Chapter 1 Section 1 Exercises

1. With $S_1 = \{\ 2, 3, 5, 7\ \},\ S_2 = \{\ 2, 4, 5, 8, 9\ \}$, and $U = \{\ 1 : 10\ \}$, compute $\overline{S}_1 \cup S_2$.

Solution.

$$\overline{S}_1 = \{\ 1, 4, 6, 8, 9, 10\ \} \quad \Rightarrow \quad \overline{S}_1 \cup S_2 = \{\ 1, 2, 4, 5, 6, 8, 9, 10\ \}.$$

2. With $S_1 = \{\ 2, 3, 5, 7\ \},\ S_2 = \{\ 2, 4, 5, 8, 9\ \}$, compute $S_1 \times S_2$ and $S_2 \times S_1$.

Solution.

$$\begin{aligned} S_1 \times S_2 = \{\ & (2,2), (2,4), (2,5), (2,8), (2,9), \\ & (3,2), (3,4), (3,5), (3,8), (3,9), \\ & (5,2), (5,4), (5,5), (5,8), (5,9), \\ & (7,2), (7,4), (7,5), (7,8), (7,9)\ \}. \\ S_2 \times S_1 = \{\ & (2,2), (2,3), (2,5), (2,7), \\ & (4,2), (4,3), (4,5), (4,7), \\ & (5,2), (5,3), (5,5), (5,7), \\ & (8,2), (8,3), (8,5), (8,7), \\ & (9,2), (9,3), (9,5), (9,7)\ \}. \end{aligned}$$

3. For $S = \{\ 2, 5, 6, 8\ \}$ and $T = \{\ 2, 4, 6, 8\ \}$, compute $|S \cap T| + |S \cup T|$.

Solution.

$$S \cap T = \{\ 2, 6, 8\ \}, \quad S \cup T = \{\ 2, 4, 5, 6, 8\ \} \quad \Rightarrow \quad |S \cap T| + |S \cup T| = 3 + 5 = 8.$$

4. What relation between two sets S and T must hold so that $|S \cup T| = |S| + |T|$.

Solution.

$$|S \cup T| = |S| + |T| - |S \cap T| = |S| + |T| \quad \Rightarrow \quad |S \cap T| = 0 \quad \Rightarrow \quad S \cap T = \varnothing.$$

Therefore, S and T are disjoint.

5. Show that for all sets S and T, $S - T = S \cap \overline{T}$.

Solution.

$$\begin{aligned} & S - T = \{\ x : x \in S \text{ and } x \notin T\ \} \\ \Rightarrow \quad & S - T = \{\ x : x \in S \text{ and } x \in \overline{T}\ \} \\ \Rightarrow \quad & S - T = S \cap \overline{T}. \end{aligned}$$