① [1, 6, 2, 4, 3, 5]

$\boxed{1}$ 6 2 4 3 5 $\longrightarrow$ 1 2 3 4 5 6

$\boxed{6}$ 2 4 3 5 $\longrightarrow$ 2 3 4 5 6

$\boxed{2}$ 4 3 5 $\longrightarrow$ 2 3 4 5

$\boxed{4}$ 3 5 $\longrightarrow$ 3 4 5

3 $\rightarrow$ 3 5 $\rightarrow$ 5

2) Our Array $A = [5,1,4,3,6,2,7,1,3]$ $n=9=3^2$

what we need

$\{\text{elements} < x\} < 3n/4 = 27/4 = 6.75$ ⟹ Since 9 is a Power of 2

$\{\text{elements} > x\} < 3n/4 = 6.75$

(A) Let's Choose the good pivots

5 → elements $< 5 = \{1,4,3,2,1,3\}$ which are $6 < 6.75$

elements $> 5 = \{6,7\}$ which are $2 < 6.75$

∴ 5 is a good pivot.

1 → elements $< 1 = \{\}$, $0 < 6.75$

elements $> 1 = \{5,4,3,6,2,7,3\}$, $7 > 6.75$

∴ 1 is not a good Pivot

we continue like this & found out

$\{2,3,3,4,5\}$ are good Pivots.

(B) Yes, in our case $\frac{n}{2} = 4.5$ & 5 elements are a good pivot

Q3. ALGORITHM

$T(n) \leftarrow$ Algorithm FindElement (A, lower, Upper)

Inputs: A - a sorted array with distinct number
lower and upper are integer indicating the lower and upper bound of array A we are working with.

OUTPUT: True or False to indicate if the array contain the element m such that A[m] = m.

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1 — IF lower > upper then
        return False
3 — mid ← (lower + upper)/2
2 — IF (A[mid] == mid) then
        Return True
    else if (A[mid] < mid) then
T(n/2) ← return FindElement(A, mid+1, upper)
    Else
        return FindElement (A, lower, mid-1)
```

$$T(n) = T(n/2) + 6 = T(n/4) + 6 + 6$$

$$= T\left(\frac{n}{8}\right) + 6 + 6 + 6 \cdots = T\left(\frac{n}{2^3}\right) + 3 \times 6$$

$$\vdots$$

$$= T\left(\frac{n}{2^K}\right) + K \cdot 6 \quad \text{Since } \frac{n}{2^K} = 1 \text{ when } 2^K = n \equiv K = \log n$$

$$= T\left(\frac{n}{n}\right) + 6\log n \equiv T(n) = 6\log n + 1 = O(\log n)$$

Since $f(n) = \log n$ and $g(n) = n$ and $\lim_{n} \frac{f(n)}{g(n)} = \frac{f'(n)}{g'(n)} = 0$

$\lim_{n \to \infty} \frac{d}{dn} \frac{\log n}{n} = 0$ therefore the algorith is little o of n $o(n)$

(4) • Choosing the median of the medians (In an unsorted array can be found in linear time) to select pivot, this guarantee to be more fast because it becomes O(n logn) for the worst.

⇒ we find the median first, then partition the array around the median element. by this the worst would be O(n logn).

Q5. The array $A = \{1, 12, 8, 7, -2, -3, 6\}$ and we will use Quick Select to find the median.
In this case the median will be ~~element at index (1+1)/2~~
the 4^{th} element on the list $\therefore$ $\boxed{K=4}$

Rules

1. Pivot $\rightarrow$ Always the leftmost element
2. L $\rightarrow$ All elements less than pivot
3. E $\rightarrow$ All elements equal to pivot
4. G $\rightarrow$ All elements greater than pivot

If $(K \leq L) \rightarrow$ Repeat the process with $A=L$ and same K

If $K > |L|+|E| \rightarrow$ Repeat the process with $A=G$ $K=K-|L|-|E|$

~~If~~ If $|L| < K \leq |L|+|E|$ return any element in E

Taking 1 as a pivot $K=4$ $A=[1, 12, 8, 7, -2, -3, 6]$

$L=[-2,-3]$ $E=[1]$ $G=[12, 8, 7, 6]$

Since $K > |L|+|E|$ i.e $4 > 2+1$, we $A=G=[12, 8, 7, 6]$

and $\boxed{K=4-2-1=1}$ $\boxed{\text{Pivot} = 12}$

$L=[8,7,6]$ $E=[12]$ $G=\{\}$

Since $K=1$ is less than $|L|$ we take L and k i.e

$\boxed{A=[8,7,6] \quad K=1 \quad P=8}$

$L=[7,6]$ $E=[8]$ $G=[\,]$ still $K \leq |L|$

$\therefore A=[7,6]$ Pivot $=7$ $K=1$

$L=[6]$ $E=[7]$ $G=[\,]$ still $K \leq |L|$

$A=[6]$ Pivot 6 $K=1$

$L=[\,]$ $E\ [6]$ $G=[\,]$ now $|L| < K \leq |L|+|E|$

$\therefore$ We consider the value in $E=[6]$ as our median

$\boxed{\Rightarrow \text{The Median is } \underline{\underline{6}}}$