

TEA: A Sequential Recommendation Framework via Temporally Evolving Aggregations

Zijian Li, Ruichu Cai*, *Member, IEEE*, Fengzhu Wu, Sili Zhang, Hao Gu, Yuxing Hao, Yuguang Yan*

Abstract—Sequential recommendation aims to choose the most suitable items for a user at a specific timestamp given historical behaviors. Existing methods usually model the user behavior sequence based on the transition-based methods like Markov Chain. However, these methods also implicitly assume that the users are independent of each other without considering the influence between users. In fact, this influence plays an important role in sequence recommendation since the behavior of a user is easily affected by others. Therefore, it is desirable to aggregate both user behaviors and the influence between users, which are evolved temporally and involved in the heterogeneous graph of users and items. In this paper, we incorporate dynamic user-item heterogeneous graphs to propose a novel sequential recommendation framework. As a result, the historical behaviors as well as the influence between users can be taken into consideration. To achieve this, we firstly formalize sequential recommendation as a problem to estimate conditional probability given temporal dynamic heterogeneous graphs and user behavior sequences. After that, we exploit the conditional random field to aggregate the heterogeneous graphs and user behaviors for probability estimation, and employ the pseudo-likelihood approach to derive a tractable objective function. Finally, we provide scalable and flexible implementations of the proposed framework. Experimental results on three real-world datasets not only demonstrate the effectiveness of our proposed method but also provide some insightful discoveries on sequential recommendation.

Index Terms—Sequential Recommendation, Conditional Random Field, Dynamic Heterogeneous Graph, Recommendation System

I. INTRODUCTION

The sequential recommendation system is achieving more and more attention because of its practicality and effectiveness [1]–[4]. In a sequential recommendation system, the users access different items at different time stamps and frequently

interact with each other. The difficulties of sequential recommendation mainly come from two aspects: the temporal dependency of historical behaviors and the nonstationarity of users. The temporal dependency of historical behaviors means that the decision of a user is influenced by the historical behaviors. And the nonstationarity of users means that the decision of a user is influenced by the social relationship with the neighbors and the user-item interactions of the neighbors. Therefore, one important challenge is how to effectively leverage the historical behaviors and the social relationship between users.

Focusing on the above challenge, numerous sequential recommendation algorithms have been proposed in recent years, which leverage the user behavior sequence and employ the Markov Chains to model the transition of items. Eskandarian et al. [1] mine the user preference and identify change-points in the sequence of user interactions by using Hidden Markov Model (as shown in Figure 1 (a)). He et al. [2] model the personalized sequential behavior by using the personalized translation vectors and the previous item embedding to predict the next item. These transition-based methods assume that the users are independent of each other, which ignores the influence between the users. Considering that the behavior of a user is easily affected by the neighbors, ignoring the dependence between users will suffer from limited performance in sequential recommendation.

Another kind of recommendation algorithm [4]–[7] focuses on analyzing the social relationships between users and user-item interactions in a static user-item graph. The typical methods include the traditional Collaborative Filtering (CF) methods [8]–[10], the deep learning enhanced approaches [11]–[14], the recently developed graph neural networks based methods [15], as well as the social-network-based methods [5], [16], [17]. These methods reveal that both the interactions among users in social networks and the user-item bipartite graphs are beneficial to the performance of recommendation system. However, almost all the aforementioned methods assume the heterogeneous graphs of the users and the items are static, which ignores the dynamical influence of the temporal interaction between items and social networks, and further results in the suboptimal performance of recommendation systems. Take Figure 1 (b) for a toy example. The existing methods without considering the dynamic user-item heterogeneous graph might recommend the v_1 in preference to v_2 since more friends of user U_1 choose v_1 .

Thus, it is essential to devise a unified framework to take advantage of both the historical behaviors of a user and dynamic interactions between the neighbors and items. Figure 1 (c) illustrates our main idea that models the temporally

Zijian Li is with the School of Computing, Guangdong University of Technology, Guangzhou China, 510006. E-mail: leizigin@gmail.com

Ruichu Cai is with the School of Computing, Guangdong University of Technology and Guangdong Provincial Key Laboratory of Public Finance and Taxation with Big Data Application, Guangzhou China, 510006. E-mail: cairuichu@gmail.com

Fengzhu Wu is with the School of Computing, Guangdong University of Technology, Guangzhou China, 510006. E-mail: fzwu97@gmail.com

Sili Zhang is with the School of Computing, Guangdong University of Technology, Guangzhou China, 510006. E-mail: zhangsili1260@gmail.com

Hao Gu is with Tencent Technology (SZ) Co., Ltd. E-mail: nickgu@tencent.com

Yuxing Hao is with Tufts University. E-mail: yhao02@tufts.edu

Yuguang Yan is with the School of Computing, Guangdong University of Technology, Guangzhou China, 510006. E-mail: ygyan@gdut.edu.cn

Manuscript received XX; revised XX; accepted XX. Date of publication XX XX, 2019; date of current version XX XX, 2019. Ruichu Cai and Zijian Li were supported in part by Natural Science Foundation of China (61876043, 61976052), Science and Technology Planning Project of Guangzhou (201902010058). (Corresponding author: Ruichu Cai and Yuguang Yan.)

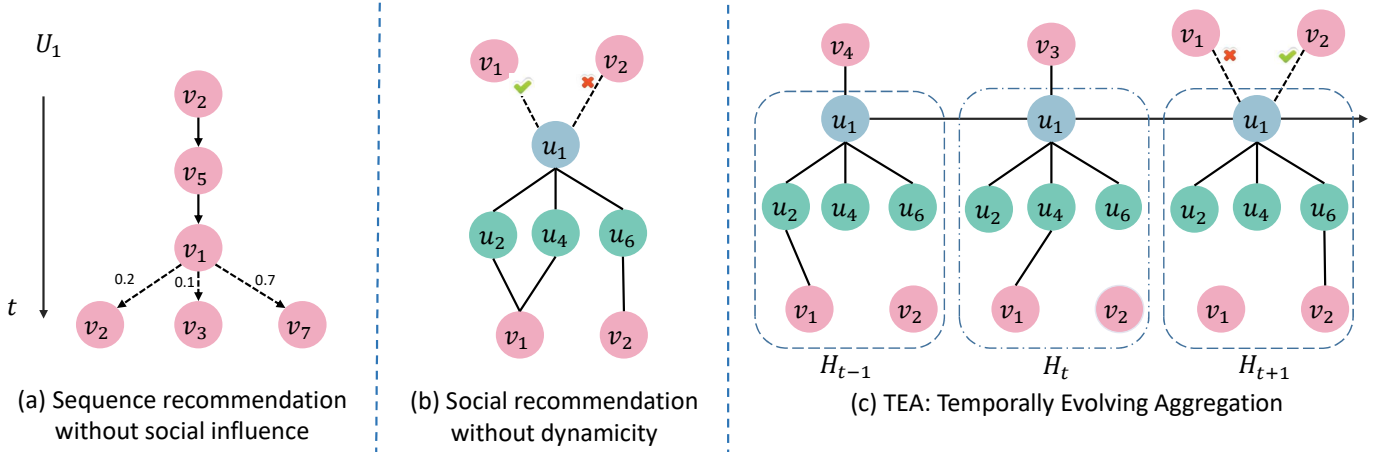


Fig. 1. Illustration of a toy sequential recommendation example, where the blue node is the target user, the green nodes are the neighbors of the target user in the social network, and the pink nodes denote the items. The black dashed lines denote the anticipated recommended results between items and target user u_1 . (a)-(b) Existing methods, which rarely consider the temporally dependent dynamics of the heterogeneous graph, lead to inaccurate prediction results. (c) On the contrary, the proposed **TEA** method simultaneously aggregates the historical user behavior sequence and the dynamic heterogeneous graph, thus resulting in more accurate predictions than existing methods.

user-item heterogeneous graphs and generates a more accurate prediction. In the figure, the decision of whether a user u_i will choose a given item v_{t+1} is controlled by two important factors: (1) the historical interactions between him or her and items; (2) the temporal dynamic heterogeneous graph, including the interactions between the neighbors and items. Hence, the goal of the proposed method is to estimate the conditional probability of v_{t+1} given a user u_i , the historical accessed item sequence $v_{1:t}$, as well as the heterogeneous graph sequence $H_{1:t+1}$, which can be formulated as $P(v_{t+1}|u_i, \mathcal{H}_{1:t+1}; v_{1:t})$.

