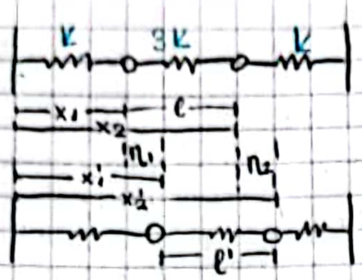
1)



$T = \frac{1}{2} m\dot{\eta}_1^2 + \frac{1}{2} m\dot{\eta}_2^2 = \frac{1}{2}\sum_{i,j} T_{ij}\dot{\eta}_i\dot{\eta}_j$

$T_{11} = m \quad T_{22} = m \; ; \; T_{12} = T_{21} = 0$

$V = \frac{1}{2}k\eta_1^2 + \frac{3}{2}k(\ell'-\ell)^2 + \frac{1}{2}k\eta_2^2 = \frac{1}{2}[k\eta_1^2 + 3k(\eta_2-\eta_1)^2 + k\eta_2^2]$

$V = \frac{1}{2}[4k\eta_1^2 + 4k\eta_2^2 - 6k\eta_1\eta_2]$

$V_{11} = 4k \; ; \; V_{22} = 4k \; ; \; V_{12} = -3k \; ; \; V_{21} = -3k$

$$\begin{vmatrix} V_{11} - \omega^2 T_{11} & V_{12} - \omega^2 T_{12} \\ V_{21} - \omega^2 T_{21} & V_{22} - \omega^2 T_{22} \end{vmatrix} = 0$$

$$\begin{vmatrix} 4k - \omega^2 m & -3k \\ -3k & 4k - \omega^2 m \end{vmatrix} = (4k - \omega^2 m)^2 - 9k^2 = 0$$

$7k^2 - 8k\omega^2 m + \omega^4 m^2 = 0$

$x^2 m^2 - 8kxm + 7k^2 = 0$

$\underbrace{\omega^2 = \frac{7k}{m} \; ; \; \frac{k}{m}}$

modos normales:

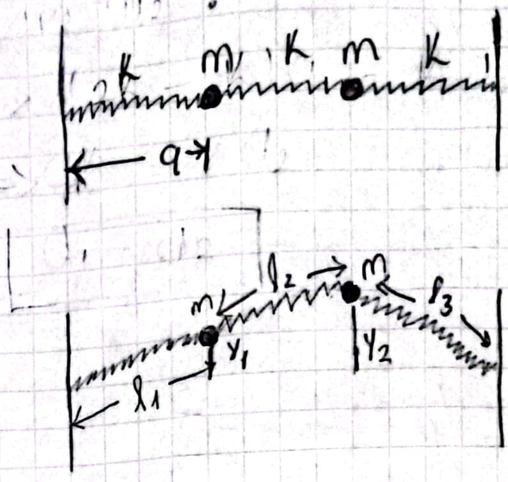
$\sum_j (V_{ij} - \omega^2 T_{ij}) a_j = 0$

$\cancel{k}\begin{pmatrix} 4 & -3 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 7k\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \rightarrow \begin{cases} 4a_1 - 3a_2 = 7a_1 \\ -3a_1 + 4a_2 = 7a_2 \\ a_2 = -a_1 \end{cases}$

$\underbrace{a_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}}$ en oposición de fase

$\underbrace{\begin{pmatrix} 4 & -3 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \rightarrow \begin{matrix} 4a_1 - 3a_2 = a_1 \\ -3a_1 + 4a_2 = a_2 \\ a_2 = a_1 \end{matrix}$

$\underbrace{a_2' = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$ en fase

② 

T_0: Tension de los resortes inicialmente

$$T = \frac{1}{2}m(\dot{y}_1 + \dot{y}_2)$$

$T_{11} = m \qquad T_{22} = m$

$T_{12} = T_{21} = 0$

$\ell_1 = \sqrt{a^2 + y_1^2}$

$\ell_2 = \sqrt{a^2 + (y_2 - y_1)^2}$

$\ell_3 = \sqrt{a^2 + y_2^2}$

Dado que $y_i \ll a$, podemos expandir y aproximar el 2do termino.

$$\ell_1 = a\left(1 + \left(\frac{y_1}{a}\right)^2\right)^{1/2} \simeq a\left(1 + \frac{1}{2}\left(\frac{y_1}{a}\right)^2\right) = a + \frac{y_1^2}{2a}$$

$$\ell_2 = a\left(1 + \frac{(y_2 - y_1)^2}{a^2}\right)^{1/2} \simeq a\left(1 + \frac{1}{2}\frac{(y_2 - y_1)^2}{a^2}\right) = a + \frac{(y_2 - y_1)^2}{2a}$$

$$\ell_3 = a\left(1 + \left(\frac{y_2}{a}\right)^2\right)^{1/2} \simeq a\left(1 + \frac{1}{2}\left(\frac{y_2}{a}\right)^2\right) = a + \frac{y_2^2}{2a}$$

