MA3111: Mathematical Image Processing
Homework 2

To submit, zip the followings into mipXXXXX.zip where XXXXX is your student ID:

- **report file: contains answers to all questions, including the requested figures in Problem 2.**
- **sharpen_by_spatial_laplacian.m: code written for Problem 2.**

Problem 1: Filter by Laplacian

In this problem, you are given two Laplacian filters $k_i(x, y), i \in \{1, 2\}$ mentioned in class and three images $f_j(x, y), j \in \{1, 2, 3\}$. Compute 2-D discrete-time convolution $(k_i \star f_j)(x, y), \forall i \in \{1, 2\}, \forall j \in \{1, 2, 3\}$, i.e. perform a total of six convolutions.

Below we specify two Laplacian filters $k_1(x, y)$ and $k_2(x, y)$ by two tables. In each table the leftmost column and the bottom row specify y and x coordinates, respectively, and all other non-empty cells contain function values. The function has zero value at locations corresponding to empty cells and locations outside the range of the table.

Laplacian filter k_1:

1		1	
0	1	-4	1
-1		1	
	-1	**0**	**1**

Laplacian filter k_2:

1	1	1	1
0	1	-8	1
-1	1	1	1
	-1	**0**	**1**

Image of a vertical edge $f_1(x, y)$ for $(x, y) \in \mathbb{Z}^2$:

$$f_1(x, y) = \begin{cases} 1 & \text{if } x \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

Image of a diagonal edge $f_2(x, y)$ for $(x, y) \in \mathbb{Z}^2$:

$$f_2(x, y) = \begin{cases} 1 & \text{if } x + y < 0 \\ 0 & \text{otherwise} \end{cases}$$

$f_3(x, y)$ for $(x, y) \in \mathbb{Z}^2$:

$$f_3(x, y) = a + by + cx + dxy$$

where $(a, b, c, d) \in \mathbb{R}^4$ are any fixed real numbers.

Problem 2: Spatial domain Laplacian sharpening

Fill in the computation in `sharpen_by_spatial_laplacian.m`, where you convolve the input image `blurry-moon.tif` with the following Laplacian filter discussed in class.

1	1	1	1
0	1	-8	1
-1	1	1	1
	-1	**0**	**1**

Do not call any of MATLAB's built-in functions like `conv(2)` and `filter(2)` to do the convolution. The output image will be

$$g(x, y) \approx f(x, y) + c\nabla^2 f(x, y).$$

For simplicity, you can keep the size of the image unchanged, and don't alter the first and last columns and rows of the image.

Try $c = 1$ and $c = -1$, and attach the corresponding output images in the report. For the best visual result, display the output image using `imshow(im)`.