高等數位訊號處理 HW1

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(1) Design a Mini-max **highpass** FIR filter such that
① Filter length = 21, ② Sampling frequency f_s = 5000Hz,
③ Pass Band 1100~2500Hz ④ Transition band: 900~1100 Hz,
⑤ Weighting function: $W(F)$ = 1 for passband, $W(F)$ = 0.5 for stop band .
⑥ Set Δ = 0.0001 in Step 5.

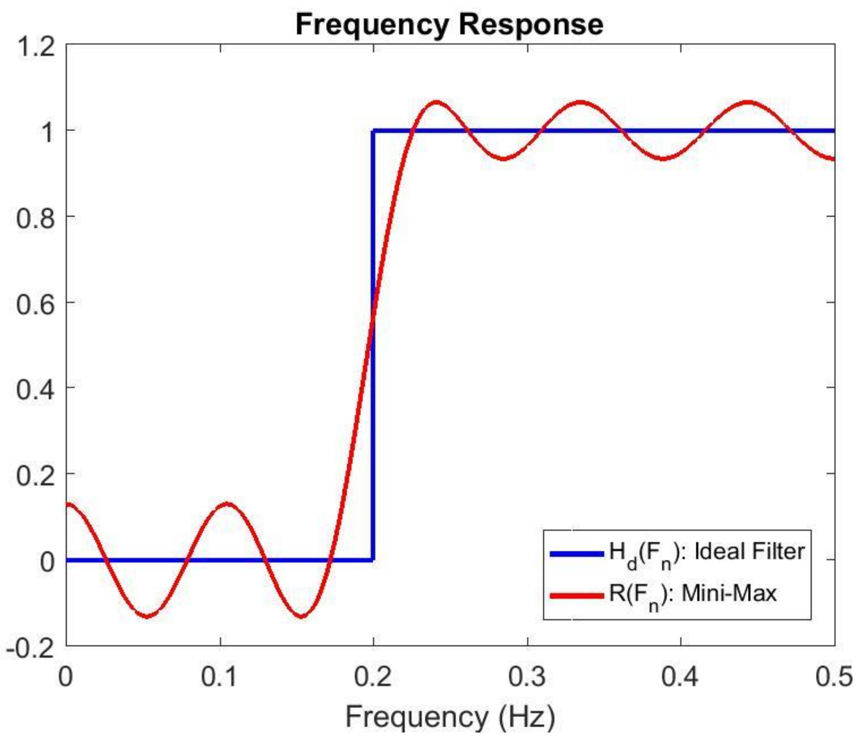
(a) the Matlab program

```
close all;
clear;
% Specifications
filter_L = 21;
SamplingRate = 5000;
StopBand = [0 1000];
TranBand=[900 1100];
weighting=[0.5 1];
delta=0.0001;

F=(0:1/SamplingRate:0.5);
Hd=F>=(StopBand(1)/SamplingRate) & F>=(StopBand(2)/SamplingRate) ;
W= (F<=(TranBand(1)/SamplingRate+eps)) * weighting(1) +
(F>=(TranBand(2)/SamplingRate)) * weighting(2);
TranBandN=TranBand/SamplingRate;
k=(filter_L-1)/2;

% Step1. Choose arbitrary k+2 extreme freq.
Fn=[(0:TranBandN(1)/round(k/2):TranBandN(1)) (TranBandN(2):(0.5-
TranBandN(2))/floor(k/2):0.5) ]; % stopband¨ú6ÂI, passband¨ú5ÂI

maxE=[0 1];
while(maxE(2)-maxE(1)>delta || maxE(2)-maxE(1)<0)
    % Step2. p.53
    FnL=round(Fn*SamplingRate)+1;
    M=[cos(2*pi*Fn'*(0:k)), (-1).^(0:k+1)'./W(FnL)'];
    S=M\Hd(FnL)';
    R=S(1:end-1)'*cos(2*pi*(0:k)'*F);
```

