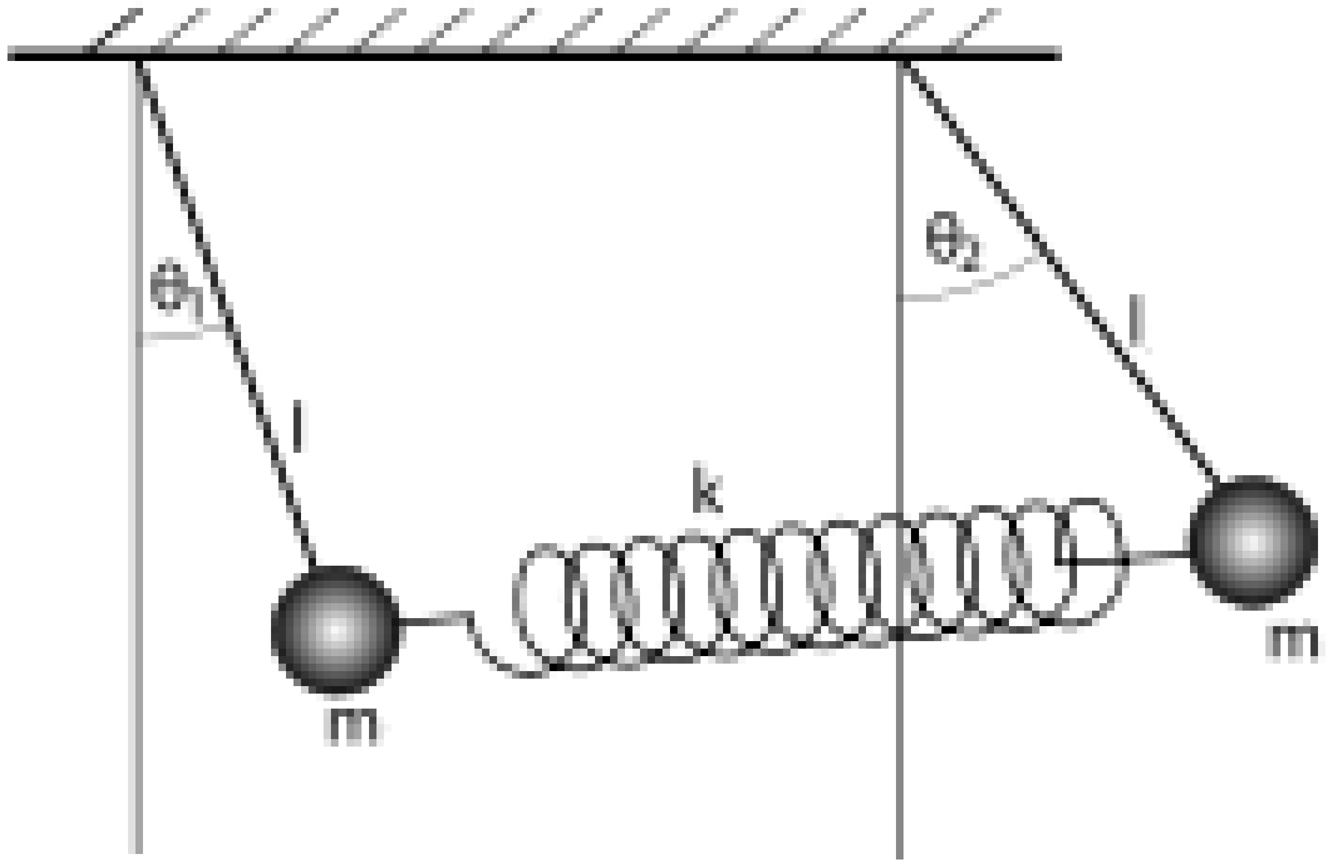
Problema



$$y_1 = -\ell_1 cos\theta_1 \qquad \dot{y}_1 = \ell_1 sen\theta_1\dot{\theta}_1$$

$$y_2 = -\ell_2 cos\theta_2 \qquad \dot{y}_2 = \ell_2 sen\theta_2\dot{\theta}_2$$

$$x_1 = x_{1e} + \ell_1 sen\theta_1 \qquad \dot{x}_1 = \ell_1 cos\theta_1\dot{\theta}_1$$

$$x_2 = x_{2e} + \ell_2 sen\theta_2 \qquad \dot{x}_2 = \ell_2 cos\theta_2\dot{\theta}_2$$

$$d_{ix} = x_{2e} - x_{1e}$$

Considere el caso de dos péndulos de igual masa m y que cuelgan, respectivamente, de dos varillas sin masa y de longitud l. Estos péndulos están acoplados por un resorte de constante elástica k.

$$T = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2)$$

$$T = \frac{1}{2}m_1(\ell_1^2 cos^2\theta_1\dot{\theta}_1^2 + \ell_1^2 sen^2\theta_1\dot{\theta}_1^2) + \frac{1}{2}m_2(\ell_2^2 cos^2\theta_2\dot{\theta}_2^2 + \ell_2 sen^2\theta_2\dot{\theta}_2^2)$$

$$T = \frac{1}{2}m_1\ell_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2\ell_2^2\dot{\theta}_2^2$$

Definimos la distancia entre las masas del pendulo:

$$d(t) = (x_2 - x_1)^2 + (y_2 - y_1)^2 = ((x_{2e} - x_{1e}) + (\ell_2 sen\theta_2 - \ell_1 sen\theta_1))^2 + (\ell_1 cos\theta_1 - \ell_2 cos\theta_2)^2$$

$$d(t)^2 = d_{ix}^2 + 2d_{ix}(\ell_2 sen\theta_2 - \ell_1 sen\theta_1) - 2\ell_1\ell_2 cos(\theta_1 - \theta_2)$$

