Bayesian Persuasion for Algorithmic Recourse

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Abstract

When subjected to automated decision-making, decision-subjects will strategically modify their observable features in ways they believe will maximize their chances of receiving a desirable outcome. In many situations, the underlying predictive model is deliberately kept secret to avoid gaming and maintain competitive advantage. This opacity forces the decision subjects to rely on incomplete information when making strategic feature modifications. We capture such settings as a game of Bayesian persuasion, in which the decision-maker sends a signal, e.g., an action recommendation, to a decision subject to incentivize them to take desirable actions. We formulate the decision-maker's problem of finding the optimal Bayesian incentive-compatible (BIC) action recommendation policy as an optimization problem and characterize the solution via a linear program. Through this characterization, we observe that while the problem of finding the optimal BIC recommendation policy can be simplified dramatically, the computational complexity of solving this linear program is closely tied to (1) the relative size of the decision-subjects' action space, and (2) the number of features utilized by the underlying predictive model. Finally, we provide bounds on the performance of the optimal BIC recommendation policy and show that it can lead to arbitrarily better outcomes compared to standard baselines.

1 Introduction

When high-stakes decisions are automated or informed by data-driven algorithms, decision-subjects will strategically modify their observable features in a way which they believe will maximize their chances of achieving better outcomes [14, 6]. Often in such settings, the decision-subject has a set of actions/interventions available to them. Each of these actions leads to some measurable effect on their observable features that impact their prediction/decision. From the decision-maker's perspective, some of these actions may be more desirable than others. Consider credit scoring as an example. Credit scores predict how likely it is for an individual applicant to pay back a loan on time. Financial institutions regularly utilize credit scores to decide whether to offer applicants a mortgage, credit card, auto loan, or other credit products, and determine the condition under which the product is offered (e.g., by setting the interest rate or credit limit). Given their knowledge of credit scoring instruments, applicants regularly attempt to improve their scores to have better chances of approval for their product of interest. For instance, a business applying for a loan may improve their score by paying off existing debt or cleverly manipulating

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¹Note that while we use credit scoring as our running example throughout, it is not the only setting our work is relevant to. Examples of strategic settings which arise as a result of decision-making include college admissions, in which a college/university (decision-maker) decides whether or not to admit a prospective student (decision-subject), hiring, in which a company decides whether or not to hire a job applicant, and lending, in which a banking institution decides to accept or reject someone applying for a loan. Oftentimes, the decision-maker is aided by automated decision-making tools in these situations (e.g., [24, 29, 17]).

their financial records to appear more profitable. While both of these interventions may improve credit score, from the perspective of the financial institution offering the loan the former is much more desirable than the latter. But how can the decision-maker incentivize decision-subjects to take such desirable actions?

One potential solution is for the decision-maker to first carefully design the predictive model in order to encourage desirable behavior, then fully reveal the model to the public (as is assumed by prior models of strategic learning, e.g., [31, 22, 12]). In many real-world situations, however, full-disclosure (i.e., revealing the exact logic of the predictive model) is infeasible or irresponsible. For instance, credit scoring formulas are closely guarded trade secrets, in part to prevent the risk of default rates surging if applicants learn how to take advantage of them.

Next, let us consider settings where full disclosure about the model is not a viable alternative (as we argued above, credit scoring is one such domain). In such settings, the decision-maker may still be interested in providing *some* information about the model to decision subjects, for instance, to provide a certain level of transparency and recourse. In particular, the decision-maker may be legally obliged to (or economically motivated to) guide decision-subjects to take desirable actions that improve their outcomes. One potential solution in such cases is for the decision-maker to recommend desirable actions for the decision-subject to take. Of course, this needs to be done carefully and credibly, otherwise self-interested decision-subjects may not follow the decision-maker's recommendations.

In this paper, we provide a model to study a strategic learning setting in which the underlying decision rule is not revealed to the decision-subjects. There are several aspects our model aims to capture. First, even though the decision rule² is not revealed to the decision-subjects, they often have prior knowledge about what the predictive model may be. Secondly, when the decision-maker provides recommendations to decision-subjects on which action to take, the recommendations should be compatible with the subjects' incentives, to ensure they will follow the recommendation. Finally, certain regulations on transparency may require decision-makers to disclose their mechanisms on how they provide recommendations, even before the decision-makers obtain their predictive model or training data.

Taken these desiderata together, we model the interaction between the decision-maker and decision-subjects by drawing from the formalism of Bayesian persuasion [20], a mathematical model of persuasion from the information design literature in economics. In general, the Bayesian persuasion setting considers a game between a sender with private information and a receiver. At the beginning of the game, the sender and receiver have a shared prior over some unknown state of nature, which will eventually be revealed to the sender. Using this belief, the sender commits to a signaling policy, a possibly stochastic mapping from states of nature to action recommendations, in order to maximize their expected payoff. Once the state of nature is revealed to the sender, they choose a signal to reveal to the receiver (according to their signaling policy), who then takes a non-contractible action which affects the payoffs of both players. In our setting, the shared prior corresponds to the decision-maker and decision-subject's belief about what the predictive model will be before training. After training, the predictive model is revealed to the decision-maker, who then recommends an action to the decision-subject based on their pre-determined action recommendation policy. Upon receiving this action recommendation, the decision-subject updates their belief about the model, and takes the action which will maximize their utility in expectation.

Our contributions Our main conceptual contribution is to cast the problem of offering recourse pathways under partial transparency as game of Bayesian persuasion between the decision-maker and decision-subjects. Our main technical contributions are as follows:

1. Using tools from Bayesian persuasion, we show that it is possible for the decision-maker

²We use the terminology "decision rule" and "predictive model" interchangeably.

to provide incentive-compatible action recommendations to incentivize rational decision subjects to take desirable actions more often than the two natural alternatives; namely (1) fully revealing the predictive model to the decision subjects, or (2) revealing no information about the model.

- 2. While the decision-maker is trivially never worse off in expectation from using an optimal set of incentive-compatible recommendations, we show that situations exist in which they are *arbitrarily better off* in expectation than if they had chosen either of the two natural alternatives.
- 3. Finally, we derive the optimal action recommendation policy for the decision-maker. While the decision-maker's optimal recommendation policy initially appears incomputable (as it involves optimizing over *continuously-many* variables), we show that the problem can naturally be cast as a linear program. We find that the computational complexity of solving this linear program is closely tied to (1) the relative size of the agent's action space, and (2) the number of features utilized by the underlying decision rule.

The outline of the paper is as follows. In Section 2, we introduce our notation and provide the relevant background on Bayesian persuasion. In Section 3, we provide intuition for how formulating the decision maker's action recommendation problem as a game of Bayesian persuasion can help them incentivize desirable actions with a simple illustrative example. Additionally, we show that using an incentive-compatible action recommendation policy can be arbitrarily better for the decision-maker than revealing the full model to the decision-subjects or revealing nothing. In Section 4, we derive the decision-maker's optimal action recommendation policy, in Section 5 we provide a simple experimental setup to illustrate the empirical performance of our methods, and we conclude in Section 6.

