Mathematical Modeling: Homework 1

1. (a) Compute the Gaussian integral

$$\int_{-\infty}^{\infty} \exp(-x^2)\ \mathrm{d}x.$$

   (b) Given $a > 0$, use dimensional analysis to show that the integral

$$\int_{-\infty}^{\infty} \exp(-ax^2)\ \mathrm{d}x$$

   must contain the factor $1/\sqrt{a}$.

2. The law governing the distance x an object falls in time t in a field of constant gravitational acceleration g with no air resistance is $x = \frac{1}{2}gt^2$.

   (a) How many independent dimensionless quantities can be formed from t, x, and g?

   (b) Rewrite the physical law in terms of dimensionless quantities.

   (c) Can the distance a body falls depend on the mass m as well? That is, can there be a physical law of the form $f(t, x, g, m) = 0$?

3. The force of air resistance F depends on the speed v, the cross-sectional area A, and the air density ρ. Suppose the physical law is governed by $g(F, v, A, \rho) = 0$. Use Pi theorem to find the physical law g.

4. In the blast wave problem we have derived that

$$r = \left(c\frac{Et^2}{\rho}\right)^{1/5}.$$

   Take $c = 1$ and use $\rho = 1.25$ kg/m^3. Some of the radius (m) vs. time (milliseconds) data for the Trinity explosion is given in the following table:

t	0.10	0.52	1.08	1.5	1.93	4.07	15.0	34.0
r	11.1	28.2	38.9	44.4	48.7	64.3	106.5	145

   Using these data, estimate the yield of the Trinity explosion in kilotons (1 kiloton equals 4.186×10^{12} joules). Compare your answer to the actual yield of approximately 21 kilotons.

5. An ecologist postulated that there is a relationship among the mass m, density ρ, volume V, and surface area S of certain animals. Discuss this conjecture in terms of dimensionless analysis.

6. In tests for fuel economy, cars are driven at constant speed V on a level highway. With no acceleration, the force of propulsion F must be in equilibrium with other forces, such as the air resistance, and so on. Assume that the variables affecting F are the velocity V, the rate C that fuel is burned (in volume per time), and the amount of energy K in a gallon of fuel (in mass per length per time-squared). Determine F as a function of V, C, and K.