(mean_squared_error(Test.LadderScore, pred2))
RMSE3 = np.sqrt(mean_squared_error(Test.LadderScore, pred3))
 	
print('第一個: RMSE = %.4f\n'%RMSE)
print('第二個: RMSE = %.4f\n'%RMSE2)
print('第三個: RMSE = %.4f\n'%RMSE3)</pre>
	第一個: RMSE = 0.4603
	
	第二個: RMSE = 0.4608
	
	第三個: RMSE = 0.5174

=> 對於連續委量預測效果的好壞,我們可以信助於 RMSE(均方根誤差,即真實值與預測值的均方根)來衡量,如果這個值越小就說明模型越優秀,即預測出來 的值會越接近於真實值很明顯

=> 我們發現模型 1 的 RMSE 相比於模型 2、3 會小一些,模型會更符合實際

f. 誤差值

<pre>df["pred"]=pd.Series(fit.predict())
abs_=(df['pred']-df['LadderScore']).abs()
 	
mae_=abs_.mean()
rmse_=((abs_**2).mean())**0.5
mape_=(abs_/df['LadderScore']).mean()
mape_</pre>
	誤差值 = 0.1834100969974436

g. Vif :

如果自變量 X 與其他自變量共線性強，那麼回歸方程的 R2 就會較高，導致 VIF 也高。一般，有自變量 VIF 值大於 10，則說明存在嚴重多重共線性，可以選擇刪除該變量或者用其他類似但 VIF 低的變量代替。

可以看到 AT 的方差膨脹因子大於 10，可以刪除該變量。

多重共線性的處理方法：

多重共線性對於線性回歸是種災難，並且我們不可能完全消除，而只能利用一些方法來減輕它的影響。

對於多重共線性的處理方式，有以下幾種思路：

1)提前篩選變量：可以利用相關檢驗來或變量聚類的方法。注意：決策樹和隨機森林也可以作為提前篩選變量的方法，但是它們對於多重共線性幫助不大，因為如果按照特徵重要性排序，共線性的變量很可能都排在前面。

2)子集選擇：包括逐步回歸和最優子集法。因為該方法是貪婪算法，理論上大部分情況有效，實際中需要結合第一種方法。

3)收縮方法：正則化方法，包括嶺回歸和 LASSO 回歸。LASSO 回歸可以實現篩選變量的功能。

4)維數縮減：包括主成分回歸(PCR)和偏最小平方法迴歸(PLS)方法。

```python
y, X = dmatrices('LadderScore~ LoggedGDPPerCapita + SocialSupport + \
                HealthyLifeExpectancy + FreedomToMakeLifeChoices + Generosity + \
                PerceptionsOfCorruption + LadderScoreInDystopia'
                , data = df, return_type='dataframe')

vif = pd.DataFrame()
vif["VIF Factor"]=[variance_inflation_factor(X.values,i) \
                  for i in range(X.shape[1])]

vif["feature"]=X.columns
vif
```

	VIF Factor	feature
0	0.000000	Intercept
1	5.104890	LoggedGDPPerCapita
2	2.972200	SocialSupport
3	4.099348	HealthyLifeExpectancy
4	1.585807	FreedomToMakeLifeChoices
5	1.180982	Generosity
6	1.367122	PerceptionsOfCorruption
7	0.000000	LadderScoreInDystopia

=> 結果顯示,所有自變量的 VIF 均低於 10,說明自變量之間並不存在多重共線性的隱患。

Step 4. 強影響點診斷

```python
#離群點檢驗
outliers = fit.get_influence()
#高槓桿值點(帽子矩)
leverage = outliers.hat_matrix_diag
#dffts值
dffits = outliers.dffits[0]
#學生化殘差
resid_stu = outliers.resid_studentized_external
#cook距離
cook = outliers.cooks_distance[0]
#covratio值
covratio = outliers.cov_ratio
#將上面的幾種異常值檢驗統計量與原始數據集合
contat1 = pd.concat([pd.Series(leverage, name = 'leverage'),pd.Series(dffits, name = 'dffits'),
pd.Series(resid_stu,name ='resid_stu'),pd.Series(cook,name = 'cook'),
pd.Series(covratio, name = 'covratio'),], axis = 1)
df_outliers = pd.concat([df,contat1], axis = 1)
df_outliers.head()
```

a. 計算異常值數值的比例

```
#計算異常值數量的比例
outliers_ratio = sum(np.where((np.abs(df_outliers.resid_stu)>2),1,0))/df_outliers.shape[0]
print('異常值數量的比例 = ',outliers_ratio)
```

```
異常值數量的比例 =  0.04697986577181208
```

b. 刪除異常值

```
#刪除異常值
df_outliers = df_outliers.loc[np.abs(df_outliers.resid_stu)<=2,]
```

c.第三次重新建模：

```
fit4 = sm.formula.ols('LadderScore~ LoggedGDPPerCapita + SocialSupport + \
                       HealthyLifeExpectancy + FreedomToMakeLifeChoices + \
                       Generosity + PerceptionsOfCorruption +\
                       LadderScoreInDystopia'\
                       ,data = df_outliers).fit()

fit4.summary()
```

