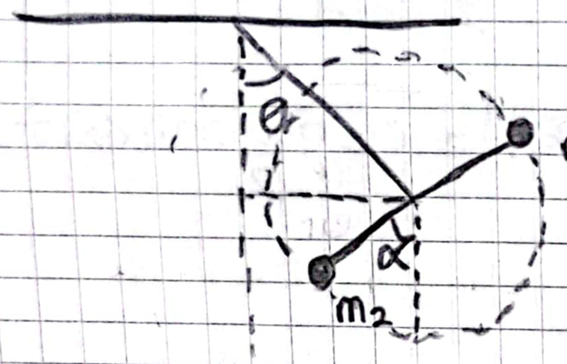
1)



Para m_1:

$(x-h) = \sqrt{r^2 - (y+k)^2}$

$$\frac{a}{2}\operatorname{sen}(\alpha_1) - l\operatorname{sen}\theta = \underbrace{\left(\frac{a^2}{4} - \frac{a^2}{4}\cos^2(\alpha_1) + al\cos(\alpha_1)\cos(\theta) - l^2\cos(\theta)\right)^{1/2}}_{\beta}$$

Para V_1

$V_1^2 = V_x^2 + V_y^2$ ∴ $x_{m_1} = -\frac{a}{2}\operatorname{sen}(\alpha_1) + l\operatorname{sen}(\theta)$

$y_{m_1} = \frac{a}{2}\cos(\alpha_1) + l\cos(\theta)$

$V_1^2 = \frac{a}{4}\cos^2(\alpha_1)\dot{\alpha}^2 + l^2\cos^2(\theta)\dot{\theta}^2 + \frac{a^2}{4}\operatorname{sen}^2(\alpha_1)\dot{\alpha}^2 + l^2\operatorname{sen}(\theta)\dot{\theta}^2$

$V_1^2 = \frac{a^2}{4}\dot{\alpha}_1^2 + l^2\dot{\theta}^2$

Para V_2:

$V_2^2 = V_x^2 + V_y^2$; $x_{m_2} = \frac{a}{2}\operatorname{sen}(\alpha_2) + l\operatorname{sen}(\theta)$

$y_{m_2} = \frac{a}{2}\cos(\alpha_2) + l\cos(\theta)$.

$V_2^2 = \frac{a^2}{4}\dot{\alpha}^2 + l^2\dot{\theta}^2$

$L = T - V \rightarrow T = T_1 + T_2$ y $V = V_1 + V_2$

$$T = \frac{1}{2} m_1 \left(\frac{a^2}{4} \dot{\alpha}_1^2 + l^2 \dot{\theta}^2 \right) + \frac{1}{2} m_2 \left(\frac{a^2}{4} \dot{\alpha}^2 + l^2 \dot{\theta}^2 \right)$$

$$U = m_1 g \left(\frac{a}{2} \cos(\alpha_1) + l\cos(\theta) \right) + m_2 g \left(\frac{a}{2} \cos(\alpha_2) + l\cos(\theta) \right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i} \right) - \frac{\partial L}{\partial \theta_i} = 0$$

$$L = \frac{a^2}{8} m_1 \dot{\alpha}^2 + \frac{1}{2} m_1 l^2 \dot{\theta}^2 + \frac{a^2}{8} m_2 \dot{\alpha}^2 + \frac{1}{2} m_2 l^2 \dot{\theta}^2 - \frac{a}{2} m_1 g \cos(\alpha_1)$$
$$- l m g \cos(\theta) - \frac{a}{2} m_2 g \cos(\alpha_2) - l m_2 g \cos(\theta)$$

Para θ:

$$\frac{\partial L}{\partial \theta} = l m_1 g \,\mathrm{sen}(\theta) + l m_2 g \,\mathrm{sen}\,\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = m_1 l^2 \dot{\theta} + m_2 l^2 \dot{\theta} \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}} \right) = m_1 l^2 \ddot{\theta} + m_1 l^2 \ddot{\theta}$$

Para α_1:

$$\frac{\partial L}{\partial \alpha_1} = \frac{a}{2} m_1 g \,\mathrm{sen}(\alpha_1)$$

$$\frac{\partial L}{\partial \dot{\alpha}_1} = \frac{a^2}{4} m_1 \dot{\alpha}_1 \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}_1} \right) = \frac{a^2}{4} m_1 \ddot{\alpha}_1$$

Para α_2:

$$\frac{\partial L}{\partial \alpha_2} = \frac{a}{2} m_2 g \,\mathrm{sen}(\alpha_2)$$

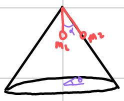
$$\frac{\partial L}{\partial \dot{\alpha}_2} = \frac{a^2}{4} m_2 \dot{\alpha}_2 \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}_2} \right) = \frac{a^2}{4} m_2 \ddot{\alpha}_2$$

