

Parallel Patterns: Convolution

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Background (1/6)

- Mathematically, convolution is an array operation where each output data element is a weighted sum of a collection of neighboring input elements.
- The weights used in the weighted sum calculation are defined by an input mask array, commonly referred to as the **convolution kernel**.



Background (2/6)

N	N[0]	N[1]	N[2]	N[3]	N[4]	N[5]	N[6]
	1	2	3	4	5	6	7

P	P[0]	P[1]	P[2]	P[3]	P[4]	P[5]	P[6]
			57				

M	M[0]	M[1]	M[2]	M[3]	M[4]
	3	4	5	4	3

3	8	15	16	15
---	---	----	----	----

$$\begin{aligned} P[2] &= N[0]*M[0] + N[1]*M[1] + N[2]*M[2] + N[3]*M[3] + N[4]*M[4] \\ &= 1*3 + 2*4 + 3*5 + 4*4 + 5*3 \\ &= 57 \end{aligned}$$



Background (3/6)

N	N[0]	N[1]	N[2]	N[3]	N[4]	N[5]	N[6]
	1	2	3	4	5	6	7

P	P[0]	P[1]	P[2]	P[3]	P[4]	P[5]	P[6]
				76			

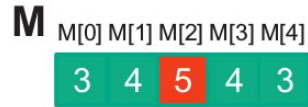
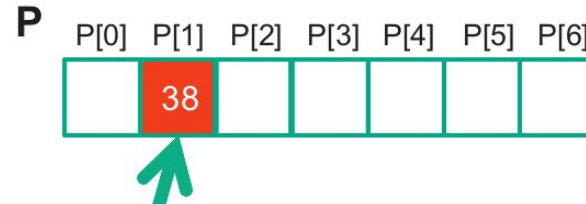
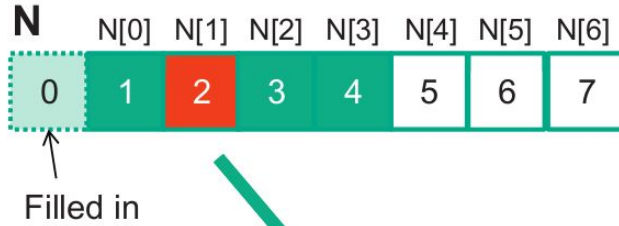
M	M[0]	M[1]	M[2]	M[3]	M[4]
	3	4	5	4	3

6	12	20	20	18
---	----	----	----	----

$$\begin{aligned} P[3] &= N[1]*M[0] + N[2]*M[1] + N[3]*M[2] + N[4]*M[3] + N[5]*M[4] \\ &= 2*3 + 3*4 + 4*5 + 5*4 + 6*3 \\ &= 76 \end{aligned}$$



Background (4/6)



$$\begin{aligned} P[1] &= 0 * M[0] + N[0]*M[1] + N[1]*M[2] + N[2]*M[3] + N[3]*M[4] \\ &= 0 * 3 + 1*4 + 2*5 + 3*4 + 4*3 \\ &= 38 \end{aligned}$$



Background (5/6)

N

1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	5	6
5	6	7	8	5	6	7
6	7	8	9	0	1	2
7	8	9	0	1	2	3

P

		321				

M

1	2	3	2	1
2	3	4	3	2
3	4	5	4	3
2	3	4	3	2
1	2	3	2	1

1	4	9	8	5
4	9	16	15	12
9	16	25	24	21
8	15	24	21	16
5	12	21	16	5



Background (6/6)

$$\begin{aligned} P_{2,2} &= N_{0,0} * M_{0,0} + N_{0,1} * M_{0,1} + N_{0,2} * M_{0,2} + N_{0,3} * M_{0,3} + N_{0,4} * M_{0,4} \\ &\quad + N_{1,0} * M_{1,0} + N_{1,1} * M_{1,1} + N_{1,2} * M_{1,2} + N_{1,3} * M_{1,3} + N_{1,4} * M_{1,4} \\ &\quad + N_{2,0} * M_{2,0} + N_{2,1} * M_{2,1} + N_{2,2} * M_{2,2} + N_{2,3} * M_{2,3} + N_{2,4} * M_{2,4} \\ &\quad + N_{3,0} * M_{3,0} + N_{3,1} * M_{3,1} + N_{3,2} * M_{3,2} + N_{3,3} * M_{3,3} + N_{3,4} * M_{3,4} \\ &\quad + N_{4,0} * M_{4,0} + N_{4,1} * M_{4,1} + N_{4,2} * M_{4,2} + N_{4,3} * M_{4,3} + N_{4,4} * M_{4,4} \\ &= 1*1 + 2*2 + 3*3 + 4*2 + 5*1 \\ &\quad + 2*2 + 3*3 + 4*4 + 5*3 + 6*2 \\ &\quad + 3*3 + 4*4 + 5*5 + 6*4 + 7*3 \\ &\quad + 4*2 + 5*3 + 6*4 + 7*3 + 8*2 \\ &\quad + 5*1 + 6*2 + 7*3 + 8*2 + 5*1 \\ &= 1 + 4 + 9 + 8 + 5 \\ &\quad + 4 + 9 + 16 + 15 + 12 \\ &\quad + 9 + 16 + 25 + 24 + 21 \\ &\quad + 8 + 15 + 24 + 21 + 16 \\ &\quad + 5 + 12 + 21 + 16 + 5 \\ &= 321 \end{aligned}$$



1D Parallel Convolution-Basic Algorithm (1/6)

```
__global__ void convolution_1D_basic_kernel(float *N, float  
    *M, float *P,  
    int Mask_Width, int Width) {  
    // kernel body  
}
```



1D Parallel Convolution-Basic Algorithm (2/6)

```
__global__ void convolution_1D_basic_kernel(float *N, float  
    *M, float *P,  
    int Mask_Width, int Width) {  
    // kernel body  
}  
  
int i = blockIdx.x*blockDim.x + threadIdx.x;
```

**Output Element
Index**



1D Parallel Convolution-Basic Algorithm

(3/6)

```
__global__ void convolution_1D_basic_kernel(float *N, float  
    *M, float *P,  
int Mask_Width, int Width) {  
    // kernel body  
}
```

```
int i = blockIdx.x*blockDim.x + threadIdx.x;
```

```
Mask_Width = 2*n + 1
```

**Mask_Width odd
number and the
convolution is
symmetric**

**Output Element
Index**



1D Parallel Convolution-Basic Algorithm (4/6)

```
__global__ void convolution_1D_basic_kernel(float *N, float *M, float *P,  
int Mask_Width, int Width) {  
  
    int i = blockIdx.x*blockDim.x + threadIdx.x;  
  
    float Pvalue = 0;  
    int N_start_point = i - (Mask_Width/2);  
    for (int j = 0; j < Mask_Width; j++) {  
        if (N_start_point + j >= 0 && N_start_point + j < Width) {  
            Pvalue += N[N_start_point + j]*M[j];  
        }  
    }  
    P[i] = Pvalue;  
}
```



1D Parallel Convolution-Basic Algorithm (5/6)

```
__global__ void convolution_1D_basic_kernel(float *N, float *M, float *P,
int Mask_Width, int Width) {

int i = blockIdx.x*blockDim.x + threadIdx.x;

float Pvalue = 0;
int N_start_point = i - (Mask_Width/2);
for (int j = 0; j < Mask_Width; j++) {
    if (N_start_point + j >= 0 && N_start_point + j < Width) {
        Pvalue += N[N_start_point + j]*M[j];
    }
}
P[i] = Pvalue;
}
```

**Memory
Bandwidth
??????**

Ratio ??????



