Study Guide For 33-142 Physics 2

Professor Ghosh

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General Knowledge

What you should know (This doesn't include all the equations from the formula sheet, just the important ones):

Vectors Example: $\vec{E} = \frac{q}{4\pi\epsilon|r|^3}\vec{r}$
An abstract tool useful for physics. It has a direction and magnitude, so you can use vectors to describe anything that depends on direction, such as electric fields.

Unit Vectors Example: $\hat{r} = \frac{\vec{r}}{|\vec{r}|}$
A vector with magnitude 1. This is quite important when describing direction. In fact, our definition of the electric field above can be condensed to: $\vec{E} = \frac{q}{4\pi\epsilon r^2}\hat{r}$

The "r" vector Just a vector describing radial direction. Conveniently, this is also $\vec{r} = r < cos(\theta), sin(\theta) >$

Coulomb (C) SI unit of measurement for charge

Epsilon (ϵ) Constant of dielectric permitivity. Larger epsilons lead to... You'll find out later in class :)

Useful Constants

$Q_{\text{proton/electron}} = \pm 1.60 * 10^{-19}\text{C}$

$\frac{1}{4\pi\epsilon_0} = 8.99 * 10^9 N \cdot m^2 \cdot C^{-2}$ Note that ϵ_0 is used here because this refers to the constant of dielectric permitivity in a vacuum (which we will frequently use)

$\hat{i}, \hat{j}, \hat{k}$ = x, y, z unit vectors: $< 1, 0, 0 >, < 0, 1, 0 >, < 0, 0, 1 >$

Important Concepts

Coulomb's Law: $\vec{F}_{\text{Coulomb}} = \frac{1}{4\pi\epsilon}\frac{Q_1Q_2}{|r|^2}\hat{r}$

Superposition Principle: A principle that describes that the net effect at a point in space is the net sum of all the individual effects from all sources at that point. This can be simplified to our description of electric fields. The net electric field at a point is the sum of all individual electric fields at that point: $\vec{E}_{net} = \sum_i \vec{E}_i$.

Conservative System: In a conservative system, all energy is conserved. This also means that there are no external forces involved, since the source and effect is all contained within the system. For example, a system that has 2 charges and only has the fields due to those two charges is conservative. However, if you enclose the effects of both but only one of those charges, then the electric field due to the charge not in your system is an "external" effect.

Useful Equations

Dot Product: Say $\vec{u} =< u_x, u_y >, \vec{v} =< v_x, v_y >$, then $\vec{u} \cdot \vec{v} = u_x * v_x + u_y * v_y = |\vec{u}||\vec{v}|cos(\theta)$

Cross Product: Say $\vec{u} =< u_x, u_y, u_z >, \vec{v} =< v_x, v_y, u_z >$, then

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} = (u_y v_z - v_y u_z)\hat{i} - (u_x v_z - v_x u_z)\hat{j} + (u_x v_y - v_x u_y)\hat{k}$$

The magnitude of the cross product is given as $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|sin(\theta)$

Magnitude of vector: Say $\vec{v} =< v_1, v_2, v_3, v_4, ...v_n >$. Then, $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2 + ...v_n^2}$

Taylor Expansion of Power Series (Binomial Expansion): $(1 + x)^{\alpha} = 1 + \alpha x$ when $x << 1$

Vectors!

So one of the most important and useful concepts in many fields, most notably physics and mathematics, is the concept of vectors. When we call something a vector $\vec{v}$, then we mean that "v" has both a magnitude, or value like speed, and direction. Basically, we give this vector a length, or value, equal to the value of "v". Here comes to play another extremely important concept in vectors. That is, since a vector is just an arrow, we can describe this vector using 2 other perpendicular vectors. This is called "decomposing" a vector. There are infinitely many ways to decompose a vector, which allows us to deal with so many situations, such as a gravitational force vector decomposed along the inclined plane. Generally, we like to represent our vectors in the Cartesian plane, or $< x_{\text{val}}, y_{\text{val}} >$.