Dep. Variable:	LadderScore	**R-squared:**	0.733
Model:	OLS	**Adj. R-squared:**	0.718
Method:	Least Squares	**F-statistic:**	48.03
Date:	Mon, 13 Jun 2022	**Prob (F-statistic):**	6.26e-28
Time:	17:09:59	**Log-Likelihood:**	-61.656
No. Observations:	112	**AIC:**	137.3
Df Residuals:	105	**BIC:**	156.3
Df Model:	6		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	-0.0614	0.104	-0.592	0.555	-0.267	0.144
LoggedGDPPerCapita	0.2540	0.084	3.015	0.003	0.087	0.421
SocialSupport	1.9191	0.667	2.876	0.005	0.596	3.242
HealthyLifeExpectancy	0.0175	0.013	1.322	0.189	-0.009	0.044
FreedomToMakeLifeChoices	2.1116	0.500	4.220	0.000	1.119	3.104
Generosity	0.2142	0.308	0.696	0.488	-0.396	0.825
PerceptionsOfCorruption	-0.9059	0.265	-3.424	0.001	-1.431	-0.381
LadderScoreInDystopia	-0.1493	0.252	-0.592	0.555	-0.649	0.351

Omnibus:	3.786	**Durbin-Watson:**	1.632
Prob(Omnibus):	0.151	**Jarque-Bera (JB):**	3.815
Skew:	-0.432	**Prob(JB):**	0.148
Kurtosis:	2.736	**Cond. No.**	1.85e+17

[1] 標準誤差假設正確指定了誤差的協方差矩陣。

[2] 最小特徵值為 1.5e-29。 這可能表明有強多重共線性問題或設計矩陣是奇特的。

d. 計算誤差

```
#總絕對深差
df_outliers["pred2"]=pd.Series(fit2.predict())
abs_= (df_outliers['pred2'] -df_outliers['LadderScore']).abs()
#mae,平均絕對溪差
mae_ = abs_.mean()
#rmse,均方根誤差
rmse_ = ((abs_**2).mean())**0.5
#mape ,平均絕對百分比課差
mape_= (abs_/df_outliers['LadderScore']).mean()
mape_
```

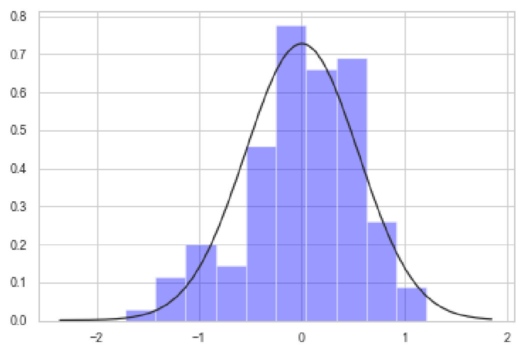
```
0.05540979663400268
```

=> 通過對比 f 和 t2,將異常值刪掉後重新建模的話效果會更理想一點具體表現為:信息準則(AIC 和 BIC)均變小,同時 RMSE(識差均方根)也有降低

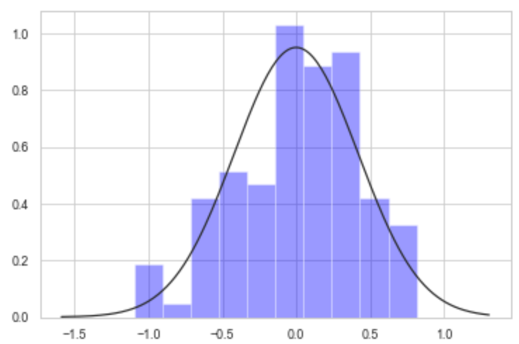
Step 5. 殘差診斷

a. KS 檢驗

```
residual = fit.resid
sns.distplot(residual,
             bins = 10,
             kde = False,
             color = 'blue',
             fit = stats.norm)
plt.show()
```

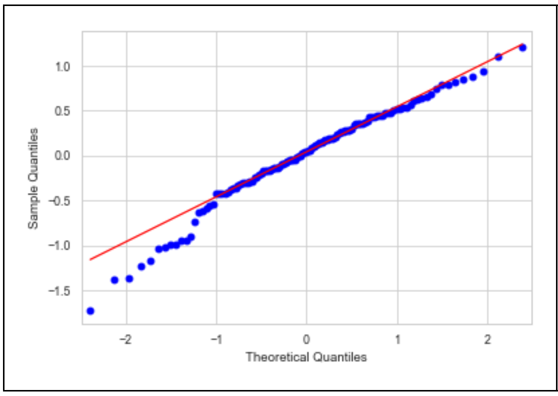


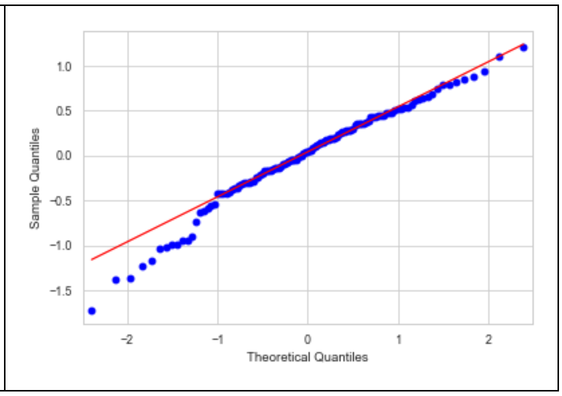

```
residual2 = fit2.resid
sns.distplot(residual2,
             bins = 10,
             kde = False,
             color = 'blue',
             fit = stats.norm)
plt.show()
```



