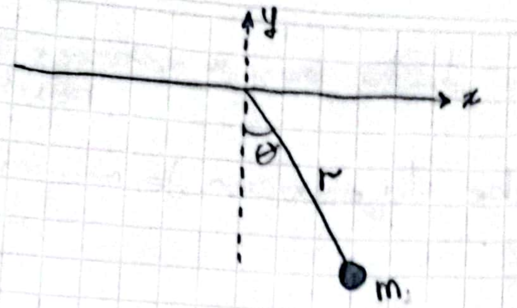
1)



donde r cambiara a tasa cte de $\dot{r} = a$

$$x = r\,\mathrm{Sen}\,\theta$$
$$y = -r\cos\theta$$

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$

$$\dot{x} = r\cos\theta\,\dot{\theta} + \dot{r}\,\mathrm{sen}\,\theta$$
$$\dot{y} = r\,\mathrm{sen}\,\theta\,\dot{\theta} - \dot{r}\cos\theta$$

$$V = mgh$$
$$V = -mg(r\cos\theta)$$

$$T = \frac{1}{2}m((r\cos\theta\dot{\theta} + \dot{r}\,\mathrm{sen}\,\theta)^2 + (r\,\mathrm{sen}\,\theta\dot{\theta} - \dot{r}\cos\theta)^2)$$

$$T = \frac{1}{2}m((r^2\cos^2\theta\,\dot{\theta}^2 + \cancel{2r\dot{r}\cos\theta\,\dot{\theta}\,\mathrm{sen}\,\theta} + \dot{r}^2\,\mathrm{Sen}^2\theta) + (r^2\,\mathrm{sen}^2\theta\,\dot{\theta}^2 - \cancel{2r\dot{r}\,\mathrm{sen}\,\theta\,\dot{\theta}\cos\theta} + \dot{r}^2\cos^2\theta))$$

$$T = \frac{1}{2}m(r^2\dot{\theta}^2 + \dot{r}^2)$$

$$L = T - V = \frac{1}{2}m(r^2\dot{\theta}^2 + \dot{r}^2) + mgr\cos\theta$$

$$L = T - V = \frac{1}{2}m(r^2\dot{\theta}^2 + \dot{r}^2) + mgr\cos\theta$$

Ahora la Energía:

$$E = \left[\frac{\partial L}{\partial \dot{r}}\dot{r} + \frac{\partial L}{\partial \dot{\theta}}\dot{\theta}\right] - L$$

$$E = m\dot{r}^2 + mr^2\dot{\theta}^2 - \frac{1}{2}m(r^2\dot{\theta}^2 + \dot{r}^2) - mgr\cos\theta$$

$$E = \frac{m(a^2 + r^2\dot{\theta}^2)}{2} - mgr\cos\theta = \text{cte ya que } \frac{\partial L}{\partial t} = 0$$

$$E = \frac{(a^2 + r^2\dot{\theta}^2)}{2} - gr\cos\theta = \text{cte}$$

Que es lo mismo que $E_{mecanica} = E_{Total} = T + V$

2) $L = \frac{1}{2}(m\dot{q}^2 - kq^2) \cdot e^{\frac{\alpha t}{m}}$ donde α y k son ctes reales positivas

a) Que situacion fisica describe la ecuacion de movimiento de la particula?

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

$$\frac{\partial L}{\partial \dot{q}} = m\dot{q}e^{\frac{\alpha}{m}t} \Rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = m(\ddot{q}e^{\frac{\alpha t}{m}} + \frac{\alpha}{m}\dot{q}e^{\frac{\alpha}{m}t})$$

$$\frac{\partial L}{\partial q} = -kqe^{\frac{\alpha t}{m}}$$

$$m\ddot{q}e^{\frac{\alpha}{m}t} + \frac{\alpha}{m}\dot{q}e^{\frac{\alpha}{m}t} - kqe^{\frac{\alpha}{m}t} = 0$$

