Masses and strong decays of open charm hexaquark states $\Sigma_c^{(*)}\Sigma_c^{(*)}$

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Abstract

Inspired by the recent discovery of the doubly charmed tetraquark state T_{cc}^+ by the LHCb Collaboration, we perform a systematic study of masses and strong decays of open charm hexaquark states $\Sigma_c^{(*)}\Sigma_c^{(*)}$. Taking into account heavy quark spin symmetry breaking, we predict several bound states of isospin I=0, I=1, and I=2 in the one boson exchange model. Moreover, we adopt the effective Lagrangian approach to estimate the decay widths of $\Sigma_c^{(*)}\Sigma_c^{(*)}\to \Lambda_c\Lambda_c$ and their relevant ratios via the triangle diagram mechanism, which range from a few MeV to a few tens of MeV. We strongly recommend future experimental searches for the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ hexaquark states in the $\Lambda_c\Lambda_c$ invariant mass distributions.

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I. INTRODUCTION

The quenched (or conventional) quark model can well describe the properties of traditional hadrons, especially the ground-state ones [1, 2]. However, a large number of states that can not be easily explained by the quenched quark model have been accumulating since $D_{s0}^*(2317)$ and X(3872) were discovered by the BaBar [3] and Belle [4] collaborations in 2003. To unveil the nature of these exotic states, many pictures, such as multiquark states, hadronic molecules, and kinetic effects, have been proposed to understand them from different perspectives, including production mechanisms, mass spectra, and decay widths [5–12]. Among them, the hadronic molecular picture is probably the most popular one since most of these exotic states are located close to the mass thresholds of conventional hadron pairs. Two-body hadronic molecules are expected to exist in three configurations, i.e., meson-meson, meson-baryon, and baryon-baryon. With more and more XYZ states, P_c states, and T_{cc} discovered in the past two decades [11], heavy hadronic molecules have attracted a lot attention.

Heavy quark spin symmetry (HQSS) plays an important role in describing heavy hadronic molecules [13–16]. HQSS dictates that the interaction between a light (anti)quark and a heavy (anti)quark is independent of the heavy (anti)quark spin in the limit of heavy quark masses. As the heavy mesons (D, D^*) and baryons (Σ_c, Σ_c^*) belong to the same HQSS doublets, using them as building blocks, one expects three hidden charm multiplets of hadronic molecules, i.e., $\bar{D}^{(*)}D^{(*)}$, $\bar{D}^{(*)}\Sigma_c^{(*)}$, and $\bar{\Sigma}_c^{(*)}\Sigma_c^{(*)}$. With the assumption that X(3872) is a \bar{D}^*D bound state, in Ref. [14], Guo et al. adopted an effective field theory approach to study the $\bar{D}^{(*)}D^{(*)}$ system and predicted a \bar{D}^*D^* bound state as the HQSS partner of X(3872). In Ref. [16], using the same approach we obtained a complete HQSS multiplet of hadronic molecules, $\bar{D}^{(*)}\Sigma_c^{(*)}$, where $P_c(4440)$ and $P_c(4457)$ are regarded as the $\bar{D}^*\Sigma_c$ bound states. Although the likely existence of $\bar{\Sigma}_c^{(*)}\Sigma_c^{(*)}$ bound states have been explored theoretically [17–20], they have not been observed experimentally.

Very recently, the LHCb Collaboration observed one open charm tetraquark state T_{cc}^+ below the D^0D^{*+} mass threshold [21], which has already been predicted by a lot of theoretical works in the diquark and anti-diquark picture [22–35] as well as in the molecular picture [36–45]. In our previous work [45], using the one boson exchange (OBE) model we have systematically studied the $D^{(*)}D^{(*)}$ system and predicted one isocalar DD^* molecule with a binding energy of about 3 MeV, which is an open charm tetraquark state and may correspond to the T_{cc}^+ state. Therefore, it is interesting to investigate the likely existence of open charm pentaquark and hexaquark states.

In Ref. [46], we obtained a complete HQSS multiplet of hadronic molecules $D^{(*)}\Sigma_c^{(*)}$, consistent with Refs. [47–49]. One should note that the minimum quark content of the $D^{(*)}\Sigma_c^{(*)}$ system is ccq, which may couple to the excited states of Ξ_{cc} . Consequently, the $D^{(*)}\Sigma_c^{(*)}$ molecules can mix with the excited doubly charmed baryons. In the present work, we investigate the mass spectrum and strong decays of $\Sigma_c^{(*)}\Sigma_c^{(*)}$, which cannot couple to conventional baryons of ccq.

Up to now, the deuteron is the only experimentally confirmed baryon-baryon bound state, whose property can be well reproduced by high precision nuclear forces [50, 51]. It is natural to expect the existence of other dibaryon states consisting of strange or charm quarks. In 1976, Jaffe predicted one famous dibaryon state $\Lambda\Lambda$ [52], who has received a lot of attention both experimentally and theoretically [53–60]. However, the existence of the $\Lambda\Lambda$ dibaryon is still controversial. In recent years, the theoretical and experimental studies of dibaryon states have been extended to systems containing more strangeness quarks [60–72].

Experimental searches for dibaryons containing charm quarks is challenging because of the high energy required and the low production of charmed baryons. On the other hand, many theoretical studies have been performed. In Ref. [73], Liu et al. predicted one bound state generated by $\Lambda_c N$ and $\Sigma_c N$ couple channels in the OBE model, which was later explored in the chiral constituent quark model [74, 75], lattice QCD simulations [76], and effective field theories [77]. Later, using the same approach, Li et al. systematically studied possible bound states made of two singly charmed baryons [17, 78, 79]. Some of these states were investigated in the quark delocalization color screening model [80, 81]. After the discovery of the doubly charmed baryon Ξ_{cc} by the LHCb Collaboration [82], Meng et al. investigated bound states made of two doubly charmed baryons [83]. Inspired by the discovery of the pentaquark states by the LHCb Collaboration [84], we studied the $\Xi_{cc}^{(*)}\Sigma_c^{(*)}$ system utilizing heavy antiquark diquark symmetry in the OBE model and effective field theory [85, 86] and obtained a complete HQSS multiplet of hadronic molecules. Recently, lattice QCD simulations obtained a series of dibaryon states with open charm, such as $\Xi_{cc}\Sigma_c$ [87] and $\Omega_{ccc}\Omega_{ccc}$ [88].