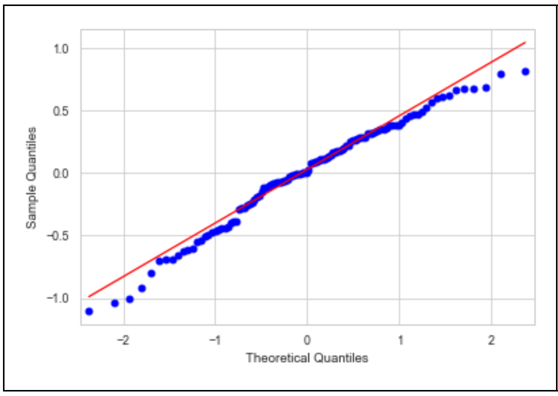
b. QQ 圖

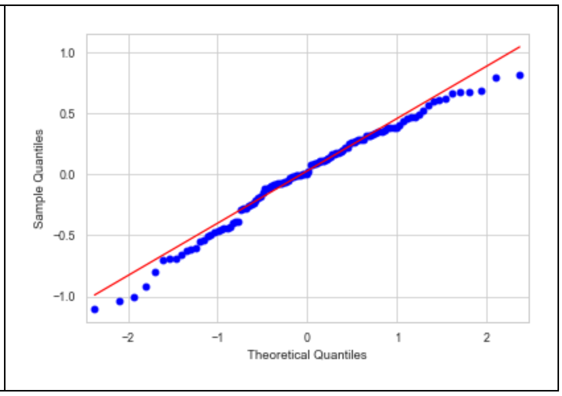
```python
pq = sm.ProbPlot(residual)

pq.qqplot(line='q')
```






```python
pq = sm.ProbPlot(residual2)

pq.qqplot(line='q')
```





c. KS 檢驗比較

```python
standard_resid=(residual-np.mean(residual))/np.std(residual)
stats.kstest(standard_resid, 'norm')
```

```
KstestResult(statistic=0.0674656950591975, pvalue=0.6260972978671546)
```

```python
standard_resid2=(residual2-np.mean(residual2))/np.std(residual2)
stats.kstest(standard_resid2, 'norm')
```

```
KstestResult(statistic=0.07566021287390895, pvalue=0.5184478479267607)
```

d. 第四次重新建模 ：對 Y 進行變換

```
import scipy.stats as stats
#找到box-cox變換得lambda係數
lamd = stats.boxcox_normmax(df_outliers.LadderScore, method = 'mle')
#對Y進行變換
df_outliers['trans_y'] = stats.boxcox(df_outliers.LadderScore, lamd)
fit5 =sm.formula.ols('trans_y~LoggedGDPPerCapita + SocialSupport + \
                    HealthyLifeExpectancy + FreedomToMakeLifeChoices + \
                    Generosity + PerceptionsOfCorruption + \
                    LadderScoreInDystopia'
                    , data = df_outliers).fit()

fit5.summary()
```

Dep. Variable:	trans_y	**R-squared:**	0.733
Model:	OLS	**Adj. R-squared:**	0.718
Method:	Least Squares	**F-statistic:**	48.06
Date:	Mon, 13 Jun 2022	**Prob (F-statistic):**	6.12e-28
Time:	16:30:32	**Log-Likelihood:**	317.70
No. Observations:	112	**AIC:**	-621.4
Df Residuals:	105	**BIC:**	-602.4
Df Model:	6		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	0.0933	0.004	26.599	0.000	0.086	0.100
LoggedGDPPerCapita	0.0080	0.003	2.820	0.006	0.002	0.014
SocialSupport	0.0680	0.023	3.014	0.003	0.023	0.113
HealthyLifeExpectancy	0.0008	0.000	1.871	0.064	-5e-05	0.002
FreedomToMakeLifeChoices	0.0758	0.017	4.479	0.000	0.042	0.109
Generosity	0.0035	0.010	0.340	0.735	-0.017	0.024
PerceptionsOfCorruption	-0.0201	0.009	-2.244	0.027	-0.038	-0.002
LadderScoreInDystopia	0.2268	0.009	26.599	0.000	0.210	0.244

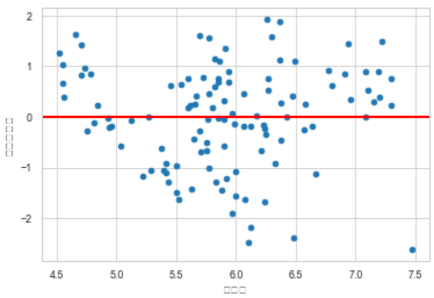
Omnibus:	5.151	**Durbin-Watson:**	1.676
Prob(Omnibus):	0.076	**Jarque-Bera (JB):**	5.168
Skew:	-0.491	**Prob(JB):**	0.0755
Kurtosis:	2.624	**Cond. No.**	4.68e+17

[1] 標準誤差假設正確指定了誤差的共變異矩陣。

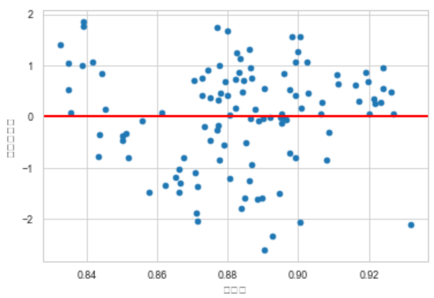
[2] 最小特徵值為 2.35e-30。 這可能表明存在強多重共線性問題或設計矩陣是奇特的。