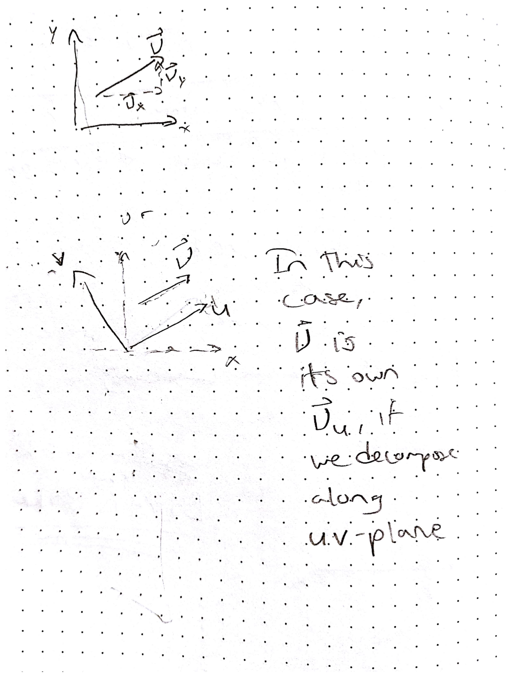

Figure 1: This image represents how you can decide whatever plane you wish to decompose the vector along. It's important to recognize that the uv-plane can be any arbitrary plane.

Now suppose I have some arbitrary vector that I know the magnitude of and direction of. But, consider I only know the direction is 30 degrees north of the positive x axis. This means that my vector should have a 30 degree angle with the x axis and it should point diagonally up-right. So, we have a vector pointing in this direction. Now, if you recall from trigonometry, we can decompose this vector so that we have an x and y component. With a 30 degree angle with the x axis, our x component is like the base of the triangle, or val $* cos(30°)$. Then our y component is val $* sin(30°)$. Then, we can represent a vector as its x and y components, as mentioned at the end of the first paragraph: $\vec{V} =< Vcos(\theta), Vsin(\theta) >$. You may see another notation: $\vec{V} = Vcos(\theta)\hat{i} + Vsin(\theta)\hat{j}$.

Verification and Approximation

In physics, an extremely important issue when dealing with new equations or formulae is the necessity to be able to verify its validity. However, if an equation is not immediately clear, there are several ideas that we apply within electricity and magnetism to understand an equation. These are all incorporated within the context of the next section.

Does it Make Sense?

The idea of whether or not an equation makes sense is quite general. But, you should know several ideas when approaching this question. One is utilizing dimensional analysis. This is a useful tool because if your equation does not have the right units, it clearly cannot be correct. By representing each variable as its SI unit form, you can perform the dimensional analysis:

Example:

$$\begin{aligned}\vec{E} =& \frac{1}{4\pi\epsilon}\frac{2Qd}{|r|^3}\hat{r}\\ =& \frac{(C)^1(m)^1}{m^3}\\ =& (C)^1(m)^1(m)^{-3}\\ =& (C)^1(m)^{-2} \quad \checkmark\text{Verified}\end{aligned}$$

So, clearly, the equation for the dipole electric field (along axis) surely holds correct units. At this point, one may think that it is more beneficial to just verify it by re-deriving the equation. However, there are several issues with this. One, it is not always true that you will encounter easily-derivable equations. In addition, deriving an equation can still lead to errors, which will not change the issue.

Following this, let us discuss the next method of evaluation. In physics, especially electricity and magnetism, the use of extremeties is a great way of understanding the equation's validity. This is because while we cannot necessarily understand the more complex picture, by going to extremities such as ∞, we can visualize and make sense of an equation.

Example:

$$\vec{E}_{\text{ring}} = \frac{1}{4\pi\epsilon}\frac{Q}{2\pi R}\frac{2\pi Rz}{(R^2+z^2)^{3/2}}\hat{z}$$

At first, the ring equation is very hard to visualize or interpret. However, let us imagine condensing the ring into a singular point, then it should behave as a point charge. In our equation, this means we let the radius R approach 0.

$$\begin{aligned}\vec{E}_{\text{ring}} &= \frac{1}{4\pi\epsilon}\frac{Qz}{(0+z^2)^{3/2}}\hat{z}\\ &= \frac{1}{4\pi\epsilon}\frac{Qz}{z^3}\hat{z}\\ &= \frac{1}{4\pi\epsilon}\frac{Q}{z^2}\hat{z} \quad \checkmark\text{Verified}\end{aligned}$$

Superposition Principle

The superposition principle is one of the most beautiful concepts in physics. It is very hard to convey just how amazing and versatile it is without some examples. To begin with, the simple idea of the superposition principle (Let us discuss this in the context of an electric field for simplicity sake) is that net electric field at any point is the sum of all individual electric fields at that point:

$$\vec{E}_{\text{net}} = \sum_i \vec{E}_i$$

This also applies to other ideas such as charges:

$$Q_{\text{Total}} = \sum_i Q_i$$

Now charges are not exactly vectors, but charges emanate electric fields, so the sum of all charges should be the total charge. Now, we can demonstrate just how beautiful the superposition principle is. Consider a very convoluted problem: I have a circle of uniform charge distribution σ but I cut out 2 adjacent holes in it. What is my electric field along the x axis?