In this work, we investigate the mass spectrum and decay patterns of the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ system dictated by HQSS. In Ref. [19], we parameterized the contact-range potential of the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ system with three parameters. However, we can not fully determine the mass spectrum of the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ system because no experimental data is available. In this work, we turn to the OBE model for help and calculate the mass spectrum. Then based on the so-obtained mass spectrum, we estimate the strong decay widths of $\Sigma_c^{(*)}\Sigma_c^{(*)}\to\Lambda_c\Lambda_c$ through the effective Lagrangian approach, which has

been widely used to explore the strong decays of hadronic molecules [89–92].

The manuscript is structured as follows. In Sec. II we present the details of the OBE model as applied to the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ system and the numerical results of their mass spectra. In Sec. III we explain the effective Lagrangian approach and calculate the decay widths of $\Sigma_c^{(*)}\Sigma_c^{(*)}\to \Lambda_c\Lambda_c$. Finally we conclude in Sec. IV

II. MASS SPECTRUM OF $\Sigma_c^{(*)}\Sigma_c^{(*)}$ SYSTEM

A. OBE potential

In this work, we employ the OBE model to derive the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ interaction, which has been successfully used to investigate hadronic molecules composed of heavy hadrons [93–96]. The OBE interaction of two heavy hadrons is generated by the exchange of light mesons, π , σ , ρ , and ω . The vector mesons, ρ and ω , provide the short-range interaction, the scalar meson σ provides the medium-range interaction, and the π meson provides the long-range interaction. Given the exploratory nature of the present work, the contributions of other mesons are neglected [97]. To derive the OBE potential, the effective Lagrangians describing the interaction between a charmed baryon and a light meson are necessary. The relevant Lagrangians are the same as those of the $\bar{D}^{(*)}\Sigma_c^{(*)}$ interaction [97], which read

$$\mathcal{L}_{S_c S_c \pi} = \frac{ig}{\sqrt{2} f_{\pi}} \vec{S}_c^{\dagger} \cdot (\vec{T} \cdot \vec{\pi} \vec{\nabla} \times \vec{S}_c) , \qquad (1)$$

$$\mathcal{L}_{S_c S_c \sigma} = g_\sigma \vec{S}_c^{\dagger} \sigma \cdot \vec{S}_c \,, \tag{2}$$

$$\mathcal{L}_{S_c S_c \rho} = g_{\rho} \vec{S}_c^{\dagger} (\vec{T} \cdot \vec{\rho}^0) \cdot \vec{S}_c - \frac{f_{\rho}}{4M} \vec{S}_{ci}^{\dagger} \vec{T} \cdot (\partial^i \vec{\rho}^j - \partial^j \vec{\rho}^i) \vec{S}_{cj},$$
(3)

$$\mathcal{L}_{S_c S_c \omega} = g_{\omega} \vec{S}_c^{\dagger} \omega^0 \cdot \vec{S}_c - \frac{f_{\omega}}{4M} \vec{S}_{ci}^{\dagger} \left(\partial^i \omega^j - \partial^j \omega^i \right) \vec{S}_{cj} ,$$
 (4)

where $\vec{S}_c = (\frac{1}{\sqrt{3}}\Sigma_c\vec{\sigma} + \vec{\Sigma}_c^*)$ denotes the superfield of Σ_c and Σ_c^* dictated by HQSS. The coupling of the π meson to Σ_c and Σ_c^* , g, was determined to be 0.84 in lattice QCD [98], which is smaller than the prediction of the quark model [73]. In this work we take g = 0.84. For the coupling to the σ meson, we estimate it using the quark model. From the nucleon and σ meson coupling of the liner sigma model, $g_{\sigma NN} = 10.2$, the corresponding coupling is determined to be $g_{\sigma} = \frac{2}{3}g_{\sigma NN} = 6.8$ [99]. The couplings of the light vector mesons contain both electric-type(g_v) and

TABLE I. Couplings of charmed baryons to light mesons. The magnetic coupling of the ρ and ω mesons are defined as $f_v = \kappa_v g_v$, and M refers to the mass scale (in units of MeV) involved in the magnetic-type couplings.

Coupling	g	g_{σ}	$g_{ ho}$	g_{ω}	$\kappa_{ ho}$	κ_{ω}	M
Value	0.84	6.8	5.8	5.8	1.7	1.7	940

magnetic-type(f_v) ones, which are related via $f_v = \kappa_v g_v$. For the ρ and ω couplings, from SU(3)-symmetry and the OZI rule, we obtain $g_\omega = g_\rho$ and $f_\omega = f_\rho$. The electric-type coupling of the singly charmed baryons to the vector mesons is determined to be $g_v = 5.8$ [73] as well as the corresponding magnetic-type coupling is estimated to be $\kappa_v = 1.7$ [100]. All the couplings are given in Table I for easy reference.

With the above Lagrangians, the OBE potentials for the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ system in coordinate space can be obtained, which read

$$V_{\pi}(\vec{r}) = \vec{T}_{1} \cdot \vec{T}_{2} \frac{g^{2}}{6f_{\pi}^{2}} \left[-\vec{a}_{1} \cdot \vec{a}_{2} \, \delta(\vec{r}) \right. \\ + \vec{a}_{1} \cdot \vec{a}_{2} \, m_{\pi}^{3} \, W_{Y}(m_{\pi}r) \\ + S_{12}(\vec{r}) \, m_{\pi}^{3} \, W_{T}(m_{\pi}r) \right],$$

$$(5)$$

$$V_{\sigma}(\vec{r}) = -g_{\sigma}^{2} \, m_{\sigma} \, W_{Y}(m_{\sigma}r) ,$$

$$(6)$$

$$V_{\rho}(\vec{r}) = \vec{T}_{1} \cdot \vec{T}_{2} \left[g_{\rho}^{2} \, m_{\rho} \, W_{Y}(m_{\rho}r) \right. \\ + \frac{f_{\rho}^{2}}{4M^{2}} \left(-\frac{2}{3} \, \vec{a}_{1} \cdot \vec{a}_{2} \, \delta(\vec{r}) \right. \\ + \frac{2}{3} \, \vec{a}_{1} \cdot \vec{a}_{2} \, m_{\rho}^{3} \, W_{Y}(m_{\rho}r) \\ - \frac{1}{3} \, S_{12}(\hat{r}) \, m_{\rho}^{3} \, W_{T}(m_{\rho}r) \right) \right],$$