Step 6. 標準化殘差檢定圖

```
#圖示法完成方差齊性的判斷
#標準化殘差與預測值之間的散點圖
plt.scatter(fit2.predict(), (fit2.resid-fit2.resid.mean())/fit2.resid.std())
plt.xlabel('預測值')
plt.ylabel('標準化殘差')
#添加水平參考線
plt.axhline(y = 0, color = 'r',linewidth = 2)
plt.show()
```




```
#圖示法完成方差齊性的判斷
#標準化殘差與預測值之間的散點圖
plt.scatter(fit3.predict(), (fit3.resid-fit3.resid.mean())/fit3.resid.std())
plt.xlabel('預測值')
plt.ylabel('標準化殘差')
#添加水平參考線
plt.axhline(y = 0, color = 'r',linewidth = 2)
plt.show()
```



a. 對數變換

```
import math
df_outliers['log_y']=df_outliers["LadderScore"].map(lambda x: math.log(x,15))
df_outliers['log_StandardErrorOfLadderScore']=df_outliers["StandardErrorOfLadderScore"].map(lambda x: math.log(x,15))
df_outliers['log_LoggedGDPPerCapita']=df_outliers["LoggedGDPPerCapita"].map(lambda x: math.log(x,15))
df_outliers['log_SocialSupport']=df_outliers["SocialSupport"].map(lambda x: math.log(x,15))
df_outliers['log_HealthyLifeExpectancy']=df_outliers["HealthyLifeExpectancy"].map(lambda x: math.log(x,15))
df_outliers['log_FreedomToMakeLifeChoices']=df_outliers["FreedomToMakeLifeChoices"].map(lambda x: math.log(x,15))
df_outliers['log_PerceptionsOfCorruption']=df_outliers["PerceptionsOfCorruption"].map(lambda x: math.log(x,15))
df_outliers['log_LadderScoreInDystopia']=df_outliers["LadderScoreInDystopia"].map(lambda x: math.log(x,15))

df_outliers.head()
```

b. 第五次重新建模 ： 使用對數建模

```
fit6_log = sm.formula.ols('log_y~LoggedGDPPerCapita + SocialSupport + \
                          HealthyLifeExpectancy + FreedomToMakeLifeChoices + \
                          Generosity + PerceptionsOfCorruption + \
                          LadderScoreInDystopia'
                          , data = df_outliers).fit()

fit6_log.summary()
```

Dep. Variable:	log_y	**R-squared:**	0.734
Model:	OLS	**Adj. R-squared:**	0.719
Method:	Least Squares	**F-statistic:**	48.38
Date:	Mon, 13 Jun 2022	**Prob (F-statistic):**	4.76e-28
Time:	16:30:33	**Log-Likelihood:**	250.80
No. Observations:	112	**AIC:**	-487.6
Df Residuals:	105	**BIC:**	-468.6
Df Model:	6		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	0.0344	0.006	5.396	0.000	0.022	0.047
LoggedGDPPerCapita	0.0152	0.005	2.931	0.004	0.005	0.025
SocialSupport	0.1215	0.041	2.964	0.004	0.040	0.203
HealthyLifeExpectancy	0.0013	0.001	1.616	0.109	-0.000	0.003
FreedomToMakeLifeChoices	0.1345	0.031	4.376	0.000	0.074	0.195
Generosity	0.0096	0.019	0.509	0.612	-0.028	0.047
PerceptionsOfCorruption	-0.0456	0.016	-2.803	0.006	-0.078	-0.013
LadderScoreInDystopia	0.0836	0.015	5.396	0.000	0.053	0.114

Omnibus:	4.721	**Durbin-Watson:**	1.663
Prob(Omnibus):	0.094	**Jarque-Bera (JB):**	4.734
Skew:	-0.470	**Prob(JB):**	0.0938
Kurtosis:	2.638	**Cond. No.**	4.68e+17

[1] 標準誤差假設正確指定了誤差的共變異矩陣。

[2] 最小特徵值為 2.35e-30。 這可能表明存在強多重共線性問題或設計矩陣是奇特的。

c. 計算誤差

```
df_outliers["pred4"]=fit6_log.predict()
abs3_ = (df_outliers['pred4'] -df_outliers['log_y']).abs()
#mae,平均絕對誤差
mae3_= abs3_.mean()
#rmse,均方根誤差
rmse3_ = ((abs3_**2).mean())**0.5
#mape,平均絕對百分比誤差
mape3_ = (abs3_/df_outliers['log_y']).mean()

print('誤差值 = ', mape3_)
```

```
誤差值 =  0.03227033524905799 4
```

Step 7. ANOVA Table

```python
import pandas as pd
import researchpy as rp
rp.summary_cont(df['LadderScore'])
```

