

Chapter 1 Section 3 Exercises

1. While passwords generally have few restrictions, they are normally not totally free. Suppose that in a certain system, passwords can be of arbitrary length but must contain at least one letter, $a - z$, and one number $0 - 9$. Construct a grammar that generates the set of such legal passwords.

Solution. The grammar G is $G = (\{\langle password \rangle, \langle letter \rangle, \langle number \rangle, \langle rest \rangle\}, \{a, b, \dots, z, 0, 1, \dots, 9\}, \{\langle password \rangle\}, P)$ where P is given by

$$\begin{aligned}\langle password \rangle &\rightarrow \langle rest \rangle \langle letter \rangle \langle rest \rangle \langle number \rangle \langle rest \rangle, \\ \langle password \rangle &\rightarrow \langle rest \rangle \langle number \rangle \langle rest \rangle \langle letter \rangle \langle rest \rangle, \\ \langle rest \rangle &\rightarrow \langle letter \rangle \langle rest \rangle \mid \langle number \rangle \langle rest \rangle \mid \lambda, \\ \langle letter \rangle &\rightarrow a \mid b \mid \dots \mid z, \\ \langle number \rangle &\rightarrow 0 \mid 1 \mid \dots \mid 9.\end{aligned}$$

2. Suppose that in some programming language, numbers are restricted as follows:

- (a) A number may be signed or unsigned.
- (b) The value field consists of two nonempty parts, separated by a decimal point.
- (c) There is an optional exponent field. If present, this field must contain the letter e , followed by a signed two-digit integer.

Solution. The grammar G is $G = (\{\langle number \rangle, \langle base \rangle, \langle rest \rangle, \langle exponent \rangle, \langle sign \rangle, \langle digit \rangle\}, \{e, 0, 1, \dots, 9\}, \{\langle number \rangle\}, P)$ where P is given by

$$\begin{aligned}\langle number \rangle &\rightarrow \langle base \rangle \mid \langle base \rangle \langle exponent \rangle, \\ \langle base \rangle &\rightarrow \langle sign \rangle \langle digit \rangle \langle rest \rangle . \langle digit \rangle \langle rest \rangle, \\ \langle rest \rangle &\rightarrow \langle digit \rangle \langle rest \rangle \mid \lambda, \\ \langle exponent \rangle &\rightarrow e \langle sign \rangle \langle digit \rangle \langle digit \rangle, \\ \langle sign \rangle &\rightarrow + \mid - \mid \lambda, \\ \langle digit \rangle &\rightarrow 0 \mid 1 \mid \dots \mid 9.\end{aligned}$$