ADA mini HW14

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Solution:

1. Reduction: we first define $-\infty$ which is much less than any number in the given sequence. Then, we insert a $-\infty$ between any two consecutive number. Then, we can solve RMSQ with this modified sequence. The answer must be some unit interval which will be the index of the answer of RMQ. For finding $-\infty$ and insert it to the original sequence, each take $O(N)$ time. Thus, reduction time is $O(N)$.

2. Example:
   Input for RMQ: $\langle 6, 8, 12, 46, 7, 18, 5\rangle$
   We set $-\infty$ as -1000
   Modified sequence: $\langle 6, -1000, 8, -1000, 12, -1000, 46, -1000, 7, -1000, 18, -1000, 5\rangle$
   Let the modified sequence be the input of RMSQ.
   Output of RMSQ: 6, 6 (The maximum-sum segment is $\{46\}$).
   Then, 6th element in the modified sequence is 46 which is the answer of RMQ.

3. More details and simple proof:

   (a) How less should $-\infty$ be: at least $-(1+$*maximum absolute value among all element*$)$.

   (b) Length of returned segment will be unit:
   Suppose there is a segment with length k which is not worse than any other segment. If one side of it is $-\infty$, we can eliminate it and get a better answer. Otherwise, its length should be at least 3. Considering any side of it, it will be some element along with a $-\infty$. Thus, we can eliminate both of them and get a better answer. Proof by contradiction. The returned segment must have unit length.