Inequality and Inequity in Network-based Ranking and Recommendation Algorithms

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ABSTRACT

Though algorithms promise many benefits including efficiency, objectivity and accuracy, they may also introduce or amplify biases. Here we study two well-known algorithms, namely PageRank and Who-to-Follow (WTF), and show under which circumstances their ranks produce *inequality* and *inequity* when applied to directed social networks. To this end, we propose a **d**irected network model with **p**referential **a**ttachment and **h**omophily (DPAH) and demonstrate the influence of network structure on the rank distributions of these algorithms. Our main findings suggest that (i) inequality is positively correlated with inequity, (ii) inequality is driven by the interplay between preferential attachment, homophily, node activity and edge density, and (iii) inequity is mainly driven by homophily. In particular, these two algorithms amplify, replicate and reduce inequity in top ranks when majorities are homophilic, neutral and heterophilic, respectively. Moreover, when inequity is amplified, minorities may improve their visibility in the rank by connecting strategically in the network. For instance, by increasing their homophily when majorities are also homophilic. These findings shed light on social and algorithmic mechanisms that hinder equality and equity in network-based ranking and recommendation algorithms.

Introduction

Online social networks and information networks have become integral parts of our everyday life. However, the opportunities offered by such networks are often constrained not only by our previous interactions[1–5], but also by algorithms. For instance, algorithms could make some people or content more visible than others via ranking or recommendations[6]. In this regard, search engines and recommender systems are increasingly used for various applications such as whom to follow, whom to cite, or whom to hire. Typically, these applications use algorithms to order items (e.g., people and academic papers) based on "importance" or "relevance", and may therefore produce social inequalities by discriminating certain individuals or groups of people in top ranks. In fact, it has been shown that recommender systems such as *Who-to-Follow* (WTF)[7] tend to increase the popularity of users who are already popular[6,8,9]. A similar effect has been found in *PageRank*[10], where nodes in high ranks stabilize their position and give little opportunity to other nodes to occupy higher positions[11]. This tendency towards the "popular" arises because these algorithms harness structural information, in particular the in- and out-degree of nodes.

However, social networks are complex systems and many other structural properties may also alter the distribution of nodes and groups in the ranking. For example, previous studies have shown that *homophily* affects the visibility of minorities in degree rankings[12] and people recommender systems[13]. Consequently, it can reinforce societal issues such as the glass ceiling effect[14–16] and the invisibility syndrome[17]. Despite these findings, little is known about the extent to which the combination of multiple structural properties can alter the visibility of minorities in top ranks from ranking and recommendation algorithms. A further complication is that debiasing ranking outcomes and making them fair is very challenging. First, *fairness* is an *essentially contested construct* that has different theoretical understandings in different contexts[18]. Second, data can be biased due to missing data[19], selective exposure[20], historical prejudices or implicit bias[21,22]. Third, biases in ranking can be mitigated in different ways[23]: by intervening on the score distribution of candidates[24], on the ranking algorithm[25], or on the ranked outcome[26]. While most of these studies tackle fairness in ranking, they do not explore the effects of networked data in ranking. This paper is a step towards this goal. Since such algorithms are so deeply involved in social, economic and political processes, we need to first understand how our connections affect them to then apply appropriate interventions towards fair results.

To this end, we propose DPAH, a network model that generates directed scale-free networks with binary-attributed nodes. It encodes two main mechanisms of edge formation found in social networks: *homophily* and *preferential attachment*[27–29]. Moreover, it allows to control for the *fraction of minorities*, *edge density*, and the *skewness of the out-degree distribution*. By using this model we systematically study how these structural properties of social networks impact the ranking of nodes in PageRank and WTF. In particular, we investigate two ranking issues, inequality and inequity, and show how they get affected by the ranking algorithm together with the type of network. We measure *inequality* by quantifying the skewness of the rank distribution of nodes that PageRank and WTF produce, and *inequity* as how well-represented the minorities are in the top of the rank compared to the proportion of minorities in the network. In this work we study both ranking issues and measure their correlation. Furthermore, we quantify them globally using the whole rank distribution, and locally within each top-k% rank. The goal is to identify both the overall inequality and inequity trend that these algorithms produce, and the tipping points where minorities start gaining visibility in the top of the rank.

As an example, consider the *directed networks* shown in Figure 1. Every column represents a network with two types of nodes, minority (orange) and majority (blue), and different levels of homophily within groups. Homophily h, is a parameter ranging from 0 to 1 and determines the tendency of two nodes of the same color to be connected. h_{MM} and h_{mm} represent homophily within majorities and minorities, respectively. When nodes are ranked using PageRank (second row), the position of the minorities in the rank varies *systematically*. For instance, when majorities are heterophilic ($h_{MM} = 0.2$, columns a and b), minorities often appear at the top (+). In contrast, when majorities are homophilic ($h_{MM} = 0.8$, columns d and e), minorities tend to appear at the tail of the rank (-). Next, we explain this systematic ranking behavior in top ranks by further varying the structure of the network.

Results

Inequality and inequity in ranking

Inequality refers to the dispersion or distribution of *importance* among *individuals*. This importance is the ranking score assigned to every node by the algorithm. We compute the *Gini* coefficient of the rank distribution to measure how far the ranking scores of individuals deviate from a totally equal distribution. As shown in Figure 2, a very low Gini score (Gini < 0.3) means that all individuals in the network (or in the top-k% for local inequality) are very similar with respect to their ranking scores. If the Gini score is extremely high (Gini ≥ 0.6), it means that only a few individuals capture most of the rank. In other words, the rank distribution is very skewed. Values in between ($0.3 \leq$ Gini < 0.6) represent moderate skewed distributions. From our example in Figure 1, we see that PageRank on average generates moderate skewed ranking distributions for all the depicted networks ($Gini_{global} \approx 0.5$). However, for very small top-k%'s, the Gini is very low. This means that the top individuals possess very similar ranking scores.

Inequity refers to *group* fairness. In particular, it measures the error distance between the fraction of minorities in the top-k% and a given fair baseline (e.g., a diversity constraint or quota). This baseline may be adjusted depending on the context of the application[23,30,31]. Here, a ranking is fair when its top-k% preserves the proportional representation of groups in the network (i.e., disparate impact fairness or realistic overview[32]). Therefore, the error represents the local inequity per top-k%, and ME the mean of these errors across all top-k% ranks or global inequity. As shown in Figure 1 last row, we measure the *local inequity* in two steps. First, we compute the fraction of minorities that appear in each top-k% rank (orange line). Second, we compute the error between the observed fraction of minorities in each top-k% rank and a fair baseline (e.g., the actual fraction of minorities in the network, in this example 20%). Then, we average these error scores across all top-k% ranks to determine the *global inequity* score (ME values). As shown in Figure 2, a fair ranking is such that $-\beta \leq ME \leq \beta$ (green region). The value of β is arbitrary, and allows for a smooth definition of "low mean error" or fairness. We set $\beta = 0.05$. If $ME > \beta$, then minorities are over-represented (blue region) in the top-k%. If $ME < -\beta$, then they are under-represented (red region), otherwise the ranking is representing very well the minorities in the top of the rank. Alternatively, we can say that the top rank (i) *replicates* the levels of inequity from the network when ME is very low, (ii) *amplifies* or worsens inequity and harms the minority group when $ME < -\beta$, and (iii) *reduces* inequity by increasing the visibility of minorities when $ME > \beta$.

Finally, we refer to the relationship between inequality and inequity as *disparity*. For example, if a ranking distribution achieves $Gini = 0.65$ and $ME = 0.5$, we say that the disparity lies in the region *III* (dark blue), i.e., high inequality and high inequity, see Figure 2.

Growth network model with homophily and directed links

In order to examine the effect of homophily on the ranking of minorities in social networks, first we need to develop realistic network models that capture not only a variety of group mixing, but also the directionality of links. Many online social networks are directed networks in their nature, including the follower-followee structure on Twitter and the hyperlink structure of the Web. Directed links are the key components of many algorithms such as PageRank and Who-to-Follow.