$$m\ddot{q} + \alpha\dot{q} - kq = 0$$

$$\ddot{q} = -\frac{\alpha}{m}\dot{q} + \frac{k}{m}q.$$

Esto muestra el comportamiento de un oscilador armonico amortiguado

b) Considere la transformacion de coordenada $Q = e^{\frac{\alpha}{2m}t}q$

$$q = \frac{Q}{e^{\frac{\alpha}{2m}t}} = Qe^{-\frac{\alpha}{2m}t}$$

$$\dot{q} = -\frac{\alpha}{2m}Qe^{-\frac{\alpha t}{2m}} + \frac{dQ}{dq}\frac{dq}{dt}\left(e^{-\frac{\alpha t}{2m}}\right)$$

$$\dot{q} = -\frac{\alpha}{2m}Qe^{-\frac{\alpha t}{2m}} + \dot{Q}e^{-\frac{\alpha t}{2m}}$$

$$L = \frac{1}{2}\left(m\left(-\frac{\alpha}{2m}Qe^{-\frac{\alpha t}{2m}} + \dot{Q}e^{-\frac{\alpha t}{2m}}\right)^2\right) - K\left(Qe^{-\frac{\alpha t}{2m}}\right)^2 e^{\frac{\alpha t}{m}}$$

$$L = \frac{1}{2}\left(m\left(\frac{\alpha^2}{4m^2}Q^2 e^{-\frac{\alpha t}{m}} - \frac{\alpha}{m}Q\dot{Q}e^{-\frac{\alpha t}{m}} + \dot{Q}^2 e^{-\frac{\alpha t}{m}}\right)\right)$$
$$- K\left(Q^2 e^{-\frac{\alpha t}{m}}\right)e^{\frac{\alpha t}{m}}$$

$$L = \frac{1}{2}\left(m\left(\frac{\alpha^2}{4m^2}Q^2 - \frac{\alpha}{m}Q\dot{Q} + \dot{Q}^2\right)\right) - KQ^2$$

$$L = \frac{1}{2}\left(\frac{\alpha^2}{4m}Q^2 - \alpha Q\dot{Q} + m\dot{Q}^2\right) - KQ^2)$$

$$L = \frac{1}{2}\left(Q^2\left(\frac{\alpha^2}{4m} - K\right) - \alpha Q\dot{Q} + m\dot{Q}^2\right)$$

- $$\frac{d}{dt}\frac{\partial L}{\partial \dot{Q}} - \frac{\partial L}{\partial Q} = 0$$

$$\frac{\partial L}{\partial Q} = Q\left(\frac{\alpha^2}{4m} - K\right) - \frac{\alpha\dot{Q}}{2}$$

$$\frac{\partial L}{\partial \dot{Q}} = -\frac{1}{2}\alpha Q + m\dot{Q}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{Q}} = -\frac{1}{2}\alpha\dot{Q} + m\ddot{Q}$$

$$\cancel{-\frac{1}{2}\alpha\dot{Q}} + m\ddot{Q} - Q\left(\frac{\alpha^2}{4m} - K\right) + \cancel{\frac{\alpha\dot{Q}}{2}} = 0$$

$$m\ddot{Q} = Q\left(\frac{\alpha^2}{4m} - K\right)$$

$$\boxed{\ddot{Q} = Q\left(\frac{\alpha^2}{4m^2} - \frac{K}{m}\right).}$$

d) La unica cantidad que se conserva es la energia ya que $\frac{\partial L}{\partial t} = 0$

e) $E = \frac{\partial L}{\partial \dot{q}} \dot{q} - L$

$$E = \cancel{-\frac{1}{2}\alpha Q\dot{Q}} + m\dot{Q}^2 - \frac{1}{2} Q^2\left(\frac{\alpha^2}{4m} - k\right) + \cancel{\frac{\alpha Q\dot{Q}}{2}} - \frac{m\dot{Q}^2}{2}$$