Por lo tanto la elongacion del resorte al cuadrado es:

$$(X_k)^2 = d(t)^2 - (d_i)^2$$

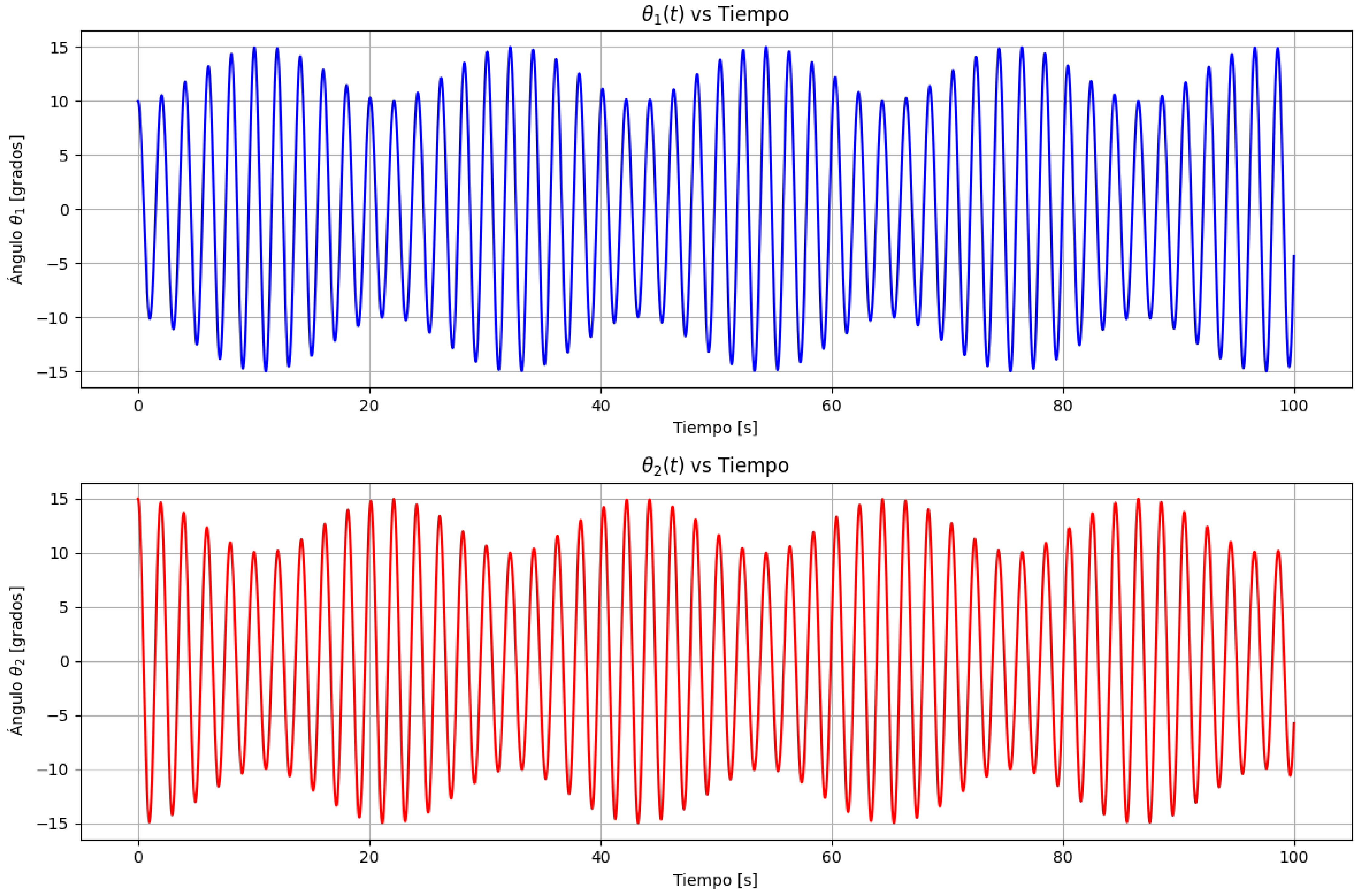
$$(X_k)^2 = d_{ix}^2 + 2d_{ix}(\ell_2 sen\theta_2 - \ell_1 sen\theta_1) - 2\ell_1\ell_2 cos(\theta_1 - \theta_2) - (d_i)^2$$

Definimos la energia potencial como

$$V = m_1 g(\ell_1 cos\theta_1) + m_2 g(\ell_2 cos\theta_2) + \frac{1}{2}k(X_k^2)$$

$$L = T - V = \frac{1}{2}m_1\ell_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2\ell_2^2\dot{\theta}_2^2 - \left(m_1g(-\ell_1 cos\theta_1) + m_2g(-\ell_2 cos\theta_2) + \frac{1}{2}k(X_k^2)\right)$$

$$\ddot{\theta}_1 = -\frac{g sen\theta_1}{\ell} + \frac{k(d_{ix}cos\theta_1 + \ell sen(\theta_1 - \theta_2)}{m_1\ell} \qquad \ddot{\theta}_2 = -\frac{g sen\theta_2}{\ell} - \frac{k(d_{ix}cos\theta_2 + \ell sen(\theta_1 - \theta_2))}{m_2\ell}$$



$$m_1 = m_2 = 1kg$$

$$\ell_1 = \ell_2 = 1m$$

$$K = 1{,}0N/m$$

$$\theta_2 = 10^\circ$$

$$\theta_1 = 5^\circ$$

$$w_1 = w_2 = 0$$

$$d_{ix} = 0{,}5m$$

$$g = -9{,}81m/s^2$$

¿Hace diferencia si el resorte conecta a las masas o si se encuentra atado a media altura de las varillas?