	Variable	N	Mean	SD	SE	95% Conf.	Interval
0	LadderScore	149.0	5.5328	1.0739	0.088	5.359	5.7067

```python
model = sm.formula.ols('LadderScore~LoggedGDPPerCapita + SocialSupport + \
                        HealthyLifeExpectancy + FreedomToMakeLifeChoices + \
                        Generosity + PerceptionsOfCorruption + \
                        LadderScoreInDystopia'
                        , data = df_outliers).fit()

result = sm.stats.anova_lm(model, type=2)
print('原始建模')
print(result)
```

```
原始建模
                             df     sum_sq    mean_sq           F  \
LoggedGDPPerCapita          1.0  39.479700  39.479700  210.213134
SocialSupport               1.0   4.534560   4.534560   24.144663
HealthyLifeExpectancy       1.0   0.930714   0.930714    4.955666
FreedomToMakeLifeChoices    1.0   6.595014   6.595014   35.115733
Generosity                  1.0   0.380142   0.380142    2.024097
PerceptionsOfCorruption     1.0   2.201754   2.201754   11.723433
LadderScoreInDystopia       1.0   0.434498   0.434498    2.313524
Residual                  105.0  19.719836   0.187808         NaN

                                PR(>F)
LoggedGDPPerCapita        8.165657e-27
SocialSupport             3.305200e-06
HealthyLifeExpectancy     2.814180e-02
FreedomToMakeLifeChoices  3.992819e-08
Generosity                1.577851e-01
PerceptionsOfCorruption   8.813401e-04
LadderScoreInDystopia     1.312588e-01
Residual                           NaN
```

```python
fit2 = smf.ols('LadderScore~ LoggedGDPPerCapita + SocialSupport + \
               HealthyLifeExpectancy + FreedomToMakeLifeChoices + \
               PerceptionsOfCorruption + LadderScoreInDystopia'
               , data = Train.drop('Generosity', axis = 1)).fit()

result2 = sm.stats.anova_lm(fit2, type=2)
print('第一次重新建模')
print(result2)
```

```
第一次重新建模
                            df      sum_sq    mean_sq           F  \
LoggedGDPPerCapita         1.0  88.128918  88.128918  275.155235
SocialSupport              1.0   8.650668   8.650668   27.009031
HealthyLifeExpectancy      1.0   4.597534   4.597534   14.354375
FreedomToMakeLifeChoices   1.0   5.644001   5.644001   17.621643
PerceptionsOfCorruption    1.0   1.247321   1.247321    3.894374
LadderScoreInDystopia      1.0   0.207096   0.207096    0.646593
Residual                 113.0  36.192543   0.320288         NaN

                               PR(>F)
LoggedGDPPerCapita         4.653080e-32
SocialSupport              9.084253e-07
HealthyLifeExpectancy      2.444682e-04
FreedomToMakeLifeChoices   5.394397e-05
PerceptionsOfCorruption    5.088927e-02
LadderScoreInDystopia      4.230218e-01
Residual                            NaN
```

```python
fit3 = smf.ols('LadderScore~ SocialSupport + HealthyLifeExpectancy + \
               FreedomToMakeLifeChoices + PerceptionsOfCorruption + \
               LadderScoreInDystopia'
               ,data = Train.drop('LoggedGDPPerCapita', axis = 1)).fit()

result3 = sm.stats.anova_lm(fit3, type=2)
print('第二次重新建模')
print(result3)
```

```
第二次重新建模
                            df      sum_sq    mean_sq           F  \
SocialSupport              1.0  85.702821  85.702821  262.274276
HealthyLifeExpectancy      1.0  14.892938  14.892938   45.576500
FreedomToMakeLifeChoices   1.0   4.784159   4.784159   14.640847
PerceptionsOfCorruption    1.0   1.829523   1.829523    5.598846
LadderScoreInDystopia      1.0   0.214852   0.214852    0.657506
Residual                 114.0  37.251543   0.326768         NaN

                               PR(>F)
SocialSupport              2.449714e-31
HealthyLifeExpectancy      6.457762e-10
FreedomToMakeLifeChoices   2.128068e-04
PerceptionsOfCorruption    1.965953e-02
LadderScoreInDystopia      4.191314e-01
Residual                            NaN
```

```python
fit4 = sm.formula.ols('LadderScore~ LoggedGDPPerCapita + SocialSupport + \
                     HealthyLifeExpectancy + FreedomToMakeLifeChoices + \
                     Generosity + PerceptionsOfCorruption +\
                     LadderScoreInDystopia'\
                     ,data = df_outliers).fit()

result4 = sm.stats.anova_lm(fit4, type=2)
print('第三次重新建模')
print(result4)
```

```
第三次重新建模
                             df     sum_sq    mean_sq           F  \
LoggedGDPPerCapita          1.0  39.479700  39.479700  210.213134
SocialSupport               1.0   4.534560   4.534560   24.144663
HealthyLifeExpectancy       1.0   0.930714   0.930714    4.955666
FreedomToMakeLifeChoices    1.0   6.595014   6.595014   35.115733
Generosity                  1.0   0.380142   0.380142    2.024097
PerceptionsOfCorruption     1.0   2.201754   2.201754   11.723433
LadderScoreInDystopia       1.0   0.434498   0.434498    2.313524
Residual                  105.0  19.719836   0.187808         NaN

                                PR(>F)
LoggedGDPPerCapita        8.165657e-27
SocialSupport             3.305200e-06
HealthyLifeExpectancy     2.814180e-02
FreedomToMakeLifeChoices  3.992819e-08
Generosity                1.577851e-01
PerceptionsOfCorruption   8.813401e-04
LadderScoreInDystopia     1.312588e-01
Residual                           NaN
```

```python
import scipy.stats as stats
lamd = stats.boxcox_normmax(df_outliers.LadderScore, method = 'mle')

df_outliers['trans_y'] = stats.boxcox(df_outliers.LadderScore, lamd)
fit5 =sm.formula.ols('trans_y~LoggedGDPPerCapita + SocialSupport + \
                    HealthyLifeExpectancy + FreedomToMakeLifeChoices + \
                    Generosity + PerceptionsOfCorruption + \
                    LadderScoreInDystopia'
                    , data = df_outliers).fit()

result5 = sm.stats.anova_lm(fit5, type=2)
print('第四次重新建模')
print(result5)
```

```
第四次重新建模
                             df    sum_sq   mean_sq           F        PR(>F)
LoggedGDPPerCapita          1.0  0.046163  0.046163  215.072395  3.644097e-27
SocialSupport               1.0  0.005971  0.005971   27.819553  7.190708e-07
HealthyLifeExpectancy       1.0  0.001571  0.001571    7.317284  7.970173e-03
FreedomToMakeLifeChoices    1.0  0.006962  0.006962   32.434535  1.132338e-07
Generosity                  1.0  0.000142  0.000142    0.661809  4.177631e-01
PerceptionsOfCorruption     1.0  0.001081  0.001081    5.035963  2.692447e-02
LadderScoreInDystopia       1.0  0.000204  0.000204    0.951058  3.316916e-01
Residual                  105.0  0.022537  0.000215         NaN           NaN
```

```python
fit6_log = sm.formula.ols('log_y~LoggedGDPPerCapita + SocialSupport + \
                        HealthyLifeExpectancy + FreedomToMakeLifeChoices + \
                        Generosity + PerceptionsOfCorruption + \
                        LadderScoreInDystopia'
                        , data = df_outliers).fit()

result6 = sm.stats.anova_lm(fit6_log, type=2)
print('第五次重新建模')
print(result6)
```

```
第五次重新建模
                             df    sum_sq   mean_sq           F        PR(>F)
LoggedGDPPerCapita          1.0  0.152210  0.152210  214.753746  3.840635e-27
SocialSupport               1.0  0.018640  0.018640   26.298873  1.343605e-06
HealthyLifeExpectancy       1.0  0.004380  0.004380    6.179798  1.449868e-02
FreedomToMakeLifeChoices    1.0  0.024059  0.024059   33.944358  6.278676e-08
Generosity                  1.0  0.000862  0.000862    1.216372  2.725953e-01
PerceptionsOfCorruption     1.0  0.005569  0.005569    7.856964  6.030677e-03
LadderScoreInDystopia       1.0  0.001054  0.001054    1.487553  2.253291e-01
Residual                  105.0  0.074420  0.000709         NaN           NaN
```