To this end, we propose a directed network model with adjustable homophily and minority size, and we refer to it as DPAH (**d**irected **p**referential **a**ttachment with **h**omophily network growth model). We generate these networks by adjusting the fraction of minorities $f_m \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$, and the in-class or group homophily $h_{MM}, h_{mm} \in \{0.0, 0.1, ..., 1.0\}$. We refer to the minority group as m, and to the majority group as M. Additionally, we assign an activity score to every node. This score is drawn from a power-law distribution and determines with what probability the existing node becomes active and creates additional links to other nodes. This means that more active nodes possess higher out-degree (see Methods for more details).

Figure 3 (a) illustrates the DPAH model. At time t, a source node p is selected with a probability proportional to its activity. Then, p connects to an existing node j with a probability related to their pair-wise homophily h_{pj} and preferential attachment that is based on k_j^{in}, the in-degree of node j. By this process, we ensure that the out-degree and in-degree distribution of nodes follow a seemingly power-law distribution that have been observed in many large social networks[33]. The algorithm stops once the network reaches an expected density. Note that source nodes can be either new nodes joining the network for the first time (e.g., node p at time t) or existing nodes (e.g., node l at time $t+1$). Once a source node connects to a target node successfully, the source node becomes available in the next rounds to become a target candidate. This means that in the beginning the model faces a cold start problem since there are no existing (target) nodes to connect to. Thus, the first 1% of new edges are between a source node (drawn from the activity distribution) and any other node with probability as in Equation (3).

How do homophily and directional links influence the ranking of minorities globally and locally?

Global disparity. We corroborate that the Gini coefficient of the rank distributions is large, $Gini \geq 0.6$ (regions I, II and III; dark colors) for both PageRank (see Figure 4) and WTF (see Figure S1 in the supplementary material). Moreover, we find that on average: (i) Balanced networks ($f_m = 0.5$) can get a fair ranking (green) when both groups possess the same homophily scores ($h_{MM} = h_{mm}$). The same applies for neutral networks ($h_{MM} = h_{mm} = 0.5$) regardless of their fraction of minorities ($f_m \leq 0.5$). (ii) When the fraction of minorities decreases ($f_m < 0.5$), groups can be fairly represented in the rank in two regimes: First, when both groups are homophilic, homophily within minorities must be higher than homophily within majorities ($h_{mm} > h_{MM} > 0.5$). Second, when both groups are heterophilic, homophily within majorities must be higher than homophily within minorities ($h_{mm} < h_{MM} < 0.5$) to balance the importance of groups.

Local disparity. We also compute inequality and inequity within each top-k% rank in order to see to what extent they change when k% increases. In the case of PageRank, we see in Figure 5 that inequality varies (i.e., from light to dark colors) in different regimes mainly due to the size of k (x-axis), and inequity due to the interplay between homophily within groups, h_{MM} and h_{mm}. In particular: (i) Only at the top-5% of the rank we see a few cases of low inequality (regions VII, VIII and IX; very light colors), this means that nodes at the very top possess very similar ranking scores, but they are very far from the rest of the population, i.e., the larger the top-k%, the higher the Gini. This holds in WTF up to roughly the top-30% (see Figure S2 in the supplementary material). Overall, PageRank converges to high inequality faster than WTF. (ii) Inequity (regions: red, blue, green) is consistent across all top-k% ranks for both algorithms. In other words, if the ranking algorithm favors or harms one group in the top-5%, it will continue to do so until converging to the fair regime (regions II, V, VIII; green). With a few exceptions, this fair regime is only reached when k is very large. For example, if a minority group is under-represented at the top-5%, it will remain under-represented at the top-80% (see $h_{mm} = 0.1$ and $h_{MM} \geq 0.7$ in Figure 5 for PageRank, and Figure S2 in the supplementary material for WTF). (iii) Minorities are often over-represented when majorities are heterophilic $h_{MM} < 0.5$; (regions III, VI, IX; blue). In contrast, minorities are often under-represented when majorities are homophilic $h_{MM} > 0.5$ (regions I, IV, VII; red). This is consistent up to $\approx$ top-80% for both algorithms.

In summary, our results suggest that the size of k does not have an influence on inequity. This means that if the algorithm amplifies inequity at the top-5%, it will also amplify inequity at larger top-k%'s. Therefore, increasing the selection pool (larger k) does not improve the representation of minorities. This can be explained by the fact that the preferential attachment mechanism disproportionately affects nodes ranking[11].

Correlation and feature importance. We compute the Spearman correlation between inequality and inequity, and conduct a random forest regression to measure the importance of each network property on both inequality and inequity values. Results are shown in Table 1 for PageRank and Table S1 (in the supplementary material) for WTF.

We find that inequality and inequity are positively correlated in both global and local regimes. In other words, the skewer the rank distribution (i.e., high Gini), the more unfair with either group (i.e., mean error far from zero), and vice versa. This correlation is stronger and more significant in PageRank than in WTF.

Global inequality (*gini*) is mainly explained by both homophily values, whereas global inequity (*me*) is mainly driven by homophily within majorities. Local inequality ($gini_k$), on the other hand, is mainly explained by the top-k% rank, and local inequity (me_k) is mainly explained by the homophily within the majority group.

These results are in agreement with what we see in previous figures; Even though majority nodes produce most of the inequality and inequity in the rank, their interplay with minority nodes can change or intensify the direction of bias. In fact, both homophily values can explain 75% (49%) of *gini*, the global inequality in PageRank (WTF), 92% (88%) of *me*, the global inequity, and 78% (74%) of me_k, the local inequity. However, the top-k% rank together with the homophily within majority nodes explain 84% (86%) of $gini_k$, the local inequality.

Table 1. 10-fold cross-validation for PageRank: We use a Random Forest Regressor to assess model performance and feature importance. R^2 values are averages (and standard deviations) across all networks. Features are ranked in descending order based on their mean importance (from left to right) and highlighted if their importance represents more than 50% of the total importance. Corr shows the Spearman correlation between inequality and inequity scores (p-values ≈ 0).

Type	Outcome	Corr	R^2	Feature importance
Global	*gini*	0.41	0.92 (0.009)	$\boldsymbol{h_{MM}} \rightarrow \boldsymbol{h_{mm}} \rightarrow f_m$
	me		0.99 (0.001)	$\boldsymbol{h_{MM}} \rightarrow h_{mm} \rightarrow f_m$
Local	$gini_k$	0.21	0.95 (0.002)	$\boldsymbol{k} \rightarrow h_{MM} \rightarrow h_{mm} \rightarrow f_m$
	me_k		0.99 (0.001)	$\boldsymbol{h_{MM}} \rightarrow h_{mm} \rightarrow k \rightarrow f_m$

How do different social mechanisms of edge formation contribute to disparity?

So far, we show that PageRank and WTF on our network model produce high inequality and a wide-range of possible inequity outcomes. How much of that inequality or inequity was a product of homophily or preferential attachment? To see the effects of these two mechanisms alone, we generate new networks by turning on and off the homophily and preferential attachment features (see Methods for the details of the models).

Figure 6 shows the inequality and inequity produced by PageRank on a variety of models: DPA (Directed Preferential Attachment), DH (Directed Homophily), Random, and DPAH (see Figure S3 in the supplementary material for WTF). Results from both algorithms show that networks whose nodes connect through preferential attachment (DPA) produce on average higher inequality compared to DH and Random. However, when preferential attachment is combined with homophily (DPAH), this inequality increases even further. Additionally, we see that WTF produces higher inequality compared to PageRank (see section A.3 in the supplementary material for more details). Inequity, on the other hand, is mainly driven by homophily. This means that, homophily (DPAH and DH) influences both, inequality and inequity in both algorithms.

Note that in Figure 6, we fixed the activity of nodes to $\gamma_M = \gamma_m = 3.0$. However, when we set these parameters to $\gamma_M = \gamma_m < 3.0$ (lower values), inequality decreases, see Figure S4 in the supplementary material. Additionally, in Figure S5 in the supplementary material, we see that edge density also plays a role in the inequality produced by PageRank and WTF. This means that, by further adjusting these two parameters (node activity and edge density), we would expect changes only to inequality since inequity is mainly affected by homophily as we saw before.