1.1 Related Work

Strategic responses to unknown predictive models. We are not the first to model the strategic interaction between decision-maker and decision-subjects when the underlying predictive model is not public knowledge. Ghalme et al. [10] compare the prediction error of a classifier when it is public knowledge to the error when decision-subjects must learn a version of it, and label this difference the "price of opacity". They show that small errors in decision-subjects' estimates of the true underlying model may lead to large errors in the performance of the model. The authors argue that their work provides formal incentives for decision-makers to adopt full transparency as a policy. However, even when decision-makers may wish to make their predictive model fully transparent, it is often not feasible due to privacy concerns, legal reasons, etc.

Bechavod et al. [1] study the effects of information discrepancy across different sub-populations of decision-subjects on their ability to improve their observable features in strategic learning settings. Like us, they do not assume the predictive model is fully known to the decision-subjects. Instead, the authors model decision-subjects as trying to infer the underlying predictive model by learning from their social circle of family and friends, which naturally causes different groups to form within the population. They show that deploying a predictive model to maximize social welfare may actually cause the quality of some groups to deteriorate, and characterize the disparity in improvements across different subgroups. In contrast to this line of work, we study a setting in which the decision-maker provides customized feedback to each decision-subject individually.

Both of these works provided strategic learning models in which the predictive model is not revealed and the decision-subjects learn an estimate of the deployed model. While their models circumvent the assumption of full information about the deployed model, they restrict the decision-subjects' knowledge to be obtained only through past data, and their response does

not correspond to utility maximization. Our work takes inspiration from Bayesian persuasion to model a fully general form of prior knowledge about the predictive model.

Persuasion and learning. The act of leveraging an information advantage in order to influence others' behavior is known as Bayesian persuasion [20] within the economics literature. In its most basic form, Bayesian persuasion can be thought of as a two-player Stackelberg game in which one player (the sender) publicly commits to a signaling policy, conditional on an unknown state of nature. The state of nature is then revealed to the sender, who then sends a signal based on their signaling policy to the other player (the receiver). The receiver uses this signal to form a posterior update over the possible states of nature, and then uses this posterior to take some action which affects the welfare of both players.

There has been much interest in persuasion in the computer science and machine learning communities over the last several years. Dughmi [8] characterize the computational complexity of computing the optimal signaling policy for several popular models of persuasion. Castiglioni et al. [2] study the problem of learning the receiver's utilities through repeated interactions. Literature on the incentivized exploration multi-arm bandit problem [26, 27, 15, 3, 30] leverages techniques in Bayesian persuasion to incentivize agents to perform bandit exploration.

Economics. Our work broadly falls under the framework of a principal-agent problem from classical microeconomics [28]. Specifically, the problem we study can be seen as an instance of the *moral hazard* problem from contract design, in which an employer (the principal) cannot verify the effort an employee (the agent) expends before offering them a contract. Our work is thematically related to Nudge theory [23], a concept from behavioral economics which proposes using indirect suggestions as a way to influence the behavior and decision-making of groups or individuals.

Algorithmic recourse. Our work, which provides action recommendations to decision-subjects, is closely related to the field of algorithmic recourse, which is concerned with providing explanations and recommendations to individuals who are unfavorably treated by automated decision-making systems [21]. A line of algorithmic recourse methods [32, 18] focus on finding recourses that are *actionable*, or realistic, for decision-subjects to take to improve their decision. In contrast, our action recommendations are "actionable" in the sense that they are interventions which promote long-term desirable behaviors while ensuring that the decision-subject is individually no worse off in expectation.

Explainability. Recent legal and regulatory frameworks, such as the General Data Protection Regulation (GDPR) [7], motivate the development of forms of algorithmic transparency suitable for real-world deployment. While this work can be thought of as providing additional transparency into the decision-making process, it does not naturally fall into the existing organizations of explanation methods (e.g., as outlined in [4]), as our policy does not simply recommend actions based on the decision rule. Rather, our goal is to incentivize actionable interventions on the decision-subjects' observable features which are desirable to the decision-maker, and we leverage persuasion techniques to ensure compliance.

Other strategic learning settings. The strategic learning literature [11, 10, 5, 25, 16, 1, 12, 13, 22] broadly studies machine learning questions in the presence of strategic decision-subjects. There has been a long line of work in strategic learning that focuses on how strategic decision-subjects adapt their input to a machine learning algorithm in order to receive a more desirable prediction, although most prior work in this literature assumes that the underlying predictive model or decision rule is fully revealed to the decision-subjects, which is typically not true in reality.

2 Setting and Background

Consider a setting in which a decision-maker assigns a predicted label $\hat{y} \in \{-1, +1\}$ (e.g., whether or not someone will repay a loan if given one) to a decision-subject with observable features

 $\mathbf{x}_0 = (x_{0,1}, \cdots, x_{0,d}) \in \mathbb{R}^d$ (e.g., amount of current debt, bank account balance, etc.) using a linear decision rule, i.e., $\hat{y} = \text{sign}\{\mathbf{x}_0^{\top}\boldsymbol{\theta} + b\}$, where $\boldsymbol{\theta} \in \mathbb{R}_+^d$ and $b \in \mathbb{R}_+$ are chosen by the decision-maker. We refer to $(\theta, b) \in \Theta$ as the decision rule, or predictive model.³ The goal of the decision-subject is to receive a positive classification (e.g., get approved for a loan), regardless of their true label y (e.g., whether they will actually end up paying it back in the future). Because of this, the decision-subject may choose to take some action a (e.g., pay off a certain amount of existing debt) from some set of possible actions \mathcal{A} in order to modify their observable features, with the goal of optimizing the prediction they receive. By taking action a, the decision-subject incurs some $cost\ c(a) \in \mathbb{R}$. This could be an actual monetary cost, but it can also represent more abstract notions of cost such as opportunity cost or the time/effort cost a decision-subject may have to pay in order to change a certain attribute about themselves. Additionally, taking an action changes a decision-subject's observable feature values from \mathbf{x}_0 to $\mathbf{x}_0 + \Delta \mathbf{x}(a)$, where $\Delta \mathbf{x}(a) \in \mathbb{R}^d_{\geq 0}$, and $\Delta \mathbf{x}_j(a)$ specifies the change in the jth observable feature as the result of taking action a. Concretely, we assume that each decision-subject has m actions $\{a_1, a_2, \dots a_m\} \in \mathcal{A}$ at their disposal in order to improve their outcomes. For convenience, we add a_{\emptyset} to \mathcal{A} to denote taking "no action" (e.g., $\Delta \mathbf{x}(a_{\emptyset}) = \mathbf{0}$, $c(a_{\emptyset}) = 0$).

As a result of taking action a, a decision-subject receives utility $u_{\rm ds}(a, \boldsymbol{\theta}, b) = {\rm sign}\{(\mathbf{x}_0 + \Delta \mathbf{x}(a))^{\top} \boldsymbol{\theta} + b\} - c(a)$. In other words, each decision-subject receives some positive (negative) utility for a positive (negative) classification, subject to some cost for taking said action. If the decision-subject had exact knowledge of the decision rule $(\boldsymbol{\theta}, b)$ used by the decision-maker, they could solve an optimization problem to determine the best action to take in order to maximize their utility. However, in many settings it is not realistic for a decision-subject to have perfect knowledge of $(\boldsymbol{\theta}, b)$. Instead, we model each decision-subject as having a $prior \ \pi(\boldsymbol{\theta}, b)$ over all possible $(\boldsymbol{\theta}, b)$ combinations, which can be thought of as "common knowledge" or "intuition" about the relative importance of each observable feature to the classifier. While we only consider the setting in which all decision-subjects share a common prior $\pi(\boldsymbol{\theta}, b)$, we believe that studying settings in which different subgroups have different priors over the decision rule being used is an interesting direction for future work. With no knowledge of $\boldsymbol{\theta}$ or b, a rational, risk-neutral decision-maker would pick an action a^* in order to maximize their expected utility with respect to their prior, i.e., $a^* \in \arg\max_{a \in \mathcal{A}} \mathbb{E}_{\pi}[u_{\mathrm{ds}}(a, \boldsymbol{\theta}, b)]$.