La ecuacion para la elongacion del resorte asumiento que dix=di

$$X_k^2 = 2d_{ix}\ell(sen\theta_2 - sen\theta_1) - 2\ell^2 cos(\theta_1 - \theta_2)$$

La ecuacion para la elongacion del resorte cuando las masas están atadas a media altura de las varillas

$$X_k^2(\frac{\ell}{2}) = d_{ix}\ell(sen\theta_2 - sen\theta_1) - \frac{\ell^2}{2} cos(\theta_1 - \theta_2)$$

Comparemos las dos ecuaciones

$$2d_{ix}\ell(sen\theta_2 - sen\theta_1) - 2\ell^2 cos(\theta_1 - \theta_2) > d_{ix}\ell(sen\theta_2 - sen\theta_1) - \frac{\ell^2}{2} cos(\theta_1 - \theta_2)$$

$X_k^2(\ell) > X_k^2(\frac{\ell}{2})$ Lo cual implica $\rightarrow$ $U(\ell) > U(\frac{\ell}{2})$

¿Como influye la relacion m1/m2 si consideramos masas diferentes?

$$m_1 = \frac{k(d_{ix}cos\theta_1 + \ell sen(\theta_1 - \theta_2))}{\ddot{\theta}_1\ell + gsen\theta_1}$$

$$m_2 = -\frac{k(d_{ix}cos\theta_2 + \ell sen(\theta_1 - \theta_2))}{\ddot{\theta}_2\ell + gsen\theta_2}$$

$$\frac{m_1}{m_2} = -\frac{k(d_{ix}cos\theta_2 + \ell sen(\theta_1 - \theta_2))(\ddot{\theta}_2\ell + gsen\theta_2)}{k(d_{ix}cos\theta_1 + \ell sen(\theta_1 - \theta_2))(\ddot{\theta}_1\ell + gsen\theta_1)}$$

¿Como influye la relacion L1/L2 si consideramos el largo de las varillas diferentes?

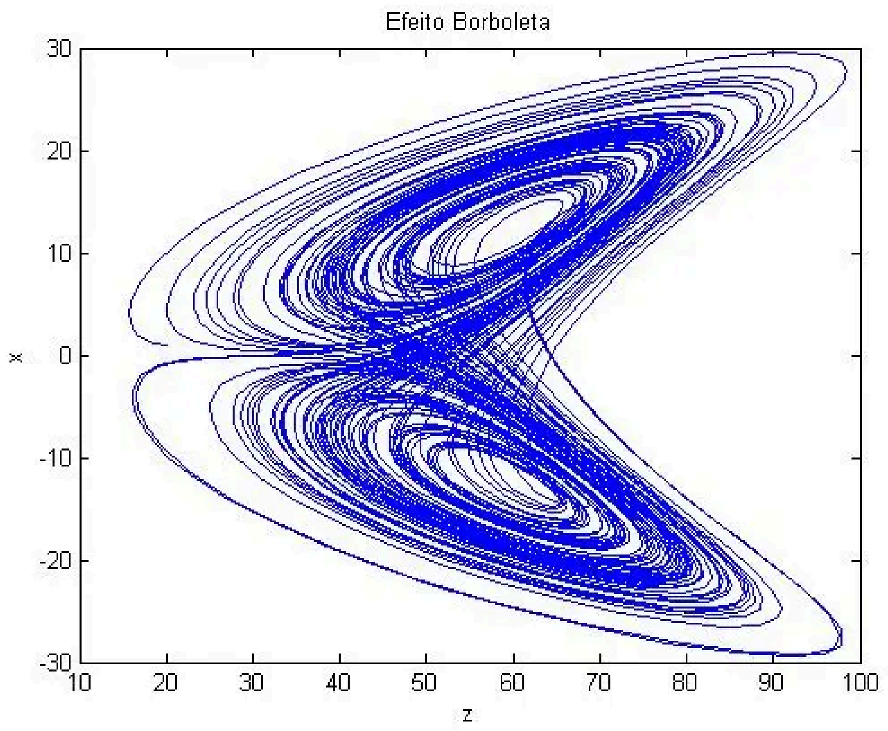
$$\ell_1 = \frac{1}{\ddot{\theta}_1}(-gsen\theta_1 + \frac{k(d_{ix}cos\theta_1 + \ell sen(\theta_1 - \theta_2)}{m_1}) \qquad \ell_2 = \frac{1}{\ddot{\theta}_2}(-gsen\theta_2 - \frac{k(d_{ix}cos\theta_2 + \ell sen(\theta_1 - \theta_2)}{m_2})$$

$$\frac{\ell_1}{\ell_2} = -\frac{\ddot{\theta}_2}{\ddot{\theta}_1}\frac{m(k(d_{ix}cos\theta_1 + \ell sen(\theta_1 - \theta_2)))}{m(k(d_{ix}cos\theta_2 + \ell sen(\theta_1 - \theta_2)))}$$