$$(7)$$

$$V_{\omega}(\vec{r}) = g_{\omega}^{2} \, m_{\omega} \, W_{Y}(m_{\omega}r) \\ + \frac{f_{\omega}^{2}}{4M^{2}} \left[-\frac{2}{3} \, \vec{a}_{1} \cdot \vec{a}_{2} \, \delta(\vec{r}) \right. \\ + \frac{2}{3} \, \vec{a}_{1} \cdot \vec{a}_{2} \, m_{\omega}^{3} \, W_{Y}(m_{\omega}r) \\ - \frac{1}{3} \, S_{12}(\hat{r}) \, m_{\omega}^{3} \, W_{T}(m_{\omega}r) \right],$$

$$(8)$$

where $\vec{T}_1 \cdot \vec{T}_2$ is the isospin factor for the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ system, and $\vec{a}_1 \cdot \vec{a}_2$ and $S_{12}(\hat{r})$ denote the spin-spin and tensor terms, respectively, and \vec{a} denotes the spin operator of the $\Sigma_c^{(*)}$ baryon. The

dimensionless functions $W_Y(x)$ and $W_T(x)$ are defined as

$$W_Y(x) = \frac{e^{-x}}{4\pi x},\tag{9}$$

$$W_T(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) \frac{e^{-x}}{4\pi x}.$$
 (10)

Since the charmed baryons and light mesons involved in our study are not point-like particles, we introduce a form factor in the interaction vertices. Here we use a monopolar form factor (for more details we refer to Refs. [45, 97])

$$F(q, m_E, \Lambda) = \frac{\Lambda^2 - m_E^2}{\Lambda^2 - q^2},$$
(11)

where m_E and q are the mass and 4-momentum of the exchanged meson. Λ is an unknown cutoff, which is often fixed by reproducing some hadronic molecular candidates that can be related to hadronic molecules of interest via symmetries. Assuming X(3872) and $P_c(4312)$ as bound states of $\bar{D}D^*$ and $\bar{D}\Sigma_c$, we can determine the corresponding cutoff of the OBE model to be 1.04 GeV [45] and 1.12 GeV [97], respectively. In addition, the cutoff needed to reproduce the binding energy of the deuteron is 0.86 GeV [101]. All these results indicate that a reasonable value for the cutoff of the OBE model is about 1 GeV. In this work, assuming that the $\Sigma_c^{(*)}$ - $\Sigma_c^{(*)}$ system is similar to the nucleon-nucleon system, we take a cutoff of Λ =0.86 GeV to calculate the binding energies of the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ dibaryon system,

With the above form factor, the functions δ , W_Y , and W_T in Eqs. (5-8) need to be changed accordingly

$$\delta(r) \to m^3 d(x, \lambda)$$
, (12)

$$W_Y(x) \to W_Y(x,\lambda)$$
, (13)

$$W_T(x) \to W_T(x,\lambda)$$
, (14)

with $\lambda = \Lambda/m$. The corresponding functions d, W_Y , and W_T read

$$d(x,\lambda) = \frac{(\lambda^2 - 1)^2}{2\lambda} \frac{e^{-\lambda x}}{4\pi},\tag{15}$$

$$W_Y(x,\lambda) = W_Y(x) - \lambda W_Y(\lambda x)$$

$$-\frac{(\lambda^2 - 1)}{2\lambda} \frac{e^{-\lambda x}}{4\pi},\tag{16}$$

$$W_T(x,\lambda) = W_T(x) - \lambda^3 W_T(\lambda x) - \frac{(\lambda^2 - 1)}{2\lambda} \lambda^2 \left(1 + \frac{1}{\lambda x} \right) \frac{e^{-\lambda x}}{4\pi}.$$
 (17)

The generic wave function of a baryon-baryon system reads

$$|\Psi\rangle = \Psi_{JM}(\vec{r})|IM_I\rangle, \qquad (18)$$

where $|IM_I\rangle$ denotes the isospin wave function and $\Psi_{JM}(\vec{r})$ are the spin and spatial wave functions. The dynamics in isospin space is embodied in the isospin factor $\vec{T}_1 \cdot \vec{T}_2$. In this work the total isospin is either 0, 1, or 2 for the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ system, then the corresponding isospin factor are -2, -1, and 1, respectively.

The spin wave function can be written as a sum over partial wave functions, which reads (in the spectroscopic notation)

$$|^{2S+1}L_{J}\rangle = \sum_{M_{S},M_{L}} \langle LM_{L}SM_{S}|JM\rangle |SM_{S}\rangle Y_{LM_{L}}(\hat{r}),$$
(19)

where $\langle LM_LSM_S|JM\rangle$ are the Clebsch-Gordan coefficients, $|SM_S\rangle$ the spin wavefunction, and $Y_{LM_L}(\hat{r})$ the spherical harmonics.

In the partial wave decomposition of the OBE potential, we encounter both spin-spin and tensor components

$$C_{12} = \vec{a}_1 \cdot \vec{a}_2 \,, \tag{20}$$

$$S_{12} = 3\vec{a}_1 \cdot \hat{r} \, \vec{a}_2 \cdot \hat{r} - \vec{a}_1 \cdot \vec{a}_2 \,, \tag{21}$$

In the present study, we consider both S and D waves. The relevant matrix elements are listed in Table IV.

B. Numerical results and discussions

For a pair of identical fermions, the total spin and isospin of $\Sigma_c^{(*)}\Sigma_c^{(*)}$ have to fulfill the condition, I+S= even. The possible combinations of spin and isospin are shown in Table II. Additionally, when deriving the OBE potential, we have assumed HQSS. As is well known, the charm quark mass is not large enough to strictly satisfy HQSS. In the present work, we take into account a HQSS breaking of the order of $\Delta=15\%$ in the following way [85]:

$$V = V_{OBE}(1 \pm \Delta). \tag{22}$$

With the above OBE potentials, the binding energies of the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ system can be obtained by solving the Schrödinger equation. With a cutoff of $\Lambda=0.86$ GeV, we obtain the binding energies

TABLE II. Binding energies of $\Sigma_c^{(*)}\Sigma_c^{(*)}$ molecules with the uncertainties originating from the breaking of HQSS and the corresponding masses, for which only central values are given. The \dagger indicates that the particular system does not bind and ? denotes the likely existence of a bound state.