Based on the above idea, we propose the **Temporally Evolving Aggregation (TEA)** in short) framework for sequential recommendation by aggregating the user behavior sequence as well as the dynamic user-item heterogeneous graph. Inspired by the sequence labeling in natural language processing [18], [19] to model the joint probability distribution), we adopt CRF to model the item decision sequence and estimate $P(v_{t+1}|u_i, \mathcal{H}_{1:t+1}; v_{1:t})$. In order to alleviate the issue of the large item space, we use the pseudo likelihood method to approximate the aforementioned conditional probability. By doing this, the training procedure can be performed by estimating the unary score and transition score in CRF, which are implemented by our designed modules. Technically, we design a *Time-Restricted User Behavior Sequence Aggregation Module* to estimate the transition score of CRF, and a *Temporal Dynamic Heterogeneous Graphs Aggregation Module* to estimate the Unary Scores of CRF. We further provide two different practical implementations based on the proposed framework. Extensive experimental studies demonstrate that our **TEA** framework outperforms the state-of-the-art recommendation methods on two published datasets and one real-world WeChat official accounts dataset.

The remainder of this paper is organized as follows. In Section II, we review related researches into recommendation systems, including social recommendation and sequential recommendation. In Section III, we define the problem of

sequential recommendation under the dynamical heterogeneous graph and further derive the objective function based on the conditional random field. In Section IV, we provide the implementation details of the proposed **TEA** model. We further analyze the connection to existing methods in Section V. And then, we present our experimental results based on two standard benchmarks and one real-world dataset in Section VI. Finally, we give our conclusion of the proposed method.

II. RELATED WORKS

In this section, we mainly discuss the existing techniques on social recommendation and sequential recommendation.

In order to effectively mine the deep demands of users, researchers set their sights on social relations, hence social recommendation has received more and more attention. One of the most important methods is Matrix Factorization (MF) [20]–[22]. Based on the traditional matrix factorization methods, Hao et al. [23] proposed a co-factorization method, which shares a common latent user-feature matrix factorized by both ratings and social relations. With the development of deep learning methods, He et al. [24] propose NeuMF by replacing the inner product with a neural architecture that can learn an arbitrary function from data. Fan et al. [25] propose a deep neural network-based model to learn non-linear features of each user from social relations and to integrate them into probabilistic matrix factorization for the social recommendation. Deng et al. [26] propose a two-phase recommendation process to utilize deep learning to calculate the impact of community effect from the interests of users' trusted friends for recommendations.

Recently, graph neural networks (GNNs) [15], [27] are widely used to aggregate node information and topological structure from social networks, hence GNNs are employed to address the social recommendation problem. In order to well aggregate the heterogeneous information, Fan et al. [5] propose the GraphRec for the social recommendation. Fu et al.

[28] leverage the metapaths [29] to obtain the heterogeneous graph embedding. Considering that the influences in the social network may be context-dependent, Song et al. [16] address the session-based social recommendation by using a dynamic-graph-attention neural network architecture. However, the aforementioned methods rarely consider the fact that different friends in social networks choose different items. In this work, considering the fact that social influence and user behaviors are time-dependent, the proposed **TEA** method focuses on aggregating the temporally evolving social influence and the user behavior sequence.

Since users usually access the items in chronological order, the users are likely to choose the items that are closely relevant to those they just accessed. Many works on sequential recommendation follow this assumption. Aiming to model the item-item transition probabilities, some traditional works borrow the idea of the Markov chain. Rendle et al. [30] bridge the Matrix Factorization (MF) and Markov Chains (MC). He et al. [2] propose TransRec to model such third-order relationships (e.g. the relationships among a user, the previously accessed item and the next item) for large-scale sequential prediction. Motivated by the advantages of sequence learning in natural language processing, many neural network-based methods are proposed to learn the sequential dynamics. Tang et al. [31] leverage convolutional neural networks to encode the sequences into the embeddings. Other works [32], [33] leverage recurrent neural networks and their variants to model the sequences of items. Kang et al. [34] further leverage attention-mechanism and propose the SASRec to balance the goal of MC-based methods and RNNs based methods. Moreover, Sun et al. [35] argue that such left-to-right unidirectional models are sub-optimal. So they propose BERT4Rec, which employs deep bidirectional self-attention to model user behavior sequences. In this paper, the proposed **TEA** leverage the Conditional Random Field (CRF) to model the translation of items, which calculates the transition score and the unary score by respectively aggregating the user behavior sequence information as well as the dynamic user-item heterogeneous graph.

III. MODEL

In this section, we begin with the problem definition of sequential recommendation. Then we derive the unified objective function under conditional probability $P(v_{t+1}|u_i, \mathcal{H}_{1:t+1}; v_{1:t})$.

A. Problem Definition

Let $U = \{u_1, u_2, \dots, u_n\}$ and $V = \{v_1, v_2, \dots, v_m\}$ denote the sets of users and items respectively, in which n is the number of users and m is the number of items. For user-item interactions, we let $\mathcal{G}^b = \{U \cup V, \mathcal{E}^b\}$ be the user-item bipartite graph with edges $(u_i, v_j) \in \mathcal{E}^b$. As for user-user relations, we let $\mathcal{G}^s = \{U, \mathcal{E}^s\}$ be the social graph with edges $(u_i, u_j) \in \mathcal{E}^s$. If we combine the bipartite graph and the social graph, we obtain the following heterogeneous graph $\mathbf{H} = \{U \cup V, \mathcal{E}^b \cup \mathcal{E}^s\}$. Let $v_{1:t}$ be the user behaviors sequence for u_i . Since we consider the temporal evolving

TABLE I
NOTATION AND DESCRIPTIONS.

| Notations | Descriptions |
|----------------------------------|--|
| U, V | User and item set. |
| m, n | The size of user set and item set. |
| $\mathcal{G}^b, \mathcal{G}_t^b$ | The bipartite graph only includes the user-item interaction and that at the t -th timestamp. |
| $\mathcal{E}^b, \mathcal{E}_t^b$ | The edges set of bipartite graph and that at the t -th time-step. |
| \mathcal{G}^s | The social networks. |
| \mathcal{E}^s | The edges among users in social network |
| \mathbf{H}_t | The heterogeneous graph that includes the social network \mathcal{G}^s and the bipartite graph \mathcal{G}_t^b at t -th time-step. |
| \mathbf{p}_i | The embedding of user u_i . |
| \mathbf{q}_j | The embedding of item v_j . |
| \mathbf{k}_j | The embedding of the j -th position in item sequences. |
| \mathbf{W}, \mathbf{b} | Weights and biases in neural networks. |
| d | The dimension number of representation. |
| Θ_f | The parameters of unary scores function. |
| Θ_g | The parameters of transition scores function. |
| \oplus | The concatenation operator of any two vectors. |
| \mathbf{x} | The observed item sequence. |
| \mathbf{y} | The label item sequence of \mathbf{x} |
| $\mathcal{N}(u_i)$ | The 1st-order neighbourhood of user u_i |
| $\mathcal{I}_t(u_i)$ | The accessed items of user u_i at t -th time-step. |
| τ | Time window for selecting the walks in the duration of $[t-\tau, t+\tau]$. |
| d | The dimension of user and item embedding. |

social influence, we let $\mathcal{H}_{t+1} = \{\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_{t+1}\}$ be the heterogeneous graph sequence, where $\mathbf{H}_t = \{U \cup V, \mathcal{E}_t^b \cup \mathcal{E}^s\}$ and \mathcal{E}_t^b is the user-item interactions in t -th time-step. For user u_i , given the behavior sequence $v_{1:t}$ and the heterogeneous graph sequence \mathcal{H}_{t+1} as well as the item v_{t+1} , our goal is to estimate the conditional probability of $P(v_{t+1}|v_{1:t}, u_i, \mathcal{H}_{t+1})$. The mathematical notation and the corresponding descriptions are summarized in Table I.

B. Methodology

We begin with the traditional Conditional Random Field (CRF), which is a probabilistic graphical model widely used in sequence labeling [18]. CRF has shown to be very effective since it can jointly model the label decision by capturing the dependencies across adjacent labels. Considering the general definition of CRF, let $\mathbf{x} = \{x_1, \dots, x_t, \dots, x_T\}$ and $\mathbf{y} = \{y_1, \dots, y_t, \dots, y_T\}$ denote the observed sequence and its corresponding labels respectively. Formally, the conditional distribution $p(\mathbf{y}|\mathbf{x})$ of Linear Chain CRF [36] is given by:

$$p(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(\mathbf{x})} \exp\left(\sum_{t=1}^T f(x_t, y_t; \Theta_f) + \sum_{t=1}^{T-1} g(y_t, y_{t+1}; \Theta_g)\right),$$

$$Z(\mathbf{x}) = \sum_{\mathbf{y}'} \exp\left(\sum_{t=1}^T f(x_t, y'_t; \Theta_f) + \sum_{t=1}^{T-1} g(y'_t, y'_{t+1}; \Theta_g)\right), \quad (1)$$

in which Θ_f and Θ_g are the trainable parameters.