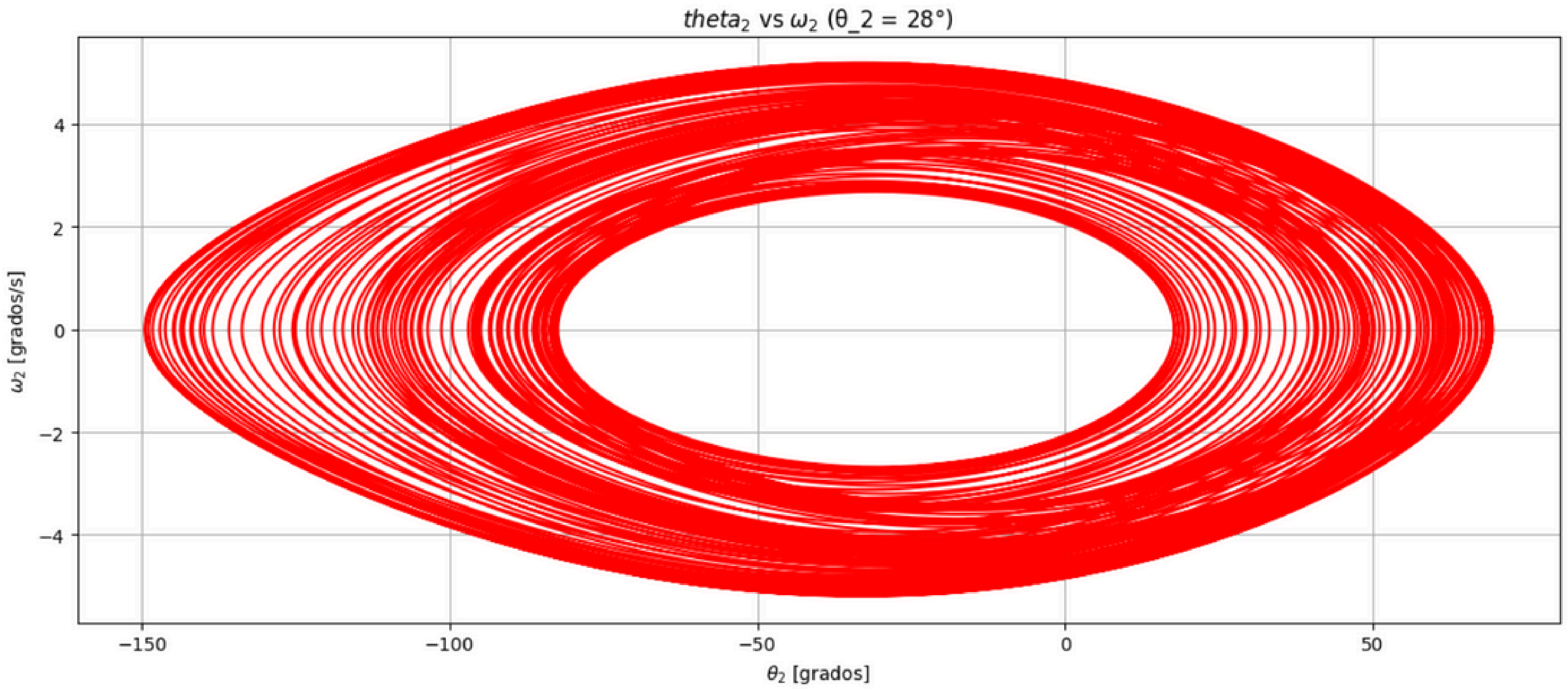
Simplificando dicha expresion:

$$\frac{\ell_1}{\ell_2} = -\frac{\ddot{\theta}_2}{\ddot{\theta}_1}\frac{d_{ix}cos\theta_1 + \ell sen(\theta_1 - \theta_2)}{d_{ix}cos\theta_2 + \ell sen(\theta_1 - \theta_2)}$$