From the decision-maker's perspective, some actions may be more desirable than others. For example, a bank may prefer that an applicant pay off more existing debt than less when applying for a loan. A college may prefer that an applicant study for the SAT instead of taking an SAT prep course. To formalize this notion of action preference, we say that the decision-maker receives some utility $u_{dm}(a) \in \mathbb{R}$ when the decision-subject takes action a. In the loan example, u_{dm} (pay off more debt) $> u_{dm}$ (pay off less debt). In the college admissions setting, u_{dm} (study for SAT) $> u_{dm}$ (take prep course).

2.1 Action Recommendation Policy

The decision-maker has an information advantage over the decision-subject, due to the fact that they know the true value of (θ, b) , whereas the decision-subject does not. The decision-maker may be able to leverage this information advantage to incentivize the decision-subject to take a more favorable action than the one they would have taken according to their prior. In order for the decision-subject to be incentivized to follow the action recommended by the decision-maker, the recommended action $\sigma \in \mathcal{A}$ needs to be Bayesian incentive-compatible.

Definition 2.1 (Bayesian incentive-compatibility). An action recommendation $\sigma = a$ is Bayesian incentive-compatible (BIC) if

$$\mathbb{E}_{\pi'}[u_{ds}(a, \boldsymbol{\theta}, b) | \sigma = a] \ge \mathbb{E}_{\pi'}[u_{ds}(a', \boldsymbol{\theta}, b) | \sigma = a], \ \forall a', \tag{1}$$

³We assume $\mathbf{x}_0 \in \mathbb{R}^d_{\geq 0}$ and $\boldsymbol{\theta} \in \mathbb{R}^d_{\geq 0}$ for ease of analysis, although our results can be easily extended to settings under which this is not the case

where the expectation is taken with respect to the decision-subject's posterior $\pi' = \pi(\boldsymbol{\theta}, b | \sigma = a)$ induced by receiving recommendation $\sigma = a$.

In other words, a recommendation $\sigma = a$ is BIC if the decision-subject's expected utility with respect to their posterior $\pi(\boldsymbol{\theta}, b | \sigma = a)$, given that the decision-maker recommends action a, is at least as high as the expected utility of taking any other action a' under the posterior. We say that an action recommendation policy $\mathcal{S}: (\boldsymbol{\theta}, b) \to \mathcal{A}$ is BIC if Condition 1 holds for all actions $\sigma \in \mathcal{A}$ recommended by the policy.

Finally, we remark that the decision-maker's action recommendation policy is implicitly assumed to be fixed and common knowledge. This is because in order for the decision-subject to perform a Bayesian update based on the observed recommendation, they must know the action recommendation policy. Additionally, the decision-maker must have the power of commitment, i.e., the decision-subject must believe that the decision-maker will select actions according to their revealed recommendation policy. In our setting, this means that the decision-maker must commit to their action recommendation policy before training their predictive model. This can be seen as a form of transparency, as the decision-maker is publicly committing to how they will use their model before they even train it. For simplicity, we assume that the decision-maker shares the same prior beliefs $\pi(\theta, b)$ as the decision-subject over the observable features before the model is trained. These assumptions are standard in the Bayesian persuasion literature (see, e.g., [20, 26]), as without them, characterizing BIC signaling policies is extremely challenging.⁴ The setting we consider is concisely summarized below.

Problem protocol: Bayesian persuasion in the strategic learning setting.

- 1. Before training, the decision-maker and decision-subject share some prior $\pi(\boldsymbol{\theta}, b)$ over the true decision rule.
- 2. After training, the decision rule is revealed to the decision-maker.
- 3. The decision-maker then uses their recommendation policy and knowledge of the true decision rule to recommend an action for the decision-subject to take.

3 Illustrative Example

As is the case in the traditional Bayesian persuasion literature [20, 19, 9], it is generally possible for the decision-maker to design an action recommendation policy such that their expected utility is higher than if they had provided no recommendation. To provide intuition for how leveraging the decision-maker's information advantage by playing a BIC recommendation policy may lead to higher expected utility for the decision-maker, we study the following example.

Consider a simple setting under which a single decision-subject has one observable feature x_0 (e.g., credit score) and two possible actions: a_{\emptyset} = "do nothing" (i.e., $\Delta x(a_{\emptyset}) = 0$, $c(a_{\emptyset}) = 0$, $u_{dm}(a_{\emptyset}) = 0$) and a_1 = "pay off existing debt" (i.e., $\Delta x(a_1) > 0$, $c(a_1) > 0$, $u_{dm}(a_1) = 1$), which in turn raises their credit score. For the sake of our illustration, we assume credit-worthiness to be a socially desirable trait, and credit scores to be a good measure of credit-worthiness. We assume the decision-maker would like to design a recommendation policy in order to maximize the chance the decision-subject plays action a_1 , regardless of whether or not the applicant will

⁴Note that while we consider the interaction between the decision-maker and one decision-subject throughout most of the paper for ease of exposition, our results extend to a heterogeneous population of decision-subjects. Under this setting, the decision-maker would publicly commit to a signaling mechanism which takes the decision subject's observable features as input and produces a probability distribution over actions for each possible (θ, b) combination.

receive the loan. In this simple setting, the decision-maker's decision rule can be characterized by a single parameter b, i.e., the decision-subject receives a positive classification if $x + b \ge 0$ and a negative classification otherwise. Note that the decision-subject does not know the exact value of b. Instead they have some prior over it, denoted as $\pi(b)$.

Based on the initial value of the decision-subject's observable feature x_0 and the true value of b, the decision-maker recommends an action $\sigma \in \{a_\emptyset, a_1\}$ for the decision-subject to take. The decision-subject then takes a possibly different action $s \in \{a_\emptyset, a_1\}$, which changes their observable feature from x_0 to $x = x_0 + \Delta x(s)$. Recall that the decision-subject's utility takes the form $u_{ds}(s, \theta, b) = \text{sign}\{(x_0 + \Delta x(s)) + b\} - c(s)$. Note that if $c(a_1) > 2$, then $u_{ds}(a_\emptyset, \theta, b) > u_{ds}(a_1, \theta, b)$ holds for any value of b, meaning that it is impossible to incentivize any rational decision-subject to play action a_1 . Therefore, in order to give the decision-maker a "fighting chance" at incentivizing action a_1 , we assume the cost of action a_1 is such that $c(a_1) < 2$.