Step 8. R 語言 CP = Python 決策樹

```
import numpy as np
import matplotlib.pyplot as plt
from sklearn import datasets, linear_model
from sklearn.metrics import mean_squared_error, r2_score,accuracy_score
from sklearn import preprocessing
from sklearn.model_selection import train_test_split

x_train,x_test,y_train,y_test = train_test_split(X,
                                                 y,
                                                 test_size = 0.3,
                                                 random_state = 5)
x_train.shape,x_test.shape
```

```
((104, 8), (45, 8))
```

```
from sklearn.tree import DecisionTreeClassifier
from sklearn import tree
from IPython.display import Image

model = DecisionTreeClassifier(criterion = 'entropy',max_depth=3)
clf = model.fit(X,y.astype('int'))

y_pred = model.predict(X)

print("決策樹預測率:"+str(model.score(X,y.astype('int'))))
```

```
決策樹預測率:0.6711409395973155
```

Step 9. AIC and BIC => Forward 、 Backward and Step-Wise

a. AIC and BIC

```
import time
from sklearn.preprocessing import StandardScaler
from sklearn.linear_model import LassoLarsIC
from sklearn.pipeline import make_pipeline

start_time = time.time()
lasso_lars_ic = make_pipeline(
    StandardScaler(), LassoLarsIC(criterion="aic", normalize=False)
).fit(X, y)

fit_time = time.time() - start_time
```

```
results = pd.DataFrame(
    {
        "alphas": lasso_lars_ic[-1].alphas_,
        "AIC criterion": lasso_lars_ic[-1].criterion_,
    }
).set_index("alphas")

alpha_aic = lasso_lars_ic[-1].alpha_
```

```
lasso_lars_ic.set_params(lassolarsic__criterion="bic").fit(X, y)
results["BIC criterion"] = lasso_lars_ic[-1].criterion_
alpha_bic = lasso_lars_ic[-1].alpha_
```

```
def highlight_min(x):
    x_min = x.min()
    return ["font-weight: bold" if v == x_min else "" for v in x]

results.style.apply(highlight_min)
```

alphas	AIC criterion	BIC criterion
0.8452906453888189	666.703705	666.703705
0.6814192343098944	543.475045	546.478991
0.6800720505260539	544.446544	550.454436
0.4580203253378148	398.357027	407.368866
0.1803493724673525	271.593594	283.609379
0.0501251064485714	246.803836	261.823567
0.0	245.292447	263.316125

b. Forward selection

```
def forward_selection(data, target, significance_level=0.05):
    initial_features = data.columns.tolist()
    best_features = []
    while (len(initial_features)>0):
        remaining_features = list(set(initial_features)-set(best_features))
        new_pval = pd.Series(index=remaining_features)
        for new_column in remaining_features:
            model = sm.OLS(target, sm.add_constant(data[best_features+[new_column]])).fit()
            new_pval[new_column] = model.pvalues[new_column]
        min_p_value = new_pval.min()
        if(min_p_value<significance_level):
            best_features.append(new_pval.idxmin())
        else:
            break
    return best_features
```

```
forward_selection(X,y)
```