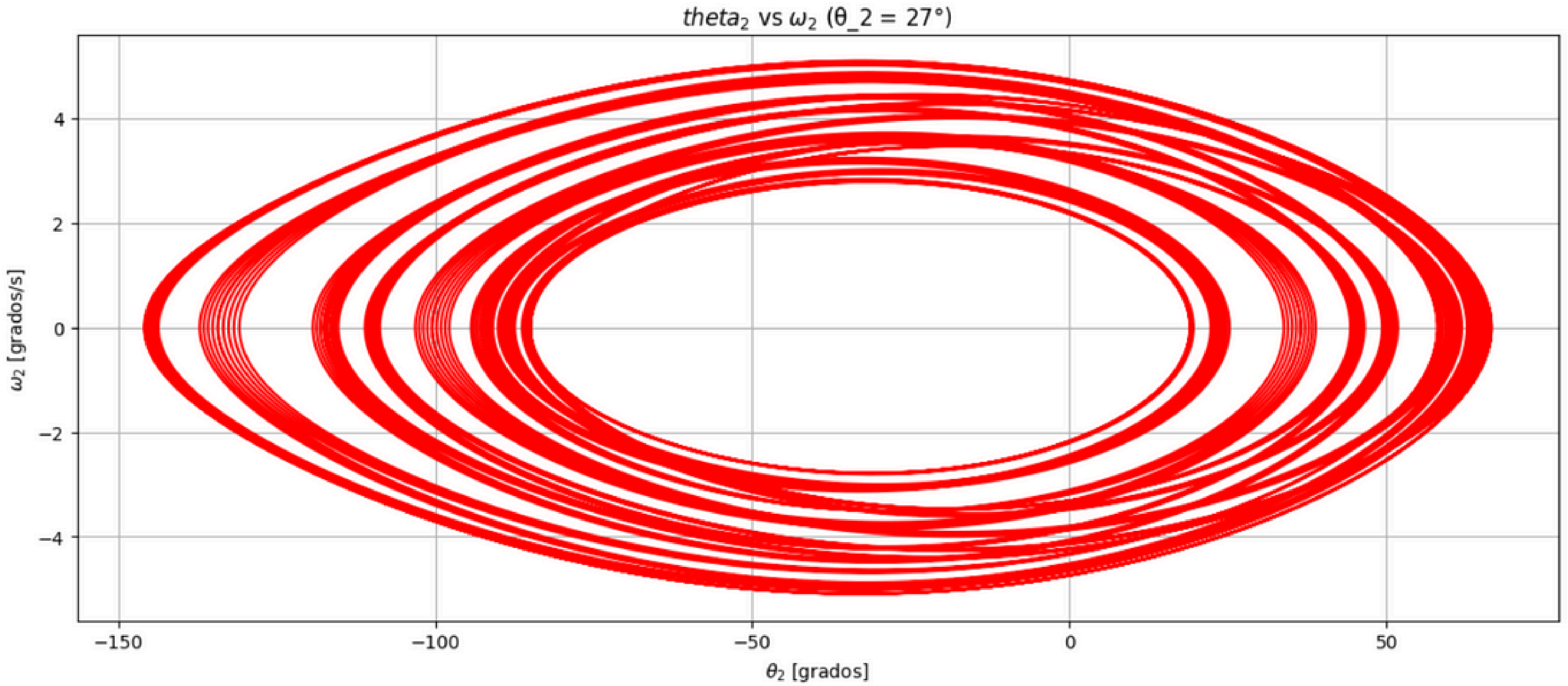
Condiciones para que el sistema sea caotico



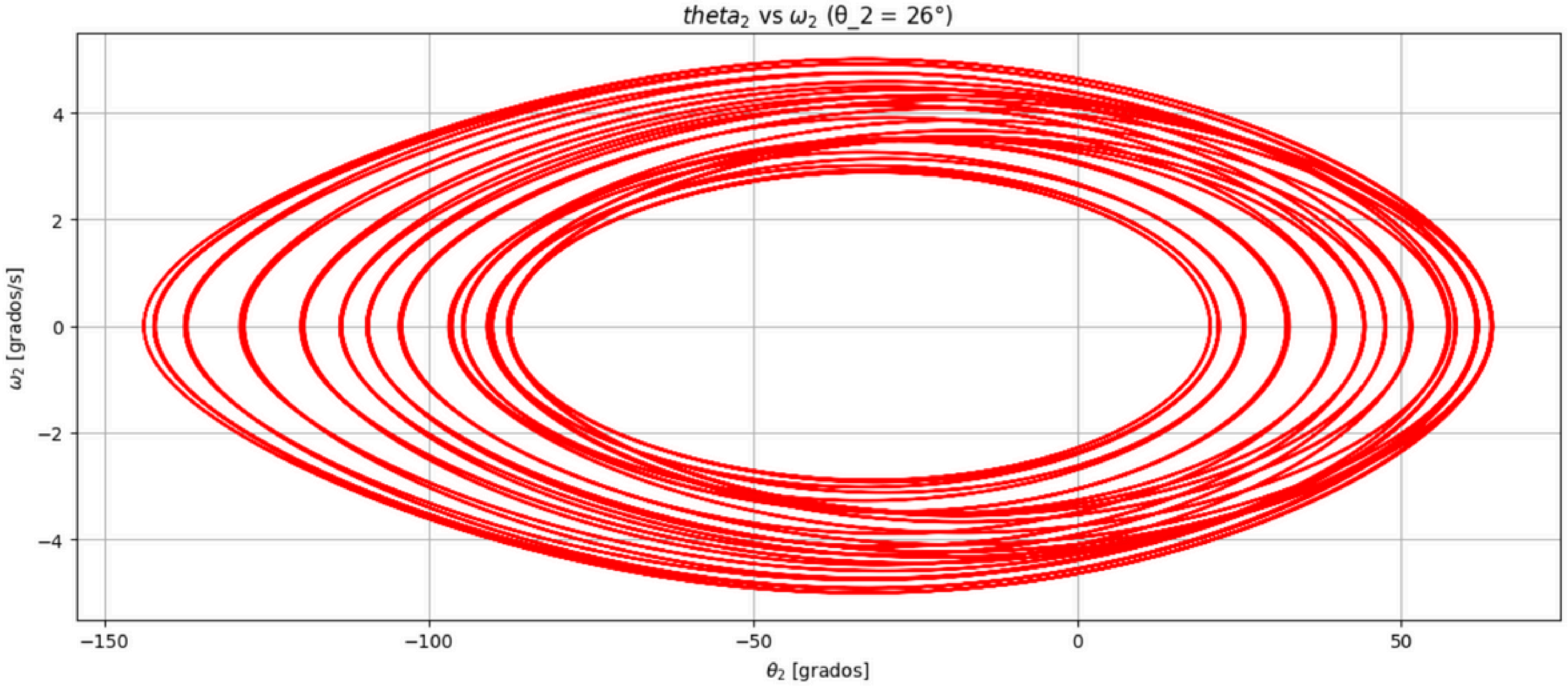
(θ_2 para un valor inicial de 28°)