We observe that in this simple setting, we can bin values of b into three different "regions", based on the outcome the decision-subject would receive if b were actually in that region. First, if $x_0 + \Delta x(a_1) + b < 0$, the decision-subject will not receive a positive classification, even if they take action a_1 . In this region, the decision-subject's initial feature value x_0 is "too low" for taking the desired action to make a difference in their classification. We refer to this region as region L. Second, if $x_0 + b \ge 0$, the decision-subject will receive a positive classification no matter what action they take. In this region, x_0 is "too high" for the action they take to make any difference on their classification. We refer to this region as region H. Third, if $x_0 + b < 0$ and $x_0 + \Delta x(a_1) + b \ge 0$, the decision-subject will receive a positive classification if they take action a_1 and a negative classification if they take action a_0 . We refer to this region as region M. Consider the following action recommendation policy.

Action recommendation policy S:

Case 1: $b \in L$. Recommend action a_1 with probability q and action a_\emptyset with probability 1-q

Case 2: $b \in M$. Recommend action a_1 with probability 1

Case 3: $b \in H$. Recommend action a_1 with probability q and action a_{\emptyset} with probability 1-q

In Case 2, the recommendation policy recommends the action (a_1) that the decision-subject would have taken had they known the true b, with probability 1. However, in Case 1 and Case 3, the decision-maker recommends, with probability q, an action (a_1) that the decision-subject would not have taken, leveraging the fact that the decision-subject does not know exactly which case they are currently in. If the decision-subject follows the decision-maker's recommendation from \mathcal{S} , then the decision-maker expected utility will increase from 0 to q if the realized $b \in L$ or $b \in H$, and will remain the same otherwise. Intuitively, if q is "small enough" (where the precise definition of "small" depends on the shared prior over b and the cost of taking action a_1), then it will be in the decision-subject's best interest to follow the decision-maker's recommendation, even though they know that the decision-maker may sometimes recommend taking action a_1 when it is not in their best interest to take that action! That is, the decision-maker may recommend that an decision-subject pay off existing debt with probability q when it is unnecessary for them to do so in order to secure a loan. We now give a criteria on q which ensures the action recommendation policy \mathcal{S} is BIC.

Theorem 3.1. Recommendation policy S is Bayesian incentive-compatible if $q = \min\{\frac{\pi(M)(2-c(a_1))}{c(a_1)(1-\pi(M))}, 1\}$ where $\pi(M)$ is an instance-specific value that depends on $\pi(b)$ and x_0 .

Proof Sketch. We show that $\mathbb{E}[u_{ds}(a_{\emptyset}, b)|\sigma = a_{\emptyset}] \geq \mathbb{E}[u_{ds}(a_1, b)|\sigma = a_{\emptyset}]$ and $\mathbb{E}[u_{ds}(a_1, b)|\sigma = a_1] \geq \mathbb{E}[u_{ds}(a_{\emptyset}, b)|\sigma = a_1]$, where the expectation is taken with respect to the decision-subject's

posterior induced by $\sigma = a$. Since these conditions are satisfied, \mathcal{S} is BIC.

Proof. Based on the decision-subject's prior over b, they can calculate

 $\pi(L) = \pi(x_0 + \Delta x(a_1) + b < 0)$, i.e., the probability the decision-subject is in region L according to the prior

 $\pi(M) = \pi(x_0 + b < 0 \text{ and } x_0 + \Delta x(a_1) + b \ge 0)$, i.e., the probability the decision-subject is in region M according to the prior

 $\pi(H) = \pi(x_0 + b \ge 0)$, i.e., the probability the decision-subject is in region H according to the prior

Case 1: $\sigma = a_0$. Given the signal $\sigma = a_{\emptyset}$, the decision-subject's posterior $\pi(\cdot | \sigma = a_{\emptyset})$ over L, M, and H will take the form

$$\pi(L|\sigma = a_{\emptyset}) = \frac{p(\sigma = a_{\emptyset}|L)\pi(L)}{p(\sigma = a_{\emptyset})} = \frac{\pi(L)}{\pi(L) + \pi(H)}$$

$$\pi(M|\sigma = a_{\emptyset}) = \frac{p(\sigma = a_{\emptyset}|M)\pi(M)}{p(\sigma = a_{\emptyset})} = 0$$

$$\pi(H|\sigma = a_{\emptyset}) = \frac{p(\sigma = a_{\emptyset}|H)\pi(H)}{p(\sigma = a_{\emptyset})} = \frac{\pi(H)}{\pi(L) + \pi(H)}$$

If the decision-subject receives signal $\sigma = a_0$, they know with probability 1 that they are *not* in region M with probability 1. Therefore, they know that taking action a_1 will not change their classification, so they will follow the decision-maker's recommendation and take action a_0 .

Case 2: $\sigma = a_1$. Given the signal $\sigma = a_1$, the decision-subject's posterior over L, M, and H will take the form

$$\pi(L|\sigma = a_1) = \frac{p(\sigma = a_1|L)\pi(L)}{p(\sigma = a_1)} = \frac{q\pi(L)}{\pi(M) + q(\pi(L) + \pi(H))} = \frac{q\pi(L)}{\pi(M) + q(1 - \pi(M))}$$

$$\pi(M|\sigma = a_1) = \frac{p(\sigma = a_1|M)\pi(M)}{p(\sigma = a_1)} = \frac{\pi(M)}{\pi(M) + q(\pi(L) + \pi(H))} = \frac{\pi(M)}{\pi(M) + q(1 - \pi(M))}$$

$$\pi(H|\sigma = a_1) = \frac{p(\sigma = a_1|H)\pi(H)}{p(\sigma = a_1)} = \frac{q\pi(H)}{\pi(M) + q(\pi(L) + \pi(H))} = \frac{q\pi(H)}{\pi(M) + q(1 - \pi(M))}$$

The decision-subject's expected utility of taking actions a_{\emptyset} and a_1 under the posterior induced by $\sigma = a_1$ are

$$\mathbb{E}[u_{ds}(a_{\emptyset}, b)|a_{1}] = \pi(H|\sigma = a_{1}) \cdot (1 - 0) + \pi(M|\sigma = a_{1}) \cdot (-1 - 0) + \pi(L|\sigma = a_{1}) \cdot (-1 - 0)$$
$$= \pi(H|\sigma = a_{1}) - \pi(M|\sigma = a_{1}) - \pi(L|\sigma = a_{1})$$

and

$$\mathbb{E}[u_{ds}(a_1,b)|a_1] = \pi(H|\sigma = a_1) \cdot (1 - c(a_1)) + \pi(M|\sigma = a_1) \cdot (1 - c(a_1)) + \pi(L|\sigma = a_1) \cdot (-1 - c(a_1))$$

In order for a_1 to be BIC,

$$\mathbb{E}[u_{ds}(a_1, b) | \sigma = a_1] \ge \mathbb{E}[u_{ds}(a_{\emptyset}, b) | \sigma = a_1].$$

Plugging in our expressions for $\mathbb{E}[u_{ds}(a_1,b)|\sigma=a_1]$ and $\mathbb{E}[u_{ds}(a_\emptyset,b)|\sigma=a_1]$, we see that

$$\pi(H|\sigma = a_1) \cdot (1 - c(a_1)) + \pi(M|\sigma = a_1) \cdot (1 - c(a_1)) + \pi(L|\sigma = a_1) \cdot (-1 - c(a_1))$$
$$> \pi(H|\sigma = a_1) - \pi(M|\sigma = a_1) - \pi(L|\sigma = a_1)$$