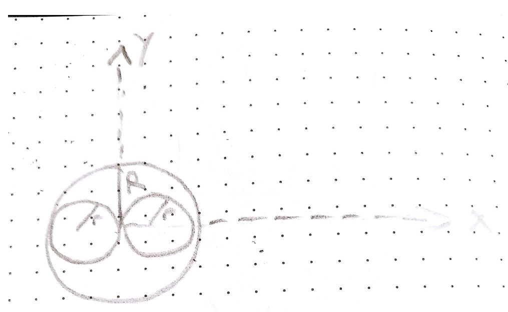

Figure 2: A visualization of the problem. The inner holes are of radius r, and the rest of the circle is filled with charge.

On its own, this problem may seem beyond the scope of our understanding. There is symmetry, but it is very convoluted. However, we know that the empty circles technically have 0 charge and the rest as some charge based on our charge density. So, we utilize our idea in the superposition principle. The effects at any point is the sum of all the effects. Now, we have 0 charge in the holes. Alone, this shape is hard to integrate over. But, we can imagine this shape as the sum of a full circle of positive charge and the sum of 2 smaller circles of negative charge. Why can we do this? Let us explain through the charge interpretation. It is as if we are adding positive charge in those circles. But at the same time, we are getting rid of those charges by creating 2 circles of negative charge (equal and opposite). So, our superposition principle says we still have the same situation.

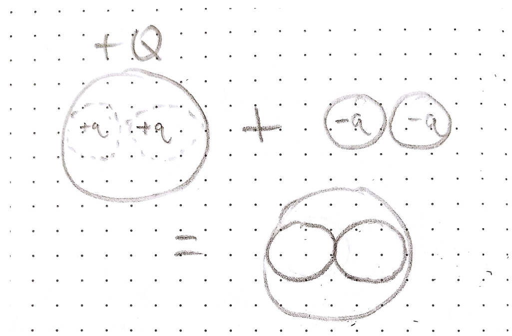

Figure 3: We can understand the holes as just being occupied simultaneously by equal and opposite charges. So, combining the positive charges with the uniform density, we have a full circle and combining the negative charges of the same density, we have 2 negatively charged circles.

So all you do is just add the electric field from the circle plus the electric field from the negative circles (position in space still matters, so it's not simply 2 times a negative circle's electric field centered at the origin). But just notice how beautifully superposition principle allows us to approach problems.

Week 1: Charges and Fields

If you wish to just learn the equations and general concepts, I'd recommend just the first page of this study guide, since it provides that. If you wish for important concepts that will aid you in thinking, then the Week by Week pages will be useful. I should have a section on vectors if that is a concept you wish to review.

Primarily, let us quickly discuss the importance of understanding the concept of an electric field. While Coulomb's Law is also very important, an important concept that arises from this class is the concept of fields. As Ghosh said, if previously there was no charge, but suddenly there is, there is suddenly a changing of the space around it. When I say the space changes, I mean that there is a field that other charged particles in the space will feel effects from. So, when we talk about the Coulomb field, we talk about the field due to a charged particle. And just like the gravitational field, electric fields due to singular charges are radial.

Interesting question: If I have 1 charge then suddenly introduce another charge a light year away, would the effect of the distant charge be instantaneous or take time? If it takes time, how long should it take?[1]

Recall that we learned what an electric field is $\vec{E}$ and Coulombs Law, $\vec{F}_{\text{Coulomb}} = \int \vec{E}\,\mathrm{d}q$. This notation is simply restating the idea of the superposition principle. In fact, we can describe $\vec{E} = \frac{1}{4\pi\epsilon}\int \frac{\mathrm{d}q}{|r|^2}\hat{r}$. When we say $\vec{E} = \frac{1}{4\pi\epsilon}\int \frac{\mathrm{d}q}{|r|^2}\hat{r}$, we are saying that what is the net electric field at this point due to all charges. However, in order for the integral to be practical, you should generalize the form. This is quite useful for more convoluted problems. However, when you deal with physical point charges, it's more useful to utilize the general principle (superposition): $\vec{E}_{\text{net}} = \sum_i \vec{E}_i$. This is why you shouldn't ever need to use an integral to deal with problems like Ghosh's example: 4 charges are placed at the edges of a box, and find the force on the charge at the bottom left corner (Which can either be envisioned as the force on that charge due to each of the other charges in the box or the net electric field due to the other charges in the box times the charge you care about).

