

# Chapter 1      Section 2      Exercises

1. How many substrings  $aab$  are in  $ww^Rw$ , where  $w = aabbab$ .

**Solution.** Because

$$w^R = babbaa, \quad ww^Rw = \underline{a}abbabbabbaa\underline{a}abbab,$$

there are two substrings  $aab$  in  $ww^Rw$ .

2. Use induction on  $n$  to show that  $|u^n| = n|u|$  for all strings  $u$  and all  $n$ .

**Proof.**

## 1. Basis

If  $n = 0$ , then

$$|u^1| = |u|, \quad 1 \cdot |u| = |u|.$$

Therefore,

$$|u^1| = 1 \cdot |u|.$$

## 2. Inductive Assumption

Assume that for  $i = 0, 1, 2, \dots, n$

$$|u^i| = i \cdot |u|.$$

## 3. Inductive Step

For  $i = n + 1$ ,

$$|u^{n+1}| = |u^n u| = |u^n| + |u| = n \cdot |u| + |u| = (n + 1)|u|.$$

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