$$E = \frac{m\dot{Q}^2}{2} - \frac{Q^2}{2}\left(\frac{\alpha^2}{4m} - k\right) = cte$$

$$\left[E + \frac{Q^2}{2}\left(\frac{\alpha^2}{4m} - k\right)\right]\frac{2}{m} = \frac{dQ}{dt}$$

$$dt = \left(\left[E + \frac{Q^2}{2}\left(\frac{\alpha^2}{4m} - k\right)\right]\frac{2}{m}\right)^{-1/2} dQ$$

3) Una partícula de masa m bajo la accion de un potencial $V(\vec{r}) = -\vec{F}\cdot\vec{r}$ donde $\vec{F}$ es constante (vector).

$$L = \frac{1}{2}m\dot{r}^2 + \vec{F}\cdot\vec{r}$$ donde $\vec{r} = r(x,y,z)$

La primera transformacion que deja invariante L es una transformacion temporal. ya que vemos que $\frac{\partial L}{\partial t} = 0$

Por lo tanto

$$\frac{dL}{dt} = \sum_{j=1}^{s}\left[\frac{\partial L}{\partial q_j}\dot{q}_j + \frac{\partial L}{\partial \dot{q}_j}\ddot{q}_j\right] + \frac{\partial L}{\partial t} \rightarrow 0$$

Sustituyendo $\frac{\partial L}{\partial q_s} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_s}\right)$ y operando se llega a

$$\frac{\partial L}{\partial \dot{q}_j}\dot{q}_j - L = cte = E(q_j, \dot{q}_j)$$

La cantidad conservada en este caso seria

$$m(\dot{x}^2) + m(\dot{y}^2) + m(\dot{z}^2) - \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2) - (\vec{F}_x, \vec{F}_y, \vec{F}_z)\cdot(x, y, z) = cte$$

$$m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2) - \vec{F}\cdot\vec{r} = cte$$

$$\boxed{\frac{m\dot{r}^2}{2} - \vec{F}\cdot\vec{r} = cte}$$

4) $V(r,v) = U(r) + n \cdot l$

$\mathbb{L} = \vec{r} \times \vec{p} = m \vec{r} \times \vec{v}$

$V(r,v) = U(r) + n \cdot \mathbb{L}$

$V(r,v) = U(r) + n \cdot (r \times v) m$

$L = T - V$

$= \frac{1}{2} m v^2 - U(r) - m \hat{n} \cdot (\vec{r} \times v)$

$V(r, v) = U(r) + n \cdot \mathbb{L}$

$\frac{\partial V}{\partial x_i} = \frac{\partial U}{\partial x_i} + \frac{\partial}{\partial x_i}(n \cdot \mathbb{L})$

$\frac{\partial}{\partial x_i}(n \cdot \mathbb{L}) = n \cdot \frac{\partial \mathbb{L}}{\partial x_i} = m \hat{n} \cdot \left\{ \frac{\partial r}{\partial x_i} \times v \right\}$

$r = x_1 \hat{e}_1 + x_2 \hat{e}_2 + x_3 \hat{e}_3 \Rightarrow \frac{\partial r}{\partial x_i} = \hat{e}_i$

$\frac{\partial}{\partial x_i}(n \cdot \mathbb{L}) = m \hat{n} \cdot (\hat{e}_i \times v)$

$F_i = -\frac{\partial V}{\partial x_i} + \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{x}_i}\right)$

$\frac{\partial V}{\partial \dot{x}_i} = \frac{\partial U}{\partial \dot{x}_i} + \frac{\partial}{\partial \dot{x}_i}(\hat{n} \cdot \mathbb{L}) = \hat{n} \cdot \frac{\partial \mathbb{L}}{\partial \dot{x}_i}$