Molecule	$I(J^P)$	B.E(MeV)	Mass(MeV)	$I(J^P)$	B.E(MeV)	Mass(MeV)	$I(J^P)$	B.E(MeV)	Mass(MeV)
$\Sigma_c \Sigma_c$	$0(0^+)$	$39.5^{+19.3}_{-16.2}$	4868.5	1(1+)	$1.6^{+2.4}_{-1.3}$	4906.4	$2(0^+)$	†	†
$\Sigma_c \Sigma_c^* - \Sigma_c^* \Sigma_c$	0(1+)	$38.4^{+18.6}_{-15.6}$	4933.6	1(2+)	$2.9^{+3.3}_{-2.1}$	4969.1	$2(1^+)$	†	†
$\Sigma_c \Sigma_c^* + \Sigma_c^* \Sigma_c$	$0(2^+)$	$0.1^{+0.7}_{\dagger}$?	1(1+)	$10.7_{-5.9}^{+7.6}$	4961.3	$2(2^+)$	$7.1_{-4.5}^{+6.3}$	4964.9
$\Sigma_c^* \Sigma_c^*$	0(0+)	$59.6^{+25.7}_{-22.2}$	4976.4	1(1+)	$13.7^{+9.0}_{-7.1}$	5022.3	$2(0^+)$	†	†
$\Sigma_c^* \Sigma_c^*$	$0(2^+)$	$13.1^{+8.4}_{-6.7}$	5022.9	1(3 ⁺)	$0.4^{+1.2}_{\dagger}$?	$2(2^{+})$	$0.4^{+1.2}_{\dagger}$?

of the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ system for isospin I=0,1, and 2, which are given in Table II. After taking into account the HQSS breaking, we find four hadronic molecules with I=0, i.e., $J^P=0^+$ $\Sigma_c\Sigma_c$, $J^P=1^+$ $\Sigma_c^*\Sigma_c^*$, $J^{PC}=0^+$ $\Sigma_c^*\Sigma_c^*$, and $J^{PC}=2^+$ $\Sigma_c^*\Sigma_c^*$. Among them, the $J^P=0^+$ $\Sigma_c\Sigma_c$ bound state corresponds to the deuteron counterpart in the charm sector. For isospin I=1, we obtain four hadronic molecules as well but with smaller binding energies than those of I=0. On the other hand, there exists only one bound state $I(J^P)=2(2^+)\Sigma_c^*\Sigma_c$ for isospin I=2. Obviously, with increasing isospin, the isospin factor decreases, which reduces the number and binding energies of $\Sigma_c^{(*)}\Sigma_c^{(*)}$ hadronic molecules.

Very recently, Dong et al. obtained three isoscalar molecules, $\Sigma_c \Sigma_c$, $\Sigma_c^* \Sigma_c$, and $\Sigma_c^* \Sigma_c^*$ in the single channel Bethe-Salpeter approach, where they did not take into account the spin-spin interaction [48]. In the same approach, Chen et al. have systematically studied the $\Sigma_c^{(*)} \Sigma_c^{(*)}$ system [102] and found bound states with binding energies larger than ours. In Ref. [102], the potentials of the $\Sigma_c^{(*)} \Sigma_c^{(*)}$ system are determined by reproducing the masses of $P_c(4440)$ and $P_c(4457)$, which are assumed as $\bar{D}^* \Sigma_c$ bound states. Since the cutoff of the OBE model in this case should be 1.12 GeV, the binding energies of the $\Sigma_c^{(*)} \Sigma_c^{(*)}$ system obtained in Ref [102] are larger than ours. A series of theoretical studies of the $\Sigma_c^{(*)} \Sigma_c^{(*)}$ system [17, 78–81, 102, 103] showed that the predictions on the existence of $\Sigma_c^{(*)} \Sigma_c^{(*)}$ molecules are rather solid, consistent with our results. However, where to search for the $\Sigma_c^{(*)} \Sigma_c^{(*)}$ hadronic molecules experimentally has not been thoroughly studied. In the following, we investigate the strong decays of likely $\Sigma_c^{(*)} \Sigma_c^{(*)}$ bound states and hope to stimulate future experimental studies.

III. STRONG DECAYS OF $\Sigma_c^{(*)}\Sigma_c^{(*)}$

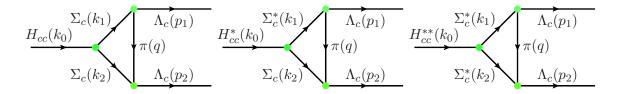


FIG. 1. Triangle diagrams of $\Sigma_c^{(*)}\Sigma_c^{(*)}$ molecules decaying into $\Lambda_c\Lambda_c$ by exchanging a π meson.

A. Effective Lagrangian approach

In this section, we further investigate the strong decays of $\Sigma_c^{(*)}\Sigma_c^{(*)}$ molecules by the effective Lagrangian approach, which has been widely applied to study the strong decays of two-body and three-body bound states [89–92, 104–106]. The transition from $\Sigma_c^{(*)}\Sigma_c^{(*)}\to\Lambda_c\Lambda_c$ is mediated by the exchange of a π meson 1 , depicted by the triangle diagrams shown in Fig. 1. The isospin and spin of the $\Lambda_c\Lambda_c$ pair are 0 and 0/1, respectively, which can help us select the quantum numbers of initial states because for strong decays isospin and angular momentum are conserved. As a result, in this work, we only study the strong decay of three isoscalar states, $J^P=0^+\Sigma_c\Sigma_c$, $J^P=1^+\Sigma_c^*\Sigma_c$, and $J^P=0^+\Sigma_c^*\Sigma_c^*$, denoted as H_{cc} , H_{cc}^* and H_{cc}^{**} in the following, respectively.

To describe the interaction vertices of Fig. 1, we need the following Lagrangians for the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ bound states and their constituents

$$\mathcal{L}_{H_{cc}\Sigma_{c}\Sigma_{c}}(x) = g_{H_{cc}\Sigma_{c}\Sigma_{c}} \int dy \Phi(y^{2}) \bar{H}_{cc}(x) \Sigma_{c}(x + \omega_{\Sigma_{c}}y) \Sigma_{c}(x - \omega_{\Sigma_{c}}y), \tag{23}$$

$$\mathcal{L}_{H_{cc}^{*}\Sigma_{c}\Sigma_{c}^{*}}(x) = g_{H_{cc}^{*}\Sigma_{c}\Sigma_{c}^{*}} \int dy \Phi(y^{2}) \bar{H}_{cc}^{*}(x) \Sigma_{c}(x + \bar{\omega}_{\Sigma_{c}^{*}}y) \Sigma_{c\nu}^{*}(x - \bar{\omega}_{\Sigma_{c}}y),$$

