1 Mathematical Preliminaries and Notation

Sets

A **set** is a collection of elements, without any structure other than membership.
The usual set operations are **union** ($\cup$), **intersection** ($\cap$), **difference** ($-$) and **complementation** defined as

$$
\begin{aligned}
S_1 \cup S_2 &= \{\ x : x \in S_1 \text{ or } x \in S_2\ \}, \\
S_1 \cap S_2 &= \{\ x : x \in S_1 \text{ and } x \in S_2\ \}, \\
S_1 - S_2 &= \{\ x : x \in S_1 \text{ and } x \notin S_2\ \}, \\
\overline{S} &= \{\ x : x \in U \text{ and } x \notin S\ \}.
\end{aligned}
$$

DeMorgan's laws

$$
\begin{aligned}
\overline{S_1 \cup S_2} &= \overline{S_1} \cap \overline{S_2}, \\
\overline{S_1 \cap S_2} &= \overline{S_1} \cup \overline{S_2}.
\end{aligned}
$$

A set S_1 is said to be a **subset** of S if every element of S_1 is also an element of S. We write this as

$$S_1 \subseteq S.$$

If $S_1 \subseteq S$, but S contains an element not in S_1, we say that S_1 is a **proper subset** of S; we write this as

$$S_1 \subset S.$$

If S_1 and S_2 have no common element, then the sets are said to be **disjoint**. We write this as

$$S_1 \cap S_2 = \varnothing.$$

A set is said to be finite if it contains a **finite** number of elements; otherwise it is **infinite**.
The set of all subsets of a set S is called the **powerset** of S and is denoted by 2^S. If S is finite, then

$$|2^S| = 2^{|S|}.$$

The sets whose elements are ordered sequences of elements from other sets are said to be the **Cartesian product** of other sets. For the Cartesian product of n sets, which itself is a set of ordered pairs, we write

$$S = S_1 \times S_2 \times \cdots \times S_n = \{\ (x_1,\ x_2,\ \cdots,\ x_n) : x_i \in S_i\ \}.$$

Suppose that $S_1,\ S_2,\ \cdots,\ S_n$ are subsets of a given set S and that the following holds:

1. The subsets $S_1,\ S_2,\ \cdots,\ S_n$ are mutually disjoint;
2. $S_1 \cup S_2 \cup \cdots \cup S_n = S$;
3. none of the S_i is empty.

Then $S_1,\ S_2,\ \cdots,\ S_n$ is called a **partition** of S.