After canceling terms and simplifying, we see that

$$-(\pi(L|\sigma=a_1) + \pi(H|\sigma=a_1))c(a_1) + \pi(M|\sigma=a_1)(2-c(a_1)) \ge 0$$

Next, we plug in for $\pi(L|\sigma=a_1)$, $\pi(M|\sigma=a_1)$, and $\pi(H|\sigma=a_1)$. Note that the denominators of $\pi(L|\sigma=a_1)$, $\pi(M|\sigma=a_1)$, and $\pi(H|\sigma=a_1)$ cancel out.

$$-q(\pi(L) + \pi(H))c(a_1) + \pi(M)(2 - c(a_1)) = -q(1 - \pi(M))c(a_1) + \pi(M)(2 - c(a_1)) \ge 0$$

Solving for q, we see that

$$q \le \frac{\pi(M)(2 - c(a_1))}{c(a_1)(1 - \pi(M))}.$$

Note that $q \geq 0$ always. Finally, in order for q to be a valid probability, we restrict q such that

$$q = \min\{\frac{\pi(M)(2 - c(a_1))}{c(a_1)(1 - \pi(M))}, 1\}.$$

This completes the proof.

Under this setting, the decision-maker will achieve expected utility $\pi(M) + q(1 - \pi(M))$. But how much better can the decision-maker do by using a BIC action recommendation policy, compared to natural alternatives? We answer this question concretely in the following section.

3.1 Performance Bounds

As we will see in Section 4, the expected utility of the decision-maker when playing the optimal BIC recommendation policy will be *no worse* than their expected utility if they had completely revealed the decision rule to the decision-subject, or if they had revealed nothing and let the decision-subject act according to the prior. In this section, we show that the decision-maker's expected utility when playing the optimal BIC recommendation policy can be *arbitrarily higher* than their expected utility from completely revealing the decision rule or revealing nothing. In particular, we prove the following theorem.

Theorem 3.2. There exists a problem instance such that the expected decision-maker utility from playing the optimal BIC recommendation policy is arbitrarily higher than the expected decision-maker utility for revealing everything or revealing nothing.

Proof. Consider the example in Section 3.

Expected utility from revealing nothing. If the decision-subject acts exclusively according to the prior, they will select action a_1 with probability 1 if $\mathbb{E}[u_{ds}(a_1,b)] \geq \mathbb{E}[u_{ds}(a_{\emptyset},b)]$ and with probability 0 otherwise. Plugging in our expressions for $\mathbb{E}[u_{ds}(a_1,b)]$ and $\mathbb{E}[u_{ds}(a_{\emptyset},b)]$, we see that the decision-subject will select action a_1 only if

$$\pi(L)(-1-c(a_1)) + \pi(M)(1-c(a_1)) + \pi(H)(1-c(a_1)) \ge \pi(L)(-1-0) + \pi(M)(-1-0) + \pi(H)(1-0)$$

Canceling terms and simplifying, we see that

$$-c(a_1)(\pi(L) + \pi(H)) + \pi(M)(2 - c(a_1)) \ge 0$$

must hold for the decision-subject to select action a_1 . Finally, substituting $\pi(L) + \pi(H) = 1 - \pi(M)$ gives us the condition $2\pi(M) - c(a_1) \ge 0$. Alternatively, if

$$\frac{\pi(M)}{c(a_1)} < \frac{1}{2},\tag{2}$$

the decision-subject will select action a_{\emptyset} with probability 1. Intuitively, this means that a rational decision-subject would take action a_1 if the ratio of $\pi(M)$ (the probability according to the prior that taking action a_1 is in the decision-subject's best interest) to $c(a_1)$ (the cost of taking action a_1) is high, and would take action a_{\emptyset} otherwise.

Expected utility from revealing everything. If the decision-maker reveals the full model to the decision-subject, they will select action a_1 when $b \in M$ and action a_\emptyset otherwise. Therefore since $u_{dm}(a_1) = 1$ and $u_{dm}(a_\emptyset) = 0$, the decision-maker's expected utility if they reveal everything is $\pi(M)$.

Expected utility from BIC policy. Recall that the decision-maker's recommendation policy from Section 3 sets $q = \min\{\frac{\pi(M)(2-c(a_1))}{c(a_1)(1-\pi(M))}, 1\}$. Under this setting, the decision-maker's expected utility is $\min\{1 \cdot \pi(M) + q \cdot (1-\pi(M)), 1\}$. Substituting in our expression for q and simplifying, we see that the decision-maker's expected utility for the BIC policy is $\min\{\frac{2\pi(M)}{c(a_1)}, 1\}$.

Suppose that $2\pi(M) = c(a_1)(1-\epsilon)$ and $c(a_1) = 2\epsilon$, for some small $\epsilon > 0$. The decision-maker's expected utility will always be 0 from revealing nothing because $\frac{2\pi(M)}{c(a_1)} = 1 - \epsilon < 1$. The decision-maker's expected utility from playing the BIC policy will be $\frac{2\pi(M)}{c(a_1)} = 1 - \epsilon$. Since $\pi(M) = \epsilon(1-\epsilon) < \epsilon$, the decision-maker's expected utility from revealing everything will be less than ϵ . Therefore, as ϵ approaches 0, the decision-maker's expected utility from revealing everything approaches 0 (the smallest value possible), and the decision-maker's expected utility from the BIC policy approaches 1 (the highest value possible). This completes the proof.

The decision-maker's expected utility as a function of their possible strategies is summarized in Table 1. Note that when $\mathbb{1}\{\pi(M) \geq \frac{c(a_1)}{2}\} = 1$, q = 1. Therefore, the decision-maker's expected utility is always as least as good as the two natural alternatives of revealing nothing about the model, or revealing everything about the model.

	Reveal nothing	BIC recommendation policy	Reveal everything
Decision-maker utility	$\mathbb{1}\{\pi(M) \ge \frac{c(a_1)}{2}\}$	$\pi(M) + q(1 - \pi(M))$	$\pi(M)$

Table 1: Decision-maker's expected utility when (1) revealing nothing about the model, (2) using the BIC recommendation policy, and (3) revealing everything about the model. See Section 3.1 for the full derivations.

4 Optimal Signaling Policy

While our example signaling policy in the previous section leads to higher expected decision-maker utility, there is no reason $a\ priori$ to believe it is optimal, or that we can expect similar results beyond the one action and one observable feature setting. We now derive the decision-maker's optimal signaling policy for the general setting with d observable features and m possible actions described in Section 2. Under the general setting, the decision-maker's optimal signaling policy can be described by the following optimization.

$$\max_{p(\sigma=a|\boldsymbol{\theta},b),\forall a\in\mathcal{A}} \mathbb{E}_{\sigma\sim\mathcal{S},(\boldsymbol{\theta},b)\sim\boldsymbol{\Theta}}[u_{dm}(\sigma)]$$
s.t.
$$\mathbb{E}_{(\boldsymbol{\theta},b)\sim\boldsymbol{\Theta}}[u_{ds}(a,\boldsymbol{\theta},b) - u_{ds}(a',\boldsymbol{\theta},b)|\sigma = a] \ge 0, \ \forall a,a'\in\mathcal{A},$$
(3)

where we omit the valid probability constraints over $p(\sigma = a|\boldsymbol{\theta}, b), a \in \mathcal{A}$ for brevity. In words, the decision-maker wants to design a signaling policy \mathcal{S} in order to maximize their expected utility, subject to the constraint that the signaling policy is BIC. At first glance, the optimization may initially seem hopeless as there are infinitely many values of $p(\sigma = a|\boldsymbol{\theta}, b)$ (one for every possible $\boldsymbol{\theta}$, b combination) that the decision-maker's optimal policy must optimize over. However, we will show that the decision-maker's optimal policy can actually be recovered by optimizing over finitely many variables.