$$\mathcal{L}_{H_{cc}^{**}\Sigma_{c}^{*}\Sigma_{c}^{*}}(x) = g_{H_{cc}^{**}\Sigma_{c}^{*}\Sigma_{c}^{*}} \int dy \Phi(y^{2}) \bar{H}_{cc}^{*}(x) \Sigma_{c}^{*\nu}(x + \omega_{\Sigma_{c}^{*}}y) \Sigma_{c\nu}^{*}(x - \omega_{\Sigma_{c}^{*}}y),$$

where $\omega_{\Sigma_c} = \frac{m_{\Sigma_c}}{m_{\Sigma_c} + m_{\Sigma_c}}$, $\omega_{\Sigma_c^*} = \frac{m_{\Sigma_c^*}}{m_{\Sigma_c^*} + m_{\Sigma_c^*}}$, $\bar{\omega}_{\Sigma_c^*} = \frac{m_{\Sigma_c^*}}{m_{\Sigma_c} + m_{\Sigma_c^*}}$, and $\bar{\omega}_{\Sigma_c} = \frac{m_{\Sigma_c}}{m_{\Sigma_c^*} + m_{\Sigma_c^*}}$ are the kinematic parameters with m_{Σ_c} and $m_{\Sigma_c}^*$ the masses of Σ_c and Σ_c^* , and $g_{H_{cc}\Sigma_c\Sigma_c}$, $g_{H_{cc}^*\Sigma_c\Sigma_c^*}$, and $g_{H_{cc}^*\Sigma_c\Sigma_c^*}$ are the couplings between the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ bound states and their constituents. The $\Phi(y^2)$ is the correlation function, which not only takes into account the distribution of the two constituent hadrons in a

¹ In principle, the exchange of ρ is also allowed. However, $\Sigma_c^{(*)}$ can only transit into $\Lambda_c \rho$ via the magnetic term, which is regarded as sub-leading for interactions involving vector mesons. As a result, we can safely neglect the ρ meson exchange in comparison with the π meson exchange.

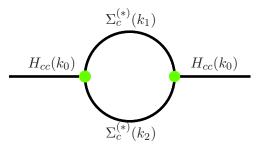


FIG. 2. Mass operators of the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ molecules

molecule but also renders the Feynman diagrams ultraviolet finite. Here we choose the correlation function in form of a Gaussian function

$$\Phi(y^2) = \operatorname{Exp}(-p_E^2/\Lambda^2),\tag{24}$$

where p_E is the Euclidean momentum and Λ is the size parameter.

To estimate the couplings between bound states and their constituents, we employ the compositeness condition [107–109]. For dibaryon bound states with total angular momentum J=0 the compositeness condition reads

$$Z_{\Sigma_c \Sigma_c} = 1 - \frac{\partial \Sigma(m_{\Sigma_c \Sigma_c}^2)}{\partial m_{\Sigma_c \Sigma_c}^2} = 0, \tag{25}$$

where $\Sigma(m_{\Sigma_c\Sigma_c}^2)$ is the self energy of a $\Sigma_c^{(*)}\Sigma_c^{(*)}$ bound states as shown in Fig. 2.

The self-energy for a vector state is expressed as $\Sigma^{\mu\nu}(p^2)$ with Lorentz indices μ and ν , which can be decomposed into two parts, longitudinal and transverse, $\Sigma^{\mu\nu}(p^2) = \hat{g}^{\mu\nu}\Sigma^T(p^2) + \frac{p^{\mu}p^{\nu}}{p^2}\Sigma^L(p^2)$, where $\hat{g}^{\mu\nu} = g^{\mu\nu} - p^{\mu}p^{\nu}/p^2$. Substituting the transverse term into Eq (25), we can also determine the couplings of a J=1 dibaryon state to their constituents.

The mass operators for the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ dibaryons read as follows:

$$M_{H_{cc}} = ig_{H_{cc}\Sigma_{c}\Sigma_{c}}^{2} \int \frac{d^{4}k_{1}}{(2\pi)^{4}} e^{-\frac{(\omega_{\Sigma_{c}}k_{2} - \omega_{\Sigma_{c}}k_{1})^{2}}{\Lambda^{2}}} \frac{1}{\not k_{1} - m_{\Sigma_{c}}} \frac{1}{\not k_{2} - m_{\Sigma_{c}}},$$

$$M_{H_{cc}^{**}}^{\mu\nu} = ig_{H_{cc}\Sigma_{c}^{*}\Sigma_{c}}^{2} \int \frac{d^{4}k_{1}}{(2\pi)^{4}} e^{-\frac{(\bar{\omega}_{\Sigma_{c}^{*}}k_{2} - \bar{\omega}_{\Sigma_{c}}k_{1})^{2}}{\Lambda^{2}}} \frac{P^{\mu\nu}(k_{1})}{\not k_{1} - m_{\Sigma_{c}^{*}}} \frac{1}{\not k_{2} - m_{\Sigma_{c}}},$$

$$M_{H_{cc}^{**}} = ig_{H_{cc}^{**}\Sigma_{c}^{*}\Sigma_{c}^{*}}^{2} \int \frac{d^{4}k_{1}}{(2\pi)^{4}} e^{-\frac{(\bar{\omega}_{\Sigma_{c}^{*}}k_{2} - \bar{\omega}_{\Sigma_{c}^{*}}k_{1})^{2}}{\Lambda^{2}}} \frac{P^{\lambda\sigma}(k_{1})}{\not k_{1} - m_{\Sigma_{c}^{*}}} \frac{P^{\lambda\sigma}(k_{2})}{\not k_{2} - m_{\Sigma_{c}^{*}}},$$

where
$$P^{\lambda\sigma}(p)=g^{\lambda\sigma}-\frac{1}{3}\gamma^{\lambda}\gamma^{\sigma}-\frac{\gamma^{\lambda}p^{\sigma}-\gamma^{\sigma}p^{\lambda}}{3p^{2}/p}-\frac{2p^{\lambda}p^{\sigma}}{3p^{2}}$$
.

In Refs. [110, 111], the above cutoff is found to be around 1 GeV. One should note, however, that the couplings of a molecular state to its components are related to its binding energy [112], and hence we take a cutoff of $\Lambda=0.86$ GeV, the same as that used to study the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ dibaryons, to determine the couplings in this work. In Table III, we present the $H_{cc}^{(**)}$ couplings to their

TABLE III. Couplings of $\Sigma_c^{(*)}\Sigma_c^{(*)}$ molecules to their components obtained with a cutoff of $\Lambda=0.86$ GeV.