Strategies towards a fair ranking

Results from both algorithms show that while the homophily within majorities is the main driver for inequality and inequity, minorities may overcome unfair rankings by connecting strategically in the network. In general, when majorities are homophilic $h_{MM} > 0.5$, minorities should increase their homophily such that $h_{mm} > h_{MM}$. When majorities are (somewhat) neutral ($h_{MM} = 0.5 \pm 0.1$), minorities may connect arbitrarily with any group without being too homophilic, otherwise they will become over-represented in the rank. Finally, when majorities are heterophilic $h_{MM} < 0.5$, one solution to achieve a fair rank is to increase the size of the minority group, and make sure that both groups behave similarly in terms of homophily ($h_{MM} \approx h_{mm}$). Otherwise, minorities will be over-represented regardless of their in-group homophily. Note that these "strategies" without algorithmic intervention may work in scenarios such as a citation or collaboration networks, but they might not work in other scenarios. In such cases, we need additional recommender systems to help under-represented groups discover those "strategic" links that will help them climb to higher ranks.

Discussion and Future Work

In this work we have proposed a systematic study to measure the inequality and inequity produced by PageRank and Who-To-Follow (WTF). Our approach disentangles the effect of network structure on the rank distributions of these two algorithms by using synthetic networks. By doing so, we control for the properties of the network and measure how these changes affect the rankings. In particular, we studied six prominent structural properties of social networks: homophily, preferential attachment, fraction of minorities, edge density, node activity and the directionality of links.

Consequently, our systematic study makes PageRank and Who-To-Follow interpretable and explainable. Our results show that the systemic bias produced by these algorithms in the rank—in particular inequity—is mainly due to *homophily imbalance* ($h_{MM} \gg h_{mm}$ or $h_{mm} \gg h_{MM}$). A potential avenue to reduce inequity is to create synthetic connections during the ranking process as it is done for correcting the class imbalance problem in supervised learning[34]. Additionally, our results show the necessary conditions to achieve a fair rank. These conditions can be implemented in ranking and recommendation algorithms to strategically adjust the importance of connections in a given network. In certain applications such as citation or collaboration networks, these strategies can be recommended through additional recommendation algorithms. For instance, recommender systems could suggest relevant articles not only based on popularity and (keyword) similarity but also based on fairness by fulfilling diversity constraints. Finally, further research can investigate other social mechanisms of edge formation that have been seen in social networks such as clustering[35], transitivity[36], and reciprocity[37]. Similarly, other structural properties such as monophily[38] and second order homophily[39] can be studied to measure their influence on ranking.

Conclusions

In this work we have investigated under which conditions PageRank and Who-To-Follow (WTF) *replicate*, *amplify* or *reduce* inequality and inequity in top ranks. In particular, given the rank distribution produced by these algorithms, we computed inequality as the dispersion among individuals in terms of ranking scores, and inequity as whether minorities are over-, under- or well-represented in top ranks compared to their representation in the network. We studied these two metrics separately and in combination to better understand the mechanisms that can explain them.

To that end, we proposed DPAH, a growth network model that allows to generate realistic scale-free directed networks with different levels of homophily, fraction of minorities, node activity, and edge density. In these networks, we found that both inequality and inequity are positively correlated and mainly driven by the homophily within majorities. This means that, when the majority group is highly homophilic, the minority group is under-represented in top ranks. Also, when the majority is highly heterophilic, the minority benefits tremendously since it is over-represented in the top-k%. However, minorities can overcome these disparities by connecting strategically with others. Thus, equity in ranking is a trade-off between homophily and the fraction of minorities.

Our systematic study makes PageRank and Who-to-Follow explainable and interpretable to help data scientists understand and estimate the disparity that these algorithms produce given the structure of networks, which is key for proposing targeted interventions. We hope our results create awareness among majority and minority groups about these disparities since they may replicate and even amplify the biases found in social networks.

Data and Methods

Synthetic networks

We propose DPAH, a **d**irected **p**referential **a**ttachment with **h**omophily network growth model, and generate networks with $n = 2000$ number of nodes, edge density of $d = 0.0015$, and adjust five network properties: fraction of minorities $f_m \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$, in-class or group homophily $h_{MM}, h_{mm} \in \{0.0, 0.1, ..., 1.0\}$, and the power-law exponents of the activity distributions $\gamma_M = \gamma_m = 3.0$. Each combination of network structure is generated 10 times, nodes are ranked using PageRank and WTF separately, and inequality and inequity scores are computed and averaged accordingly for each algorithm.

Directed network

We define a directed network as: Let $G = (V, E, C)$ be a node-attributed unweighted graph with $V = \{v_1, ..., v_n\}$ being a set of n nodes, $E \subseteq V \times V$ a set of e directed edges, and $C = \{c_1, ..., c_n\}$ a list of binary class labels where each element c_i represents the class membership of node v_i. The fraction of minorities f_m captures the relative size of the minority class—with respect to C—in the network. We refer to the minority group as m, and to the majority group as M. A network is *balanced* when all class labels have the same number of nodes ($f_m = 0.5$), otherwise it is *unbalanced* ($f_m < 0.5$).

In order to generate directed links, inspired by the activity-driven network model[40], we assign an activity score to each node that determines with what probability the existing node becomes active and creates additional links to other nodes. It has been shown that in empirical networks the activity of the nodes follows a power-law distribution[40]. Therefore, we assign an activity to each node drawn from a power-law distribution. Note that each group possess its own activity distribution and they are defined by its power-law exponent γ_M and γ_m for majority and minority nodes, respectively.

Then, the probability of connecting a source (active) node v_i to a target node v_j (or in other words the probability of connecting to v_j given the source node v_i) is explained by any of the following three mechanisms of edge formation.

Preferential Attachment (DPA)

Also known as the *rich-get-richer* effect or *cumulative advantage* in social networks[29,41]. It indicates that nodes tend to connect to popular nodes. We define popularity as the in-degree of the node. Therefore, the probability that a source node v_i connects to a target node v_j is proportional to the *in-degree* of the target node v_j.

$$P(i \to j) = P(j|i) = \frac{k_j^{in}}{\sum_l k_l^{in}} \tag{1}$$

Homophily (DH)
It is the tendency of individuals to connect (or interact) with similar others[28,42]. Thus, the probability that a source node v_i connects to a target node v_j is driven by the homophily between their classes c_i and c_j. We assign a homophily value to each dyad based on pre-defined homophily parameters within majorities and minorities, h_{MM} and h_{mm}, respectively. Homophily values range from 0.0 to 1.0. If the homophily value is high, that means that nodes of the same class are attracted to each other more often than nodes of different attributes. Nodes of the same class with homophily $h_{aa} = 0.5$ are referred to as *neutral* (i.e., they connect randomly to either class), otherwise they are *heterophilic* if $h_{aa} < 0.5$ (i.e., more likely to connect to the other class), or *homophilic* when $h_{aa} > 0.5$ (i.e., more likely to connect to the same class).

$$P(i \to j) = P(j|i) = \frac{h_{ij}}{\sum_l h_{il}} \tag{2}$$

Preferential Attachment with Homophily (DPAH)
We propose DPAH[1], a directed growth network model with adjustable homophily and fraction of minorities. This mechanism combines DPA and DH, and is an extension of the BA-Homophily model[12].

$$P(i \to j) = P(j|i) = \frac{h_{ij} k_j^{in}}{\sum_l h_{il} k_l^{in}} \tag{3}$$

Note that DPA and DH are especial cases of DPAH where only the in-degree mechanism varies. This means that, the out-degree distribution remains the same as in DPAH: it is driven by the activity model. Additionally, we include a random model where both source and target nodes are chosen at random (i.e., Erdős-Rényi model). Table 2 shows the parameters adjusted in each model. Number of nodes n and edge density d are arbitrary in the sense that they are not part of the edge formation mechanism. Thus, we fix them to make a fair comparison across all models.

Table 2. Model parameters: Check marks denote that a given model (column) requires a particular parameter (row): number of nodes n, fraction of minorities f_m, edge density d, in-group homophily h_{aa}, and the power-law exponent of the activity distribution γ. Sub-indices M and m refer to the majority and minority groups, respectively. The difference between DH and DPAH is the preferential attachment (in-degree) mechanism. All models produce directed networks.