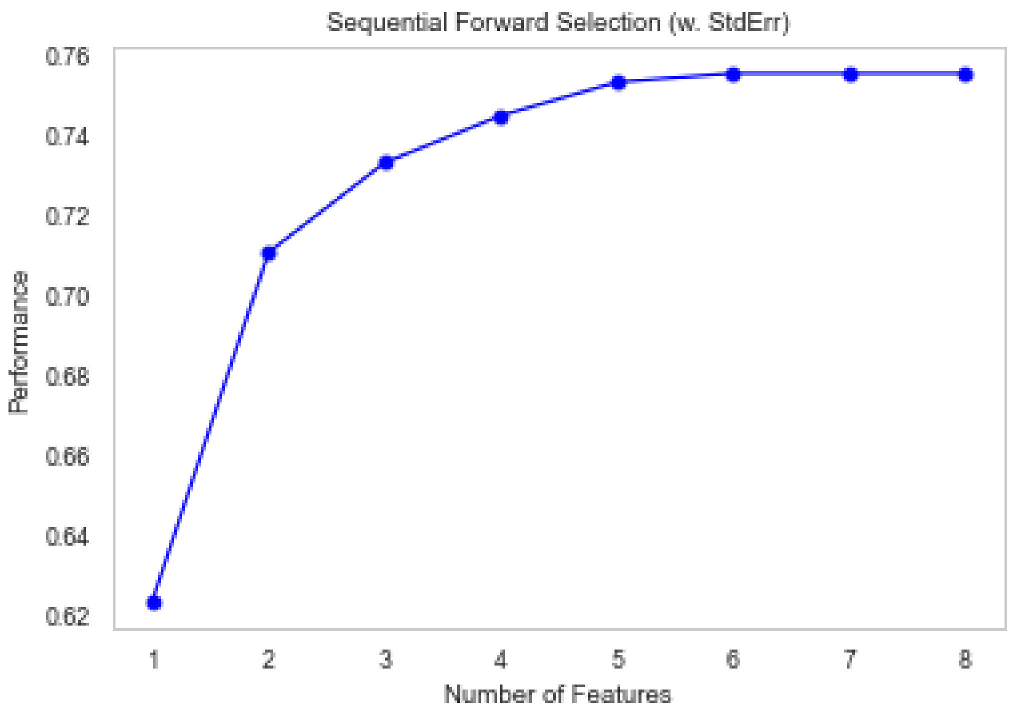
```
#importing the necessary libraries
import mlxtend
from mlxtend.feature_selection import SequentialFeatureSelector as SFS
from sklearn.linear_model import LinearRegression
# Sequential Forward Selection(sfs)
sfs = SFS(LinearRegression(),
          k_features=8,
          forward=True,
          floating=False,
          scoring = 'r2',
          cv = 0)

sfs.fit(X,y)
sfs.k_feature_names_
```

```
('Intercept',
 'LoggedGDPPerCapita',
 'SocialSupport',
 'HealthyLifeExpectancy',
 'FreedomToMakeLifeChoices',
 'Generosity',
 'PerceptionsOfCorruption',
 'LadderScoreInDystopia')
```

```
df_SFS_results = pd.DataFrame(sfs.subsets_ ).transpose()
df_SFS_results
```

	feature_idx	cv_scores	avg_score	feature_names
1	(1,)	[0.6237203782313991]	0.62372	(LoggedGDPPerCapita,)
2	(1, 4)	[0.7109512261766451]	0.710951	(LoggedGDPPerCapita, FreedomToMakeLifeChoices)
3	(1, 2, 4)	[0.7334321909172843]	0.733432	(LoggedGDPPerCapita, SocialSupport, FreedomToM...
4	(1, 2, 4, 6)	[0.7452626912764992]	0.745263	(LoggedGDPPerCapita, SocialSupport, FreedomToM...
5	(1, 2, 3, 4, 6)	[0.75363454970928]	0.753635	(LoggedGDPPerCapita, SocialSupport, HealthyLif...
6	(1, 2, 3, 4, 5, 6)	[0.7558471374226853]	0.755847	(LoggedGDPPerCapita, SocialSupport, HealthyLif...
7	(0, 1, 2, 3, 4, 5, 6)	[0.7558471374226855]	0.755847	(Intercept, LoggedGDPPerCapita, SocialSupport,...
8	(0, 1, 2, 3, 4, 5, 6, 7)	[0.7558471374226856]	0.755847	(Intercept, LoggedGDPPerCapita, SocialSupport,...

```
fig = plot_sfs(sfs.get_metric_dict(), kind='std_err')

plt.title('Sequential Forward Selection (w. StdErr)')
plt.grid()
plt.show()
```



c. Backward elimination

```
def backward_elimination(data, target,significance_level = 0.05):
    features = data.columns.tolist()
    while(len(features)>0):
        features_with_constant = sm.add_constant(data[features])
        p_values = sm.OLS(target, features_with_constant).fit().pvalues[1:]
        max_p_value = p_values.max()
        if(max_p_value >= significance_level):
            excluded_feature = p_values.idxmax()
            features.remove(excluded_feature)
        else:
            break
    return features
```

```
backward_elimination(X,y)
```

```
['Intercept',
 'LoggedGDPPerCapita',
 'SocialSupport',
 'HealthyLifeExpectancy',
 'FreedomToMakeLifeChoices',
 'PerceptionsOfCorruption',
 'LadderScoreInDystopia']
```

```
sbs = SFS(LinearRegression(),
         k_features=8,
         forward=False,
         floating=False,
         cv=0)

sbs.fit(X, y)
sbs.k_feature_names_
```

```
('Intercept',
 'LoggedGDPPerCapita',
 'SocialSupport',
 'HealthyLifeExpectancy',
 'FreedomToMakeLifeChoices',
 'Generosity',
 'PerceptionsOfCorruption',
 'LadderScoreInDystopia')
```

```
df_SBS_results = pd.DataFrame(sbs.subsets_ ).transpose()
df_SBS_results
```