1D Parallel Convolution-Basic Algorithm (6/6)

```
__global__ void convolution_1D_basic_kernel(float *N, float *M, float *P,
int Mask_Width, int Width) {

int i = blockIdx.x*blockDim.x + threadIdx.x;

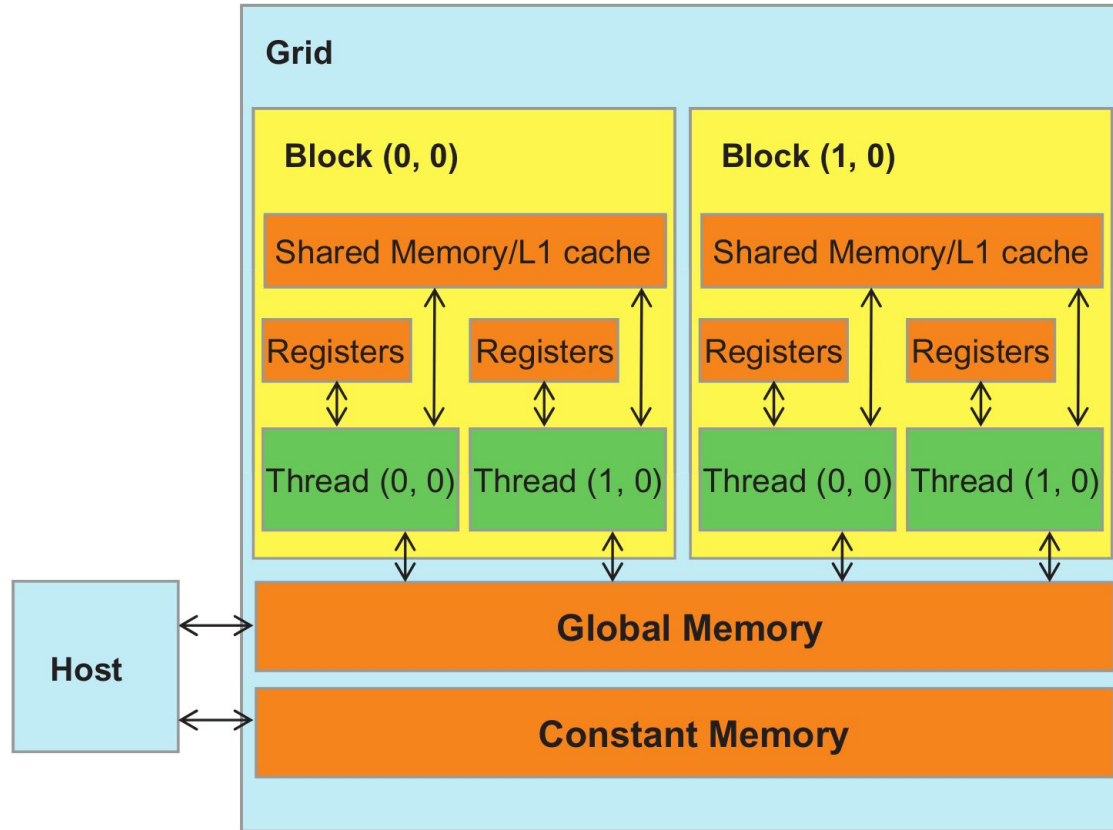
float Pvalue = 0;
int N_start_point = i - (Mask_Width/2);
for (int j = 0; j < Mask_Width; j++) {
    if (N_start_point + j >= 0 && N_start_point + j < Width) {
        Pvalue += N[N_start_point + j]*M[j];
    }
}
P[i] = Pvalue;
}
```

**Memory
Bandwidth
????? Bad**

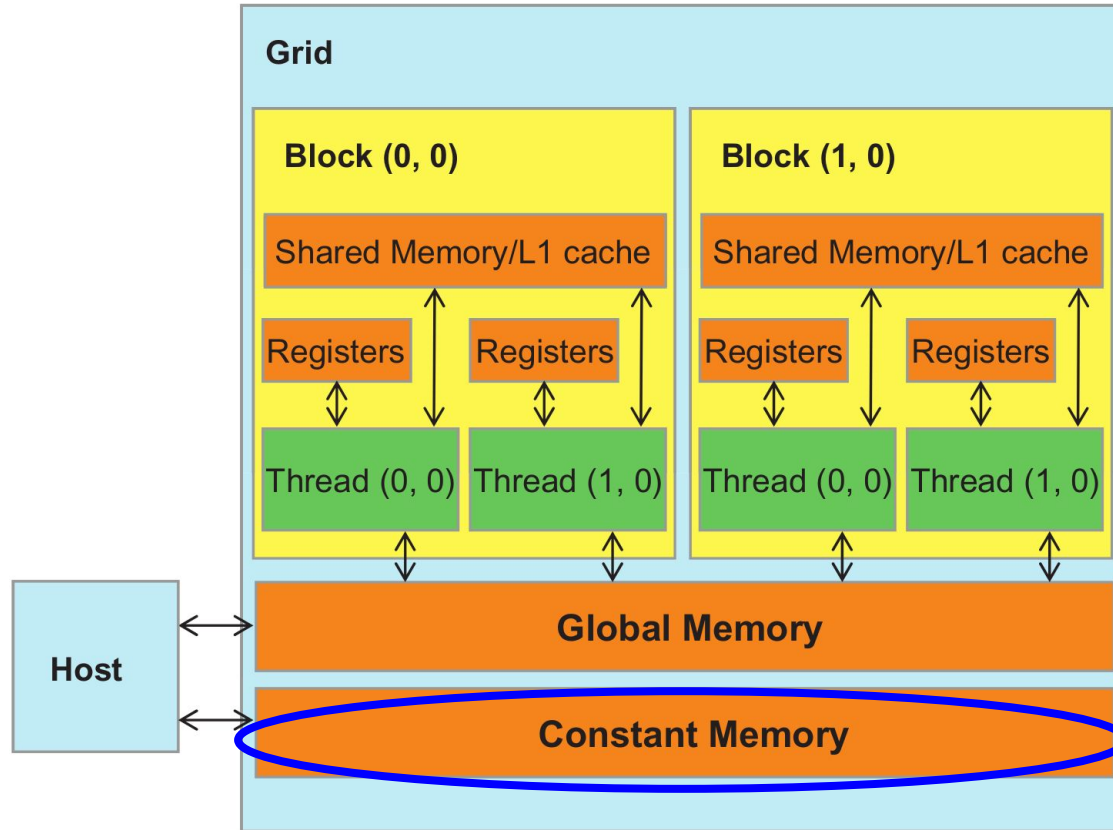
**Ratio ??????
1.0**



Constant Memory and Caching (1/11)