Couplings	$g_{H_{cc}\Sigma_c\Sigma_c}$	$g_{H_{cc}^*\Sigma_c^*\Sigma_c}$	$g_{H_{cc}^{**}\Sigma_{c}^{*}\Sigma_{c}^{*}}$
Value	4.3	5.2	7.7

components. Because the binding energy of H_{cc}^{***} is larger than those of H_{cc}^{*} and H_{cc} , $g_{H_{cc}^{***}\Sigma_{c}^{*}\Sigma_{c}^{*}}$ is larger than $g_{H_{cc}^{*}\Sigma_{c}^{*}\Sigma_{c}}$ and $g_{H_{cc}\Sigma_{c}\Sigma_{c}}$ as well.

The other vertices of the triangle diagrams of Fig 1 can be classified into two categories, $\Sigma_c \to \Lambda_c \pi$ and $\Sigma_c^* \to \Lambda_c \pi$. The Lagrangians describing these interactions are given by

$$\mathcal{L}_{\pi\Lambda_c\Sigma_c} = -i\frac{g_{\pi\Lambda_c\Sigma_c}}{f_{\pi}} \bar{\Lambda}_c \gamma^{\mu} \gamma_5 \partial_{\mu} \vec{\phi}_{\pi} \cdot \vec{\tau} \Sigma_c,
\mathcal{L}_{\pi\Lambda_c\Sigma_c^*} = \frac{g_{\pi\Lambda_c\Sigma_c^*}}{f_{\pi}} \bar{\Lambda}_c \partial^{\mu} \vec{\phi}_{\pi} \cdot \vec{\tau} \Sigma_{c\mu}^*, \tag{27}$$

where $f_{\pi}=132$ MeV and the couplings $g_{\pi\Lambda_c\Sigma_c}$ and $g_{\pi\Lambda_c\Sigma_c^*}$ can be determined by fitting to experimental data. From the decay widths of $\Gamma(\Sigma_c \to \Lambda_c\pi)=1.89$ MeV and $\Gamma(\Sigma_c^* \to \Lambda_c\pi)=15.0$ MeV [113], we obtain the couplings $g_{\pi\Lambda_c\Sigma_c}=0.55$ and $g_{\pi\Lambda_c\Sigma_c^*}=0.97$, consistent with other works [73, 114]. In addition, we find that the two couplings approximately satisfy the relationship $g_{\pi\Lambda_c\Sigma_c^*}=\sqrt{3}g_{\pi\Lambda_c\Sigma_c}$, given by the quark model [73].

With the above Lagrangians, the amplitudes of $H_{cc}^{(**)} \to \Lambda_c \Lambda_c$ can be easily written down

$$\mathcal{M}_{H_{cc}} = i \frac{g_{H_{cc}\Sigma_{c}\Sigma_{c}}g_{\pi\Lambda_{c}\Sigma_{c}}^{2}}{f_{\pi}^{2}} \int \frac{d^{4}q}{(2\pi)^{4}} \bar{u}_{\Lambda_{c}} \gamma_{\mu} \gamma_{5} q^{\mu} \frac{1}{\not{k}_{1} - m_{1}} \frac{1}{q^{2} - m_{\pi}^{2}} \frac{1}{\not{k}_{2} - m_{1}} \gamma_{\nu} \gamma_{5} q^{\nu} F^{2}(q, m_{\pi}, \Lambda) \bar{u}_{\Lambda_{c}}^{T}, \tag{28}$$

$$\mathcal{M}_{H_{cc}^{*}} = i \frac{g_{H_{cc}^{*}\Sigma_{c}^{*}\Sigma_{c}}g_{\pi\Lambda_{c}\Sigma_{c}}g_{\pi\Lambda_{c}\Sigma_{c}^{*}}}{f_{\pi}^{2}} \int \frac{d^{4}q}{(2\pi)^{4}} \bar{u}_{\Lambda_{c}} q^{\mu} \frac{P^{\mu\nu}(k_{1})\varepsilon_{\nu}(k_{0})}{\not{k}_{1} - m_{1}} \frac{1}{q^{2} - m_{\pi}^{2}} \frac{1}{\not{k}_{2} - m_{2}} \gamma_{\alpha} \gamma_{5} q^{\alpha} F^{2}(q, m_{\pi}, \Lambda) \bar{u}_{\Lambda_{c}}^{T},$$

$$\mathcal{M}_{H_{cc}^{**}} = i \frac{g_{H_{cc}^{**}\Sigma_{c}^{*}\Sigma_{c}^{*}} g_{\pi\Lambda_{c}\Sigma_{c}^{*}}^{2}}{f_{\pi}^{2}} \int \frac{d^{4}q}{(2\pi)^{4}} \bar{u}_{\Lambda_{c}} q^{\mu} \frac{P^{\mu\nu}(k_{1})}{\not{k}_{1} - m_{1}} \frac{1}{q^{2} - m_{\pi}^{2}} \frac{P_{\nu\alpha}(k_{2})}{\not{k}_{2} - m_{1}} q^{\alpha} F^{2}(q, m_{\pi}, \Lambda) \bar{u}_{\Lambda_{c}}^{T},$$

where \bar{u}_{Λ_c} and $\bar{u}_{\Lambda_c}^T$ represent the spinors of the final-state $\Lambda_c\Lambda_c$ pair, and $F(q, m_E, \Lambda)$ is the form factor

$$F(q, m_E, \Lambda) = \frac{\Lambda^2 - m_E^2}{\Lambda^2 - q^2},\tag{29}$$

which not only removes ultraviolet divergence of the loop diagram, but also takes into account the off-shell effects. The m_E is the mass of the exchanged particle. The cutoff is expressed by $\Lambda = m_E + \alpha \Lambda_{QCD}$, where Λ_{QCD} is around 200-300 MeV and the dimensionless parameter α is around 1 [115]. Thus we vary the cutoff from 0.4 GeV to 0.6 GeV to estimate the uncertainties induced.

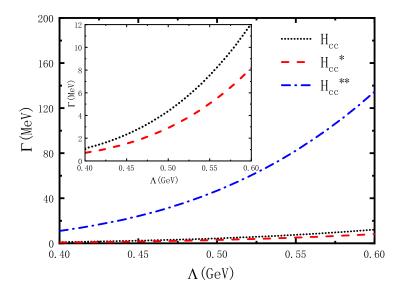


FIG. 3. Decay widths of $H_c^{(**)} \to \Lambda_c^+ \Lambda_c^+$ as functions of the cutoff Λ .