Unimos: $\ddot{\theta} = g \,\mathrm{sen}\,\theta$

$\ddot{\alpha}_1 = 2g\,\mathrm{sen}(\alpha_1)/a$

$\ddot{\alpha}_2 = 2g\,\mathrm{sen}(\alpha_2)/a$

2)



$z_{m_1} = r\cos\alpha$
$x_{m_1} = r\,\text{sen}\,\theta\,\text{sen}\,\alpha$

$z_{m_2} = \ell - r\cos\alpha$
$y_{m_2} = -r\,\text{sen}\,\alpha\cos\theta$

$z_{m_1} = r\cos\alpha$
$x_{m_1} = r\,\text{sen}\,\theta\,\text{sen}\,\alpha + r\cos\theta\,\text{sen}\,\alpha\,\theta$

$z_{m_2} = -r\cos\alpha$
$y_{m_2} = r\,\text{sen}\,\alpha\,\text{sen}\,\theta\,\theta - r\,\text{sen}\,\alpha\cos\theta$

$$V^2 = V_x^2 + V_y^2 + V_z^2$$

$$V^2 = x^2 + y^2 + z^2$$

$V_{m_1}^2\ r^2 + r^2\text{sen}^2\alpha\,\theta^2 \qquad V_{m_2}^2 = r^2\cos^2\alpha$

$$T = \frac{1}{2}m_1 r^2 + \frac{1}{2}m_1 r^2\text{sen}^2\alpha\,\theta^2 + \frac{1}{2}m_2 r^2\cos^2\alpha$$

$$V = m_1 g r\cos\alpha + m_2 g\,\ell - r m_2 g\cos\alpha$$

Ahora construímos los lagrangianos para r y θ

$$\frac{d}{dt}\left(\frac{\partial L}{\partial r}\right) - \frac{\partial L}{\partial r} = 0 \rightarrow \frac{d}{dt}\left(m_1 r^2\,\text{sen}^2\alpha\,\theta\right) \rightarrow 2m_1 r\,\text{sen}^2\alpha\,\theta + m_1 r^2\,\text{sen}^2\alpha\,\theta = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \theta}\right) - \frac{\partial L}{\partial \theta} = 0 \rightarrow \frac{d}{dt}\left(m_1 r + m_2\cos^2\alpha\, r\right) - \left(m_1 r\,\text{sen}^2\alpha\,\theta^2 - m_1 g\cos\alpha - m_2 g\cos\alpha\right) = 0$$

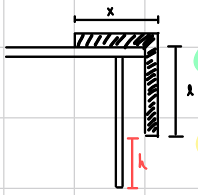
$$m_1 r + m_2\cos^2\alpha\, r - m_1 r\,\text{sen}^2\alpha\,\theta^2 + \cos\alpha\, g(m_1+m_2) = 0$$

Así, tendríamos las aceleraciones

$$\theta = \frac{2\theta}{r}$$

$$r = -g\cos\alpha(m_1+m_2) + m_1 r\,\text{sen}^2\alpha\,\frac{\theta^2}{m_1} + m_2\cos^2\alpha$$

4)



masa = M
Longitud = L

L = x + ℓ
h = altura inicial

$\lambda = \frac{M}{L}$

$y(0) = h$

$v(0) = 0$

$y(t) = ?$

Sabemos que $\Delta x = \Delta y$

$T = \frac{1}{2} m \left(\frac{dy}{dt}\right)^2$

$U = m g y$

$F = \frac{d}{dt}(m v)$

$\cancel{\lambda} y g = \frac{d}{dt}(\cancel{\lambda} y y)$

$y g = y^2 + y y$

$y = \frac{dy}{dt} = \frac{dy}{dy}\frac{dy}{dt} = y \frac{dy}{dy} = \frac{d}{dy}\left(\frac{1}{2} y^2\right)$

$g y = y^2 + \frac{1}{2} y \frac{dy^2}{dy}$

Haciendo estos reemplaces, tenemos una ecuación diferencial lineal

$\frac{dy^2}{dy} + \underbrace{\frac{2}{y}}_{Q(y)} y^2 = \underbrace{2g}_{P(y)}$, $M(y) = e^{\int \frac{2}{y} dy}$

$= y^2$

$\int \cancel{\frac{d}{dy}} (y^2 \ y^2) \cancel{dy} = \int 2 g y^2 \, dy$

$y^2 \ y^2 = \frac{2}{3} g \cdot y^3 + C$

Usamos las condiciones iniciales para hallar c

$t = 0$ $\qquad h^2 \ 0 = \frac{2}{3} g h^3 + c$

$c = -\frac{2}{3} g h^3$

Así, reemplazando c y despejando tenemos

$y = \sqrt{\frac{2g(x^3 - h^3)}{3y^2}}$

En este problema, la energía no se conserva