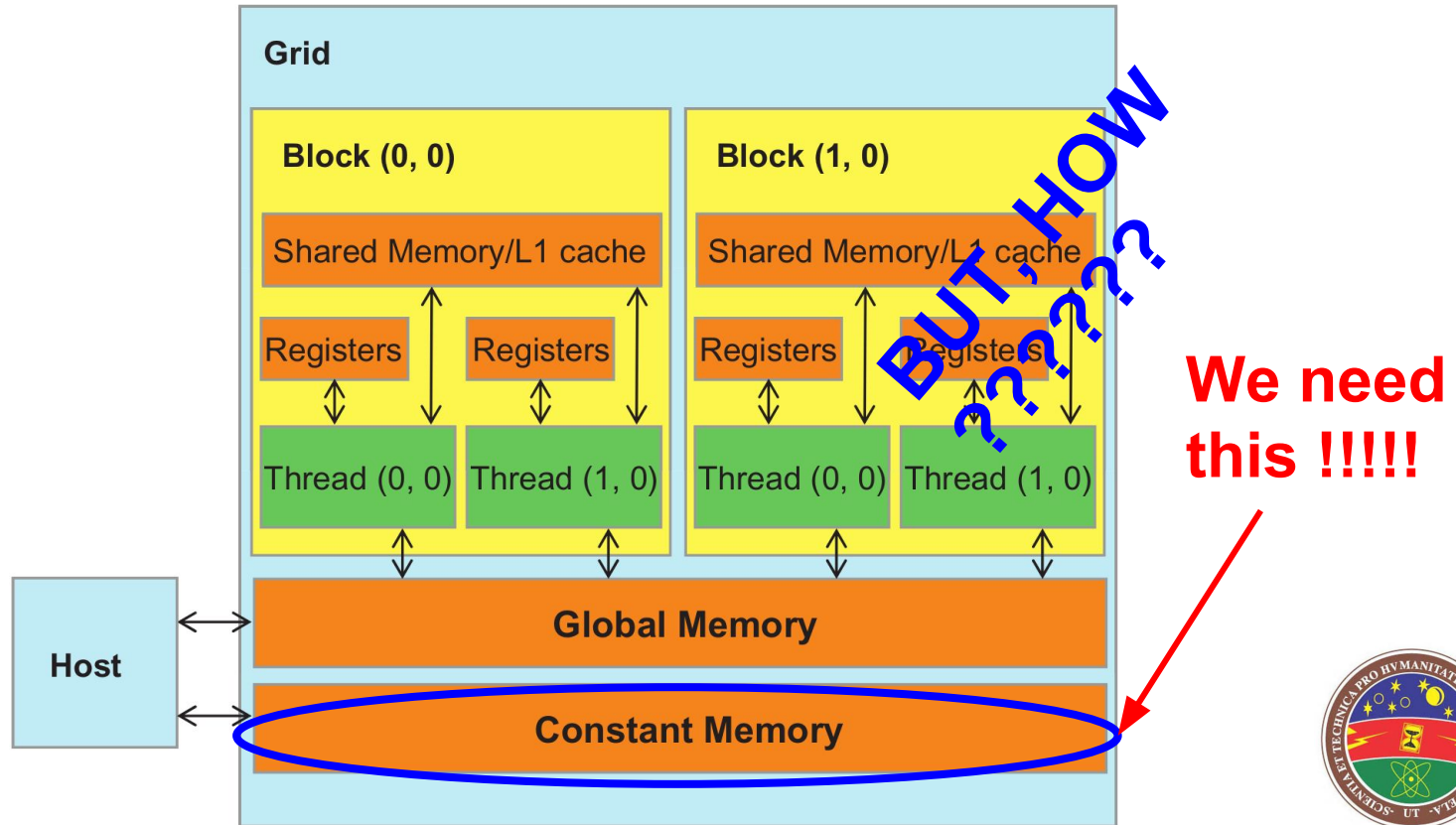
Constant Memory and Caching (2/11)



We need this !!!!!



Constant Memory and Caching (3/11)



Constant Memory and Caching (4/11)

```
#define MAX_MASK_WIDTH 10  
__constant__ float M[MAX_MASK_WIDTH];
```

Host Code

Constant Memory and Caching (5/11)

```
#define MAX_MASK_WIDTH 10  
__constant__ float M[MAX_MASK_WIDTH];
```

Host Code

**And ... The DATA
???? In device ????**



Constant Memory and Caching (6/11)

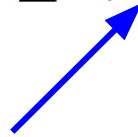
```
#define MAX_MASK_WIDTH 10
```

```
__constant__ float M[MAX_MASK_WIDTH];
```

Host Code

```
cudaMemcpyToSymbol(M, h_M, Mask_Width*sizeof(float));
```

**Special Memory Copy
Function ...**



Constant Memory and Caching (7/11)

```
#define MAX_MASK_WIDTH 10
```

```
__constant__ float M[MAX_MASK_WIDTH];
```

Host Code

```
cudaMemcpyToSymbol(M, h_M, Mask_Width*sizeof(float));
```

**Special Memory Copy
Function ...**

cudaMemcpyToSymbol(dest, src, size)



Constant Memory and Caching (8/11)

```
__global__ void convolution_1D_basics_kernel(float *N, float *P, int Mask_Width,
int Width) {

    int i = blockIdx.x*blockDim.x + threadIdx.x;

    float Pvalue = 0;
    int N_start_point = i - (Mask_Width/2);
    for (int j = 0; j < Mask_Width; j++) {
        if (N_start_point + j >= 0 && N_start_point + j < Width) {
            Pvalue += N[N_start_point + j]*M[j];
        }
    }
    P[i] = Pvalue;
}
```