There are three important components in the above CRF model: the partition function $Z(\mathbf{x})$, the unary scores function $f(x_t, y_t)$ and the transition scores function $g(y_t, y_{t+1})$. The partition function $Z(\mathbf{x})$ is a normalization factor in order to obtain a probability. The unary scores function $f(x_t, y_t)$ is used to estimate the probability of y_t given the observed

x_t . And the transition scores function $g(y_t, y_{t-1})$ is used to estimate the probability of y_t given y_{t-1} .

The three components framework provides us a unified solution to aggregate both the historical behaviors of users and the dynamic social influence from the social networks. Following the formulation of CRF, the purpose of our model is to estimate the conditional distribution as follow:

$$P(v_{1:t+1}|u_i, \mathcal{H}_{t+1}) = \frac{1}{Z(\mathcal{H}_{t+1}, u_i)} \exp\left(\sum_{t=1}^T f(\mathbf{H}_{t+1}, u_i, v_{t+1}; \Theta_f) + \sum_{t=1}^{T-1} g(v_{t+1}, v_t; \Theta_g)\right), \quad (2)$$

$$Z(\mathcal{H}, u_i) = \sum_{S_{t+1}^{u_i}} \exp\left(\sum_{t=1}^T f(\mathbf{H}_{t+1}, u_i, v_{t+1}; \Theta_f) + \sum_{t=1}^{T-1} g(v_{t+1}, v_t; \Theta_g)\right),$$

in which $f(\mathbf{H}_{t+1}, u_i, v_{t+1}; \Theta_f)$ denotes the aggregation of temporal evolving social influence, $g(v_{t+1}, v_t; \Theta_g)$ denotes the aggregation of user behaviors. In specific, $f(\mathbf{H}_{t+1}, u_i, v_{t+1}; \Theta_f)$ describes the relationship between the the dynamic heterogeneous graph \mathbf{H}_{t+1} and the available item v_{t+1} and $g(v_{t+1}, v_t; \Theta_g)$ models the dependency between the available item v_{t+1} and the user behavior sequence.

However, it is almost impossible to calculate $Z(\mathcal{H}, u_i)$ since the sequence length is too large. In order to address this issue, we employ the pseudo likelihood method as an effective approximation method [37], [38], and further derive the following estimation of the conditional probability:

$$P(v_{1:t+1}|u_i, \mathcal{H}_{t+1}) \approx PL(v_{1:t+1}|u_i, \mathcal{H}_{t+1}) = \prod_t P(v_{t+1}|v_{1:t}, u_i, \mathcal{H}_{t+1}). \quad (3)$$

Combining Equation (2) and Equation (3), we further derive the following estimation of the conditional probability $P(v_{t+1}|v_{1:t}, u_i, \mathcal{H}_{t+1})$:

$$P(v_{t+1}|v_{1:t}, u_i, \mathcal{H}_{t+1}) = \frac{\exp(f(\mathbf{H}_{t+1}, u_i, v_{t+1}; \Theta_f) + g(v_{t+1}, v_{1:t}; \Theta_g))}{\sum_{v_j \in V} \exp(f(\mathbf{H}_{t+1}, u_i, v_j; \Theta_f) + g(v_j, v_{1:t}; \Theta_g))}. \quad (4)$$

Finally, we can obtain the objective function of our proposed model as follows:

$$\mathcal{L}_{crf} = \frac{1}{n} \sum_{i=1}^n \sum_{t=1}^T \log P(v_{t+1}|v_{1:t}, u_i, \mathcal{H}_{t+1}). \quad (5)$$

The aforementioned objective function is usually impractical because the size of the item set is very large and the computation cost is unaffordable. Inspired by [39], we employ

the negative sampling strategy to obtain the tractable unified objective function of sequential recommendation as follows:

$$\mathcal{L}_{crf} = \frac{1}{n} \sum_{i=1}^n \sum_{t=1}^{T-1} \log \sigma(f(\mathbf{H}_{t+1}, u_i, v_{t+1}; \Theta_f) + g(v_{t+1}, v_{1:t}; \Theta_g)) + \sum_{k=1}^{n_s} [\log \sigma(-f(\mathbf{H}_{t+1}, u_i, v_k; \Theta_f) - g(v_k, v_{1:t}; \Theta_g))], \quad (6)$$

where σ is the sigmoid activation function and v_k is the negative item uniformly sampled from the whole item set V .

The objective function enjoys an appealing physical meaning. It provides the insights of how to design the model for sequential recommendation: $f(\mathbf{H}_{t+1}, u_i, v_{t+1}; \Theta_f)$ models the information of temporal evolving heterogeneous graph in the forms of the unary energy function; meanwhile $g(v_{t+1}, v_{1:t}; \Theta_g)$ not only models the alternative item v_t but also the user behavior sequence in the form of the pairwise energy function.

IV. IMPLEMENTATION OF TEMPORALLY EVOLVING AGGREGATION FRAMEWORK

In this section, we provide the implementation details of the proposed temporally evolving aggregation model. As illustrated in Figure 2(a), our implementation takes both the aggregation of user behavior sequences and the aggregation of temporally dependent heterogeneous graphs into consideration and employs the GRU cells [40] and CRF layers to predict the final results. The details of the two aggregation modules are presented in Figure 2 (b) and Figure 2 (c) respectively. We will give detailed descriptions of these two aggregation modules in the following subsections.

A. Time-Restricted User Behavior Sequence Aggregation for the Transition Scores

In this subsection, we will introduce the technical details of $g(v_{t+1}, v_{1:t}; \Theta_g)$. Given user u_i and the corresponding behavior sequence $v_{1:t}$, we aim to calculate the user-specific item transition score.

1) *User Behavior Sequence Aggregation*: Considering that the future behavior of a user is not only influenced by the latest accessed items but also the items that the user has accessed before, the user behavior sequence aggregation block should consider both the transition between items and the long-term dependency of items. Inspired by the great success of the self-attention mechanism [41] in various tasks like machine translation, we propose an extension of the self-attention mechanism to model the personalized item transition and long-term dependency by simultaneously leveraging the item information and the position information. Formally, given the j -th candidate item, we calculate the weights of each historical item as follows:

$$a_{\tau j} = \text{softmax}\left(\frac{\mathbf{W}_Q(\mathbf{q}_j + \mathbf{k}_j)(\mathbf{W}_K(\mathbf{q}_\tau + \mathbf{k}_\tau))^\top}{\sqrt{d}}\right), \tau < j, \quad (7)$$