By rewriting the BIC constraints as integrals over Θ and applying Bayes' rule, our optimization over $p(\sigma = a | \theta, b), a \in \mathcal{A}$ takes the following form

$$\max_{p(\sigma=a|\boldsymbol{\theta},b),\forall a\in\mathcal{A}} \mathbb{E}_{\sigma\sim\mathcal{S},(\boldsymbol{\theta},b)\sim\boldsymbol{\Theta}}[u_{dm}(\sigma)]$$
s.t.
$$\int_{\boldsymbol{\Theta}} p(\sigma=a|\boldsymbol{\theta},b)\pi(\boldsymbol{\theta},b)(u_{ds}(a,\boldsymbol{\theta},b)-u_{ds}(a',\boldsymbol{\theta},b))d\boldsymbol{\Theta} \geq 0, \ \forall a,a'\in\mathcal{A}.$$

Note that if $u_{ds}(a, \boldsymbol{\theta}, b) - u_{ds}(a', \boldsymbol{\theta}, b)$ is the same for some "region" $R \subseteq \boldsymbol{\Theta}$ (which we formally define below), we can pull $u_{ds}(a, \boldsymbol{\theta}, b) - u_{ds}(a', \boldsymbol{\theta}, b)$ out of the integral and instead sum over the different regions. Intuitively, a region can be thought of as the set of all $(\boldsymbol{\theta}, b)$ pairs that are indistinguishable from a decision-subject's perspective because they lead to the exact same utility for any possible action the decision-subject could take. Based on this idea, we formally define a region of $\boldsymbol{\Theta}$ as follows.

Definition 4.1 (Region). A region $R \subseteq \Theta$ is the set of all $(\theta, b) \in \Theta$ such that $u_{ds}(a, \theta, b) - u_{ds}(a', \theta, b) = u_{ds}(a, \theta', b') - u_{ds}(a', \theta', b')$, $\forall a, a' \in \mathcal{A}$, $(\theta, b), (\theta', b') \in R$. We denote the set of all regions by \mathcal{R} .

After pulling the decision-subject utility function out of the integral, our optimization takes the following form:

$$\max_{p(\sigma=a|\boldsymbol{\theta},b),\forall a\in\mathcal{A}} \quad \mathbb{E}_{\sigma\sim\mathcal{S},(\boldsymbol{\theta},b)\sim\boldsymbol{\Theta}}[u_{dm}(\sigma)]$$
s.t.
$$\sum_{R\in\mathcal{R}} (u_{ds}(a,R) - u_{ds}(a',R)) \int_{(\boldsymbol{\theta},b)\in R} p(\sigma=a|\boldsymbol{\theta},b)\pi(\boldsymbol{\theta},b)dR \geq 0, \ \forall a,a'\in\mathcal{A}.$$

Now that the decision-subject's utility $u_{ds}(\cdot)$ no longer depends on $(\boldsymbol{\theta}, b)$, we can integrate $p(\sigma = a|\boldsymbol{\theta}, b)\pi(\boldsymbol{\theta}, b)$ over each region R. We denote p(R) as the probability that the true $(\boldsymbol{\theta}, b) \in R$ according to the decision-subject prior.

$$\max_{p(\sigma=a|R), \forall a \in \mathcal{A}, R \in \mathcal{R}} \quad \mathbb{E}_{\sigma \sim \mathcal{S}, (\boldsymbol{\theta}, b) \sim \boldsymbol{\Theta}}[u_{dm}(\sigma)]$$
s.t.
$$\sum_{R \in \mathcal{R}} p(\sigma = a|R)\pi(R)(u_{ds}(a, R) - u_{ds}(a, R)) \ge 0, \ \forall a, a' \in \mathcal{A}.$$

Since it is possible to write the constraints in terms of $p(\sigma = a|R)$, $\forall a \in \mathcal{A}, R \in \mathcal{R}$, it suffices to optimize directly over these quantities. The final step is to rewrite the objective. For completeness, we include the constraints which make each $\{p(\sigma = a_1|R), p(\sigma = a_2|R), \ldots, p(\sigma = a_m|R)\}$, $\forall R$ a valid probability distribution.

Theorem 4.2 (Optimal signaling policy). The decision-maker's optimal signaling policy can be characterized by the following linear program:

$$\max_{p(\sigma=a|R), \forall a \in \mathcal{A}, R \in \mathcal{R}} \quad \sum_{a \in \mathcal{A}} \sum_{R \in \mathcal{R}} p(R) p(\sigma=a|R) u_{dm}(a)$$

$$s.t. \quad \sum_{R \in \mathcal{R}} p(\sigma=a|R) p(R) (u_{ds}(a,R) - u_{ds}(a',R)) \ge 0, \ \forall a, a' \in \mathcal{A}$$

$$\sum_{a \in \mathcal{A}} p(a|R) = 1, \ \forall R, \quad p(a|R) \ge 0, \ \forall R \in \mathcal{R}, a \in \mathcal{A}.$$

Note that the feasible region of the optimization problem will always be non-empty, as the decision-maker can always recommend the action the decision-subject would play according to the prior.

4.1 Size of $|\mathcal{R}|$

We have shown that the problem of determining the decision-maker's optimal signaling policy can be transformed from an optimization over infinitely many $p(\sigma = a|\theta, b)$ variables into an optimization over finitely many regions $R \in \mathcal{R}$. However, computing $\pi(R)$, $\forall R \in \mathcal{R}$ may require integrating over 2^m regions of Θ in the worst-case. (Recall that m is the number of actions available to the decision-subject.) In this section, we show that $|\mathcal{R}|$ is generally much smaller for a special ordering over actions (as shown in Figure 1). This action scheme captures the set of actions that only affect one observable feature and can reasonably be thought of as coming from a finite set. We believe this action scheme reasonably reflects many real-world settings (e.g., consider a setting in which an decision-subject must choose between paying off some amount of debt and opening a new credit card, when strategically modifying their observable features before applying for a loan).

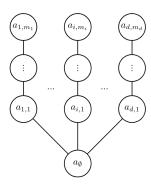


Figure 1: Graphical respresentation of special ordering over the actions available to each decision-subject. Each branch corresponds to an observable feature and each node corresponds to a possible action the decision-subject may take. The root corresponds to taking no action (denoted by a_{\emptyset}). Nodes further away from the root on branch i correspond to higher $\Delta \mathbf{x}_i$, i.e., $\Delta \mathbf{x}_i(a_{\emptyset}) \prec \Delta \mathbf{x}_i(a_{i,1}) \prec \ldots \prec \mathbf{x}_i(a_{i,m_i})$.