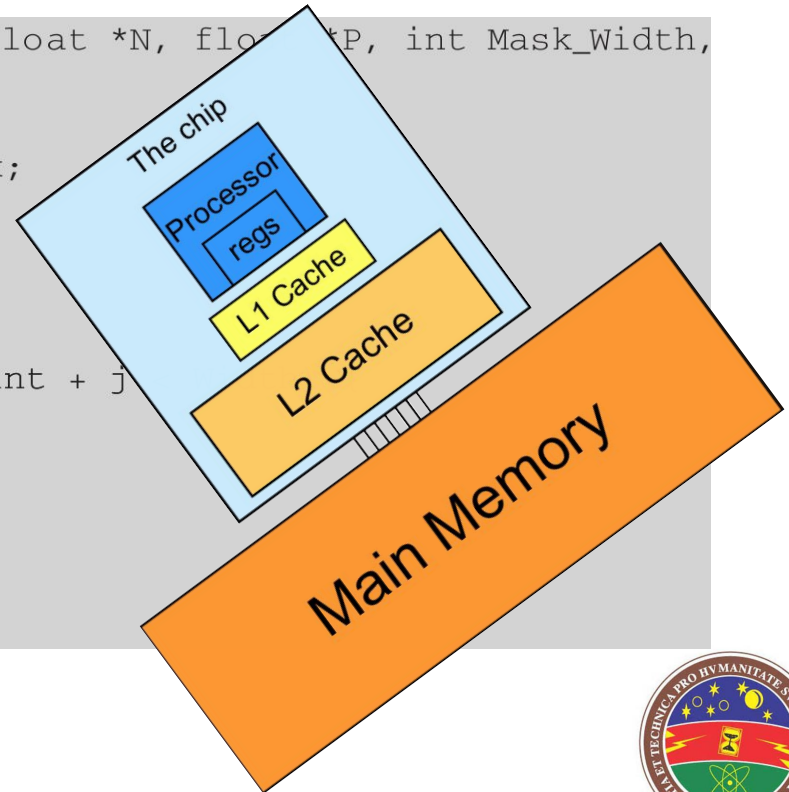


Constant Memory and Caching (9/11)

```
__global__ void convolution_1D_basics_kernel(float *N, float *P, int Mask_Width,
int Width) {

    int i = blockIdx.x*blockDim.x + threadIdx.x;

    float Pvalue = 0;
    int N_start_point = i - (Mask_Width/2);
    for (int j = 0; j < Mask_Width; j++) {
        if (N_start_point + j >= 0 && N_start_point + j < Width) {
            Pvalue += N[N_start_point + j]*M[j];
        }
    }
    P[i] = Pvalue;
}
```



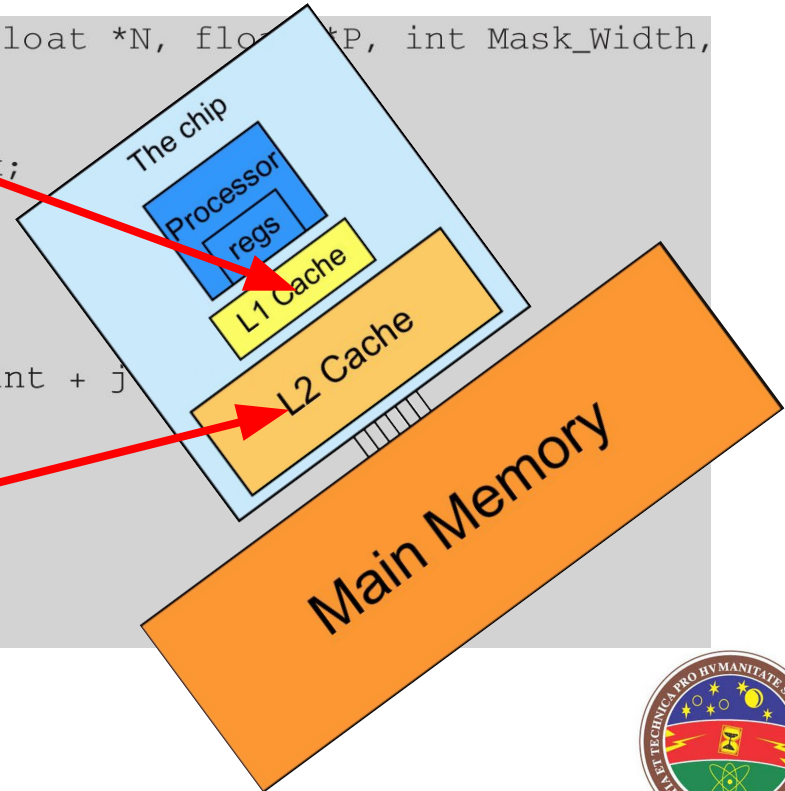
Constant Memory and Caching (10/11)

```
__global__ void convolution_1D_basic_kernel(float *N, float *P, int Mask_Width,
int Width) {

    int i = blockIdx.x*blockDim.x + threadIdx.x;

    float Pvalue = 0;
    int N_start_point = -(Mask_Width/2);
    for (int j = 0; j < Mask_Width; j++) {
        if (N_start_point + j >= 0 && N_start_point + j < Width)
            Pvalue += N[N_start_point + j]*M[j];
    }
    P[i] = Pvalue;
}
```

