8.5

39. [10 points] For the red-black tree in Figure 1, insert the keys 8, 7, and 5 (in this order) and redraw the tree after any necessary re-coloring and rebalancing. Clearly label the red nodes with R. Show the tree after key 8 is inserted, another tree after 7 is inserted, and a final tree after 5 is inserted (if you show other intermediate steps, then clearly label these three trees).

Recurrence Relations:

40. For the following two questions, use the Master Method to find the closed-form solution of the following recurrence equations: **(you MUST show your work)**.

$$T(n) = \begin{cases} c & \text{if } n < d \\ aT(n/b) + f(n) & \text{if } n \geq d \end{cases}$$

Case 1. if $f(n)$ is $O(n^{\log_b a - \varepsilon})$, then $T(n)$ is $\Theta(n^{\log_b a})$

Case 2. if $f(n)$ is $\Theta(n^{\log_b a} \log^k n)$, then $T(n)$ is $\Theta(n^{\log_b a} \log^{k+1} n)$

Case 3. if $f(n)$ is $\Omega(n^{\log_b a + \varepsilon})$, then $T(n)$ is $\Theta(f(n))$, provided $af(n/b) \leq \delta f(n)$ for some $\delta < 1$.

(a) [5 points]

$T(n) = 3T(n/3) + b\,n^2$

5

$a = 3$

$b = 3$

$\log_b a = \log_3 3 = 1$

$f(n) = bn^2 = \Omega(n^{1+\varepsilon})$ where $\varepsilon = 1$

$\Rightarrow T(n)$ is $\Theta(bn^2)$

$\left(\frac{n^2}{3} \leq \delta 3n^2 \Rightarrow \frac{1}{9} \leq \delta < 1\right.$

(b) [5 points]

$T(n) = 4T(n/2) + b\,n^2$

4

$a = 4$

$b = 2$

$\log_b a = \log_2 4 = 2$

$f(n) = bn^2 = \Theta(n^2 \lg^0 n)$ where $K = 0$

$\therefore T(n)$ is $\Theta(n^2 \log n)$

41. [5 points] For recurrence equation $T(n) = 8\,T(n/2) + n^2$, prove that T(n) is $O(n^3)$ using the guess-and-test method (**Hint:** use as your guess $(n^3 - n^2)$, which is $O(n^3)$).

4

$T(n) = c8\,T(n/2) + n^2 \leq c8\left(\frac{n^3}{2^3} - \frac{n^2}{2^2}\right) + n^2 = n^3 - 2n^2 + n^2 = c(n^3 - n^2) \leq (n^3 - n^2) = O(n^3$

42. [5 points] Let A be an array of n integers. What is the time complexity of the following algorithm? Give your answer as a summation that specifies the precise number of times that the statement A[i] ← A[i] + A[j] + A[k] is executed.

```
for i ← 0 to n − 1 do
    for j ← 0 to i do
        for k ← 0 to i do
            A[i] ← A[i] + A[j] + A[k]
```

$O(n)$

$O(n^2) \leftarrow O(1 + 2 + \dots + n) = \frac{n(n+1)}{2} = O($

$O(n^4) \leftarrow O(1 + 2^2 + \dots + n^2) = \frac{n^2(n^2+1)}{2} = O($

$O(n^4)$

$O(n^4)$

5