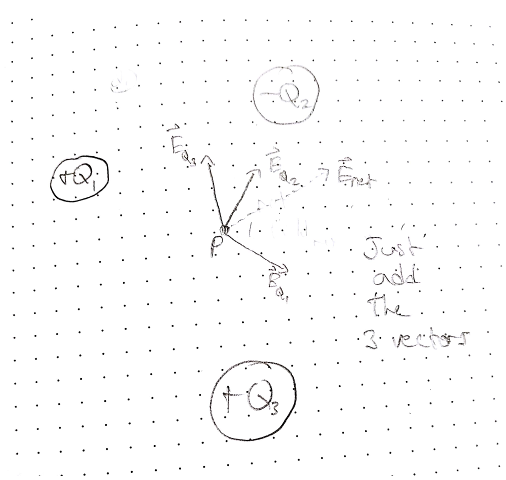

Figure 4: This image represents the idea of superposition. As you can see, the electric field at any point can be represented by the sum of the electric field of every individual electric field at that point (in this case, the sum of electric fields due to individual charges).

[1] Electric fields are essentially a result of travelling photons, so we should expect it to take 1 year for our charge to experience the effect of the distant charge

Extra Mini Subsection: Dipoles

Another very important concept in physics is the concept of a dipole. Why is this important? Well, as you'll later learn, many induced charge distributions in materials can be generalized to be a dipole, which tremendously simplifies the problem. In physics, we aim to turn something convoluted into something very simple and beautiful. Now, for a dipole, the general question is, how do I determine the electric field of a dipole. What's even more interesting is that the net charge of a dipole is 0 Coulombs, since there is a positive and negative charge contained in a dipole. However, even still, there should still be a net electric field. To calculate this, we apply the superposition principle again. Calculate the electric field due to one charge and add it to the electric field due to the other charge. For the purposes of this class, we only should be able to determine the electric field along the dipole's axis and the electric field along the perpendicular axis stemming from its midpoint.

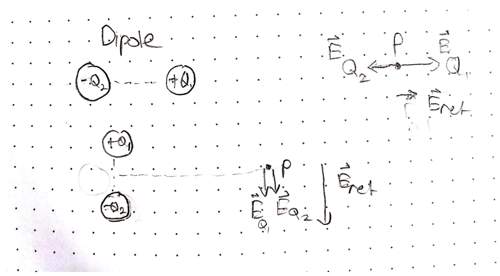

Figure 5: This image demonstrates the general idea you should have when deriving the electric field due to a dipole.

So, let us proceed by the derivation. Call our distance between the two poles of the dipole to be "d" and the absolute value of the charge of each pole to be "Q". We also want to center our origin to be the center of the dipole, and r is my distance from the center to the point P (Let $r >> d$). Lastly, I am also calculating the electric field along the positive x axis as per the image above. For this study guide, I will only be deriving the along axis electric field. Let the perpendicular problem be something for you to think about or do on your own time. Proceeding, we can describe the electric field:

$$\vec{E}_{Q_1} = \frac{1}{4\pi\epsilon}\frac{Q}{(r-\frac{d}{2})^2}\hat{x} \tag{1}$$

$$\vec{E}_{Q_2} = \frac{1}{4\pi\epsilon}\frac{-Q}{(r+\frac{d}{2})^2}\hat{x} \tag{2}$$

Now, we apply the superposition principle:

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\epsilon}\frac{Q}{(r-\frac{d}{2})^2}\hat{x} + \frac{1}{4\pi\epsilon}\frac{-Q}{(r+\frac{d}{2})^2}\hat{x} \tag{3}$$

$$= \frac{1}{r^2}\left(\frac{1}{4\pi\epsilon}\frac{Q}{(1-\frac{d}{2r})^2} + \frac{1}{4\pi\epsilon}\frac{-Q}{(1+\frac{d}{2r})^2}\right)\hat{x} \tag{4}$$

Why did I pull out an r^2 from the denominator? Since my $r >> d$, I know that $\frac{d}{2r} << 1$. So, I can apply the Binomial Expansion. Proceeding, we have:

$$\begin{aligned}
\vec{E}_{\text{net}} =& \frac{1}{4\pi\epsilon}\frac{1}{r^2}((Q)(1-\frac{d}{2r})^{-2} + (-Q)(1+\frac{d}{2r})^{-2})\hat{x} && (5)\\
=& \frac{1}{4\pi\epsilon}\frac{1}{r^2}((Q)(1+\frac{d}{r}) + (-Q)(1-\frac{d}{r}))\hat{x} && (6)\\
=& \frac{1}{4\pi\epsilon}\frac{1}{r^2}(2Q(\frac{d}{r}))\hat{x} && (7)\\
=& \frac{1}{4\pi\epsilon}\frac{2Qd}{r^3}\hat{x} && (8)
\end{aligned}$$