	Random	**DPA**	**DH**	**DPAH**
n	✓	✓	✓	✓
f_m	✓	✓	✓	✓
d	✓	✓	✓	✓
h_{MM}	-	-	✓	✓
h_{mm}	-	-	✓	✓
γ_M	-	✓	✓	✓
γ_m	-	✓	✓	✓

Ranking and Recommendation algorithms
There exist a variety of ranking and recommendation algorithms that follow different strategies depending on the nature of the problem. For instance, in information systems, items such as content, Web pages, and products are ranked to recommend users what to read or buy[43]. In social networks, however, people are ranked to identify their hierarchy or importance[44–46], and recommended to other users in order to establish new connections[47–50]. These rankings and recommendations are based on algorithms that often rely on whom we are already connected with. In this work, we focus on two such algorithms widely used in practice[51]: PageRank[10] and Who-to-Follow (WTF)[7]. While PageRank determines the global ranking of nodes in comparison with all other nodes, WTF deals with ranking nodes in a node level and thus remains a local measure. For that reason, we focus on these two algorithms to capture both dimensions.

PageRank
It was invented to rank all web pages in the Web[10], and has been used in several applications[51]. For example, to study citation and co-authorship networks[52–54]. PageRank assigns an importance score to every single node in a network. This score takes

[1]DPAH stands for: **D**irected network with **P**referential **A**ttachment and **H**omophily.

into account the number and quality of incoming links of each node. The PageRank of node i is defined as follows:

$$PR(i) = (1-\alpha) + \alpha \sum_{j \in N_i} \frac{PR(j)}{k_j^{out}} \tag{4}$$

where $i \in G$, N_i represents all neighbors of i (e.g., all nodes i points to), and k_j^{out} the out-degree of node j. The damping factor α, or probability of following links using a Random Walker, is set to 0.85 as suggested by Brin and Page[55]. We use the `fast-pagerank`[56] python package to compute the PageRank score of all nodes using sparse adjacency matrices.

Who-To-Follow (WTF)

This recommendation algorithm was created and used by Twitter to suggest new people to follow[7]. It is based on SALSA[57] which in turn is based on Personalized PageRank[58]. In a nutshell, for each user u, the algorithm looks for its *circle of trust*, which is the result of an egocentric random walk (similar to personalized PageRank)[7]. Then, based on this circle-of-trust, the algorithm ranks all users that are not yet friends with u but are connected through the circle of trust. Then, we take the top-k of these (recommended) users, and add up the counter of being selected as a recommendation to each of them. This is done for every node u in the network. At the end, the rank of each node encodes the *number of times a user was suggested as a recommendation* across all nodes in the network. Thus, the WTF score for each node is defined as follows:

$$WTF(i) = \sum_{j \in V} \mathbb{1}_{SALSA(j)}(i) \tag{5}$$

where $SALSA(j)$ refers to the top-k users the SALSA algorithm recommends to node j. In this work we select the top-10 users as recommendations. $\mathbb{1}_A(x)$ denotes the indicator function or boolean predicate function to test set inclusion (i.e., whether $x \in A$).

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Author contributions statement

L.E.N., F.K., C.W. and M.S. devised the research project. L.E.N. conceived the experiments and analyzed the datasets. F.K. performed analytical derivations. L.E.N., F.K., C.W., and M.S. wrote the paper. All authors reviewed the manuscript.

Additional information

Competing Interests

The author(s) declare no competing interests.

Availability of materials and data

The code and datasets generated during and/or analyzed during the current study are available in the GitHub repository, `https://github.com/gesiscss/Homophilic_Directed_ScaleFree_Networks`.

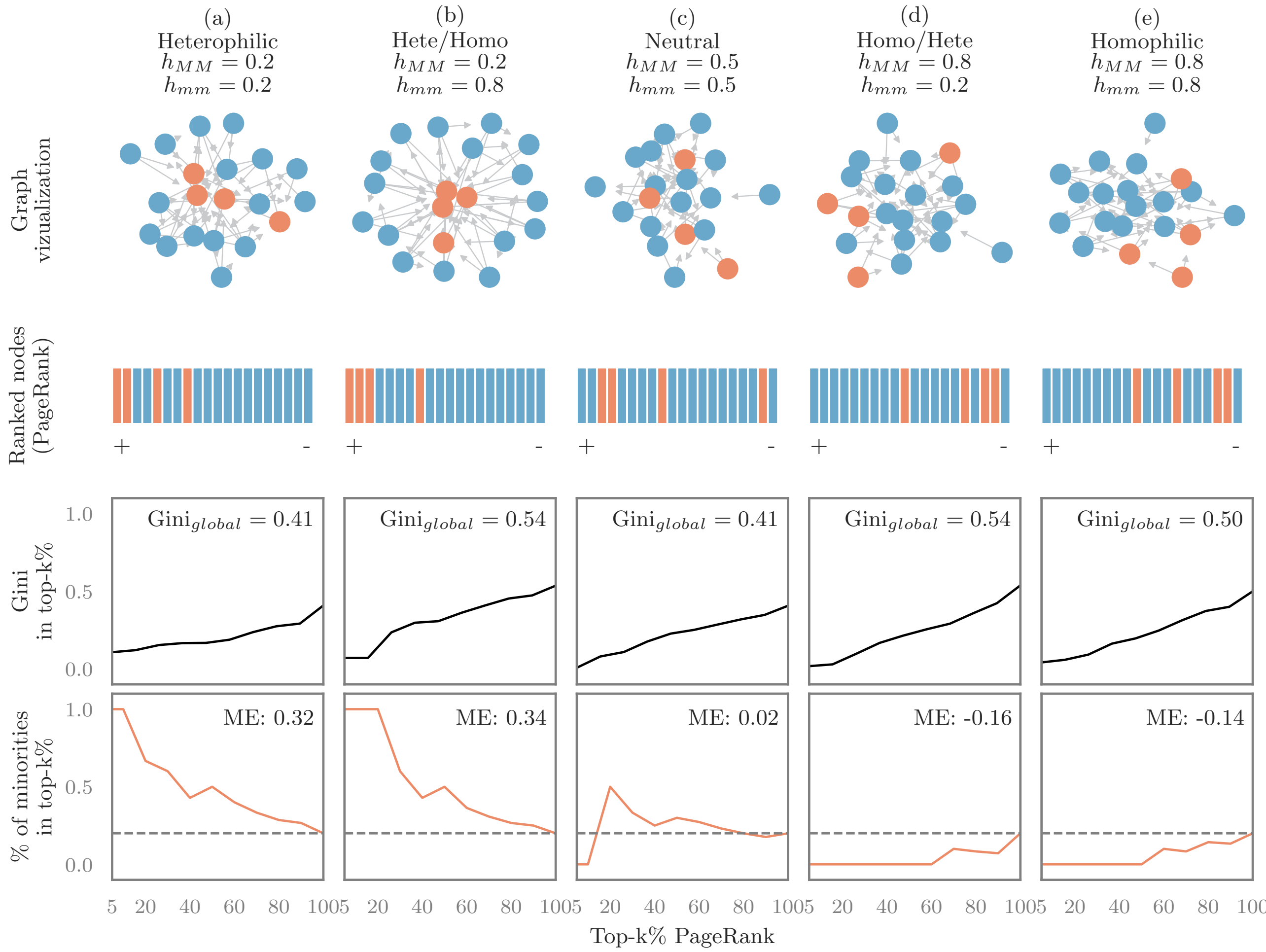

Figure 1. Inequality and inequity: Every column represents a network with certain level of homophily. All networks contain 20 nodes: 20% belong to the minority group (orange), and 80% to the majority group (blue). Edges follow a preferential attachment with homophily mechanism. The top row shows the graph and the level of homophily within groups (*MM*: majorities and *mm*: minorities). The second row shows all nodes in descending order (from + to -) based on their PageRank scores. The third row represents the rank *inequality*: Gini coefficients of the rank distribution for every top$-k\%$ (black line). Gini_{global} refers to the Gini coefficient of the entire rank distribution (i.e., at top-100%). We see that the lower the *k*, the lower the Gini of the rank distribution. The bottom row represents the rank *inequity*: Percentage of minorities found in each top-$k\%$ of the rank distribution (orange line). *ME* is the mean error of these percentages compared to a fair baseline or diversity constraint (i.e., how much the orange line deviates from the dotted line across all top-k's). Here we see three main patterns: (a,b) When the majority group is heterophilic, minorities are on average over-represented, $ME > 0.02$. (d,e) When majorities are homophilic, minorities are on average under-represented, $ME < -0.02$. (c) When both groups are neutral, the observed fraction of minorities is almost as expected, $ME \approx 0$.