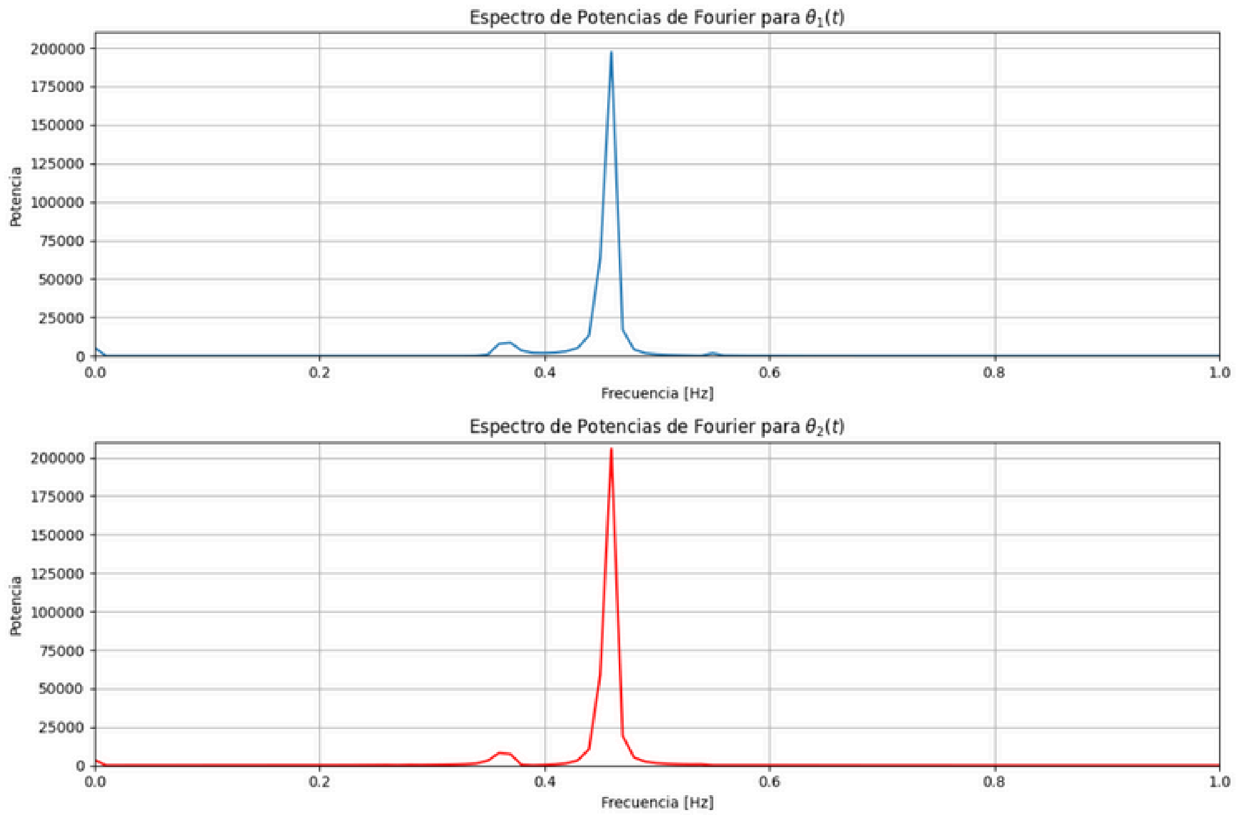
(Cambio en θ_2 a un valor inicial de 27°)



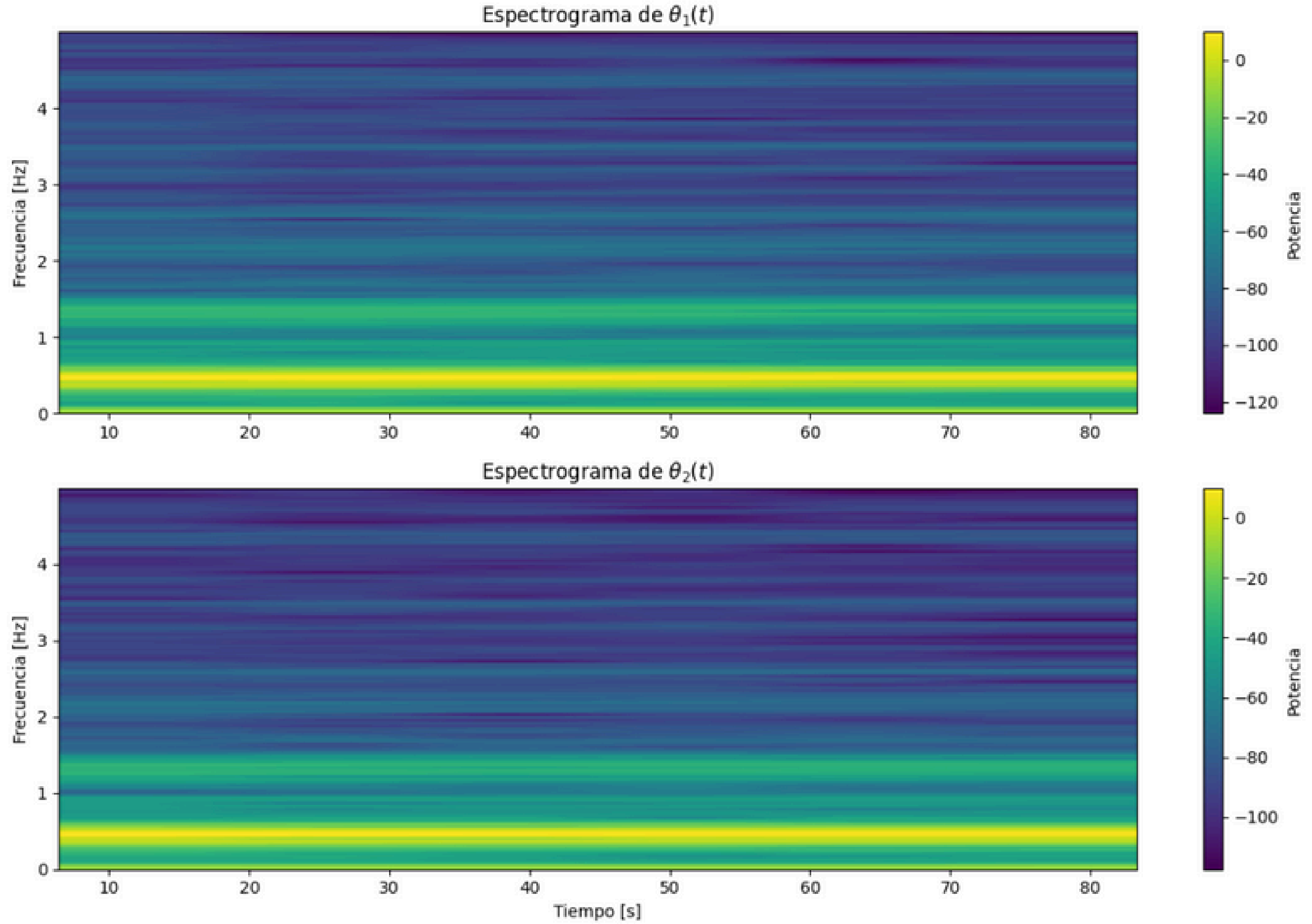
[Cambio en θ_2 a un valor inicial de 26°]