Recall that this applies to the situation described above. We can generalize this idea by creating a dipole moment vector, defined $\vec{\rho} = Q\vec{d}$, where $\vec{d}$ points from the negative pole to the positive pole of the dipole. This pole direction is the direction of the electric field along its axis around the dipole (Where the electric field between the poles is simply from the positive end to the negative end (opposite direction)). Then our electric field along the axis is condensed to:

$$\frac{1}{4\pi\epsilon}\frac{2\vec{\rho}}{r^3}$$

Now I want to go back to discussing the mathematical applications of integrals to physics. As was previously discussed, we can represent our electric field as

$$\vec{E} = \frac{1}{4\pi\epsilon}\int \frac{\mathrm{d}q}{|r|^2}\hat{r}$$

The idea behind this description of the electric field is that I want to sum up all the effects of every infinitesimal small piece of charge dq. However, you cannot simply integrate on dq because every dq may have a different position, leading to a dependence on the r. In many cases, you can represent dq by d$q = \rho * dV$. ρ would be the charge density as Q/V, or charge per volume. However, depending on the symmetry, you can have a different representation for that charge distribution. For example, in a thin rod of charge Q and length L, you can utilize the charge distribution in terms of its length (since this rod is thin and uniform and symmetric), $\rho = Q/L$. By understanding this, you can apply this to many problems. One example I will cover is the electric field due to an infinite plate of charge of uniform charge density ρ. Note that just because it has a fixed density does not mean that it will have infinite charge because $\rho * \infty$ makes no physical sense (we are doing physics, after all. So infinity is purely a mathematical tool). Thus, let me describe the electric field due to each individual charge at a distance d from the plate:

$$\vec{E} = \frac{1}{4\pi\epsilon}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\rho}{|r^2|}\hat{r}\,\mathrm{d}A$$

This expression is a tad too complicated for our purposes, since it involves understanding of the double integral. However, we can apply an interesting symmetry trick. A plane of charge is simply an infinite amount of infinite rods on top of each other. So, we can apply our expression for the infinite rod here and integrate along only one direction. Let us choose our rods to be lined up so that they line up along the z axis:

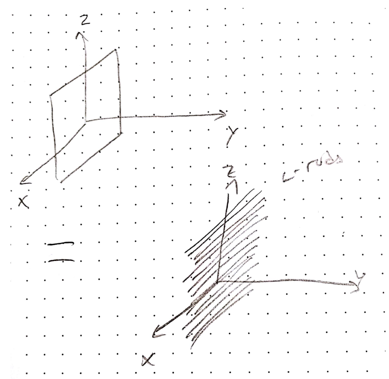

Figure 6: The image is a graphical representation of this method of integrating along an infinite plane. Understand that there are many ways of approaching this problem (i.e. you can integrate along an **infinite** plane by splitting it into an infinite number of concentric rings.

Our expression for the electric field due to an infinite rod is

$$\vec{E}_{infrod} = \frac{1}{4\pi\epsilon}\frac{2\rho}{r}\hat{r}$$

Recall that ρ is the same as $\frac{\mathrm{d}q}{\mathrm{d}A}$. So we still need to integrate along the z direction. We can apply another interesting thing about symmetry. Since the rods are infinite, we can imagine each rod as a point charge having the electric field of the rod, each charge placed along the midpoint. Then, our r is very simple: $r = \sqrt{y^2 + d^2}$, where there is no x dependence. Finally, let us rewrite our expression:

$$\vec{E} = \frac{1}{4\pi\epsilon}\int_{-\infty}^{\infty}\frac{2\rho}{\sqrt{y^2+d^2}}\hat{r}\,\mathrm{d}y$$

We also know that our $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{1}{\sqrt{y^2+d^2}} < d,\ y >$. However, again, because of symmetry, we have that our y components should cancel out since the plane is symmetric along positive and negative y axis. So, we only care about the z component: $\vec{z} = \frac{d}{\sqrt{y^2+d^2}}\hat{z}$. Let us once again rewrite our integral:

$$\vec{E} = \frac{1}{4\pi\epsilon}\int_{-\infty}^{\infty}\frac{2\rho d}{y^2+d^2}\,\mathrm{d}y$$

You can then use trigonemtric substitution. As a result of some calculations, the expression condenses to:

$$\Big|_{-\pi/2}^{\pi/2} 2\rho\theta = \frac{\rho}{2\epsilon}$$

Next, we will discuss a brief introduction to Gauss' Law. A more in depth teaching can be found in the section called "Maxwell's Equations". Here, we will spend more time discussing particular ways of approaching problems. Gauss' Law states that the total electric field flux through a surface is equivalent to the charge enclosed. The simpler equation is as given:

$$\oiint_{\mathrm{d}S}\vec{E}\cdot\hat{n}\,\mathrm{d}S = \frac{1}{\epsilon}\sum_i q_i$$