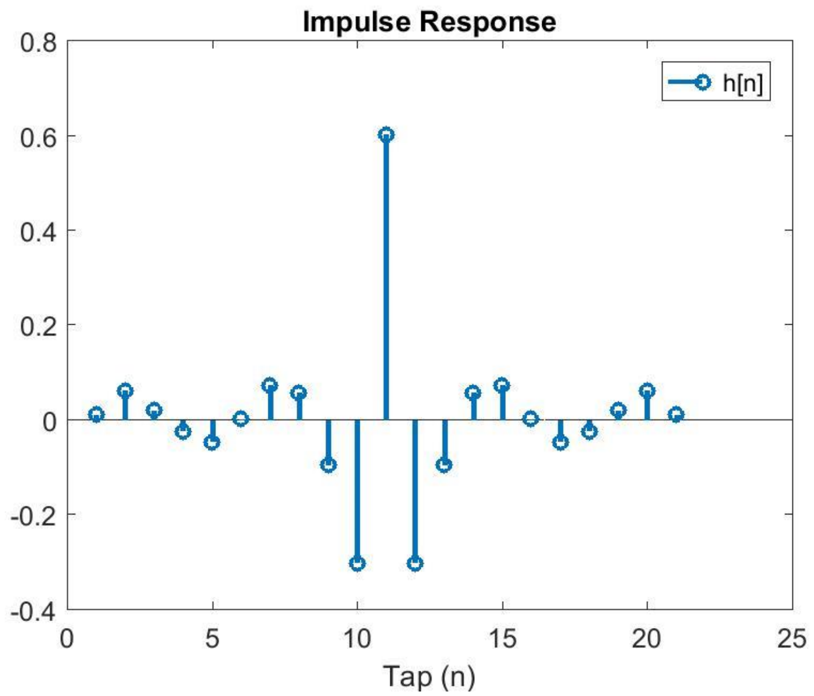
```
    % Step3. Compute err(F) for 0<F<0.5, excluding the transition band
    Err=(R-Hd).*W;

    % Step4. Find k+2 extremes
    ex=[];    % extremes not at boundaries
    exB=[]; % extremes at boundaries
    if (Err(1)*(Err(2)-Err(1))<0)
        exB=[exB [Err(1);0]];
    end

    if (Err(end)*(Err(end)-Err(end-1))>0)
        exB=[exB [Err(end);0.5]];
    end

    for n=(2:length(F)-1)
        if ((Err(n)-Err(n-1))*(Err(n)-Err(n+1))>0)
            if(F(n)==TranBandN(1) || F(n)==TranBandN(2))
                exB=horzcat(exB ,[Err(n); F(n)]); % extremes at transition band
            else
                ex=horzcat(ex, F(n)); % extremes not at boundaries
            end
        end
    end
    % choose exact k+2 extremes
    % choose boundary extreme points with larger error
    exNum=size(ex,2);
    while (exNum<k+2)
        [value,loca]=max(abs(exB(1,:)));
        ex=horzcat(ex, exB(2,loca));
        exB(1,loca)=0;
        exNum=exNum+1;
    end
    Fn = sort(ex);
    % Step5. Set max error
    maxE=horzcat(max(abs(Err)), maxE);
end
maxE=maxE(end-2:-1:1);
%Step6. even symmetry
h=[S(end-1:-1:2)/2;S(1);S(2:end-1)/2];
```

```
figure(1)
plot(F,Hd,'b','LineWidth',2);
hold on
plot(F,R,'r','LineWidth',2);
title('Frequency Response')
xlabel('Frequency (Hz)')
legend('H_d(F_n): Ideal Filter','R(F_n): Mini-Max','Location','southeast')
set(gca,'FontSize',12)
saveas(figure(1),'Frequency Response.jpg')

figure(2)
stem(h,'LineWidth',2)
xlabel('Tap (n)')
title('Impulse Response')
legend('h[n]')
set(gca,'FontSize',12)
saveas(figure(2),'Impulse Response.jpg')
```

(b) the frequency response



(c) the impulse response $h[n]$



(d) the maximal error for each iteration

Iteration	1	2	3	4	5	6	7
Max(|error|)	0.3945	1.6292	0.6266	0.0901	0.0657	0.0654	0.0654

(2) (a) What are the two most important applications of the Fourier transform?

(b) From the view point of implementation, what are the disadvantages of the discrete Fourier transform?