With the amplitudes of $H_{cc}^{(**)} \to \Lambda_c \Lambda_c$ determined, one can obtain the corresponding partial decay widths as

$$\Gamma = \frac{1}{2J+1} \frac{1}{8\pi} \frac{|\vec{p}|}{m_{H_{cc}}^{2}} |\bar{\mathcal{M}}|^{2}, \tag{30}$$

where J is the total angular momentum of the $H_{cc}^{(**)}$ molecule, the overline indicates the sum over the polarization vectors of final states, $|\vec{p}|$ is the momentum of either final state in the rest frame of $\Sigma_c^{(*)}\Sigma_c^{(*)}$ and $m_{H_{cc}^{(**)}}$ is the mass of the corresponding molecule. At last, we have to multiply a factor 1/2 to the above result to take into account the statistics of identical particles.

B. Numerical results and discussions

In Fig .3, we present the decay widths of $H_{cc}^{(**)} \to \Lambda_c \Lambda_c$ as functions of the cutoff Λ , where the masses of $H_{cc}^{(**)}$ molecules are the central values of Table II. With the cutoff varying from 0.4 to 0.6 GeV, the decay widths of H_{cc} and H_{cc}^* molecules change from 1.1 to 12.1 MeV and 0.7 to 8.2 MeV, respectively, which are close to each other and relatively narrow. On the other hand, the width of the H_{cc}^{**} molecule can be very large, which changes from 11.0 to 134.6 MeV. For a cutoff of 0.5 GeV, the decay widths of H_{cc} , H_{cc}^* , and H_{cc}^{**} molecules are 4, 3, and 47 MeV, respectively.

One should note that there also exist uncertainties for the masses of the $H_{cc}^{(**)}$ molecules in our OBE model. To take into account the impact of mass (binding energy) uncertainties of $H_{cc}^{(**)}$ molecules on the decay widths, we show the decay widths of $H_{cc}^{(**)}$ molecules as functions of

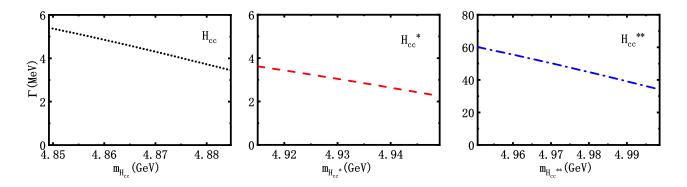


FIG. 4. Decay widths of $H_c^{(**)} \to \Lambda_c^+ \Lambda_c^+$ as functions of the masses of $H_c^{(**)}$, obtained with a cutoff of $\Lambda = 0.5$ GeV.

their masses in Fig. 4, where the ranges of masses are obtained from the upper and lower limits of binding energies of Table II, and the cutoff is taken to be 0.5 GeV. One can see that the decay widths of H_{cc} and H_{cc}^* only vary by several MeV, while the decay width of H_{cc}^{**} varies by tens of MeV, which shows that the decay width of $H_{cc}^{(**)}$ molecules are not very sensitive to their masses.

All of the $H_{cc}^{(**)}$ molecules can decay into $\Lambda_c\Lambda_c$, which indicates that all of them could be detected in the $\Lambda_c\Lambda_c$ mass distributions. In Fig .5 we show the ratios of the decay widths of the H_{cc}^{**} and H_{cc}^{**} molecules to that of the H_{cc} molecule, and the corresponding ratios are around 10 and 1, respectively, which tells that three peaks will appear in the $\Lambda_c\Lambda_c$ invariant mass spectrum, two narrow structures and one rather wide structure. In addition, we find that these ratios are insensitive to the cutoff used.

IV. SUMMARY AND OUTLOOK

Inspired by the recent discovery of the doubly charmed tetraquark sate T_{cc} , we performed a systematic study of the mass spectrum and strong decays of doubly charmed hexquark states composed of $\Sigma_c^{(*)}\Sigma_c^{(*)}$. We adopted the one-boson exchange model to calculate the binding energies of the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ system, where the cutoff is fixed by reproducing the binding energy of the deuteron. After considering breaking of HQSS, we found four bound states with isospin 0, i.e., $J^P = 0^+\Sigma_c\Sigma_c$, $J^P = 1^+\Sigma_c^*\Sigma_c$, $J^P = 0^+\Sigma_c^*\Sigma_c^*$, and $J^P = 2^+\Sigma_c^*\Sigma_c^*$, four bound states with isospin 1, $J^P = 1^+\Sigma_c\Sigma_c$, $J^P = 1^+\Sigma_c^*\Sigma_c$, $J^P = 2^+\Sigma_c^*\Sigma_c$, and $J^P = 1^+\Sigma_c^*\Sigma_c^*$, and one bound state with isospin 2, $J^P = 2^+\Sigma_c^*\Sigma_c$. Among them, the $J^P = 0^+\Sigma_c\Sigma_c$ state could be regarded as the deuteron counterpart with double charm, which is much more bound than the deuteron. In addition, we

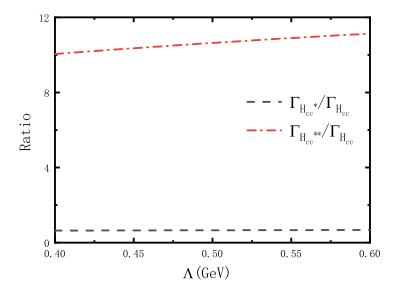


FIG. 5. Ratios between decay widths of $H_c^{(**)} \to \Lambda_c^+ \Lambda_c^+$ as functions of the cutoff Λ .

found that with the increase of total isospin, the number of $\Sigma_c^{(*)}\Sigma_c^{(*)}$ bound states decreases and the binding energies of I=1 dibaryons are smaller than those of I=0.