The memory is
marked to be
used from L1 or
L2 Cache



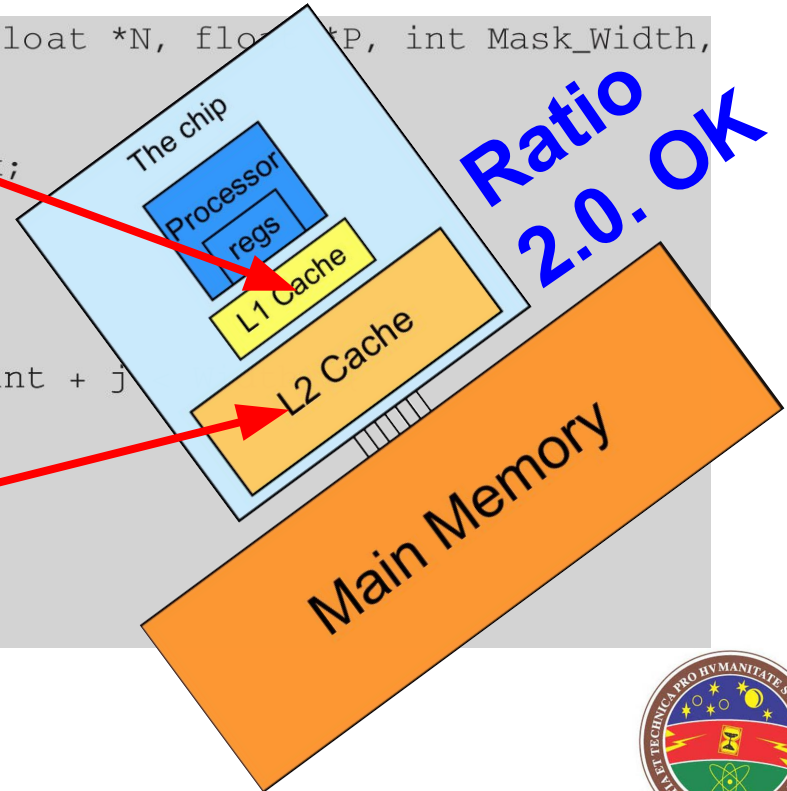
Constant Memory and Caching (11/11)

```
__global__ void convolution_1D_basic_kernel(float *N, float *P, int Mask_Width,
int Width) {

    int i = blockIdx.x*blockDim.x + threadIdx.x;

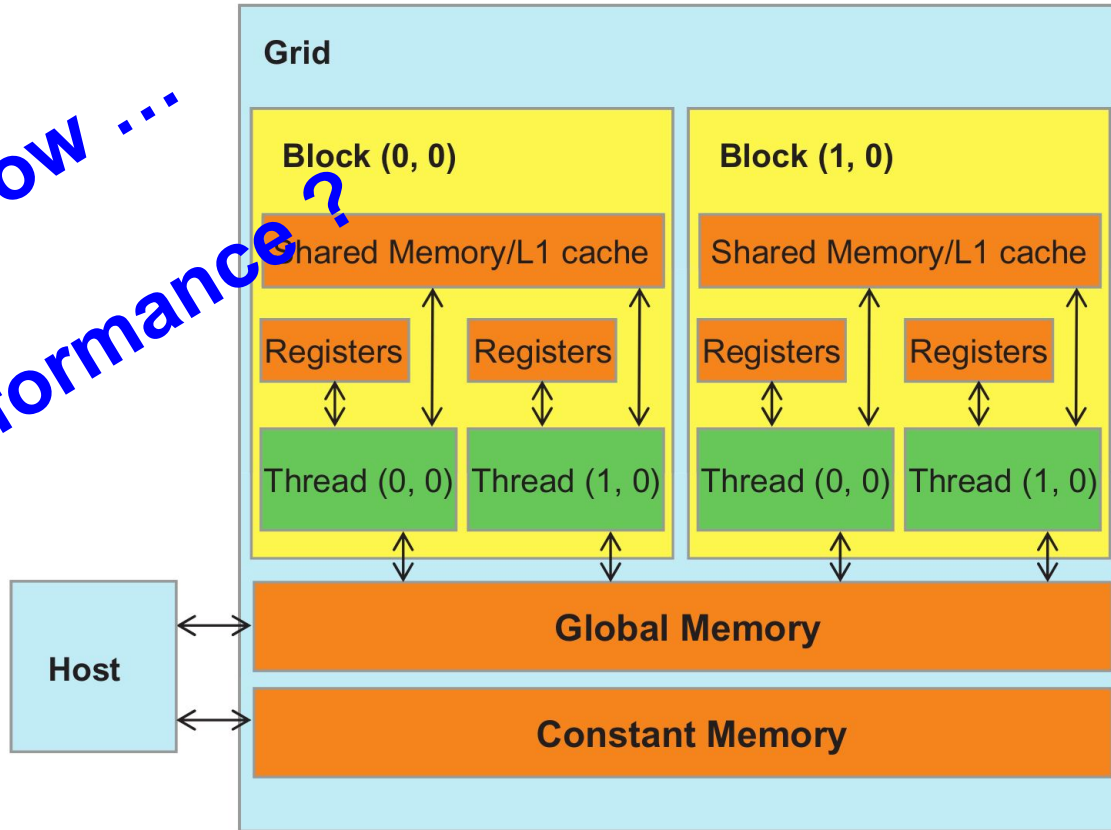
    float Pvalue = 0;
    int N_start_point = i - Mask_Width/2;
    for (int j = 0; j < Mask_Width; j++) {
        if (N_start_point + j >= 0 && N_start_point + j < Width)
            Pvalue += N[N_start_point + j]*M[j];
    }
    P[i] = Pvalue;
}
```

Aggressively cache
the constant memory
variables during
kernel execution.



Tiled 1D Convolution With Halo Elements(1/25)

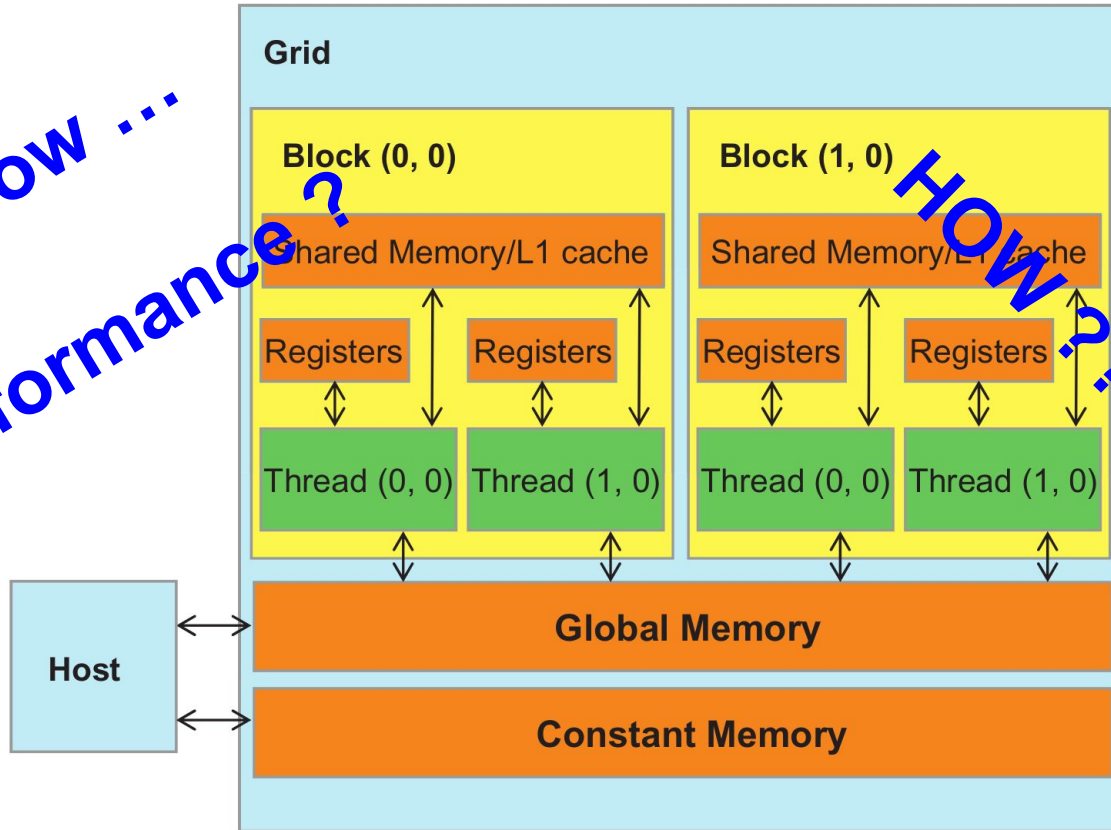
And, now ...
More performance ?