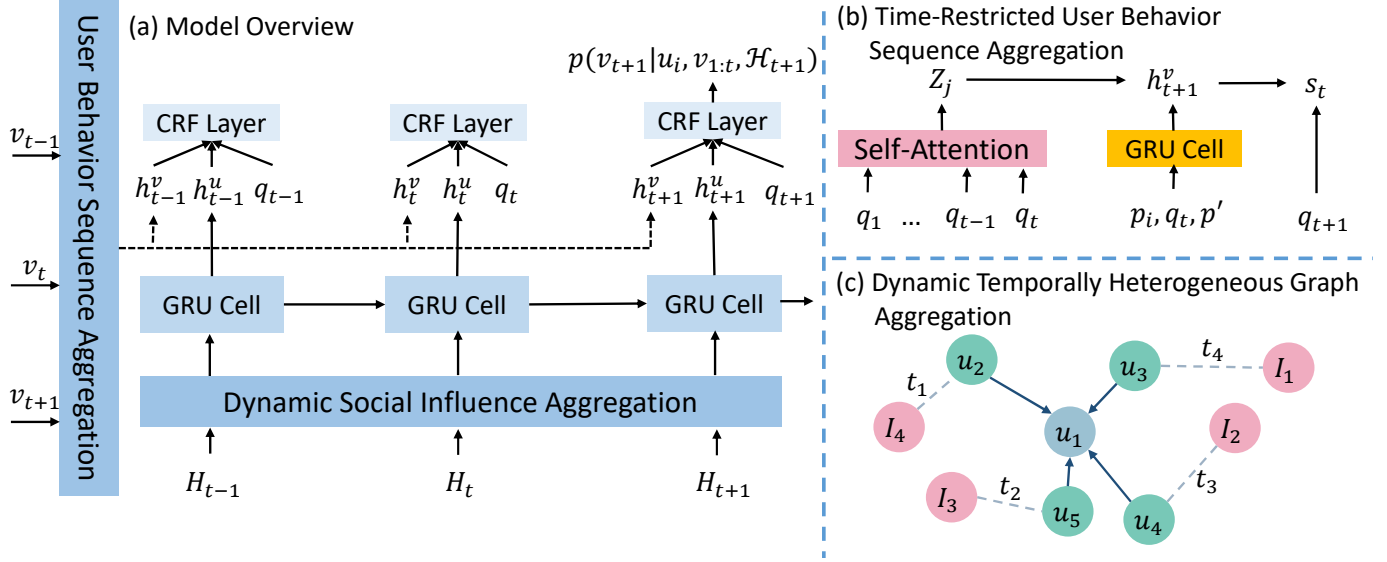


Fig. 2. The framework of the temporally evolving aggregation model for the sequential recommendation. (a) The overview of the proposed model, the temporally dependent heterogeneous graphs aggregated representation h_{t+1}^u , the user behavior aggregated representation h_{t+1}^v and the item embedding q_{t+1} are fed into the CRF layer and $P(v_{t+1}|u_i, \mathcal{H}_{t+1}; v_{1:t})$ is estimated. (b) The time-restricted user behavior sequence aggregation block is based on the user behavior sequence aggregation and the time-restricted aggregation. Note that the GRU used in this module is different from that in (a). (c) The dynamic temporally heterogeneous graph aggregation block, which is based on the bipartite graph aggregation and social network aggregation, takes \mathcal{H}_t as input, the arrows denote the message passing direction.

where q_j is the embedding of item v_j , k_j is the position embedding at j -th position of the input sequence, W_Q, W_K are trainable projection parameters and \sqrt{d} is the scaling factor, and d is the dimension of the embedding. As a result, we can calculate the historical item aggregated representation as follows:

$$\mathbf{z}_j = \sum_{\tau=1}^{\tau=j-1} a_{\tau j} \mathbf{W}_V (\mathbf{q}_\tau + \mathbf{k}_\tau), \quad (8)$$

in which W_V are trainable projection parameters.

2) *Time-Restricted Aggregation*: Since the temporal interactions between users and items are very sparse, for the users that contain limited social relationships and items interactions, it is hard to obtain a ideal user embedding for the sparse social substructure, and it is also difficult to obtain a debiased item embedding. Therefore, it is a challenging task to well aggregate the information from the users to the items and vice verse. Fortunately, we find that the users that select the same items usually share the same interests and intent. Inspired by this intuition, we further proposed the time-restricted aggregation module.

First, we selected the walk with three nodes (e.g., USER-ITEM-USER) with the restriction of time window τ . In detail, given the interaction (u_i, v_t) , we find the other users that select the same item in the time window of $[t-\tau, t+\tau]$, where τ is the window size. In our experimental implementation, we choose $\tau = 60$ days. Therefore, we can collect the τ -restricted walks for example $u_i - v_t - u'$. Sequentially, we employ another GRU to aggregate the information from the dense substructures to the sparse substructures, which can be formalized as follow:

$$\mathbf{h}_{u_i}, \mathbf{h}_{v_t}, \mathbf{h}_{u'} = \text{GRU}(\mathbf{p}_i, \mathbf{q}_t, \mathbf{p}'; \mathcal{W}_R), \quad (9)$$

in which we take the walk $u_i - v_t - u'$ as input and $\mathbf{h}_{u_i}, \mathbf{h}_{v_t}, \mathbf{h}_{u'}$ are the output of GRU of each timestamp; \mathcal{W}_R are the trainable parameters.

3) *Calculate the Transition Scores*: In order to well perform the personalized user behavior sequence aggregation, we further add the user embedding \mathbf{p}_i into the transformed item representation. Formally, we can calculate the transition score s_t as follows:

$$s_t = \left(\mathbf{W}_g^{(3)} [\mathbf{h}_t^{v_j} \oplus \mathbf{h}_{u_i} \oplus \mathbf{h}_{v_t} \oplus \mathbf{p}_i] \right)^T \mathbf{q}_j, \quad (10)$$

$$\mathbf{h}_t^{v_j} = \mathbf{p}_i + \mathbf{W}_g^{(2)} \left(\text{ReLU}(\mathbf{W}_g^{(1)} \mathbf{z}_j + \mathbf{b}_g^{(1)}) \right) + \mathbf{b}_g^{(2)},$$

in which $\mathbf{W}_g^{(1)}, \mathbf{W}_g^{(2)}, \mathbf{b}_g^{(1)}, \mathbf{b}_g^{(2)}$ are the trainable parameters. For convenience, we let $\Theta_g = \{\mathbf{W}_Q, \mathbf{W}_K, \mathbf{W}_V, \mathbf{W}_g^{(1)}, \mathbf{W}_g^{(2)}, \mathbf{W}_g^{(3)}, \mathbf{b}_g^{(1)}, \mathbf{b}_g^{(2)}, \mathbf{p}, \mathbf{q}, \mathbf{k}, \omega_R\}$ be the trainable parameters of $g(v_{t+1}, v_{1:t}; \Theta_g)$.

B. Dynamic Temporal Heterogeneous Graphs Aggregation for the Unary Scores

In this part, we will introduce the details of the dynamic temporally heterogeneous graphs aggregation $f(\mathbf{H}_{t+1}, u_i, v_{t+1}; \Theta_f)$, which is used to calculate the unary scores. The dynamic temporally heterogeneous graphs aggregation contains the bipartite graph aggregation and the social network aggregation.

1) *Bipartite Graph Aggregation*: In this part, we aim to obtain the aggregated of the bipartite graph at t -th time-step. Given user u_i and the heterogeneous graph sequence \mathcal{H}_{t+1} , we first obtain the user-specific representation \mathbf{h}_t of \mathcal{H}_t . Specifically, we employ two different aggregated strategies and raise two variants of the proposed method: the GraphSAGE

[42] based method (named TEA-S) and the graph attention networks [43] based method (named TEA-A). More experimental details will be introduced in the next section.

As for the TEA-S variation, we can obtain the user-specific representation $\hat{\mathbf{h}}_t$ as follows:

$$\hat{\mathbf{h}}_t = \text{ReLU}(\mathbf{W}_A \text{MEAN}(\mathbf{q}_k, \forall k \in \mathcal{I}_t(\mathcal{N}(u_i)))) , \quad (11)$$

where \mathbf{W}_A are the trainable parameters and $\mathcal{I}_t(\mathcal{N}(u_i))$ denotes the items interacted by u_i 's neighbors at between t -th and $t+1$ -th time-step; and MEAN denotes the average pooling operation.

As for the TEA-A variation, we aggregate the item information to the user with the help of the graph attention mechanism, which can be formulated as:

$$\hat{\mathbf{h}}_t = \text{ReLU} \left(\sum_{j \in \mathcal{I}_t(\mathcal{N}(u_i))} \alpha_{ij} \mathbf{q}_j \right), \quad (12)$$

where α_{ij} is the weight of user u_i and item v_j and is defined as

$$\alpha_{ij} = \frac{\exp(\text{LeakyReLU}(\mathbf{w}_A^\top [\mathbf{W}_A \mathbf{q}_t \oplus \mathbf{W}_A \mathbf{q}_j]))}{\sum_{k \in \mathcal{I}_t(\mathcal{N}(u_i))} \exp(\text{LeakyReLU}(\mathbf{w}_A^\top [\mathbf{W}_A \mathbf{q}_t \oplus \mathbf{W}_A \mathbf{q}_k]))}, \quad (13)$$

in which \mathbf{q}_t is the embedding of the item interacted by u_i at t -th time-step and \oplus is the concatenation operation. \mathbf{w}_A and \mathbf{W}_A are trainable parameters. And *LeakyReLU* is the leaky version of a rectified linear unit.

In order to model temporally dependent heterogeneous graphs propagation, we feed $\hat{\mathbf{h}}_t$ into the Gated Recurrent Unit [40]. The GRU cell operation at the t -th time-step can be formulated as:

$$\mathbf{h}_t = \text{GRU}(\hat{\mathbf{h}}_t, \mathbf{h}_{t-1}; \mathcal{W}_G), \quad (14)$$

in which \mathcal{W}_G denotes all trainable parameters of the GRU cell.

2) *Social Network Aggregation*: To propagate the information of neighbors' interests, we further aggregate the information from the social network. For simplicity, we only formulate the GraphSAGE aggregation as follows:

$$\mathbf{h}_s = \text{ReLU}(\mathbf{W}_S \text{MEAN}(\mathbf{p}_k, \forall k \in \mathcal{N}(u_i))), \quad (15)$$

where \mathbf{W}_S is the trainable parameters.