In general, the LP which characterizes the optimal signaling policy (Theorem 4.2) optimizes over $\Theta(md|\mathcal{R}|)$ variables, where m is an upper-bound on m_i , $\forall i \in \{1, ..., d\}$. In order to characterize the size of $|\mathcal{R}|$, we provide an alternative definition of a region of Θ below, which can be derived by simply plugging in the decision-subject utility function into Definition 4.1 and canceling terms.

Definition 4.3 (Region, revisited). A region R of Θ is the set of all $(\boldsymbol{\theta}, b) \in \Theta$ such that $sign\{(\mathbf{x}_0 + \Delta \mathbf{x}(a))^\top \boldsymbol{\theta} + b\} - sign\{(\mathbf{x}_0 + \Delta \mathbf{x}(a'))^\top \boldsymbol{\theta} + b\} = sign\{(\mathbf{x}_0 + \Delta \mathbf{x}(a))^\top \boldsymbol{\theta}' + b'\} - sign\{(\mathbf{x}_0 + \Delta \mathbf{x}(a))^\top \boldsymbol{\theta}' + b'\} + sign\{(\mathbf{x}_0$

In other words, $(\boldsymbol{\theta}, b)$ and $(\boldsymbol{\theta'}, b')$ belong to the same region if the difference in their predictions for any two actions a and a' is the same. An immediate consequence of this alternative definition is that a region R can essentially be uniquely characterized by the set of actions $A_R \subseteq \mathcal{A}$ which receive a positive classification when $(\boldsymbol{\theta}, b) \in R$. Thus, if taking action a results in a positive classification for some $(\boldsymbol{\theta}, b)$ and a negative classification for some $(\boldsymbol{\theta'}, b')$, the only way for $(\boldsymbol{\theta}, b)$ and $(\boldsymbol{\theta'}, b')$ to be in the same region is if taking any action in \mathcal{A} results in a positive classification for $(\boldsymbol{\theta}, b)$ and a negative classification for $(\boldsymbol{\theta'}, b')$. We refer to this region as the "all or nothing" region, which is the union of two convex sets (i.e., the set of $(\boldsymbol{\theta}, b)$ such that all actions result in the decision-subject receiving a positive classification and the set of $(\boldsymbol{\theta}, b)$ such that no action results in the decision-subject receiving a positive classification). Besides this special case, if $(\boldsymbol{\theta}, b)$ and $(\boldsymbol{\theta'}, b')$ produce different classifications for the same action, they are in different regions by Definition 4.3. Armed with this alternative definition of regions over $\boldsymbol{\Theta}$, we are now ready to characterize $|\mathcal{R}|$ for the setting described in Figure 1.

Theorem 4.4. For the setting described in Figure 1, there are $|\mathcal{R}| = \Theta(m_*^d)$ regions (i.e., $|\mathcal{R}| = \Omega(m_*^d)$ and $|\mathcal{R}| = \mathcal{O}(m_*^d)$), where d is the number of decision-subject observable features, and $m_* = \max_i m_i$ is an upper bound on the number of actions the decision-subject has at their disposal to improve each observable feature.

Proof. In order to characterize the number of regions $|\mathcal{R}|$, we use the notion of a dominated action a.

Definition 4.5 (Dominated Action). An action a is dominated by some other action a' if $\Delta \mathbf{x}(a) \leq \Delta \mathbf{x}(a')$ and $\Delta \mathbf{x}_i(a) < \Delta \mathbf{x}_i(a')$ for at least one index $i \in \{1, \ldots, d\}$.

For example, in Figure 1, action $a_{i,j}$ is dominated by action $a_{i,k}$, $\forall i, 1 \leq j < k \leq m_i$. Note that in general if action a' dominates action a, then it is always possible to write $\Delta \mathbf{x}_i(a')$ in terms of $\Delta \mathbf{x}_i(a)$, i.e., $\Delta \mathbf{x}_i(a') = \Delta \mathbf{x}_i(a) + \boldsymbol{\delta}(a, a')$, where $\boldsymbol{\delta}(a, a') \succeq \mathbf{0}_d$ and $\delta_i(a, a') > 0$ for some $i \in \{1, \ldots, d\}$.

Lemma 4.6. If action a is dominated by action a', then $a' \in A_R$ for any region R where $a \in A_R$.

Proof. The decision-subject receives a positive classification for some $(\boldsymbol{\theta}, b)$ if $(\mathbf{x}_0 + \Delta \mathbf{x}(a))^{\top} \boldsymbol{\theta} + b \geq 0$. If a' dominates a, then $\Delta \mathbf{x}(a')$ can be written as $\Delta \mathbf{x}(a') = \Delta \mathbf{x}(a) + \boldsymbol{\delta}(a, a')$, where $\boldsymbol{\delta}(a, a') \succeq \mathbf{0}_d$. Therefore, if $(\mathbf{x}_0 + \Delta \mathbf{x}(a))^{\top} \boldsymbol{\theta} + b \geq 0$ (i.e., when $a \in A_R$), then $(\mathbf{x}_0 + \Delta \mathbf{x}(a'))^{\top} \boldsymbol{\theta} + b = (\mathbf{x}_0 + \Delta \mathbf{x}(a))^{\top} \boldsymbol{\theta} + \delta(a, a')^{\top} \boldsymbol{\theta} + b \geq 0$ (and $a' \in A_R$), since $\boldsymbol{\delta}(a, a') \succeq \mathbf{0}_d$ and $\boldsymbol{\theta} \in \mathbb{R}_+^d$.

 $|\mathcal{R}| = \Theta(m_*^d)$ then follows directly from the fact that each action only affects one observable feature, and the decision-subject has at most m_* actions at their disposal to modify a given feature.

Under this setting, when the number of observable features d is small, the decision-maker can essentially be thought of as optimizing over a number of variables that is polynomial in the number of actions m. However, as d grows large, the decision-maker's optimization becomes intractable.

Now that the space of regions \mathcal{R} is specified, it is possible to calculate $\pi(R)$ for $R \in \mathcal{R}$, given $\pi(\boldsymbol{\theta},b)$ without enumerating all possibilities. Recall that a region R is characterized by the set of actions $A_R \subseteq \mathcal{A}$ that receive a positive classification whenever $(\boldsymbol{\theta},b) \in R$. Each action $a \in \mathcal{A}$ is associated with a hyperplane in $\boldsymbol{\Theta}$ -space, $(\mathbf{x}_0 + \Delta \mathbf{x}(a))^{\top} \boldsymbol{\theta} + b = 0$. The decision-subject receives a positive classification when taking action a for all $(\boldsymbol{\theta},b)$ such that $(\mathbf{x}_0 + \Delta \mathbf{x}(a))^{\top} \boldsymbol{\theta} + b \geq 0$ and a negative classification otherwise. Therefore, each region R is a convex subspace of $\boldsymbol{\Theta}$, and can be calculated as $\pi(R) = \int_{(\boldsymbol{\theta},b)\in R} \pi(\boldsymbol{\theta},b) dR$.