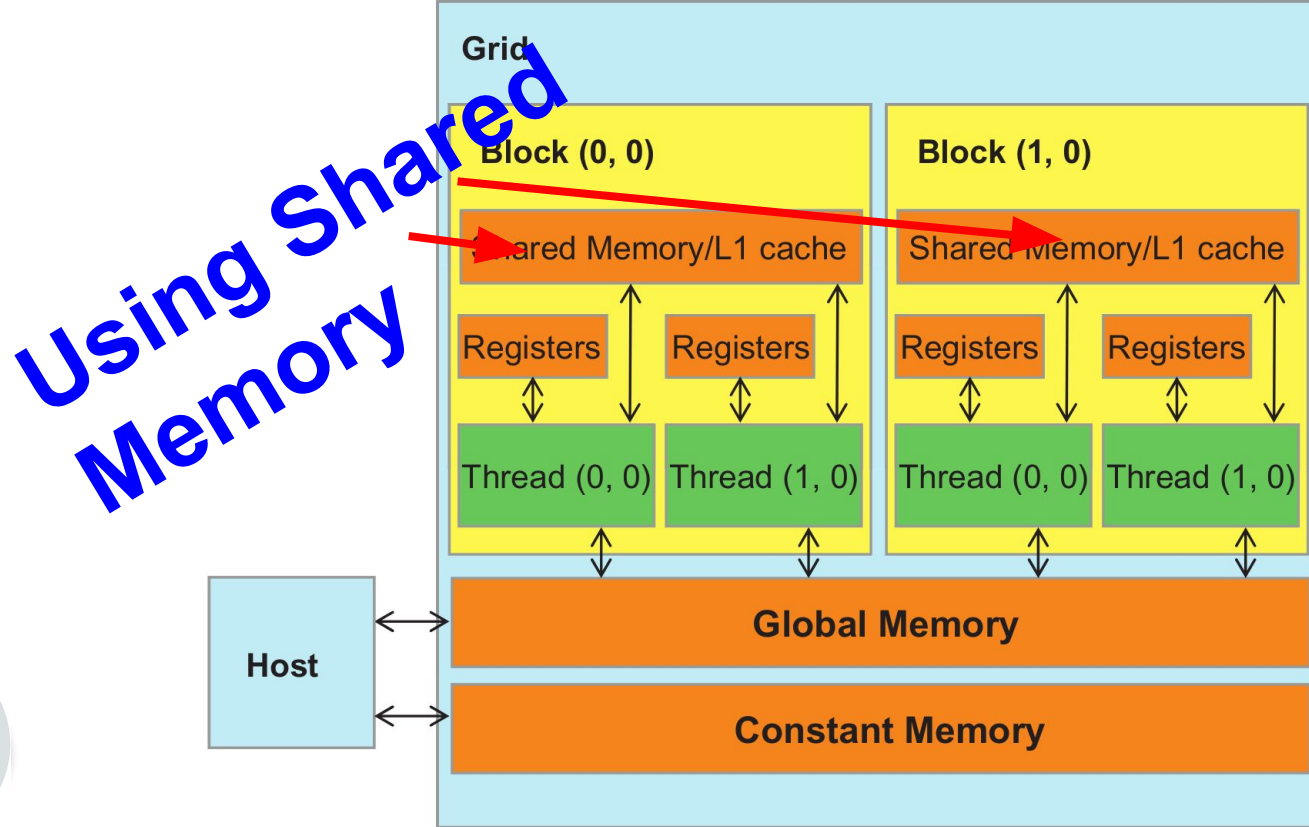
Tiled 1D Convolution With Halo Elements(2/25)

And, now ...
More performance?

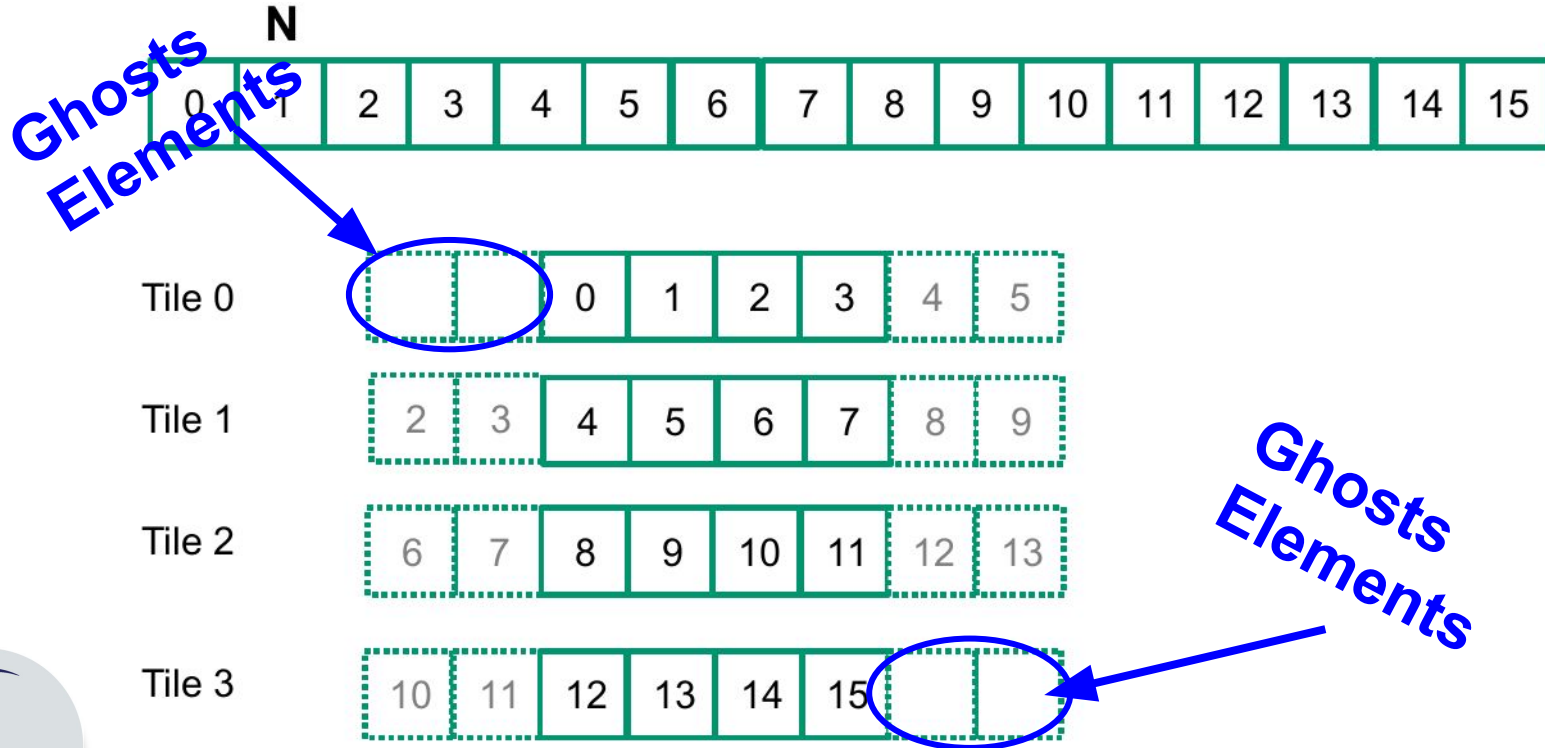
How???



