Chapter 1 Section 2 Exercises

1. How many substrings aab are in ww^Rw, where $w = aabbab$.

Solution. Because

$$w^R = babbaa, \qquad ww^Rw = \underline{aab}babbabbaa\underline{aab}bab,$$

there are two substrings aab in ww^Rw.

2. Use induction on n to show that $|u^n| = n|u|$ for all strings u and all n.

Proof.

1. **Basis**

   If $n = 0$, then
   $$|u^1| = |u|, \qquad 1 \cdot |u| = |u|.$$
   Therefore,
   $$|u^1| = 1 \cdot |u|.$$

2. **Inductive Assumption**

   Assume that for $i = 0, 1, 2, \cdots, n$
   $$|u^i| = i \cdot |u|.$$

3. **Inductive Step**

   For $i = n + 1$,
   $$|u^{n+1}| = |u^n u| = |u^n| + |u| = n \cdot |u| + |u| = (n+1)|u|.$$

Thus, $|u^n| = n|u|$ for all strings u and all n.

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3. The reverse of a string, introduced informally above, can be defined more precisely by the recursive rules

$$a^R = a,$$

$$(wa)^R = aw^R,$$

for all $a \in \Sigma, w \in \Sigma^*$. Use this to prove that

$$(uv)^R = v^R u^R,$$

for all $u, v \in \Sigma^+$.

Proof.

1. **Basis**

   For $|v| = 1$, suppose that $v = a, a \in \Sigma$

   $$(uv)^R = (ua)^R = au^R = v^R u^R.$$

2. **Inductive Assumption**

   Suppose that for $|v| = 1, 2, \cdots, n$

   $$(uv)^R = v^R u^R.$$

3. **Inductive Step**

   For $i = n + 1$, suppose that $v = wa, a \in \Sigma$

   $$(uv)^R = (uwa)^R = a(uw)^R = aw^R u^R = v^R u^R.$$

Thus,

$$(uv)^R = v^R u^R,$$

for all $u, v \in \Sigma^+$.

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