3) *Calculate the Unary Scores*: Based on the aforementioned aggregation, we fuse the time-dependent representation \mathbf{h}_t and time-independent representation \mathbf{h}_s into one vector and calculate the social influence score s_f , i.e., the output of unary scores function $f(\cdot)$. It is formulated as:

$$s_f = \mathbf{h}_t^{u_i \top} \mathbf{q}_j, \quad (16)$$

$$\mathbf{h}_t^{u_i} = \mathbf{W}_f^{(2)} \text{ReLU}(\mathbf{W}_f^{(1)} [\mathbf{h}_t \oplus \mathbf{h}_s] + \mathbf{b}_f^{(1)}) + \mathbf{b}_f^{(2)},$$

in which $\mathbf{W}_f^{(1)}, \mathbf{W}_f^{(2)}, \mathbf{b}_f^{(1)}, \mathbf{b}_f^{(2)}$ are trainable parameters. In summary, we let $\Theta_f = \{\mathbf{W}_A, \mathbf{W}_S, \mathbf{w}_G, \mathbf{W}_f^{(1)}, \mathbf{W}_f^{(2)}, \mathbf{b}_f^{(1)}, \mathbf{b}_f^{(2)}, \mathbf{p}, \mathbf{q}\}$ be the trainable parameters of $f(\mathbf{H}_{t+1}, u_i, v_{t+1}; \Theta_f)$.

C. Model Summarization

The total loss of our proposed model is summarized as follow:

$$\mathcal{L} = \mathcal{L}_{crf} + \gamma \mathcal{L}_{reg}, \quad (17)$$

where \mathcal{L}_{reg} is the L2 normalization on all parameters and γ is a trade-off hyper-parameter.

Based on this objective function, our model is trained by the following procedure:

$$(\hat{\Theta}_g, \hat{\Theta}_f) = \arg \min_{\Theta_g, \Theta_f} \mathcal{L}. \quad (18)$$

All parameters are jointly optimized using the Adam [44] algorithm.

In the testing, we estimate the probability of $P(v_{t+1}|v_{1:t}, u_i, \mathcal{H}_{t+1})$ as follows:

$$P(v_{t+1}|v_{1:t}, u_i, \mathcal{H}_{t+1}) = \sigma(f(\mathbf{H}_{t+1}, u_i, v_{t+1}; \hat{\Theta}_f) + g(v_{t+1}, v_{1:t}; \hat{\Theta}_g)). \quad (19)$$

V. CONNECTIONS TO EXISTING MODELS

We will discuss the connections to the existing transition-based sequential recommendation methods. Most of the existing works of transition-based sequential recommendation methods [2], [30] are based on Markov Chains. These methods mainly consider two important factors: (1) the interactions between users and items to capture the inherent intent of users, (2) the sequential dynamics between items to capture the relationships between items. Thus, we find that our method is more general and some of the existing works can be taken as special cases of ours. The detailed discussions for each work are as follows.

Regarding the work FPMC [30], it simplifies the huge state space problem by introducing the basket of items and consequently ignores the sequence information of historical items in each basket. In the contrast, our method utilizes the historical item sequence by using the self-attention mechanism with position embedding and is more general than FPMC.

Regarding the work TransRec [2], it models the personalized sequential behavior by using the personalized translation vectors and the previous item embedding to predict the next items but ignores the long-term dependencies since it only considers the relationships between any two items. Moreover, TransRec addresses the problem of the huge state space of items by introducing the subspace, while our method utilizes the negative sampling strategy. Thus, our method is more feasible and efficient to capture the dynamic social influence of the target users.

VI. EXPERIMENT

In this section, we experimentally evaluate the performance of our method on three datasets against the state-of-the-art compared methods. The preprocessed scripts and the source code can be found at ¹.

¹<https://github.com/DMIRLAB-Group/TEA>

TABLE II
STATISTICS OF THE DATASETS.

| Dataset | Epinions | Yelp | Wechat |
|----------------|----------|-----------|-----------|
| # users | 22,167 | 270,770 | 568,100 |
| # items | 296,278 | 184,134 | 242,702 |
| # interactions | 798,620 | 3,602,495 | 9,422,722 |
| # social links | 355,813 | 5,974,526 | 5,667,864 |
| density | 0.0121% | 0.0072% | 0.0068% |
| social density | 0.0724% | 0.0081% | 0.0018% |

A. Datasets

We evaluate our proposed TEA framework on two public datasets (Epinions and Yelp) and a large-scale private dataset (WeChat Official Accounts Dataset). The statistics of datasets are summarized in Table II. The brief information of the datasets is as follows:

- **Epinions²**: A benchmark dataset for the recommendation. In Epinions, a user can rate and give comments on items. Besides, a user can also select other users as their trusters, and we use the trust graphs (which are composed of the trust relationships) as the network information.
- **Yelp³**: An online review platform where users review local businesses (e.g., restaurants and shops). The user-item interactions and the social networks are extracted in the same way as Epinions.
- **WeChat Official Accounts Dataset**: WeChat is a Chinese multi-purpose messaging, social media, and mobile payment application developed by Tencent. And WeChat official accounts dataset is one of the functions. On the WeChat Official Account platform, users can read and share articles. This dataset is constructed by user-article clicking records and user-user social networks on this platform.

We preprocess the datasets following the approach in [2]. Specifically, for all these datasets, we follow the previous works [34], [35] and treat a rating or review as implicit feedback. We further use the timestamps to determine the sequence order of actions. We discard users and items with fewer than 5 associated actions. In cases where star ratings are available, we take the item with a rating higher than 3 as users' positive feedback.

For data splitting, we employ the widely used leave-one-out evaluation [10], [24]. We hold out the latest interaction of each user as the test set and select the second latest interaction as the validation set. The remaining data are used for training.

B. Implementation Details

We use PyTorch to implement our model and deploy it on RTX 2080 GPU. Hyper-parameter settings for all three datasets are as follows: embedding dimension $d = 64$, batch size $B = 1024$, dropout rate $p_{\text{drop}} = 0.5$, L2 regularization weight $\gamma = 5e-4$, negative sampling size $n_s = 50$, sequence truncation length $L_s = 50$, neighbor truncation length $L_n = 20$, and learning rate $\eta = 0.01$. We train all the methods with five different random seeds and report the means and standard deviations.

²http://www.trustlet.org/extended_epinions.html

³<https://www.kaggle.com/yelp-dataset/yelp-dataset>

C. Evaluation Metrics

We evaluate all the models with two widely used Top-N metrics: Hit Rate@ K (HR@ K) and Normalized Discounted Cumulative Gain@ K (NDCG@ K). HR measures the percentage that recommended items contain at least one correct item interacted by the user, while NDCG considers the positions of correct recommended items. In the context of sequential recommendation, since we only test on the latest item in a user behavior sequence, HR is identical to recall and proportional to precision [34].

Since it is time-consuming to rank all items for each user during the evaluation, we followed the strategy in [34]. Specifically, for each user, we randomly sample 100 negative items and rank these items with the ground-truth item. HR and NDCG are estimated based on the ranking results. We report the experiment results for $K = 5/10/20$.

D. Compared Methods

We compare our proposed models (TEA-S and TEA-A) based on TEA framework with three kinds of baselines: the matrix factorization based models, the graph neural networks based models, and the sequence recommendation methods.

Matrix Factorization based Methods:

- **BPRMF [10]**: A general learning framework for personalized ranking recommendation uses implicit feedback.
- **NeuMF [24]**: It replaces the inner product with a multi-layer perception (MLP) to learn the user-item interaction function.
- **SocialMF [7]**: It considers the social information and propagation of social information into the matrix factorization model.
- **SoRec [23]**: It performs co-factorization on the user-item rating matrix and user-user social relations matrix.

Graph Neural Network based Methods:

- **GraphRec [5]**: It uses the graph neural network to combine user behavior information and social network information into the recommendation task. For fairness, we discard the opinion/rate embedding in our implementation.
- **LightGCN [45]**: A state-of-the-art graph-based collaborative filtering method. It explicitly integrates a bipartite graph structure into the embedding learning process to model the high-order connectivity in the user-item interaction graph.
- **DGRec [16]**: A session-based recommendation method that combines the user action-temporal information and the social information via recurrent neural networks and dynamic graph attention networks.