$= m \hat{n} \cdot \frac{\partial}{\partial \dot{x}_i}(r \times v) = m \hat{n} \cdot \left\{ r \times \frac{\partial v}{\partial \dot{x}_i} \right\}$

$\frac{\partial V}{\partial \dot{x}_i} = m \hat{n} \cdot (r \times \hat{e}_i)$

$\frac{d}{dt}\left(\frac{\partial V}{\partial \dot{x}_i}\right) = m \hat{n} \cdot (v \times \hat{e}_i)$

$F_i = -\frac{\partial U}{\partial x_i} - m \hat{n} \cdot (\hat{e}_i \times v) + m \hat{n} \cdot (v \times \hat{e}_i)$

$\boxed{F_i = -\frac{\partial U}{\partial x_i} - 2 m \hat{n} \cdot (\hat{e}_i \times v)}$

hallamos las componentes F_x, F_y, F_z a partir de $F = m\ddot{q}$:

$$\hat{\mathbb{V}} \times \hat{e}_r = \begin{pmatrix} x & y & z \\ \dot{r}_x & \dot{r}_y & \dot{r}_z \\ e_x & e_y & e_z \end{pmatrix}$$

$$\hat{\mathbb{V}} \times \hat{e}_r = (\dot{r}_y e_z - \dot{r}_z e_y)\hat{x} + (\dot{r}_x e_z - \dot{r}_z e_x)\hat{y} + (\dot{r}_x e_y - \dot{r}_y e_x)\hat{z}$$

$$m \cdot (\mathbb{V} \times \hat{e}_r) = n_x(\dot{r}_y - \dot{r}_z) + n_y(\dot{r}_x - \dot{r}_z) + n_z(\dot{r}_x - \dot{r}_y)$$

//

$$\ddot{r}_x = -\frac{\partial U}{\partial x} - 2mn_x(\dot{r}_y - \dot{r}_z)/m$$

$$\ddot{r}_y = -\frac{\partial U}{\partial y} - 2mn_y(\dot{r}_x - \dot{r}_z)/m$$

$$\ddot{r}_z = -\frac{\partial U}{\partial z} - 2mn_z(\dot{r}_x - \dot{r}_y)/m$$

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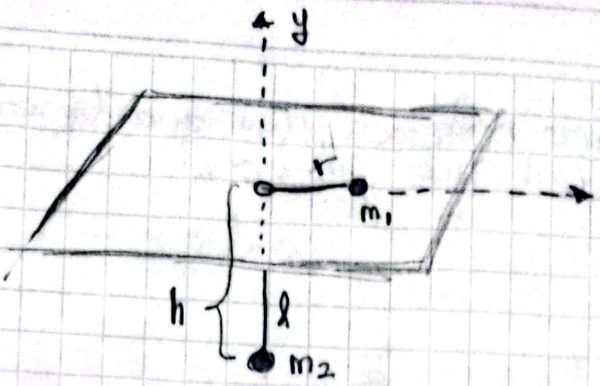
$$\frac{\partial L}{\partial t} = 0$$ Por lo tanto se conserva la energía

5)



$h = l - r$

$x = r\cos\theta$ $\qquad$ $\dot{x} = \dot{r}\cos\theta - \dot{\theta} r \,\text{sen}\,\theta$

$y = r\,\text{sen}\,\theta$ $\qquad$ $\dot{y} = \dot{r}\,\text{sen}\,\theta + \dot{\theta} r\cos\theta$

$$L = \frac{1}{2} m_1 \dot{r}_1^2 + \frac{1}{2} m_2 \dot{r}_2 - mg(l-r)$$

$$\dot{r}_1^2 = (\dot{r}\cos\theta - \dot{\theta} r\,\text{sen}\,\theta)^2 + (\dot{r}\,\text{sen}\,\theta + \dot{\theta} r\cos\theta)^2 =$$