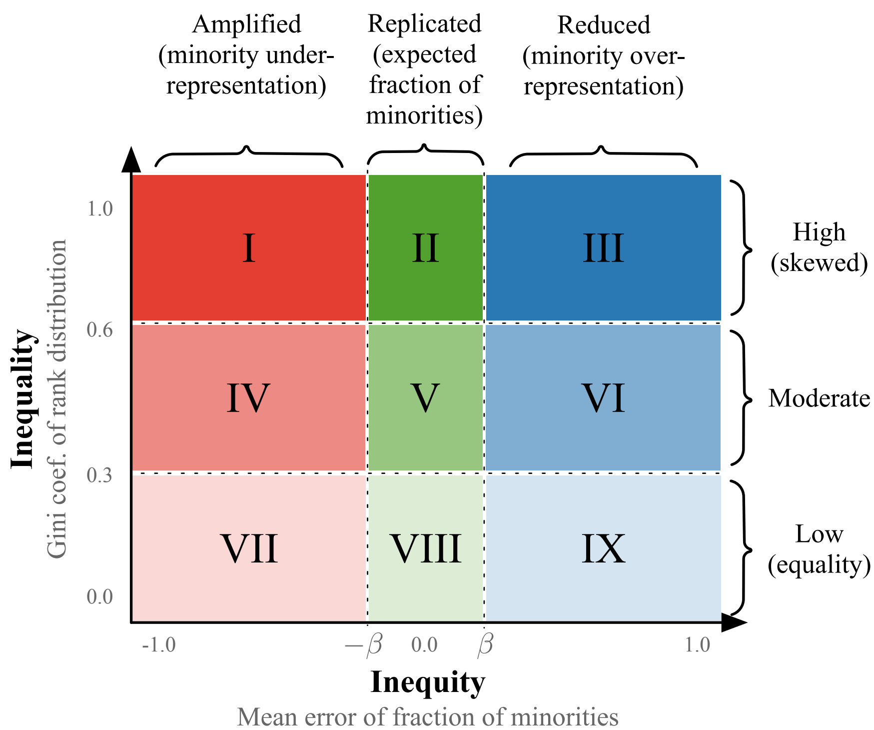

Figure 2. Regions of disparity. We measure *inequality* (y-axis) as the skewness of the rank distribution, and *inequity* (x-axis) as the mean differences between the proportional representation of groups in top-k% ranks and the network. Highly skewed distributions lie in regions I to III (darker colors), and fair rankings, where minorities are well represented in the top ranks, lie in regions II, V, VIII (green). We set $\beta = 0.05$ which is arbitrary and allows for a flexible region of *group fairness*.

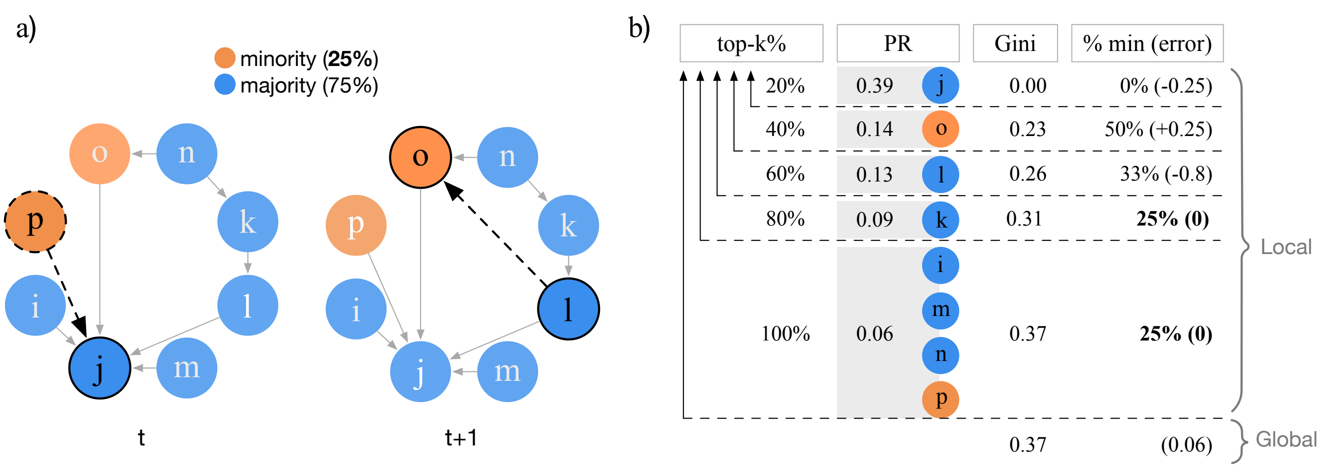

Figure 3. DPAH model and ranking of nodes. a) Illustration of the directed network model with preferential attachment and homophily (DPAH). At time t, a source node p is drawn from a power-law (activity) distribution and joins the network for the first time. Then, a target node j is drawn based on the in-degree distribution and the pair-wise homophily h_{pj}. At time $t+1$, a new edge is added between already existing nodes $l \rightarrow o$ based on the same mechanism. The algorithm repeats until a desired edge density is fulfilled. b) The PageRank score of each node is shown under *PR*. Nodes in each top-k% of the rank are grouped based on the unique PageRank scores. In this example, the top-60% of nodes concentrate most of the PageRank and their scores are somewhat similar (i.e., low Gini). Also, the ranking is fair from top-80% onwards, since they capture the same fraction of minorities as in the population, 25%. Local values are measured per top-k%, and global values are measured using the whole distribution for inequality (Gini), and the average across all top-k% ranks for inequity (mean error).

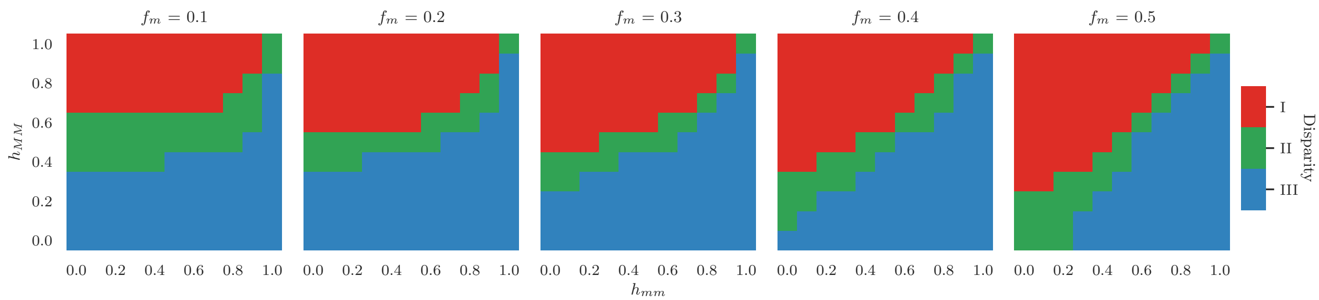

Figure 4. The effects of homophily and fraction of minorities in the global disparity of PageRank. Columns represent the fraction of minorities in the network, x-axis indicates the homophily within minorities, and y-axis the homophily within majorities. Colors denote the region where the disparity lies in according to our interpretation in Figure 2. First, we see that, on average, there is never low global inequality (i.e., regions IV to IX —lighter colors— do not appear). This makes sense because these are scale-free networks. Second, depending on the level of homophily within groups, minorities on average can be under-represented (region I, red), or over-represented (region III, blue), or well-represented (region II, green). For example, in the balanced case $f_m = 0.5$, minorities are on average under-represented when $h_{MM} > h_{mm} \geq 0.3$.