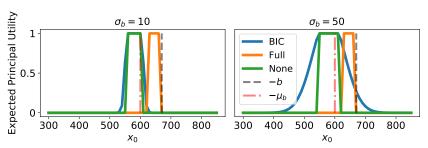
5 Experiments

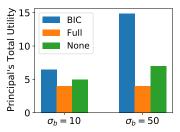
We provide preliminary experiments for the simple setup described in Section 3 to demonstrate the utility that the decision-maker gains from the optimal BIC action recommendation policy over other baselines. The baseline policies we consider are (1) revealing the full decision rule b, and (2) revealing no information about b.

To contextualize this simple synthetic setup, consider a banking institution deciding whether approve a loan application from an applicant based on credit score $x_0 \in [300, 850]$ with a simple threshold classifier. The bank approves the application $(\hat{y} = 1)$ if $x_0 + b > 0$ and rejects $(\hat{y} = -1)$ otherwise. Here, we assume the ground-truth threshold value used by the decision-maker to be 670 (i.e. b = -670), which is typically considered as a decent credit score. Recall that $a_{\emptyset} =$ "do nothing" and $a_1 =$ "pay off existing debt" and set the utility of the decision-maker to be $u_{dm}(a_1) = 1, u_{dm}(a_{\emptyset}) = 0$, as, for the sake of our illustration, we assume credit score to be a good measure of credit-worthiness. Finally, we assume the prior to be $\pi(b) \sim \mathcal{N}(\mu_b, \sigma_b^2)$.

In Figure 2^5 , we show how the optimal BIC action recommendation policy obtains higher decision-maker utility compared to the baseline policies. In particular, Figure 2a shows the decision-maker's expected utility for different credit scores x_0 the decision-subjects have under two scenarios, each with the standard deviation of the prior distribution of $\sigma_b = 10$ (left) and $\sigma_b = 50$ (right) but the same mean of $\mu_b = -600$. As expected, the full-information policy (orange) yields peak decision-maker expected utility around the true b, while the no-information

⁵In this setting, the decision-subject cost of taking action a_1 is set to $c(a_1) = 0.5$, and $\Delta x(a_1) = 40$ (i.e., action a_1 improves an applicant's credit score by 40 points).





(a) Expected decision-maker utility when $\mu_b = -600$, b = -670, for two different scores for each case in varying prior standard deviation, $\sigma_b = 10$ (left) and $\sigma_b = 50$ (right).

(b) Expected decision-maker utility value summed up across Figure (a).

Figure 2: (a) decision-maker's expected utility across different decision-subject scores(x_0), and (b) decision-maker's total utility computed by summing up expected utility across different scores of decision-subjects, for three policies: our BIC action recommendation policy (BIC, blue), policy revealing the full decision rule (Full, orange), and policy revealing no information (None, green). As the decision-subject's uncertainty about the true threshold b (measured by σ_b) increases, the advantage of BIC policy becomes more visible.

policy (green) does so around the prior mean μ_b . As the decision-subjects' uncertainty about the true b increases (i.e., the standard deviation of the prior distribution increases from 10 to 50), the decision-maker benefits from the BIC action recommendation policy for a broader range of credit scores unlike other baselines.

To better measure the total amount of decision-maker utility yielded by each policy, we assume a uniform distribution of the decision-subjects' credit scores in the population and take the sum of expected decision-maker utility values across different scores (equivalent to the area under the curve of the decision-maker's expected utility in Figure 2a). We plot these "total" utility values in Figure 2b, and as expected, the larger σ_b is, the more visible the comparative advantage of our method compared to the baselines. Intuitively, this is because the more uncertain a decision-subject is about the true decision rule being deployed, the more the decision-maker can leverage this uncertainty to persuade the decision-subject to take desirable actions.

As action a_1 becomes more cost-prohibitive (or less effective), we expect the decision-maker's utility to decrease, as there is less incentive for the decision-subjects to take the action⁶. As shown in Figure 3, we indeed observe such a trend as the cost of a_1 increases (left) and $\Delta x(a_1)$ decreases (right). Nevertheless, our BIC action recommendation policy (blue) yields strictly higher decision-maker utility values compared to the baselines across all conditions.

6 Conclusion and Future Work

In this work, we propose a way in which a decision-maker can leverage their information advantage over decision-subjects in order to incentivize them to take more desirable actions. In order to do so, we effectively cast the algorithmic recourse problem as a game of Bayesian persuasion. We show that the decision-maker's optimal signaling policy takes the form of a linear program with $\Theta(md|\mathcal{R}|)$ variables, and we provide a characterization of $|\mathcal{R}|$ for a special action structure which captures several real-world settings. While solving this LP requires keeping track of a number of variables which may be exponential in the input size, the initial characterization of the optimal action recommendation policy requires optimizing over infinitely many variables. There are several exciting directions for future work, which we outline below.

⁶In this setting, we set $\mu_b = -650$ and $\sigma_b = 50$ so that the decision-subjects are considered to have a reasonable estimate of the true threshold b = -670.

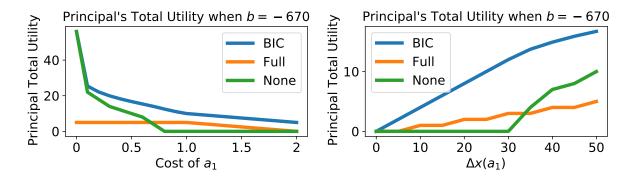


Figure 3: Decision-maker's total utility across different cost (left) and Δx (right), for our BIC action recommendation policy (**BIC**, blue), policy revealing the full decision rule (**Full**, orange), and policy revealing no information (**None**, green). When taking the action becomes cost-prohibitive (high cost) or less effective (small Δx), the decision-maker's utility decreases as there is less incentive for the decision-subject to take the action. Nevertheless, BIC action recommendation policy yields strictly higher decision-maker utility compared to the baselines.

Characterizing the number of regions for more general action spaces. While we characterized the number of regions for settings in which the action ordering takes a special structure (namely, each action affects only one observable feature), it may be possible to characterize the number of regions for more complicated settings. In order to do this, new algorithmic techniques may be required.

NP-Hardness proof. The fact that the linear program for the decision-maker's optimal signaling policy requires optimizing over an exponential number of variables in the worst-case provides strong evidence to suggest that the problem of recovering the decision-maker's optimal BIC recommendation policy is NP-Hard. This evidence is further supported by hardness results for similar persuasion settings [8]. It would be interesting to formalize this intuition by providing a formal reduction to a problem in NP.

Public persuasion. Throughout this work, we assumed that the recommendations received by each decision-subject were *private*. However, if a decision-subject is given access to recommendations for multiple individuals, it may be possible for them to reconstruct the underlying model. While out of the scope of this project, it would be interesting to study models of *public* persuasion in the

Experiments on semi-synthetic datasets. We illustrated our methods on a purely synthetic dataset inspired by our example provided in Section 3. It would be interesting to evaluate our methods in a more realistic regime, such as one based on the Taiwan credit dataset [33].

Other simplifying assumptions. Finally, we made several assumptions in order to gain the insights offered by our analysis. In particular, we consider *linear* decision rules and assumed all decision-subject parameters (cost function, initial observable features, etc.) were known to the decision-maker. It would be interesting to extend our work to settings with non-linear decision rules, or settings in which not all of the decision-subjects' parameters are known to the decision-maker.

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