Ans:

(a) spectrum analysis & change convolution to multiplication

(b) 無法反應出頻率隨時間的變化，以及複數運算，運算量較大、儲存量也大，且無理數，無法實現。

(3) Suppose that $x[n] = y(0.0002n)$ and the length of $x[n]$ is 25000 and $X[m]$ is the FFT of $x[n]$. Find m_1 and m_2 such that $X[m_1]$ and $X[m_2]$ correspond to the 300Hz and -100Hz components of $y(t)$, respectively.

Ans:

$$N = 25000, f_s = \frac{1}{\Delta_t} = \frac{1}{0.0002} = 5000Hz, \frac{f_s}{N} = \frac{1}{5}$$

(a) 300Hz

$$Y\left(m_1 \frac{f_s}{N}\right) = Y(300) \rightarrow m_1 \times \frac{1}{5} = 300 \rightarrow m_1 = 1500$$

(b) -100Hz

$$\boldsymbol{Y\left((m_2-N)\frac{f_s}{N}\right)=Y(-100)\ \rightarrow\ (m_2-N)\times\frac{1}{5}=-100}$$

$$\boldsymbol{\rightarrow\ m_2=25000-500=24500}$$

(4) Why ① the transition band and ② the weighting function are important in Minimax FIR digital filter design?

Ans:

For transition band, according to different design methods:

- **Mini-Max :** 只要 **Extreme Point** 不選在 **Transition Band**，就能保證收斂。
- **MSE :** 當 **Transition Band** 加寬，**Mean Square Error** 越小。
- **Frequency Sampling:** 可利用 **Linear programing** 來選擇適當的 **Transition Band** 來減少誤差。
- **For all filter with fixed length N, supposed we desire:**

$$\boldsymbol{10\delta_1\delta_2=10^{-\frac{3N}{2}\Delta F}}$$

  - $\textbf{Passband Ripple} \leq\ \boldsymbol{\delta_1}$
  - $\textbf{Stopband Ripple} \leq\ \boldsymbol{\delta_2}$
  - $\textbf{Width of Transition Band} \leq\ \boldsymbol{\Delta F}$

It can be seen that we can sacrifice the frequency response of transition band to obtain higher accuracies of passband and stopband.

The wider transition band → the more accuracy.

For weighting function, it can also improve the accuracy in filter design accordance with to our demand. For example, if we set the weighting function: W(f) = 1 in the passband and 0 < W(f) < 1 in the stopband, the error in passband will smaller than stopband. Conversely, we set the stopband more important than passband: W(f) = 1 in the stopband and 0 < W(f) < 1 in the passband, the error in stopband will be smaller.

(5) Estimate the length of the digital filter if both the passband ripple and the stopband ripple are smaller than 0.02, the sampling interval Δ_t = 0.0001, and the transition band is from 2000Hz to 2200Hz.

Ans:

$$\mathrm{N}=\boldsymbol{\frac{2}{3}\frac{1}{\Delta F}log_{10}\left(\frac{1}{10\delta_1\delta_2}\right)}$$

- $\textbf{Passband Ripple} \leq\ \boldsymbol{\delta_1}$
- $\textbf{Stopband Ripple} \leq\ \boldsymbol{\delta_2}$

- $\mathbf{Width\ of\ Transition\ Band} \leq \Delta F$
- $\Delta F = (f_1 - f_2)/f_s = (f_1 - f_2)T$

$$\Delta F = (2200 - 2000) \times 0.0001 = 0.02$$

$$\mathrm{N} = \frac{2}{3}\frac{1}{0.02} log_{10}\left(\frac{1}{10 \times 0.02 \times 0.02}\right) \approx 79.93$$

推得 **N** 大約為 **80** 點

(6) Make a comparison among the methods of MSE, Minimax, and frequency sampling for FIR filter design and show their advantages and disadvantages.

Ans:

	MSE	Minimax	Frequency sampling
Design Method	Inner product, matrix inverse	Recursive (complicated)	Inverse FFT (easiest)
Advantages	Smallest average error	Smallest maximum error	Easiest to design
Disadvantages	May have large maximum error	Must be symmetric function, and the most complex to design	Hard to add weighting, with aliasing and Gilb's phenomenon, and not optimization