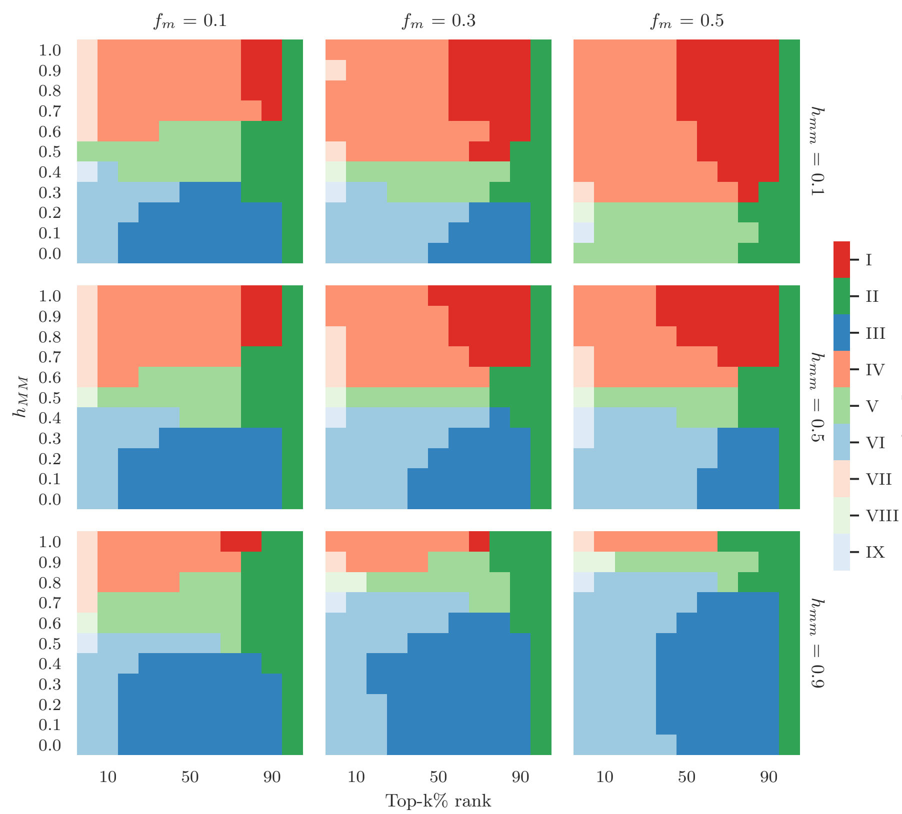

Figure 5. The effects of homophily and fraction of minorities in the local disparity of PageRank. Columns represent the fraction of minorities (10%, 30% and 50%) and rows show homophily within minorities (from top to bottom: heterophilic, neutral and homophilic). The x-axis denotes the top-k% rank and the y-axis shows homophily within majorities. Colors refer to the regions of disparity introduced in Figure 2. One can see that the minority suffers most (red) when the majority is homophilic and the minority is either heterophilic or neutral. Moreover, inequality is lowest (very light colors) only for a few cases at top-5%. This means that the top best ranked nodes are very similar and their ranks are far from the majority of nodes (i.e., due to preferential attachment). Moreover, inequity remains mostly consistent regardless of top-k%. In other words, if the ranking algorithm favors one group in the top-5% (e.g., red or blue), it will continue to do so until entering the fair regime (green).

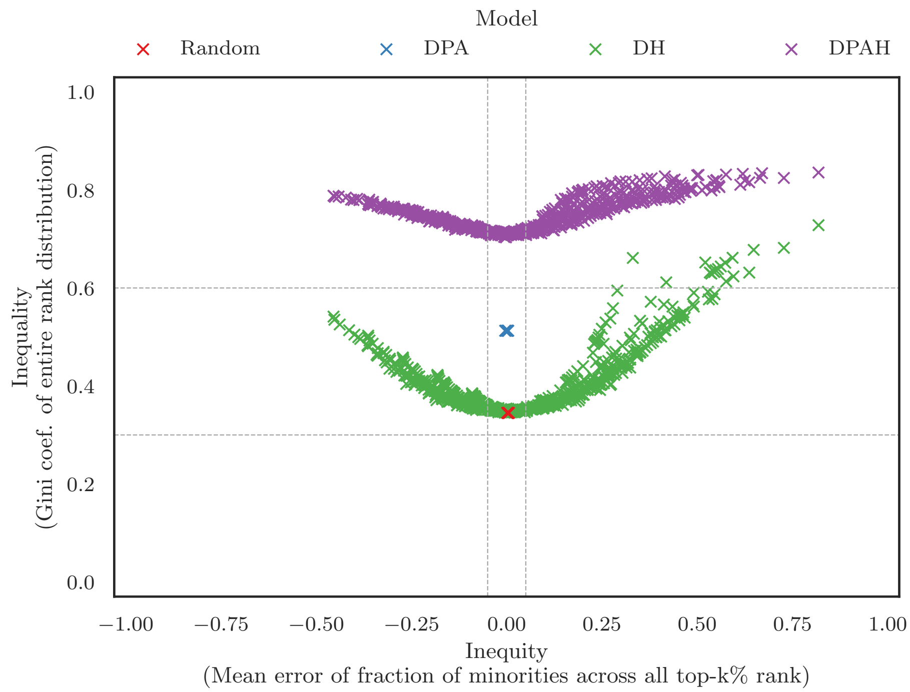

Figure 6. The effects of homophily and preferential attachment in the global disparity of PageRank. We generated directed networks using four different models of edge formation. DPA: only preferential attachment. DH: only homophily. DPAH: our proposed model that combines DPA and DH. Random: a baseline where nodes are connected randomly. We see the following patterns: (i) Homophily (DH) produces a moderate-to-high level of inequality ($0.3 < Gini < 0.8$), while preferential attachment (DPA) produces a consistent moderate inequality ($Gini \approx 0.5$). When both mechanisms are combined (DPAH), the rank inequality increases even further ($0.7 < Gini < 0.9$). (ii) Random and Preferential attachment (DPA) are always fair ($ME = 0$), while in the cases where homophily is involved (DH and DPAH) inequity is often high ($|ME| > \varepsilon$). Thus, in general preferential attachment is the main driver of inequality, while homophily influences both inequality and inequity.

Supplementary Material

A Analytics

A.1 Derivation of the probability of having an internal link

Let $K_a^{in}(t)$ and $K_a^{out}(t)$ be the sum of the in- and out-degrees of nodes from group a at time t. The overall growth of the network follows a DPAH process. Thus, the evolution of in-degree and out-degree follows:

$$\begin{cases} K_a^{in}(t)+K_b^{in}(t)=K^{in}(t)=m\times t \\ K_a^{out}(t)+K_b^{out}(t)=K^{out}(t)=m\times t \end{cases} \tag{1}$$

where m is the number of new links in the network at each time step t. In each time step, a node v_i is chosen. That results in m new out-going links from v_i and m new incoming links to node v_j. We set $m=1$. Thus, in each time step only one edge is created.

Let us denote the relative fraction of group size for each group as f_a and f_b, and the activation probability as η. The activation probability is independent of the node attribute. Thus, the behavior of the network is similar to what we have shown before[12]; only the total number of links is different. We can show that in the limit of $\Delta t \to 0$, for each group, the in-degree growth function follows the following:

$$\frac{dK_a^{in}}{dt}=(m\times\eta)\left(f_a\left(1+\frac{h_{aa}K_a^{in}(t)}{h_{aa}K_a^{in}(t)+h_{ab}K_b^{in}(t)}\right)+f_b\left(\frac{h_{ba}K_a^{in}(t)}{h_{bb}K_b^{in}(t)+h_{ba}K_a^{in}(t)}\right)\right) \tag{2}$$

$$\frac{dK_b^{in}}{dt}=(m\times\eta)\left(f_b\left(1+\frac{h_{bb}K_b^{in}(t)}{h_{bb}K_b^{in}(t)+h_{ba}K_a^{in}(t)}\right)+f_a\left(\frac{h_{ab}K_b^{in}(t)}{h_{aa}K_a^{in}(t)+h_{ab}K_b^{in}(t)}\right)\right) \tag{3}$$

We focus on the case of links within group a. The same analysis applies for group b.

