[Yesterday 6:44 AM] Prem Nair

Math Review

Learn Mathematical Induction

Example. Please follow this style and prove the result in Channel Intro and bring to the first day of the class.

Prove by induction 1^2 + 2^2 + ... + n^2 = n(n+1)(2n+1)/6.

Proof.

Step 1. Prove base case(s).

Let n = 1.

Then LHS = 1^2 = 1 and RHS = 1(1+1)(2+1)/6 = 1.

Hence base case is proved.

Step 2. State induction hypothesis.

Assume the result is true for n = k,

1^2 + 2^2 + ... + k^2 = k(k+1)(2k+1)/6

Step 3. Induction step. **//This is the important step. Doing step 1 and 2 will get you 0 credit in the exam.**

Prove the result for n = k+1.

//At this point write down the LHS for n = k+1

LHS = 1^2 + 2^2 + ... + k^2 + (k+1)^2 **//25% credit**

//Use induction hypothesis

LHS = k(k+1)(2k+1)/6 + (k+1)^2 **//50% credit**

//Simplify LHS

k(k+1)(2k+1)/6 + (k+1)^2 = (k+1)[k(2k+1)/6 + (k+1)]

= (k+1)[2k^2 + k + 6k + 6]/6 **//70% credit**

= (k+1)[2k^2 + 7k + 6]/6 **//90% credit**

= (k+1)(k+2)(2k+3)/6 = RHS **//100% credit**

Note:

Step 3. Induction step. **//This is the important step. Doing step 1 and 2 will get you 0 credit in the exam.**

Prove the result for n = k+1.

//At this point write down the LHS for n = k+1

LHS = 1^2 + 2^2 + ... + k^2 + (k+1)^2 **//25% credit**

//Use induction hypothesis

LHS = k(k+1)(2k+1)/6 + (k+1)^2 **//50% credit**

//Simplify LHS

k(k+1)(2k+1)/6 + (k+1)^2 = RHS **//You never proved the result, You will get 50%**

[Yesterday 6:46 AM] Prem Nair

One More Example of induction

Theorem. For all $n \geq 0$, $n^5 - n$ is divisible by 10.

Proof.

Base cases $n = 0$ and $n = 1$. (we need two base cases. You will see the reason. Keep reading!)

Clearly, the result holds.

Induction hypothesis.

Assume the result is true for all values of n in the interval [0, m].

$n^5 - n$ is divisible by 10 for $0 \leq n \leq m$.

Induction step.

We need to prove the result for the next value. That is, we need to

prove that $(m + 1)^5 - (m + 1)$ is divisible by 10.

$(m + 1)^5 - (m + 1) = [(m - 1) + 2]^5 - [(m - 1) + 2]$

$= (m - 1)^5 + \mathbf{10\,(m - 1)^4 + 40\,(m - 1)^3 + 80\,(m - 1)^2 + 80\,(m - 1)} + \mathbf{32} - (m + 1) - \mathbf{2}$

(everything in bold is a multiple of 10)

$= (m - 1)^5 - (m - 1) +$ **10 (polynomial in m).**

By induction hypothesis, $(m - 1)^5 - (m - 1)$ is dividable by 10. Hence the proof.

(Since we are using m - 1 and not m, we need two base cases.)

Problems

Problem 1: Prove by induction. 1^3 + 2^3 + ...+ n^3 = [n(n+1)/2]^2
Problem 2: What is the sum? 7 + 12 + 17 + ...+ 1087. You must use correct formula.
Problem 3: What is the sum? 1 + 1/3 + 1/9 + 1/27 + ... You must use correct formula.
Problem 4: Simplify the expression x - 3y/8 + 5z/7 into a fraction. That is expression_1/expression_2 form.
Problem 5: What is log 3 to the base 9. Can you find it without a calculator.