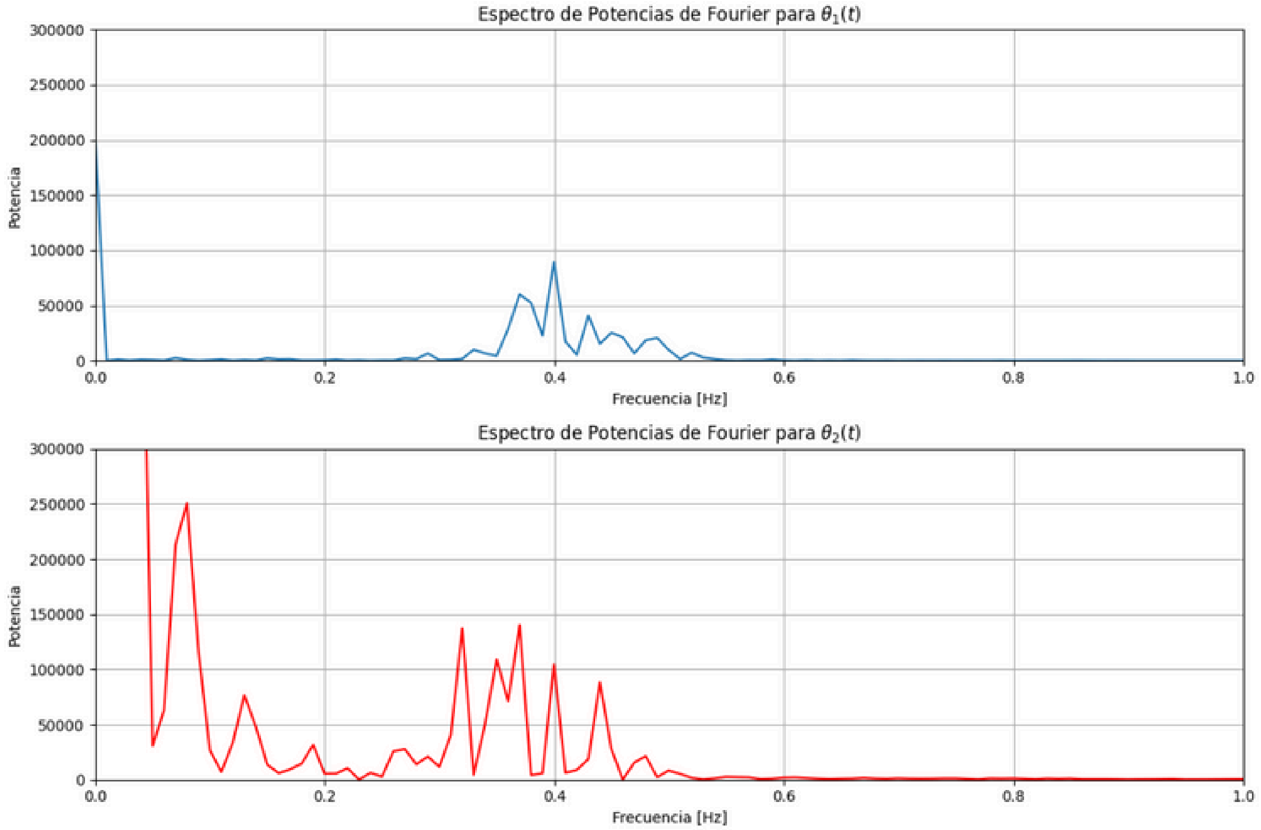
(Espectro de potencias de fourier para amplitudes pequeñas. Grafico para θ_1 y θ_2)