Para hallar los desplazamientos de los resortes hacemos $\ell_i - a = x_i$

$$x_1 = \frac{y_1^2}{2a} \quad ; \quad x_2 = \frac{(y_2 - y_1)^2}{2a} \quad ; \quad x_3 = \frac{y_2^2}{2a}$$

Debido a que $y_i \ll a$ podemos suponer que los desplaz. sean los suficientes pequeños como para que la tension T_0 no cambie.

$$\Rightarrow V = \frac{1}{2} K X^2 \longrightarrow \frac{1}{2} T_0 X$$

$$V = \frac{1}{2a} T_0 \left(y_1^2 + y_2^2 + (y_2 - y_1)^2 \right) = \frac{1}{2a} T_0 (2y_1^2 + 2y_2^2 - 2y_1 y_2)$$

$$V_{11} = \frac{2T_0}{a} \; ; \; V_{22} = \frac{2T_0}{a} \; ; \; V_{12} = V_{21} = -\frac{T_0}{a}$$

$$\begin{vmatrix} \frac{2T_0}{a} - \omega^2 m & -\frac{T_0}{a} \\ -\frac{T_0}{a} & \frac{2T_0}{a} - \omega^2 m \end{vmatrix} = 0$$

$$\left(\frac{2T_0}{a} - \omega^2 m\right)^2 - \left(\frac{T_0}{a}\right)^2 = 0$$

$$\frac{2T_0}{a} - \omega^2 m = \pm \frac{T_0}{a}$$

$$\frac{2T_0}{a} \mp \frac{T_0}{a} = \omega^2 m$$

$$\frac{T_0}{am} = \omega^2 \quad \wedge \quad \frac{3T_0}{am} = \omega^2$$

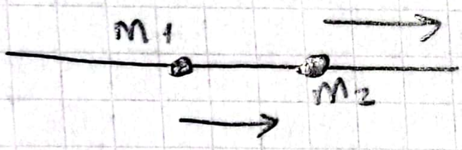
$$\omega_1 = \sqrt{\frac{T_0}{am}} \qquad \omega_2 = \sqrt{\frac{3T_0}{am}}$$

Los modos se hallan:

$$\sum_j (V_{ij} - \omega^2 T_{ij}) a_{ij} = 0$$

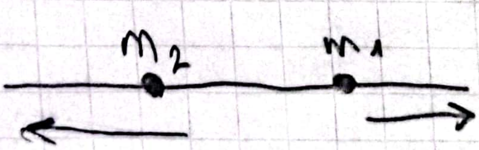
$$i=1 : \left(\frac{2T_0}{a} - \frac{T_0}{a}\right) a_1 + \frac{T_0}{a} a_2 = 0$$

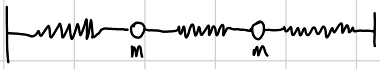
$$a_1 = a_2$$



$$i=2 : -\frac{T_0}{a} a_1 + \left(\frac{2T_0}{a} - \frac{3T_0}{a}\right) a_2 = 0$$

$$a_1 = -a_2$$





$$V = \frac{1}{2} k\left(n_1^2 + n_2^2 - 2k n_1 n_2\right) - \overbrace{\frac{2\epsilon q^2}{\ell^3}}^{\mu}\left(n_2 - n_1\right)$$

$$T = \frac{1}{2} m n_1^2 + \frac{1}{2} m n_2^2$$

$$\det\left| V - \omega^2 T \right| = 0$$

$$\det\begin{vmatrix} 2K + \mu - \omega^2 m & -K + \mu \\ -K + \mu & 2K + \mu - \omega^2 m \end{vmatrix} = 0$$

$$(2K + \mu - \omega^2 m)^2 = (K + \mu)^2$$

$$2K + \mu - \omega^2 m = \pm\; K + \mu$$

$$\omega^2 = \frac{2K + \mu \pm (K + \mu)}{m}$$

$$\omega_1 = \sqrt{\frac{K}{m}} \qquad \omega_2 = \sqrt{\frac{3K + 2\mu}{m}}$$

$$\begin{pmatrix} 2K + \mu & -K - \mu \\ -K - \mu & 2K + \mu \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \omega_1^2 m \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$(2K + \mu) a_1 + (-K - \mu) a_2 = \omega_1^2 a_1$$

$$(-K - \mu) a_1 + (2K + \mu) a_2 = \omega_1^2 a_2$$

→ Resolviendo, sería:

Primer modo de oscilación

$a_1 = a_2$ → Estan en fase

Segundo modo de oscilación

$a_2 = -a_1$ → Van en direcciones opuestas