To find the charge enclosed, you should understand what this means. If I have a charge density and I construct a surface, you should know that to find the charge enclosed, you should integrate through using the density, since the density represents charge per volume (traditionally). However, for non 3-dimensional problems, you may find these densities to represent charge per area or even charge per length. In addition, you should also understand that when i say $\oiint_{\mathrm{d}S} \vec{E} \cdot \mathrm{d}A$, I am saying I want the electric flux through my **entire** surface (If you want a better understanding of this, refer to the "Maxwell's Equations" section). This integral is very difficult to integrate over. However, looking at this, you may realize that you need to understand how the electric field looks in order to approach this problem. After all, even if you have the charge enclosed, if you don't know how the electric field looks, how can you either evaluate the integral or simplify the integral to solve for the electric field? A prime example can be given by calculating the electric field due to an infinite plate. We know that the electric field should look horizontal everywhere and have the same magnitude along the vertical (it's infinite, after all). So, if we want to find the electric field a distance x from the plate, we should construct a surface that not only encloses charge but also has a constant electric field going through all its faces:

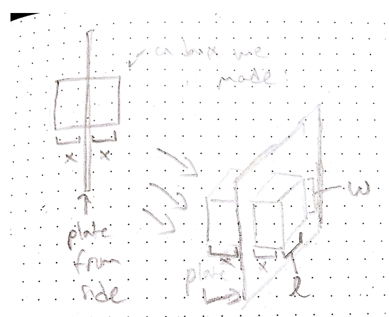

Figure 7: The image is a little blurry, but it represents the box that we constructed around our plate. The box is our surface. As you can see, it stretches out by x on both sides and has a cross-sectional length of l and height of w.

Now we can use Gauss' Law. Let's say our charge density is σ, as charge per area. Since our plate is infinitely thin, our charge enclosed is from the part of the plate that we are boxing in, which is just $l \times w$. So our charge enclosed is $\sigma * l * w$. So our righthand side is just

$$\frac{1}{\epsilon} \sum_i q_i = \frac{\sigma l w}{\epsilon}$$

Now our surface is truly useful. Because we know that the electric field should be constant along the vertical since it's infinite, and should also be horizontal everywhere since it's infinite, we can simplify the integral on the left side. So, since the integral on the left is the surface integral, we have to do it for every face of our box. Through every face except the left and right face, we clearly should have no electric field flux. After all, we know the electric field is horizontal everywhere, so the dot product of the horizontal electric field with the perpendicular area vectors of those faces should be 0 (Remember, area vectors, or $\hat{n}\,\mathrm{d}A$) is a vector representing $\mathrm{d}A$ pointing perpendicular to and out of the surface). So, we are left with having to calculate the electric flux through the left and right faces. But, we know that since the plate is symmetric, it should have the same electric field a distance x from either side, so the electric field should be constant through both faces. Notice also that the electric field going through the left and right faces are perpendicular to those faces, so our equation simplifies:

$$\oiint_{\mathrm{d}S} \vec{E} \cdot \hat{n}\,\mathrm{d}A = E * 2A.$$

You may ask, why does our equation simplify like that? Basically, since our electric field is constant based on our box. So, you can properly see it through this derivation (S_1 is left face and S_2 is right face. Also there are 6 faces total since we made a box):

$$\begin{aligned}\oiint_{\mathrm{d}S} \vec{E}\cdot\hat{n}\,\mathrm{d}A &= \iint_{S_1} \vec{E}\cdot\hat{n}\,\mathrm{d}A + \iint_{S_2} \vec{E}\cdot\hat{n}\,\mathrm{d}A + \iint_{S_3} \vec{E}\cdot\hat{n}\,\mathrm{d}A + \iint_{S_4} \vec{E}\cdot\hat{n}\,\mathrm{d}A + \iint_{S_5} \vec{E}\cdot\hat{n}\,\mathrm{d}A + \iint_{S_6} \vec{E}\cdot\hat{n}\,\mathrm{d}A \\ &= \iint_{S_1} \vec{E}\cdot\hat{n}\,\mathrm{d}A + \iint_{S_2} \vec{E}\cdot\hat{n}\,\mathrm{d}A = \iint_{S_1} E\,\mathrm{d}A + \iint_{S_2} E\,\mathrm{d}A \\ &= E(\iint_{S_1} \mathrm{d}A + \iint_{S_2} \mathrm{d}A)\end{aligned}$$