Let p_{aa} be the probability to establish a link between two nodes of group a. The probability for an incoming or existing node from group a to link to a node of the same group is given by:

$$p_{aa}(t)=f_a\frac{h_{aa}K_a^{in}(t)}{h_{aa}K_a^{in}(t)+h_{ab}K_b^{in}(t)} \tag{4}$$

In the simple network growth model, the total degree of the groups increases linearly over time.

$$\begin{cases} K_a^{in}(t)=C\times(m\times\eta)\times t \\ K_b^{in}(t)=(2-C)\times(m\times\eta)\times t \\ K_a^{out}(t)=(m\times\eta)\times t \\ K_b^{out}(t)=(m\times\eta)\times t \end{cases} \tag{5}$$

Denoting C as the in-degree growth factor of the minority group.

A.2 Calculating Homophily from empirical network

We can calculate homophily in empirical networks using the information about in-group links. First, the total number of edges in a directed network follows:

$$e=e_{aa}+e_{ab}+e_{ba}+e_{bb} \tag{6}$$

To calculate e_{aa}, the number of links within class a, we can simply argue that it depends on p_{aa}, the probability of connecting two nodes belonging to class a, multiplied by the probability of the arrival or source node to be of class a, denoted by f_a, the fraction of nodes in class a, as shown in Equation (4).

Our network model grows linearly in time. That means, the in-degree growth for each group is linear. Let us assume that the in-degree growth rate of group a is denoted by C_a:

$$K_a^{in}(t)=C_aK^{in}(t) \tag{7}$$

Since the in-degree growth remains constant over time, we can calculate C_a in the empirical network by summing all in-degrees of the group

$$C_a(empirical) = \frac{K_a^{in}}{K^{in}} \tag{8}$$

Equation (4) can be rewritten as

$$p_{aa} = f_a \frac{h_{aa}C_a}{h_{aa}C_a + h_{ab}(1-C_a)} \tag{9}$$

In empirical networks, p_{aa} represents the probability of a directed edge from class a to class a. This probability is proportional to the number of edges from a to a, normalized by the total number of directed edges that start from a:

$$p_{aa} = \frac{e_{aa}}{e_{aa} + e_{ab}} \tag{10}$$

We can then calculate Equation (10) in the empirical network. Finally we use maximum-likelihood estimate to find the best values for h_{aa} and h_{bb} in Equation (9).

Note that in-degree growth rate C has an sub-linear relationship to the exponent of the in-degree distribution σ and the exponent of the in-degree growth θ[12]. Thus, another method to retrieve empirical homophily is to fist estimate the exponents of the in-degree distributions for minority and majorities (σ_a and σ_b) and plug that into the equation.

$$p_{aa} = \frac{f_a^2 h_{aa}(1-\theta_b)}{f_a h_{aa}(1-\theta_b) + f_b h_{ab}(1-\theta_a)} \tag{11}$$

$$p_{bb} = \frac{f_b^2 h_{bb}(1-\theta_a)}{f_b h_{bb}(1-\theta_a) + f_a h_{ba}(1-\theta_b)} \tag{12}$$

where $\sigma_a = -(\frac{1}{\theta_a} + 1)$ and $\sigma_b = -(\frac{1}{\theta_b} + 1)$.

A.3 WTF produces higher inequality compared to PageRank

In the main manuscript we see that WTF produces skewer rank distributions compared to PageRank. To understand this behavior, we need to first understand how the algorithms work. PageRank scores reflect the *global* importance of nodes in the network, and this global importance is mostly determined by in-degree[59]. On the other hand, the WTF score of a node is the number of times the node appears in the top-10 recommendation across all nodes in the network. This top-10 is determined by the circle-of-trust of each node, similar to a Personalized PageRank. This means that this top-10 contains the most visited nodes by a random walker that always starts at the node who is getting the recommendation. Thus, that (local) top-10 will be highly influenced by in-degree too. However, since the WTF score counts the number of times a node appears as a recommendation, it is likely that the highest WTF scores refer to high degree nodes due to preferential attachment. Therefore, the high inequality produced by WTF can be explained by the fact that WTF combines a local random walk with a global count.

B Additional Tables and Figures

Table S1. 10-fold cross-validation for WTF: We use a Random Forest Regressor to assess model performance and feature importance. R^2 values are averages (and standard deviations) across all networks. Features are ranked based on their mean importance (from left to right) and highlighted if their importance represents more than 50% of the total importance. Features with a mark (*) are less important than random. Corr shows the disparity as the Spearman correlation between inequality and inequity scores (p-values ≈ 0).

Type	Outcome	Corr.	R^2	Feature importance
Global	$gini$	0.29	0.35 (0.03)	$h^*_{MM} \rightarrow h^*_{mm} \rightarrow f^*_m$
	me		0.92 (0.01)	$\boldsymbol{h_{MM}} \rightarrow h_{mm} \rightarrow f_m$
Local	$gini_k$	0.06	0.86 (0.00)	$\boldsymbol{k} \rightarrow h^*_{MM} \rightarrow h^*_{mm} \rightarrow f^*_m$
	me_k		0.85 (0.01)	$\boldsymbol{h_{MM}} \rightarrow \boldsymbol{h_{mm}} \rightarrow k \rightarrow f^*_m$

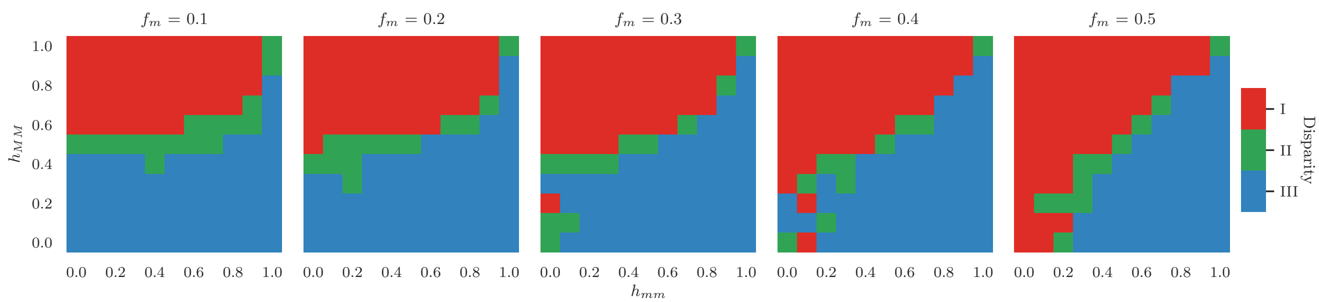

Figure S1. The effects of homophily and fraction of minorities in the global disparity of WTF Columns represent the fraction of minorities in the network, x-axis indicates the homophily within minorities, and y-axis the homophily within majorities. Colors denote the region where the disparity lies in according to our interpretation (see Figure 8 in main article). As in the case of PageRank (cf. Figure 4 in main article), we see that, on average, there is never low global inequality. Also, depending on the level of homophily within groups, minorities on average can be under-represented (region I, red), or over-represented (region III, blue). Note that the fair case (region II, green) rarely occurs.

