

# Chapter 1      Section 2      Exercises

1. How many substrings  $aab$  are in  $ww^Rw$ , where  $w = aabbab$ .

**Solution.** Because

$$w^R = babbaa, \quad ww^Rw = \underline{aabb}abbabbaa\underline{aabb}ab,$$

there are two substrings  $aab$  in  $ww^Rw$ .

2. Use induction on  $n$  to show that  $|u^n| = n|u|$  for all strings  $u$  and all  $n$ .

**Proof.**

## 1. Basis

If  $n = 0$ , then

$$|u^1| = |u|, \quad 1 \cdot |u| = |u|.$$

Therefore,

$$|u^1| = 1 \cdot |u|.$$

## 2. Inductive Assumption

Assume that for  $i = 0, 1, 2, \dots, n$

$$|u^i| = i \cdot |u|.$$

## 3. Inductive Step

For  $i = n + 1$ ,

$$|u^{n+1}| = |u^n u| = |u^n| + |u| = n \cdot |u| + |u| = (n + 1)|u|.$$

Thus,  $|u^n| = n|u|$  for all strings  $u$  and all  $n$ .

■

3. The reverse of a string, introduced informally above, can be defined more precisely by the recursive rules

$$\begin{aligned} a^R &= a, \\ (wa)^R &= aw^R, \end{aligned}$$

for all  $a \in \Sigma, w \in \Sigma^*$ . Use this to prove that

$$(uv)^R = v^R u^R,$$

for all  $u, v \in \Sigma^+$ .

**Proof.**

### 1. Basis

For  $|v| = 1$ , suppose that  $v = a, a \in \Sigma$

$$(uv)^R = (ua)^R = au^R = v^R u^R.$$

### 2. Inductive Assumption

Suppose that for  $|v| = 1, 2, \dots, n$

$$(uv)^R = v^R u^R.$$

### 3. Inductive Step

For  $i = n + 1$ , suppose that  $v = wa, a \in \Sigma$

$$(uv)^R = (uwa)^R = a(uw)^R = aw^R u^R = v^R u^R.$$

Thus,

$$(uv)^R = v^R u^R,$$

for all  $u, v \in \Sigma^+$ .

■

**4. Prove that  $(w^R)^R = w$  for all  $w \in \Sigma^*$ .**

**Proof.**

### 1. Basis

For  $|w| = 0$ , which means that  $w = \lambda$ ,

$$(\lambda^R)^R = (\lambda)^R = \lambda = w.$$

## 2. Inductive Assumption

Suppose that for  $|w| = 1, 2, \dots, n$

$$(w^R)^R = w.$$

## 3. Inductive Step

For  $i = n + 1$ , suppose that  $w = va, a \in \Sigma$

$$(w^R)^R = ((va)^R)^R = (av^R)^R = (v^R)^R a = va = w.$$

Thus,  $(w^R)^R = w$  for all  $w \in \Sigma^*$ .

■

5. Let  $L = \{ab, aa, baa\}$ . Which of the following strings are in  $L^*$ :  $abaabaaabaa$ ,  $aaaabaaaa$ ,  $baaaaabaaaab$ ,  $baaaaabaa$ ? Which strings are in  $L^4$ ?

**Solution.**

$$abaabaaabaa = ab|aa|baa|ab|aa \in L^5;$$

$$aaaabaaaa = aa|aa|baa|aa \in L^4;$$

$$baaaaabaaaab \notin L^*;$$

$$baaaaabaa = baa|aa|ab|aa \in L^4.$$

6. Let  $\Sigma = \{a, b\}$  and  $L = \{aa, bb\}$ . Use set notation to describe  $\overline{L}$ .

**Solution.**

$$\overline{L} = \{\lambda, a, b, ab, ba\} \cup \{w \in \{a, b\}^+ : |w| \geq 3\}.$$