Sequential Recommendation Methods:

- **TransRec [2]**: A sequential recommendation method that models each user as a translation vector to capture the transition from the current item to the next.
- **SASRec [34]**: It leverages the Transformer [41] to capture users' sequential behaviors.
- **ASASRec [47]**: An improved version of SASRec with an adversarial training strategy.

TABLE III

THE PERFORMANCE EVALUATION OF THE COMPARED METHODS ON EPINIONS DATASET. THE VALUE PRESENTED ARE AVERAGED OVER 5 REPLICATED WITH DIFFERENT RANDOM SEEDS.

| Model Class | Models | HR@5 | NDCG@5 | HR@10 | NDCG@10 | HR@20 | NDCG@20 |
|----------------------------|---------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Matrix Factorization based | BPRMF [10] | 38.72±0.10 | 29.66±0.12 | 47.53±0.10 | 32.50±0.07 | 57.21±0.22 | 34.95±0.13 |
| | NeuMF [24] | 41.35±0.59 | 31.13±0.69 | 51.15±0.43 | 34.31±0.64 | 60.93±0.34 | 36.78±0.59 |
| | SocialMF [7] | 41.78±0.16 | 32.57±0.29 | 50.01±0.18 | 35.23±0.29 | 58.23±0.14 | 37.31±0.25 |
| | SoRec [23] | 40.81±0.33 | 31.14±0.30 | 49.61±0.16 | 33.99±0.24 | 58.42±0.19 | 36.22±0.25 |
| Graph Neural Network based | GraphRec [5] | 39.50±0.35 | 30.16±0.27 | 48.94±0.42 | 33.21±0.21 | 58.87±0.29 | 35.72±0.20 |
| | LightGCN [45] | 42.59±0.07 | 32.20±0.09 | 51.92±0.08 | 35.22±0.07 | 60.54±0.09 | 37.41±0.08 |
| | DGRec [16] | 40.36±0.25 | 30.52±0.16 | 49.67±0.14 | 33.53±0.15 | 59.26±0.19 | 35.95±0.15 |
| Sequence based | DMAN [46] | 35.15±0.27 | 27.06±0.33 | 45.01±0.06 | 30.23±0.24 | 55.85±0.27 | 32.98±0.30 |
| | TransRec [2] | 44.79±0.12 | 36.09±0.21 | 52.51±0.11 | 38.58±0.17 | 60.98±0.11 | 40.72±0.07 |
| | SASRec [34] | 43.32±0.20 | 33.97±0.20 | 51.88±0.20 | 36.74±0.20 | 60.31±0.20 | 38.87±0.18 |
| | ASAS [47] | 44.97±0.34 | 35.59±0.29 | 53.44±0.29 | 38.33±0.27 | 61.41±0.29 | 40.35±0.28 |
| Ours | TEA-A | 47.84±0.04 | 38.40±0.41 | 55.99±0.04 | 41.04±0.41 | 63.51±0.29 | 42.95±0.38 |
| | TEA-S | 48.13±0.25 | 38.65±0.18 | 56.10±0.17 | 41.24±0.15 | 63.58±0.08 | 43.13±0.11 |

- DMAN [46]: It effectively utilizes the sequential data by segmenting the overall behavior sequence and maintaining the long-term interests of users.

Model Variants:

- TEA-S: We use the GraphSAGE based aggregation method in the bipartite graph aggregation.
- TEA-A: We use the Graph Attention mechanism based aggregation method in the bipartite graph aggregation.
- TEA-RS: We remove the time-restricted aggregation and use the GraphSAGE based aggregation method in the bipartite graph aggregation.
- TEA-RA: We remove the time-restricted aggregation and use the Graph Attention mechanism based aggregation method in the bipartite graph aggregation.

E. Results

Tables III, IV and V present the recommendation performance of all the methods on the three datasets, respectively. We do not report the performance of LightGCN, DMAN on WeChat Official Accounts dataset because of the limitation of memory.

First, by modeling social influence, the performances of social-aware methods (SocialMF, SoRec, GraphRec, and DGRec) are improved compared with that of BPRMF in most cases, which is consistent with previous works. This observation indicates that social information reflects users' interests effectively. Second, the sequence based methods (DGRec, TransRec, SAS, and ASAS) also perform comparably well. These improvements reflect the importance of temporal information on recommendation tasks. Third, DGRec and our proposed methods (including TEA-S and TEA-A) that combine social information and temporal information achieve much better performance, especially on large datasets. At last, our proposed TEA-S and TEA-A consistently outperform all the compared methods on both public and real-world datasets with an average improvement of 3.15% on HR@10 and 8.38% on NDCG@10 against the best competitor. The significant improvements validate the effectiveness of aggregating the user behavior sequence and the influence between the users. We also observe that performance of TEA-A is slightly

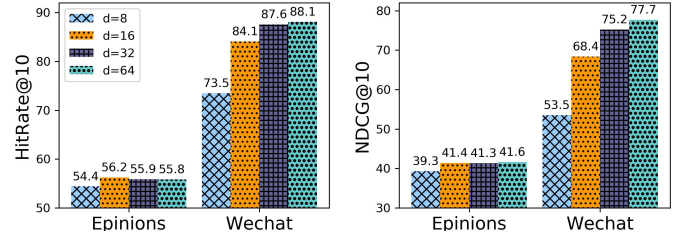


Fig. 3. The sensitivity of the embedding dimension d .

lower than that of TEA-S, indicating that the graph attention mechanism is difficult to handle the high sparsity of temporally evolving heterogeneous graphs.

F. The Sensitivity of Hyper-parameters

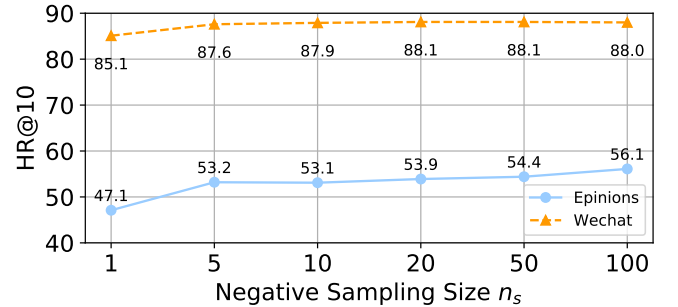


Fig. 4. The sensitivity of the negative sampling size n_s .

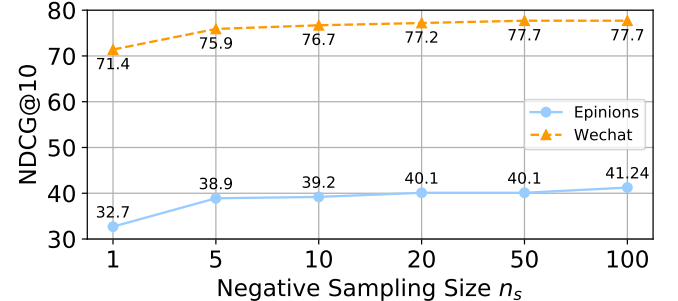


Fig. 5. The sensitivity of the negative sampling size n_s .

TABLE IV

THE PERFORMANCE EVALUATION OF THE COMPARED METHODS ON YELP DATASET. THE VALUE PRESENTED ARE AVERAGED OVER 5 REPLICATED WITH DIFFERENT RANDOM SEEDS.

| Model Class | Models | HR@5 | NDCG@5 | HR@10 | NDCG@10 | HR@20 | NDCG@20 |
|----------------------------|---------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Matrix Factorization based | BPRMF [10] | 66.33±0.27 | 52.46±0.16 | 76.51±0.26 | 55.77±0.16 | 84.59±0.22 | 57.82±0.15 |
| | NeuMF [24] | 70.38±0.26 | 56.14±0.28 | 79.35±0.12 | 59.06±0.24 | 86.14±0.12 | 60.79±0.22 |
| | SocialMF [7] | 64.82±0.24 | 49.69±0.24 | 76.27±0.28 | 53.42±0.21 | 84.99±0.28 | 55.63±0.19 |
| | SoRec [23] | 70.41±0.10 | 54.55±0.10 | 81.45±0.04 | 58.15±0.07 | 89.03±0.04 | 60.08±0.06 |
| Graph Neural Network based | GraphRec [5] | 68.37±0.23 | 51.44±0.27 | 81.55±0.17 | 55.74±0.18 | 90.61±0.17 | 58.05±0.16 |
| | LightGCN [45] | 73.04±0.21 | 57.10±0.21 | 84.39±0.07 | 60.80±0.19 | 92.08±0.07 | 62.76±0.17 |
| | DGRec [16] | 76.22±0.24 | 60.18±0.28 | 86.57±0.18 | 63.55±0.26 | 92.93±0.08 | 65.18±0.16 |
| Sequence based | DMAN [46] | 72.93±0.33 | 57.45±0.16 | 83.64±0.34 | 60.94±0.29 | 91.03±0.25 | 62.82±0.26 |
| | TransRec [2] | 75.81±0.15 | 60.63±0.16 | 80.19±0.20 | 64.00±0.15 | 93.13±0.12 | 65.78±0.15 |
| | SASRec [34] | 69.28±0.39 | 53.18±0.43 | 81.66±0.08 | 57.21±0.37 | 90.36±0.08 | 59.43±0.34 |
| | ASASRec [47] | 72.97±0.13 | 56.76±0.10 | 84.53±0.04 | 60.53±0.09 | 92.18±0.04 | 62.48±0.07 |
| Ours | TEA-A | 80.38±0.25 | 65.42±0.36 | 88.99±0.14 | 68.23±0.33 | 94.11±0.10 | 69.54±0.21 |
| | TEA-S | 80.43±0.18 | 65.59±0.26 | 88.97±0.08 | 68.37±0.23 | 94.09±0.07 | 69.68±0.21 |