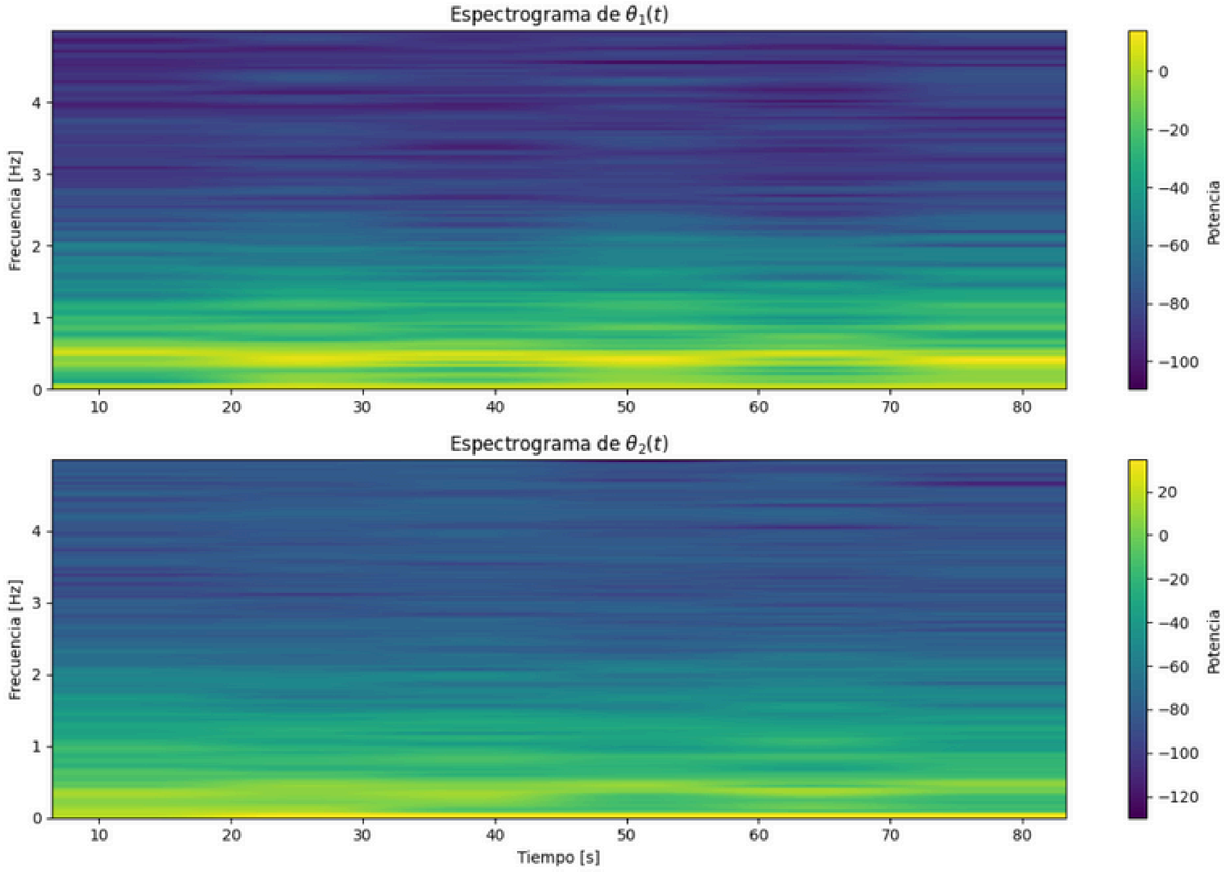
(Espectrograma para amplitudes pequeñas. Grafico para θ_1 y θ_2)



(Espectro de potencias de fourier para amplitudes grandes. Grafico para θ_1 y θ_2)



(Espectrograma para amplitudes grandes. Grafico para θ_1 y θ_2)



Realizando las aproximaciones especificadas nuestras equivalencias quedan de la siguiente forma:

$$y_1 = \ell_1$$
$$y_2 = \ell_2$$
$$x_1 = x_{1e} + \ell_1\theta_1$$
$$x_2 = x_{2e} + \ell_2\theta_2$$
$$d_{ix} = x_{2e} - x_{1e}$$

$$\dot{y}_1 = 0$$
$$\dot{y}_2 = 0$$
$$\dot{x}_1 = \ell_1\dot{\theta}_1$$
$$\dot{x}_2 = \ell_2\dot{\theta}_2$$

$$T = \frac{1}{2}m_1(\ell_1\dot{\theta}_1)^2 + \frac{1}{2}m_2(\ell_2\dot{\theta}_2)^2$$

$$V = -mg(\ell_2 + \ell_1) + \frac{1}{2}k(\ell_2\theta_2 - \ell_1\theta_1)^2$$

$$L = \frac{1}{2}m_1(\ell_1\dot{\theta}_1)^2 + \frac{1}{2}m_2(\ell_2\dot{\theta}_2)^2 + mg(\ell_2 + \ell_1) - \frac{1}{2}k(\ell_2\theta_2 - \ell_1\theta_1)^2$$

$$\ddot{\theta}_1 = \frac{k(\ell_1^2\theta_1 - \ell_1\ell_2\theta_2)}{m_1\ell_1^2}$$

$$\ddot{\theta}_1 = \frac{k}{m_1}(\theta_1 - \theta_2)$$

$$\ddot{\theta}_2 = \frac{k(\ell_2^2\theta_2 - \ell_2\ell_1\theta_1)}{m_2\ell_2^2}$$

$$\ddot{\theta}_2 = \frac{k}{m_2}(\theta_2 - \theta_1)$$