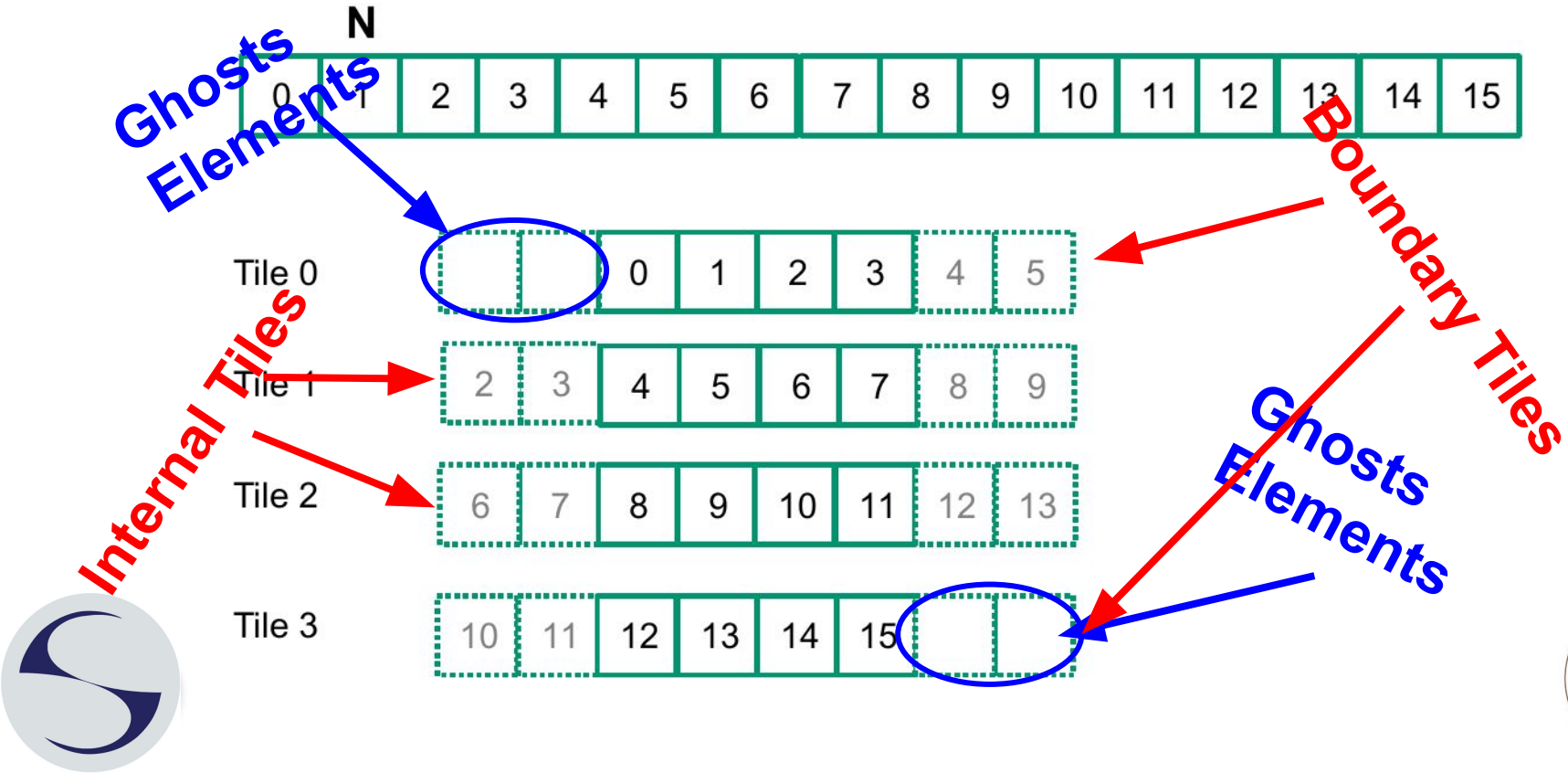
Tiled 1D Convolution With Halo Elements(3/25)



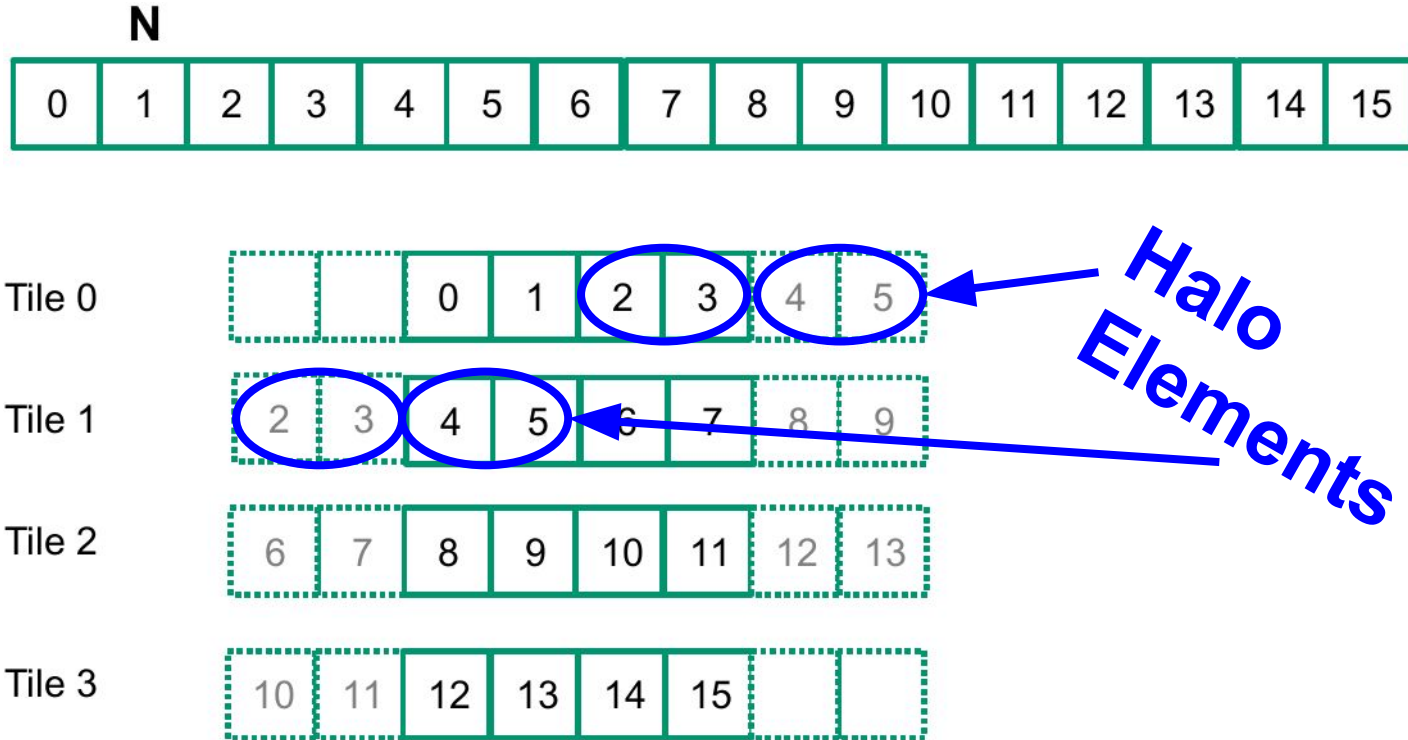
Tiled 1D Convolution With Halo Elements(4/25)