TABLE V

THE PERFORMANCE EVALUATION OF THE COMPARED METHODS ON WECHAT DATASET. THE VALUE PRESENTED ARE AVERAGED OVER 5 REPLICATED WITH DIFFERENT RANDOM SEEDS.

| Model Class | Models | HR@5 | NDCG@5 | HR@10 | NDCG@10 | HR@20 | NDCG@20 |
|----------------------------|---------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Matrix Factorization based | BPRMF [10] | 62.33±0.12 | 56.38±0.12 | 68.55±0.18 | 58.38±0.14 | 75.60±0.10 | 60.16±0.14 |
| | NeuMF [24] | 68.58±0.16 | 61.66±0.16 | 75.26±0.18 | 63.82±0.16 | 82.53±0.14 | 65.65±0.15 |
| | SocialMF [7] | 68.25±0.13 | 61.30±0.42 | 74.62±0.07 | 63.36±0.39 | 81.44±0.04 | 65.09±0.37 |
| | SoRec [23] | 73.66±0.04 | 66.43±0.11 | 79.60±0.02 | 68.36±0.10 | 85.56±0.03 | 69.86±0.09 |
| Graph Neural Network based | GraphRec [5] | 66.04±0.33 | 52.17±0.29 | 76.80±0.25 | 55.66±0.26 | 85.72±0.18 | 57.93±0.24 |
| | LightGCN [45] | - | - | - | - | - | - |
| | DGRec [16] | 74.99±0.22 | 63.94±0.24 | 82.29±0.14 | 66.31±0.24 | 88.52±0.09 | 67.89±0.20 |
| Sequence based | DMAN [46] | - | - | - | - | - | - |
| | TransRec [2] | 73.54±0.17 | 64.87±0.16 | 80.06±0.14 | 66.99±0.15 | 86.03±0.14 | 68.50±0.14 |
| | SASRec [34] | 76.37±0.35 | 65.97±0.35 | 83.76±0.21 | 68.37±0.29 | 89.94±0.11 | 69.94±0.23 |
| | ASASRec [47] | 78.28±0.23 | 68.13±0.25 | 85.19±0.21 | 70.38±0.19 | 90.81±0.14 | 71.80±0.13 |
| Ours | TEA-A | 82.06±0.25 | 73.47±0.26 | 87.61±0.16 | 75.28±0.21 | 92.18±0.17 | 76.44±0.13 |
| | TEA-S | 83.23±0.19 | 76.12±0.21 | 88.12±0.13 | 77.70±0.18 | 92.42±0.17 | 78.79±0.16 |

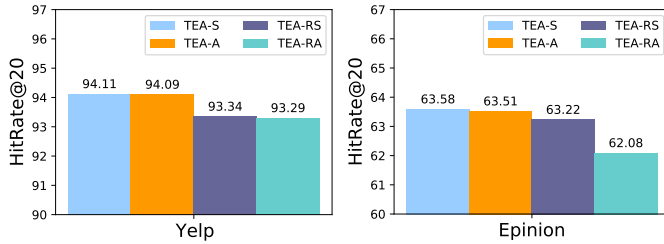


Fig. 6. The Experiment results of Fig. 7. The Experiment results of ablation studies on Yelp dataset. ablation studies on Epinion dataset.

1) *Embedding Dimension.*: In Figure 3 we analyze the sensitivity of the embedding dimension d by showing HR@10 and NDCG@10 of our proposed TEA-S with d varying from 8 to 64. We can observe that our model significantly benefits from a larger dimension when the dataset is large. A small embedding dimension ($d = 16$) is enough for TEA-S to achieve the best performance on Epinions.

2) *Sensitivity of the Number of Negative Samples.*: Figures 4 and 5 shows the sensitivity of the number of negative samples n_s in Equation (6) by showing HR@10 and NDCG@10 of our proposed TEA-S with n_s varying from 1 to 100. The variant with $n_s = 5$ performs comparably well, though using $n_s \geq 10$ still boosts performance especially on the large-scale dataset, which means that using more negative samples is

helpful to estimate the item transition probability. The variant with $n_s = 100$ achieves similar performance to the default setting $n_s = 50$, which indicates that our model is stable with n_s .

G. Ablation Study

In order to evaluate the effectiveness of the time-restricted aggregation, we remove the aforementioned aggregation module and obtain the variants **TEA-RS** and **TEA-RA**. experimental results are shown in Figures 6 and 7. From these results, we can find that the models with time-restricted aggregation achieve a better performance, especially the results on the Yelp dataset. We also find that the promotion in Epinion dataset is not so remarkable, this is since the social networks in Epinion are much denser than that of Yelp. To some extent, the experiment results reflect that the proposed time-restricted aggregation can mitigate the drawbacks of sparse social networks and user-item interactions.

VII. CONCLUSION

This paper presents a temporally evolving aggregations framework for the sequential recommendation. Beginning from the original conditional random field, we derive the unified objective function for the sequential recommendation, which leverages the social influence between users and

the dynamic user-item heterogeneous graph. The proposed framework provides the insights and principles of designing the sequential recommendation model. We further provide two different implementations of the proposed framework. Experimental results on three real-world datasets show that the TEA framework outperforms state-of-the-art methods.

VIII. ACKNOWLEDGMENTS

We would like to thank Lingling Yi and Li Li from WeChat for their help and supports on this work.