	avg_score	cv_scores	feature_idx	feature_names
8	0.755847	[0.7558471374226856]	(0, 1, 2, 3, 4, 5, 6, 7)	(Intercept, LoggedGDPPerCapita, SocialSupport,...

d. Step-Wise

```python
def stepwise_selection(data, target,SL_in=0.05,SL_out = 0.05):
    initial_features = data.columns.tolist()
    best_features = []
    while (len(initial_features)>0):
        remaining_features = list(set(initial_features)-set(best_features))
        new_pval = pd.Series(index=remaining_features)
        for new_column in remaining_features:
            model = sm.OLS(target, sm.add_constant(data\
                                        [best_features+[new_column]])).fit()
            new_pval[new_column] = model.pvalues[new_column]
        min_p_value = new_pval.min()
        if(min_p_value<SL_in):
            best_features.append(new_pval.idxmin())
            while(len(best_features)>0):
                best_features_with_constant = sm.add_constant(data[best_features])
                p_values = sm.OLS(target, best_features_with_constant).fit().pvalues[1:]

                max_p_value = p_values.max()
                if(max_p_value >= SL_out):
                    excluded_feature = p_values.idxmax()
                    best_features.remove(excluded_feature)
                else:
                    break
        else:
            break
    return best_features
```

```python
stepwise_selection(X,y)
```

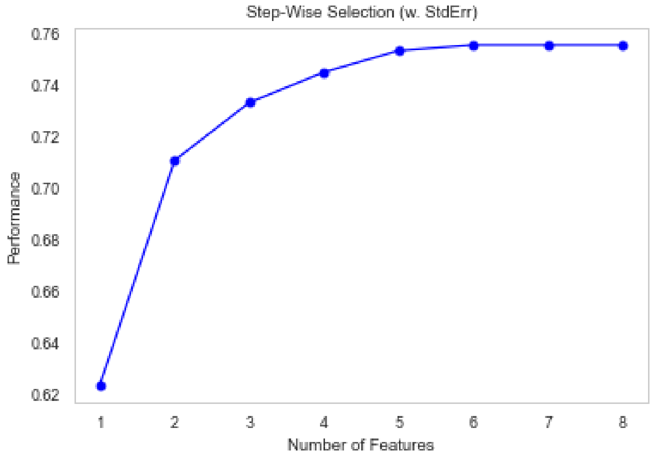
```python
sffs = SFS(LinearRegression(),
          k_features=8,
          forward=True,
          floating=True,
          cv=0)
sffs.fit(X, y)
sffs.k_feature_names_
```

```
('Intercept',
 'LoggedGDPPerCapita',
 'SocialSupport',
 'HealthyLifeExpectancy',
 'FreedomToMakeLifeChoices',
 'Generosity',
 'PerceptionsOfCorruption',
 'LadderScoreInDystopia')
```

```python
df_SFFS_results = pd.DataFrame(sffs.subsets_ ).transpose()
df_SFFS_results
```

	feature_idx	cv_scores	avg_score	feature_names
1	(1,)	[0.6237203782313991]	0.62372	(LoggedGDPPerCapita,)
2	(1, 4)	[0.7109512261766451]	0.710951	(LoggedGDPPerCapita, FreedomToMakeLifeChoices)
3	(1, 2, 4)	[0.7334321909172843]	0.733432	(LoggedGDPPerCapita, SocialSupport, FreedomToM...
4	(1, 2, 4, 6)	[0.7452626912764992]	0.745263	(LoggedGDPPerCapita, SocialSupport, FreedomToM...
5	(1, 2, 3, 4, 6)	[0.75363454970928]	0.753635	(LoggedGDPPerCapita, SocialSupport, HealthyLif...
6	(1, 2, 3, 4, 5, 6)	[0.7558471374226853]	0.755847	(LoggedGDPPerCapita, SocialSupport, HealthyLif...
7	(0, 1, 2, 3, 4, 5, 6)	[0.7558471374226855]	0.755847	(Intercept, LoggedGDPPerCapita, SocialSupport,...
8	(0, 1, 2, 3, 4, 5, 6, 7)	[0.7558471374226856]	0.755847	(Intercept, LoggedGDPPerCapita, SocialSupport,...

```python
fig = plot_sfs(sffs.get_metric_dict(), kind='std_err')

plt.title('Step-Wise Selection (w. StdErr)')
plt.grid()
plt.show()
```



三、 參考資料及工作分配表

1. 參考資料：

[Kaggle : World Happiness Expanatory Data Analysis]

[維基百科 ： 世界幸福報告]

[世界幸福報告官網]

2. 工作分配表：

	資料整理	Word 檔製作	程式碼	PPT 製作	報告
王常騰		✓			
溫宏岳	✓		✓		✓
張俊翔				✓	
陳曦	✓				
胡家宏	✓	✓			