$E \cdot \hat{n}$ for S_1 and S_2 is just E because for the left and right faces, they are pointing in the same direction (electric field points outwards horizontal, and area vectors also point outwards), so the dot product is just as if you multiply their magnitudes. In addition, as I said before, the electric field is constant here so we can just pull it out. The expression: $\iint_{S_1} \mathrm{d}A$ is the same as asking for the area of that face. Continuing on, we should now finalize our calculations. We know that our area of those two faces is simply $A = lw$. Thus, our expression condenses to:

$$E * 2lw = \frac{\sigma lw}{\epsilon}$$

Notice that we have beautifully simplified our originally complex looking Gauss' Law to a simple equation. Now, we can solve the the electric field at a distance x from it (We don't define x because we want to calculate the electric field for any arbitrary x from the plate). So, our final expression for electric field is:

$$E = \frac{\sigma}{2\epsilon}$$

Notice how it is the same as the way we calculated it earlier by performing the very convoluted integral

$$\vec{E} = \frac{1}{4\pi\epsilon}\int_{-\infty}^{\infty} \frac{2\rho d}{y^2 + d^2} dy$$

Now, I will discuss the idea behind conductors. The basic idea of a conductor is that a material that is a conductor will have electrons that are free to move. In addition, know that every metal is a conductor. So, what does it mean for electrons to be able to move freely? This means that if I apply an electric field on those electrons, they will move, creating a charge distribution that will cancel out the electric field. But, why does it cancel? Think of it like this: If I have an electric field, surely my electrons will move. But, by moving, the net charge must remain constant, so there will be some positive charge on the opposite end. If my electric field is 0, I wont accelerate.:

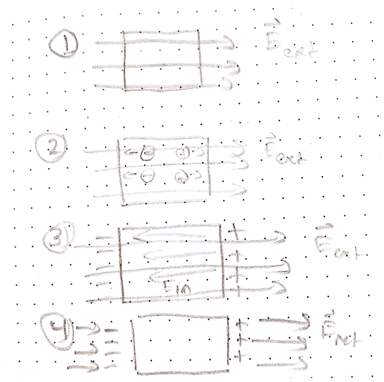

Figure 8: This image demonstrates the general process by which a metal maintains a 0 net electric field inside. The charges move and build up a net charge along the surface that opposes the external field

Note that the movement of the charge creates a charge distribution on the surface of the metal. Note that this is the net representation. In reality, I still have negative and positive charges in the metal itself. However, they all cancel, so the net effect is as if I only have charge on the surface. This charge build up will keep building up as long as my net electric field inside is non-zero, so it will eventually cancel out. Note, the charge on either side must be the same because the net charge should still be 0 (since my metal was neutrally charged to begin with). This also means that if I had a net charge Q, the sum of my surface charges must have the net charge Q. After all, my charges have only moved. I haven't added or taken away any charge by applying my external electric field.

Interesting result: If you want to think of energy, you may wonder: If my external field is gone, didn't I lose energy? That energy has been distributed along the outside. If you look at the distribution, you may notice that it is like a dipole, where the electric field outside of the metal due to this distribution is the same direction as the external field. As a result, we have a stronger electric field outside.

Speaking of energy, that leads to the final point in Week 1. That is, energy and electric potential (voltage). There is a very special feature of coulomb force and gravitational fields. That is, they are conservative forces. When I say conservative, I mean that if I were to change my position, I can always return that energy by reversing the process. Aka, if I move from point A to point B, then from point B to point A, I should have a 0 net change in energy. In addition, all Coulomb electric fields (fields due to charges) are conservative. This not only means what we mentioned earlier with point A and point B, but also that no matter what path I take from point A to point B, I will have the same change in energy. With electric fields, we define the electric potential, or energy per charge, as the voltage. This is defined by:

$$V = -\int_A^B \vec{E} \cdot \mathrm{d}\vec{s}$$

The equation represents the idea that along my path, I take the dot product of the electric field and the direction I am moving (essentially says I want to multiply the part of my field along my path by the direction). Notice its similarity to the definition of energy: $E = \int \vec{F} \cdot \mathrm{d}\vec{s}$. It is essential to realize that electric potential only carries meaning when you describe its potential in its difference with respect to its potential at another point. This is why we define it with the integral.