Tiled 1D Convolution With Halo Elements(5/25)



Tiled 1D Convolution With Halo Elements(6/25)



Tiled 1D Convolution With Halo Elements(7/25)

```
__shared__ float N_ds[FILE_SIZE + MAX_MASK_WIDTH - 1];
```

**In the last
example is 4**



Tiled 1D Convolution With Halo Elements(8/25)

```
__shared__ float N_ds[FILE_SIZE + MAX_MASK_WIDTH - 1];
```

En el ejemplo
anterior es igual
a 4

Y este es igual a
5



Tiled 1D Convolution With Halo Elements(9/25)

```
__shared__ float N_ds[FILE_SIZE + MAX_MASK_WIDTH - 1];
```

En el ejemplo anterior es igual a 4

Y este es igual a 5

$n = \text{Mask_Width} / 2$

Mask_Width es impar. Para el ejemplo $n = 2$



Tiled 1D Convolution With Halo Elements(10/25)

```
int halo_index_left = (blockIdx.x - 1)*blockDim.x + threadIdx.x;  
if (threadIdx.x >= blockDim.x - n) {  
    N_ds[threadIdx.x - (blockDim.x - n)] =  
        (halo_index_left < 0) ? 0 : N[halo_index_left];  
}
```

**We map the index
using previous
block (tile)**



Tiled 1D Convolution With Halo Elements(11/25)

```
int halo_index_left = (blockIdx.x - 1) * blockDim.x + threadIdx.x;  
if (threadIdx.x >= blockDim.x - n) {  
    N_ds[threadIdx.x - (blockDim.x - n)] =  
        (halo_index_left < 0) ? 0 : N[halo_index_left];  
}
```

**We map the index
using previous
block (tile)**

**This to load the left
halo elements into
shared memory**



Tiled 1D Convolution With Halo Elements(12/25)

```
int halo_index_left = (blockIdx.x - 1)*blockDim.x + threadIdx.x;  
if (threadIdx.x >= blockDim.x - n) {  
    N_ds[threadIdx.x - (blockDim.x - n)] =  
        (halo_index_left < 0) ? 0 : N[halo_index_left];  
}
```

**We pick up the last
n threads**



Tiled 1D Convolution With Halo Elements(13/25)

```
int halo_index_left = (blockIdx.x - 1)*blockDim.x + threadIdx.x;  
if (threadIdx.x >= blockDim.x - n) {  
    N_ds[threadIdx.x - (blockDim.x - n)] =  
        (halo_index_left < 0) ? 0 : N[halo_index_left];  
}
```

**We check for
ghosts elements**

**We pick up the last
n threads**



Tiled 1D Convolution With Halo Elements(14/25)

**We need to load the
center elements**



Tiled 1D Convolution With Halo Elements(15/25)

**We need to load the
center elements**

`N_ds[n + threadIdx.x] = N[blockIdx.x*blockDim.x + threadIdx.x];`



Tiled 1D Convolution With Halo Elements(16/25)

**We need to load the
center elements**

```
N_ds[n + threadIdx.x] = N[blockIdx.x*blockDim.x + threadIdx.x];
```

**The first n elements
were loaded before**




Tiled 1D Convolution With Halo Elements(17/25)

We need to load right
halo elements



Tiled 1D Convolution With Halo Elements(18/25)

We need to load right halo elements




```
int halo_index_right=(blockIdx.x+1)*blockDim.x+threadIdx.x;
if (threadIdx.x < n) {
    N_ds[n+blockDim.x+threadIdx.x] =
        (halo_index_right >= Width) ? 0 : N[halo_index_right];
}
```



Tiled 1D Convolution With Halo Elements(19/25)

We need to load right halo elements



```
int halo_index_right=(blockIdx.x+1)*blockDim.x+threadIdx.x;
if (threadIdx.x < n) {
    N_ds[n+blockDim.x+threadIdx.x] =
        (halo_index_right >= Width) ? 0 : N[halo_index_right];
}
```

Similar to load left halo elements



Tiled 1D Convolution With Halo Elements(20/25)

We need to load right halo elements

```
int halo_index_right=(blockIdx.x+1)*blockDim.x-threadIdx.x;
if (threadIdx.x < n) {
    N_ds[n + blockDim.x + threadIdx.x] =
        (halo_index_right >= Width) ? 0 : N[halo_index_right];
}
```

Check yourself ... :)

Similar to load left halo elements



Tiled 1D Convolution With Halo Elements(21/25)

Now we have all data
into shared memory



Tiled 1D Convolution With Halo Elements(22/25)

*Now we have all data
into shared memory*

*We can make the
calculations*

```
float Pvalue = 0;
for(int j = 0; j < Mask_Width; j++) {
    Pvalue += N_ds[threadIdx.x + j]*M[j];
}
P[i] = Pvalue;
```



Tiled 1D Convolution With Halo Elements(23/25)

Now we have all data
into shared memory

We can make the
calculations

Don't forget syncthreads

```
float Pvalue = 0;
for(int j = 0; j < Mask_Width; j++) {
    Pvalue += N_ds[threadIdx.x + j]*M[j];
}
P[i] = Pvalue;
```



Tiled 1D Convolution With Halo Elements(24/25)

```
__global__ void convolution_1D_basic_kernel(float *N, float *P, int M,
int Width) {

    int i = blockIdx.x*blockDim.x + threadIdx.x;
    __shared__ float  N_ds[TILE_SIZE + MAX_MASK_WIDTH -1];

    int n = Mask_Width/2;

    int halo_index_left = (blockIdx.x - 1)*blockDim.x + threadIdx.x;
    if (threadIdx.x >= blockDim.x - n) {
        N_ds[threadIdx.x - (blockDim.x - n)] =
            (halo_index_left < 0) ? 0 : N[halo_index_left];
    }

    N_ds[n + threadIdx.x] = N[blockIdx.x*blockDim.x + threadIdx.x];

    int halo_index_right = (blockIdx.x + 1)*blockDim.x + threadIdx.x;
    if (threadIdx.x < n) {
        N_ds[n + blockDim.x + threadIdx.x] =
            (halo_index_right >= Width) ? 0 : N[halo_index_right];
    }

    __syncthreads();

    float Pvalue = 0;
    for(int j = 0; j < Mask_Width; j++) {
        Pvalue += N_ds[threadIdx.x + j]*M[j];
    }
    P[i] = Pvalue;
}
```



TODO (25/25)

- Everything you need to have fun is on the Homework.



THANKS

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