REFERENCES

- [1] F. Eskandarian and B. Mobasher, "Modeling the dynamics of user preferences for sequence-aware recommendation using hidden markov models," *arXiv preprint arXiv:1905.06863*, 2019.
- [2] R. He, W.-C. Kang, and J. McAuley, "Translation-based recommendation," in *Proceedings of the eleventh ACM conference on recommender systems*, 2017, pp. 161–169.
- [3] J. Tang, X. Hu, H. Gao, and H. Liu, "Exploiting local and global social context for recommendation," in *Proceedings of the Twenty-Third international joint conference on Artificial Intelligence*, 2013, pp. 2712–2718.
- [4] J. Tang, S. Wang, X. Hu, D. Yin, Y. Bi, Y. Chang, and H. Liu, "Recommendation with social dimensions," in *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 30, no. 1, 2016.
- [5] W. Fan, Y. Ma, Q. Li, Y. He, E. Zhao, J. Tang, and D. Yin, "Graph neural networks for social recommendation," in *The World Wide Web Conference*, 2019, pp. 417–426.
- [6] H. Ma, D. Zhou, C. Liu, M. R. Lyu, and I. King, "Recommender systems with social regularization," in *Proceedings of the fourth ACM international conference on Web search and data mining*, 2011, pp. 287–296.
- [7] M. Jamali and M. Ester, "A matrix factorization technique with trust propagation for recommendation in social networks," in *Proceedings of the fourth ACM conference on Recommender systems*, 2010, pp. 135–142.
- [8] Y. Hu, Y. Koren, and C. Volinsky, "Collaborative filtering for implicit feedback datasets," in *2008 Eighth IEEE International Conference on Data Mining*. Ieee, 2008, pp. 263–272.
- [9] Y. Koren, "Factorization meets the neighborhood: a multifaceted collaborative filtering model," in *Proceedings of the 14th ACM SIGKDD international conference on Knowledge discovery and data mining*, 2008, pp. 426–434.
- [10] S. Rendle, C. Freudenthaler, Z. Gantner, and L. Schmidt-Thieme, "Bpr: Bayesian personalized ranking from implicit feedback," in *Proceedings of the Twenty-Fifth Conference on Uncertainty in Artificial Intelligence*, 2009, pp. 452–461.
- [11] H. Liu, L. Jing, J. Wen, Z. Wu, X. Sun, J. Wang, L. Xiao, and J. Yu, "Deep global and local generative model for recommendation," in *Proceedings of The Web Conference 2020*, 2020, pp. 551–561.
- [12] H. Wang, F. Zhang, M. Zhao, W. Li, X. Xie, and M. Guo, "Multi-task feature learning for knowledge graph enhanced recommendation," in *The World Wide Web Conference*, 2019, pp. 2000–2010.
- [13] Z.-H. Deng, L. Huang, C.-D. Wang, J.-H. Lai, and S. Y. Philip, "Deepcf: A unified framework of representation learning and matching function learning in recommender system," in *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 33, no. 01, 2019, pp. 61–68.
- [14] P. Covington, J. Adams, and E. Sargin, "Deep neural networks for youtube recommendations," in *Proceedings of the 10th ACM conference on recommender systems*, 2016, pp. 191–198.
- [15] P. W. Battaglia, J. B. Hamrick, V. Bapst, A. Sanchez-Gonzalez, V. Zambaldi, M. Malinowski, A. Tacchetti, D. Raposo, A. Santoro, R. Faulkner et al., "Relational inductive biases, deep learning, and graph networks," *arXiv preprint arXiv:1806.01261*, 2018.
- [16] W. Song, Z. Xiao, Y. Wang, L. Charlin, M. Zhang, and J. Tang, "Session-based social recommendation via dynamic graph attention networks," in *Proceedings of the Twelfth ACM International Conference on Web Search and Data Mining*, 2019, pp. 555–563.
- [17] J. Yu, H. Yin, J. Li, M. Gao, Z. Huang, and L. Cui, "Enhance social recommendation with adversarial graph convolutional networks," *IEEE Transactions on Knowledge and Data Engineering*, 2020.
- [18] R. Panchendrarajan and A. Amasesan, "Bidirectional lstm-crf for named entity recognition," in *Proceedings of the 32nd Pacific Asia Conference on Language, Information and Computation*, 2018.
- [19] Z. Hao, D. Lv, Z. Li, R. Cai, W. Wen, and B. Xu, "Semi-supervised disentangled framework for transferable named entity recognition," *Neural Networks*, vol. 135, pp. 127–138, 2021.
- [20] A. Mnih and R. R. Salakhutdinov, "Probabilistic matrix factorization," *Advances in neural information processing systems*, vol. 20, pp. 1257–1264, 2007.
- [21] L. Baltrunas, B. Ludwig, and F. Ricci, "Matrix factorization techniques for context aware recommendation," in *Proceedings of the fifth ACM conference on Recommender systems*, 2011, pp. 301–304.
- [22] X. He, H. Zhang, M.-Y. Kan, and T.-S. Chua, "Fast matrix factorization for online recommendation with implicit feedback," in *Proceedings of the 39th International ACM SIGIR conference on Research and Development in Information Retrieval*, 2016, pp. 549–558.
- [23] H. Ma, H. Yang, M. R. Lyu, and I. King, "Sorec: social recommendation using probabilistic matrix factorization," in *Proceedings of the 17th ACM conference on Information and knowledge management*, 2008, pp. 931–940.
- [24] X. He, L. Liao, H. Zhang, L. Nie, X. Hu, and T.-S. Chua, "Neural collaborative filtering," in *Proceedings of the 26th international conference on world wide web*, 2017, pp. 173–182.
- [25] W. Fan, Q. Li, and M. Cheng, "Deep modeling of social relations for recommendation," in *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 32, no. 1, 2018.
- [26] S. Deng, L. Huang, G. Xu, X. Wu, and Z. Wu, "On deep learning for trust-aware recommendations in social networks," *IEEE transactions on neural networks and learning systems*, vol. 28, no. 5, pp. 1164–1177, 2016.
- [27] T. N. Kipf and M. Welling, "Semi-supervised classification with graph convolutional networks," *arXiv preprint arXiv:1609.02907*, 2016.
- [28] X. Fu, J. Zhang, Z. Meng, and I. King, "Mgann: metapath aggregated graph neural network for heterogeneous graph embedding," in *Proceedings of The Web Conference 2020*, 2020, pp. 2331–2341.
- [29] C. Shi, B. Hu, W. X. Zhao, and S. Y. Philip, "Heterogeneous information network embedding for recommendation," *IEEE Transactions on Knowledge and Data Engineering*, vol. 31, no. 2, pp. 357–370, 2018.
- [30] S. Rendle, C. Freudenthaler, and L. Schmidt-Thieme, "Factorizing personalized markov chains for next-basket recommendation," in *Proceedings of the 19th international conference on World wide web*, 2010, pp. 811–820.
- [31] J. Tang and K. Wang, "Personalized top-n sequential recommendation via convolutional sequence embedding," in *Proceedings of the Eleventh ACM International Conference on Web Search and Data Mining*, 2018, pp. 565–573.
- [32] B. Hidasi and A. Karatzoglou, "Recurrent neural networks with top-k gains for session-based recommendations," in *Proceedings of the 27th ACM international conference on information and knowledge management*, 2018, pp. 843–852.
- [33] M. Quadriana, A. Karatzoglou, B. Hidasi, and P. Cremonesi, "Personalizing session-based recommendations with hierarchical recurrent neural networks," in *proceedings of the Eleventh ACM Conference on Recommender Systems*, 2017, pp. 130–137.
- [34] W.-C. Kang and J. McAuley, "Self-attentive sequential recommendation," in *2018 IEEE International Conference on Data Mining (ICDM)*. IEEE, 2018, pp. 197–206.
- [35] F. Sun, J. Liu, J. Wu, C. Pei, X. Lin, W. Ou, and P. Jiang, "Bert4rec: Sequential recommendation with bidirectional encoder representations from transformer," in *Proceedings of the 28th ACM international conference on information and knowledge management*, 2019, pp. 1441–1450.
- [36] X. Ma and E. Hovy, "End-to-end sequence labeling via bi-directional lstm-cnns-crf," in *Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, 2016, pp. 1064–1074.
- [37] J. Besag, "Statistical analysis of non-lattice data," *Journal of the Royal Statistical Society: Series D (The Statistician)*, vol. 24, no. 3, pp. 179–195, 1975.
- [38] T. Ma, C. Xiao, J. Shang, and J. Sun, "Cgnf: Conditional graph neural fields," 2018.
- [39] T. Mikolov, I. Sutskever, K. Chen, G. Corrado, and J. Dean, "Distributed representations of words and phrases and their compositionality," *arXiv preprint arXiv:1310.4546*, 2013.
- [40] K. Cho, B. van Merriënboer, Ç. Gülçehre, D. Bahdanau, F. Bougares, H. Schwenk, and Y. Bengio, "Learning phrase representations using rnn encoder-decoder for statistical machine translation," in *EMNLP*, 2014.

- [41] A. Vaswani, N. Shazeer, N. Parmar, J. Uszkoreit, L. Jones, A. N. Gomez, L. Kaiser, and I. Polosukhin, "Attention is all you need," in NIPS, 2017.
- [42] W. L. Hamilton, R. Ying, and J. Leskovec, "Inductive representation learning on large graphs," in Proceedings of the 31st International Conference on Neural Information Processing Systems, 2017, pp. 1025–1035.
- [43] P. Veličković, G. Cucurull, A. Casanova, A. Romero, P. Lio, and Y. Bengio, "Graph attention networks," arXiv preprint arXiv:1710.10903, 2017.
- [44] D. P. Kingma and J. Ba, "Adam: A method for stochastic optimization," arXiv preprint arXiv:1412.6980, 2014.
- [45] X. He, K. Deng, X. Wang, Y. Li, Y. Zhang, and M. Wang, "Lightgcn: Simplifying and powering graph convolution network for recommendation," in Proceedings of the 43rd International ACM SIGIR Conference on Research and Development in Information Retrieval, 2020, pp. 639–648.
- [46] Q. Tan, J. Zhang, N. Liu, X. Huang, H. Yang, J. Zhou, and X. Hu, "Dynamic memory based attention network for sequential recommendation," Proceedings of the AAAI Conference on Artificial Intelligence, vol. 35, no. 5, pp. 4384–4392, May 2021. [Online]. Available: <https://ojs.aaai.org/index.php/AAAI/article/view/16564>
- [47] J. Manotumruksa and E. Yilmaz, "Sequential-based adversarial optimisation for personalised top-n item recommendation," in Proceedings of the 43rd International ACM SIGIR Conference on Research and Development in Information Retrieval, 2020, pp. 2045–2048.