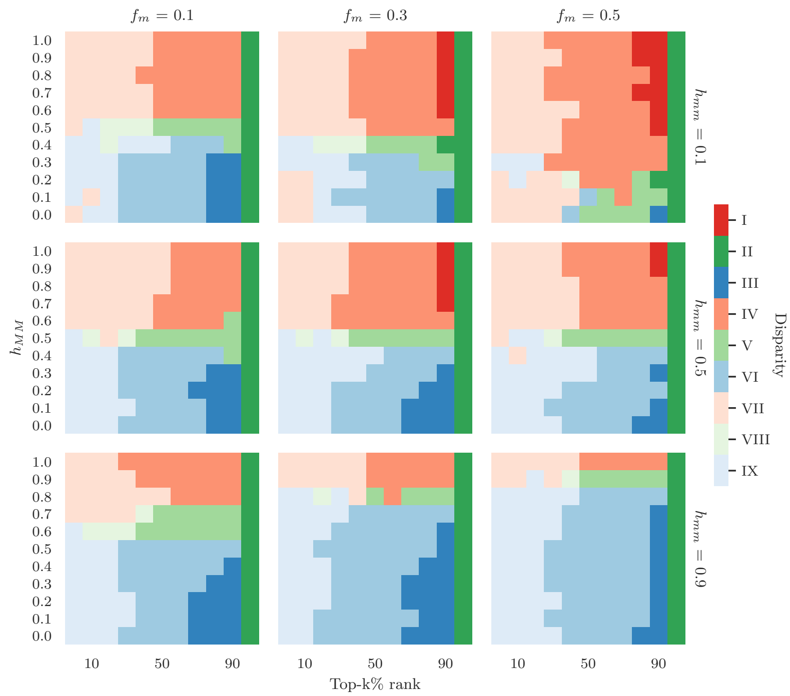

Figure S2. The effects of homophily and fraction of minorities in the local disparity of WTF. Columns represent the fraction of minorities (10%, 30% and 50%) and rows show homophily within minorities (from top to bottom: heterophilic, neutral and homophilic). The x-axis denotes the top-k% rank and the y-axis shows homophily within majorities. Colors refer to the regions of disparity (see Figure 8 in main article). As in the case of PageRank (cf. Figure 5 in main article), we see that the minority suffers most (red) when the majority is homophilic and the minority is either heterophilic or neutral. Also, inequity remains mostly consistent regardless of top-k%. In contrast to PageRank (up to top-5%), WTF manages to capture nodes with very similar ranking scores (roughly) up to the top-30% (i.e., Gini is low, regions VII, VIII, IX).

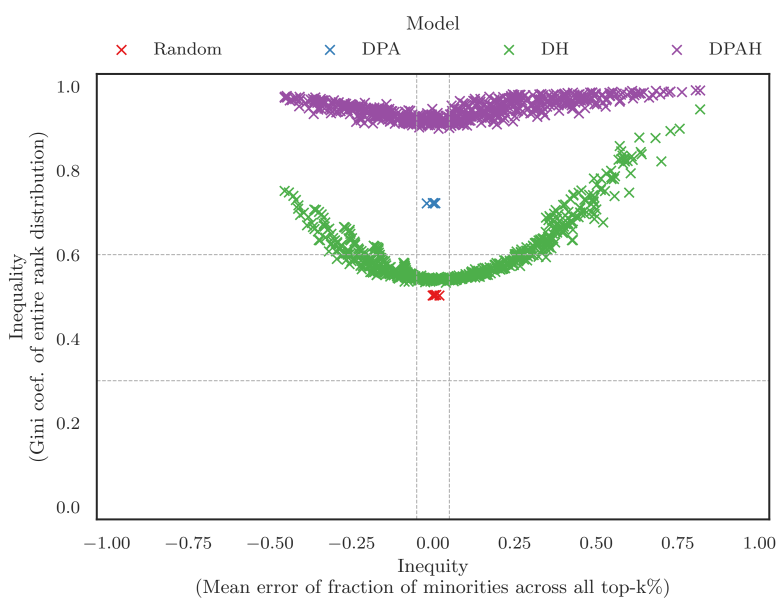

Figure S3. The effects of homophily and preferential attachment in the global disparity of WTF We generated directed networks using four different models of edge formation. DPA: only preferential attachment. DH: only homophily. DPAH: our proposed model that combines DPA and DH. Random: a baseline where nodes are connected randomly. Compared to PageRank (cf. Figure 2 in main article), all models generate higher inequality (y-axis), whereas inequity remains similar.

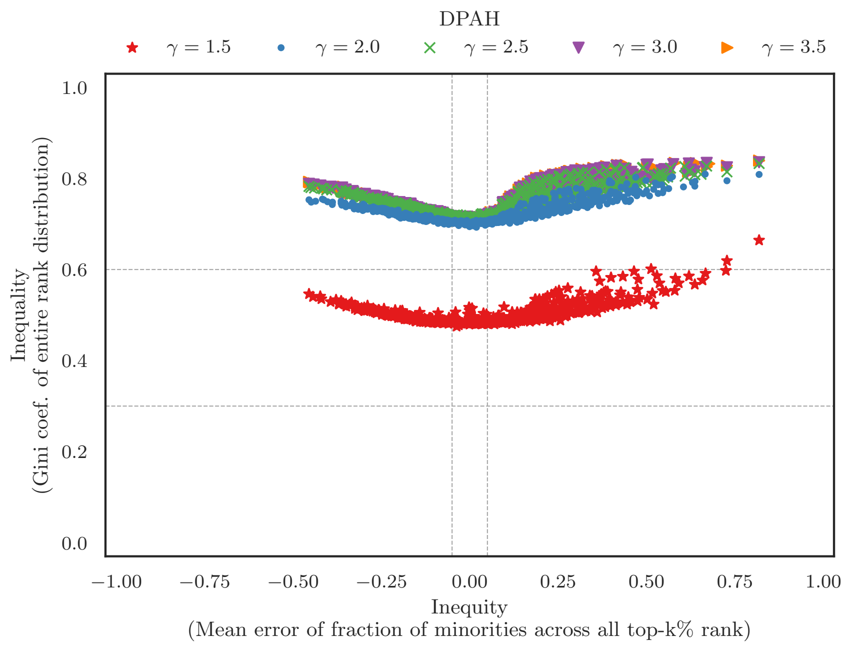

Figure S4. The effects of node activity in the global disparity of PageRank. We generate DPAH networks by varying f_m, h_{MM}, h_{mm}, and fixing edge density $d = 0.0015$ and 10 epochs. Each color represents the activity of nodes as the out-degree exponents of the networks $\gamma_M = \gamma_m \in \{1.5, 2.0, 2.5, 3.0, 3.5\}$. We see that by reducing the out-degree exponent in the DPAH networks (from $\gamma = 3.5$ to $\gamma = 1.5$), we reduce inequality (vertical axis), and inequity remains stable.

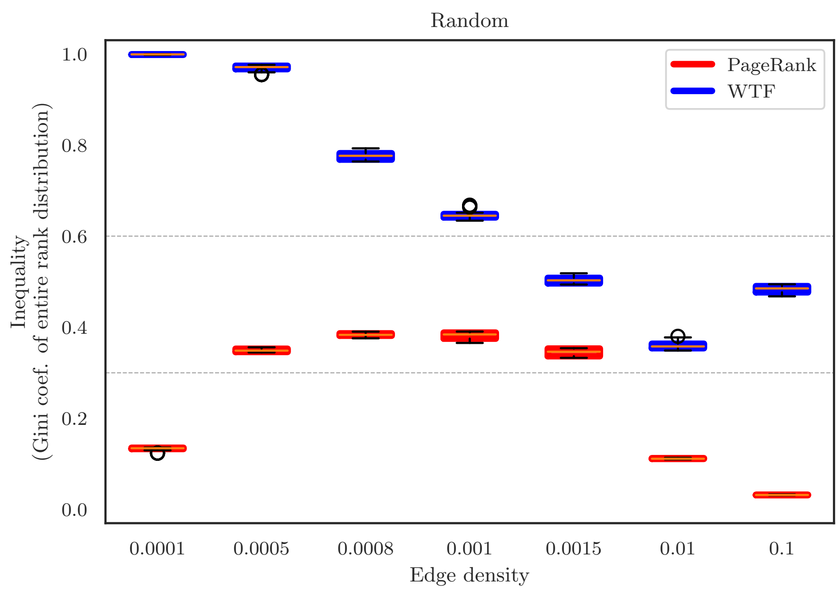

Figure S5. Global inequality on Random networks as a function of edge density. We generate directed Erdős-Rényi networks to demonstrate how the global inequality (y-axis) varies with respect to the edge density (x-axis) of the network. For each density value we generate networks with different fractions of minorities and 10 epochs. Note that $d = 0.0015$ corresponds to the Random networks used in the main experiments. Inequality computed on the PageRank distribution is shown in red, while the inequality on WTF is shown in blue. We see different trends for each algorithm. First, the inequality (Gini coefficient) of PageRank is very low when the edge density is extreme (i.e., either too low or too high). This means that in these regimes most nodes are similarly important regardless of the magnitude of their degrees. Second, the inequality of WTF is in general negatively correlated with density (i.e., the lower the density, the higher the inequality[60]). However, in the extreme case of denser networks (i.e., $d = 0.1$), inequality raises. Recall that ranking *inequity* is very close to zero ($ME \approx 0$) in random networks. Further studies are required to analytically understand the limits of inequality with respect to density.