Now, another important concept is the idea of reference points. When I say reference point, I say: what is my potential with respect to that point. This is also equivalent to treating that reference point as having 0 potential. You may question: But if I use the equation and do $\int_A^B \vec{E} \cdot \mathrm{d}\vec{s}$, clearly I will end up with $V(B) - V(A)$, and $V(A)$ may not necessarily be 0. However, if I say my refernce point is A, the equation can be looked at in this form:

$$(V(B) - V(A)) - 0$$

If you notice, with this representation, it is saying, if I take my voltage at A to be 0, what is my voltage at B? It must be the voltage with respect to, hence $V(B) - V(A)$. Another important point about reference points. Choose your reference points wisely. After all, for any problem, you should define your reference point and do all calculations with the same reference point.

Week 2

Week 3

Week 4

Week 5

Week 6

Maxwell's Equations

This section goes in depth in explaining the implications and intuitions for understanding and using Maxwell's Equations. First, I will write down the 4 equations governing electric and magnetic fields. Note: **Boldfaced** variables simply mean that they're vectors:

$$\begin{aligned}
\oiint_{\partial\Omega} \mathbf{E}\cdot\mathbf{n}\,\mathrm{d}S &= \frac{1}{\epsilon}\iiint_{\Omega}\rho\,\mathrm{d}V \\
\oiint_{\partial\Omega} \mathbf{B}\cdot\mathbf{n}\,\mathrm{d}S &= 0 \\
\oint_{\partial\Sigma} \mathbf{E}\cdot\mathrm{d}\mathbf{s} &= -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma}\mathbf{B}\cdot\mathrm{d}S \\
\oint_{\partial\Sigma} \mathbf{B}\cdot\mathrm{d}\mathbf{s} &= \iint_{\Sigma}\mathbf{J}\cdot\mathrm{d}\mathbf{S} + \frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma}\mathbf{E}\cdot\mathrm{d}\mathbf{S}
\end{aligned}$$

Gauss' Law for Electric Fields

$$\oiint_{\partial\Omega} \mathbf{E}\cdot\mathbf{n}\,\mathrm{d}S = \frac{1}{\epsilon}\iiint_{\Omega}\rho\,\mathrm{d}V$$

When we learn physics, we don't aim to just learn to utilize and memorize the equations. Rather, by understanding intuitively how the concepts are incorporated and how the different operators in the equation work together, you can understand exactly what is being modeled, which also allows you to be able to construct models for more situations.

With that being said, let us discuss Gauss' Law. The expression is very complex, but let us dumb it down to a very simple series of explanations. When we see a loop in the double integral, that means a closed surface integral. This is the same as saying: I want to find the flux through the entire surface (all faces). In the expression, we have

$$\mathbf{E}\cdot\mathbf{n}\,\mathrm{d}S$$

Know that $\mathbf{n}$ is the unit vector pointing orthogonal (perpendicular) to the surface. In addition, when we do a dot product $\mathbf{E}\cdot\mathbf{n}$, we are saying what is the component of the electric field going perpendicular to the surface. This is multiplied by $\mathrm{d}S$, or a small piece of area on the surface. So we can visualize this as similar to saying

$$\mathbf{E}\cdot\mathrm{d}\mathbf{A}$$

which we know to be the definition of the electric flux. Then, the left hand side of Gauss' Law simply says: I want to calculate the entire electric flux through a closed surface. In Gauss' Law, this closed surface is the surface you construct. On the right side, we have

$$\frac{1}{\epsilon}\iiint_{\Omega}\rho\,\mathrm{d}V$$

ρ is our charge density, so by doing a triple integral over our surface, we are saying: What is the charge enclosed by our surface? So the main idea of Gauss' Law is that it relates the electric flux to the charge enclosed. Notice that to utilize Gauss' Law, you should know what the electric field looks like. This is so that you can know how to use ideas like symmetry to get a simple surface where the electric field goes through at a constant angle (hopefully perpendicular) and is constant. If it is constant, then the integral expression can be eliminated if you know the surface area of your surface.

Gauss' law for Magnetic Fields

$$\oiint_{\partial\Omega} \mathbf{B} \cdot \mathbf{n}\, \mathrm{d}S = 0$$

Looking at the left side, this is simply the same expression for Gauss' Law for electric fields except replaced with magnetic flux. However, notice how the right hand side is 0. When we think about magnetic fields, we will always have loops. After all, unlike electric fields, magnetic fields always have a north and south pole, so the magnetic field cannot go off indefinitely without ever returning. As a result, when we talk about the closed surface integral, or the flux through the entire surface, we include both the magnetic field going out and the magnetic field coming back. These always cancel out (unless there are magnetic monopoles[2]), which leads to the 0.

[2]Magnetic Monopoles are an active field of research. In fact, this version of Gauss' Law indicates that if there were monopoles, we would have a non-zero magnetic flux. There is no evidence to suggest that magnetic monopoles cannot exist.