$$\dot{r}_1^2 = \dot{r}^2\cos^2\theta - \cancel{2\dot{r}\dot{\theta} r\cos\theta\,\text{sen}\,\theta} + \dot{\theta}^2 r^2\,\text{sen}^2\theta + \dot{r}^2\,\text{sen}^2\theta$$
$$+ \cancel{2\dot{r}\dot{\theta} r\,\text{sen}\,\theta\cos\theta} + \dot{\theta}^2 r^2 \cos^2\theta)$$

$$\dot{r}_1^2 = \dot{r}^2 + \dot{\theta}^2 r^2.$$

$$\dot{r}_2^2 = \dot{r}$$

$$L = \frac{1}{2} m_1 (\dot{r}^2 + \dot{\theta}^2 r^2) + \frac{1}{2} m_2 \dot{r}^2 - m_2 g(l-r).$$

$$\frac{\partial L}{\partial \dot{r}} = m_1\dot{r} + m_2\dot{r} \rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = m_1\ddot{r} + m_2\ddot{r} = \ddot{r}(m_1 + m_2)$$

$$\frac{\partial L}{\partial r} = m_1\dot{\theta}^2 r + m_2 g$$

$$\ddot{r} = \frac{m_1\dot{\theta}^2 r + m_2 g}{(m_1 + m_2)} \rightarrow \text{Para } r.$$

$$\frac{\partial L}{\partial \dot{\theta}} = m_1 \dot{\theta} r^2 \rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = m_1\ddot{\theta} + 2m_1\dot{\theta} r\dot{r}$$

$$\ddot{\theta} = -2\dot{\theta}\dot{r}r$$

b) Para este sistema podemos ver que tenemos la coordenada cíclica θ tal que $\frac{\partial L}{\partial \theta} = 0$.

$$\frac{d}{dt}(m_1 r^2 \dot{\theta}) = 0$$

$$\boxed{m_1 r^2 \dot{\theta} = cte.}$$

1^{ra} Cantidad conservada es el momento conjugado a la coordenada θ

dado que $\frac{\partial L}{\partial t} = 0$

$$\dot{E} = \left(\dot{r}^2(m_1+m_2) + m_1\dot{\theta}^2 r^2 - \frac{1}{2}m_1(\dot{r}^2 + \dot{\theta} r^2) - \frac{1}{2}m_2\dot{r}^2 + m_2 g(l-r) \right)$$

$$E = \frac{2\dot{r}^2 m_1}{2} + \frac{2\dot{r}^2 m_2}{2} + m_1\dot{\theta}^2 r^2 - \frac{m_1\dot{r}^2}{2} - \frac{m_1 \dot{\theta} r^2}{2} - \frac{1}{2}m_2\dot{r}^2 + mgl - r$$

$$E = \dot{r}^2\frac{m_1}{2} + \dot{r}^2\frac{m_2}{2} + \frac{m_1\dot{\theta}^2 r^2}{2} + mg(l-r)$$

$$\boxed{E = \frac{\dot{r}^2(m_1+m_2)}{2} + \frac{m_1\dot{\theta}^2 r^2}{2} + m_1 g(l-r) = T + V = cte}$$

2^{da} Cantidad Conservada Energía

c) Encuentre la posición de equilibrio del sistema

Debe cumplirse $\ddot{r} = 0$ en la posición equilibrio

Por lo tanto $m_1 r\dot{\theta}^2 = -m_2 g$

$$\boxed{r_{eq} = -\frac{m_2 g}{m_1 \dot{\theta}^2}}$$

Debido a que $\ddot{\theta} = 0$

$$\ddot{\theta} = -2\dot{r}r\theta = 0$$

$$\dot{\theta} = cte.$$

Cualquier punto es posic equilibrio siempre y cuando $\dot{\theta} = cte$

d) $E_{inicial} = \frac{1}{2} m_1 V_{i_1}^2 + \frac{1}{2} m_2 V_{i_2}^2 - mg(l-a)$ ($\frac{1}{2} m_1 V_{i_1}^2 \to 0$, $\frac{1}{2} m_2 V_{i_2}^2 \to 0$)

