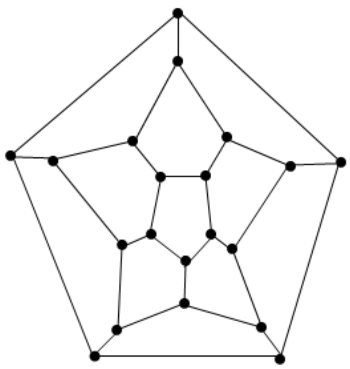
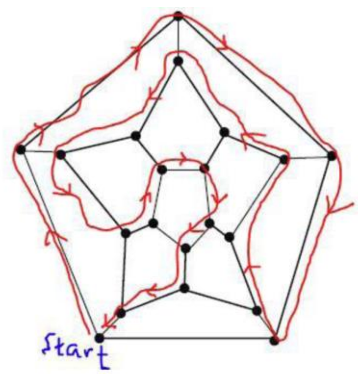
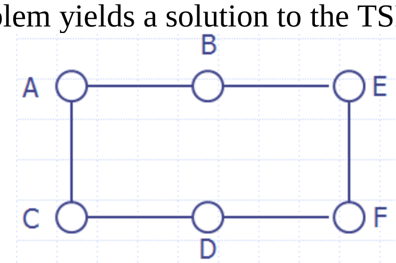
Lab 12
Due Tuesday 2 pm

1. Does the following graph have a Hamiltonian cycle? If so, draw a spanning cycle.
   Solution: Yes





2. Illustrate the proof that the HamiltonianCycle problem is polynomial reducible to TSP by considering the following Hamiltonian graph—an instance of HamiltonianCycle—and transforming it to a TSP instance in polynomial time so that a solution to the HC problem yields a solution to the TSP problem, and conversely.



   Solution: The slides for Lesson 12 give a full explanation of a nearly identical problem.

3. Show that TSP is NP-complete. (Hint: use the relationship between TSP and HamiltonianCycle discussed in the slides. You may assume that the HamiltonianCycle problem is NP-complete.)

   Solution: We know from the slides that

$$\textsc{HamiltonianCycle} \xrightarrow{\text{poly}} \text{TSP}$$

   We assume HamiltonianCycle is NP-complete. Let R be any NP problem. We must show

$$R \xrightarrow{\text{poly}} \text{TSP}$$

But we have

$$R \xrightarrow{\text{poly}} \textsc{HamiltonianCycle} \text{ and } \textsc{HamiltonianCycle} \xrightarrow{\text{poly}} \text{TSP}$$

By transitivity of polynomial reducibility, we have

$$R \xrightarrow{\text{poly}} \text{TSP}$$

We have shown that TSP is NP-complete.

4. Show that the worst case for VertexCoverApprox can happen by giving an example of a graph G which has these properties:
    a. G has a smallest vertex cover of size *s*
    b. VertexCoverApprox outputs size 2*s as its approximation to optimal size.

Solution. Consider the following disconnected graph with two edges and four vertices. The smallest vertex cover has size 2 but the Vertex Cover Approx algorithm outputs a vertex cover of size 4.