All the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ bound states can decay into $\Lambda_c\Lambda_c$ by exchange of a π meson via the triangle diagrams. We used the effective Lagrangian approach to estimate the decay widths of $\Sigma_c^{(*)}\Sigma_c^{(*)}\to\Lambda_c\Lambda_c$. Due to the conservation law of isospin and spin we only studied three isoscalar states, i.e., $J^P=0^+\Sigma_c\Sigma_c$, $J^P=1^+\Sigma_c^*\Sigma_c$, and $J^P=0^+\Sigma_c^*\Sigma_c^*$, which can decay into $\Lambda_c\Lambda_c$. We found that the decay widths of $J^P=0^+\Sigma_c\Sigma_c$ and $J^P=1^+\Sigma_c^*\Sigma_c$ are several MeV and that of $J^P=0^+\Sigma_c^*\Sigma_c^*$ is around half a hundred MeV, which depends strongly on the cutoff. We also found that the decay widths of $\Sigma_c^{(*)}\Sigma_c^{(*)}\to\Lambda_c\Lambda_c$ are weakly dependent on the masses of $H_{cc}^{(**)}$ molecules. The ratios of the decay widths of H_{cc}^{**} and H_{cc}^* to that of H_{cc} are about 10 and 1, respectively, which are only weakly dependent on the cutoff. We encourage our experimental colleagues to search for the doubly charmed hexaquark states $\Sigma_c^{(*)}\Sigma_c^{(*)}$ in the $\Lambda_c\Lambda_c$ invariant mass distributions, which may be scrutinized at LHC, J-PARC, and RHIC in future.

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TABLE IV. Relevant partial wave matrix elements for the $\Sigma_c^{(*)}\Sigma_c^{(*)}$ system.

State	J^P	Partial wave	$ec{a}_1 \cdot ec{a}_2$	$3\vec{a}_1 \cdot \hat{r} \vec{a}_2 \cdot \hat{r} - \vec{a}_1 \cdot \vec{a}_2$
$\Sigma_c \Sigma_c J$	r = 0	1S_0	-3	0
$\sum_{c} \sum_{c} J$	r = 1	${}^{3}S_{1}$ - ${}^{3}D_{1}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \sqrt{8} \\ \sqrt{8} & -2 \end{pmatrix}$
$\sum_{c}\sum_{c}^{*} J$	T=1	3S_1 - 3D_1 - 5D_1	$\begin{pmatrix} -\frac{5}{2} & 0 & 0 \\ 0 & -\frac{5}{2} & 0 \\ 0 & 0 & \frac{3}{2} \end{pmatrix}$	$\begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{3}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{3}{2} \\ -\frac{3}{\sqrt{2}} & -\frac{3}{2} & -\frac{3}{2} \end{pmatrix}$
$\sum_{c}\sum_{c}^{*}J$	T=2	${}^{5}S_{2}$ - ${}^{3}D_{2}$ - ${}^{5}D_{2}$	$\begin{pmatrix} \frac{3}{2} & 0 & 0 \\ 0 & -\frac{5}{2} & 0 \\ 0 & 0 & \frac{3}{2} \end{pmatrix}$	$\begin{pmatrix} 0 & 3\sqrt{\frac{3}{10}} & 3\sqrt{\frac{7}{10}} \\ 3\sqrt{\frac{3}{10}} & -\frac{1}{2} & -\frac{3}{2}\sqrt{\frac{3}{7}} \\ 3\sqrt{\frac{7}{10}} & -\frac{3}{2}\sqrt{\frac{3}{7}} & \frac{9}{14} \end{pmatrix}$
$\sum_{c}^{*} \sum_{c}^{*} J$	r = 0	$^{1}S_{0}$ - $^{5}D_{0}$	$\begin{pmatrix} -\frac{15}{4} & 0\\ 0 & -\frac{3}{4} \end{pmatrix}$	$\begin{pmatrix} 0 & -3 \\ -3 & -3 \end{pmatrix}$
$\sum_{c}^{*}\sum_{c}^{*} J$	' = 1	${}^{3}S_{1}$ - ${}^{3}D_{1}$ - ${}^{5}D_{1}$ - ${}^{7}D_{1}$	$ \begin{pmatrix} -\frac{11}{4} & 0 & 0 & 0 \\ 0 & -\frac{11}{4} & 0 & 0 \\ 0 & 0 & -\frac{3}{4} & 0 \\ 0 & 0 & 0 & \frac{9}{4} \end{pmatrix} $	$ \begin{pmatrix} 0 & \frac{17}{5\sqrt{2}} & 0 & -\frac{3\sqrt{7}}{5} \\ \frac{17}{5\sqrt{2}} & -\frac{17}{10} & 0 & \frac{3}{5}\sqrt{\frac{2}{7}} \\ 0 & 0 & -\frac{3}{2} & 0 \\ -\frac{3\sqrt{7}}{5} & \frac{3}{5}\sqrt{\frac{2}{7}} & 0 & -\frac{108}{35} \end{pmatrix} $
$\sum_{c}^{*}\sum_{c}^{*} J$	T=2	$^{5}S_{2}$ - $^{1}D_{2}$ - $^{3}D_{2}$ - $^{5}D_{2}$ - $^{7}D_{2}$	$ \begin{pmatrix} -\frac{3}{4} & 0 & 0 & 0 & 0 \\ 0 & -\frac{15}{4} & 0 & 0 & 0 \\ 0 & 0 & -\frac{11}{4} & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{9}{4} \end{pmatrix} $	$ \begin{pmatrix} 0 & -\frac{3}{\sqrt{5}} & 0 & 3\sqrt{\frac{7}{10}} & 0 \\ -\frac{3}{\sqrt{5}} & 0 & 0 & 3\sqrt{\frac{2}{7}} & 0 \\ 0 & 0 & \frac{17}{10} & 0 & \frac{6}{5}\sqrt{\frac{3}{7}} \\ 3\sqrt{\frac{7}{10}} & 3\sqrt{\frac{2}{7}} & 0 & \frac{9}{14} & 0 \\ 0 & 0 & \frac{6}{5}\sqrt{\frac{3}{7}} & 0 & -\frac{27}{35} \end{pmatrix} $
$\sum_{c}^{*}\sum_{c}^{*} J$	T=3	$^{7}S_{3}$ - $^{3}D_{3}$ - $^{5}D_{3}$ - $^{7}D_{3}$	$ \begin{pmatrix} \frac{9}{4} & 0 & 0 & 0 \\ 0 & -\frac{11}{4} & 0 & 0 \\ 0 & 0 & -\frac{3}{4} & 0 \\ 0 & 0 & 0 & \frac{9}{4} \end{pmatrix} $	$ \begin{pmatrix} 0 & -\frac{3\sqrt{3}}{5} & 0 & \frac{5\sqrt{3}}{9} \\ -\frac{3\sqrt{3}}{5} & -\frac{17}{35} & 0 & \frac{16}{35} \\ 0 & 0 & \frac{12}{7} & 0 \\ \frac{5\sqrt{3}}{9} & \frac{16}{35} & 0 & \frac{99}{70} \end{pmatrix} $

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