$E_{inicial} = -mg(l-a)$

$E_{Final} = \frac{1}{2} m_1 V_{f_1}^2 + \frac{1}{2} m_2 V_{f_2}^2 - m_2 g l$

Debido a la conservacion de Energia

$E_{inicial} = E_{Final}$

$-m_2 g(l-a) = \frac{1}{2} m_1 V_{f_1}^2 + \frac{1}{2} m_2 V_{f_2}^2 - m_2 g l$

Debido a que la cuerda es ideal $V_{f_1} = V_{f_2}$.

$-m_2 g(l-a) = \frac{1}{2} m_1 V_{f_2}^2 + \frac{1}{2} m_2 V_{f_2}^2 - m_2 g l$

$-m_2 g(l-a) = \frac{V_{f_2}^2}{2}(m_1 + m_2) - m_2 g l$

$\boxed{\sqrt{\frac{2 m_2 g a}{m_1 + m_2}} = V_{f_2}}$ Velocidad de m_2 cuando $a = 0$

6) Encuentre las ec. movimiento

$F = ma$

$F_x = -K_1(x-a)$

$F_y = -K_2(y-a)$

$F_x = ma_x$

$\Rightarrow -K_1(x-a) = m\ddot{x}$

$\Rightarrow \ddot{x} + \omega_1^2(x-a) = 0$

$F_y = ma_y$

$\Rightarrow -K_2(y-a) = m\ddot{y}$

$\Rightarrow \ddot{y} + \omega^2(y-a)$

Las ecuaciones de movimiento:

$x(t) = A_x \cos(\omega_1 t + \phi_x) + a$

$y(t) = A_y \cos(\omega_2 t + \phi_y) + a$

ϕ_x y ϕ_y son los desfases iniciales

$\omega_1 = \sqrt{\frac{K_1}{m}}$ es la frec. angular en x

$\omega_2 = \sqrt{\frac{K_2}{m}}$ es la frec. angular en y.

Para angulos pequeños

$X = x - a$ y $Y = y - a$

X y Y son las desviaciones de la masa respecto a la posicion de equilibrio.

$\Rightarrow \ddot{X} + \omega_1^2 X = 0$

$\Rightarrow \ddot{Y} + \omega_2^2 Y = 0$

$K_1 = K_2$

$\Rightarrow \omega_1^2 = \omega_2^2 = \omega^2$

Caso $K_1 \neq K_2$

$\omega_1^2 \neq \omega_2^2$

$$\mathcal{L} = T - V$$

$$\mathcal{L} = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) - \frac{1}{2}\left[k_1 (x-a)^2 + k_2 (y-a)^2\right]$$

$$\frac{\partial L}{\partial t} = 0 \Rightarrow E = \frac{\partial L}{\partial \dot{x}} \dot{x} + \frac{\partial L}{\partial \dot{y}} \dot{y} - L = cte$$

$$E = \frac{m}{2} (\dot{x}^2 + \dot{y}^2) + \frac{k_1}{2} (x-a)^2 + \frac{k_2}{2} (y-a)^2$$

$$E_x = \frac{m}{2} \dot{x}^2 + \frac{k_1}{2} (x-a)^2$$

$$E_x = \frac{P_x^2}{2m} + \frac{k_1}{2} (x-a)^2$$

$$E_y = \frac{m}{2} \dot{y}^2 + \frac{k_2}{2} (y-a)^2$$

$$E_y = \frac{P_y^2}{2m} + \frac{k_2}{2} (y-a)^2$$

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{r} = (x, y, 0)$$
$$P = (P_x, P_y, 0)$$

$$\vec{L} = \hat{k} (x P_y - y P_x)$$

$$L_z = x P_y - y P_x$$