Prof. Emdad Khan

September 2019
Lab#10

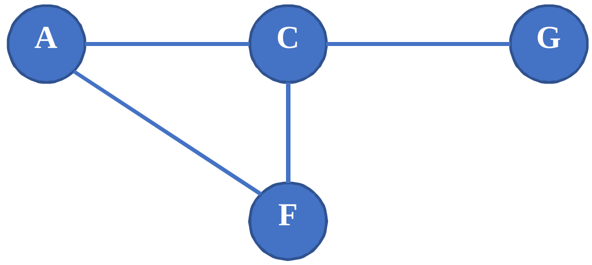
Group 1

Group members:
Asad Ali Kanwal
Aser Ahmad Ibrahim Ahmad
Jean Wilbert Volcy
Zayed Hassan

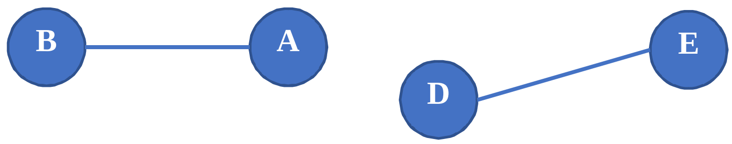
1. Problem 1

A. $U = \{A, B\}$:

B. $W = \{A, C, G, F\}$:



C. $Y = \{A, B, D, E\}$:



D. The subset $X = \{A, B, F\}$ satisfies the conditions: $X \subseteq V$ and $H = G\{X\}$.

E. The following two sets (R & S) of vertices satisfy the described conditions:
$R = G[V_1] = \{D, E, I\}$, and
$S = G[V_2] = \{A, B, F, C, G, H\}$.

2. Problem 2

   The implementation is in the submitted folder.

   A. Please check `Graph.java → pathExists` method.
   B. Please check `NumComponents.java`.
   C. Please check `HasCycle.java`.

3. Problem 3

A. The graph G contains n vertices and $m = n - 1$ edges. It contains subsets $V_1 \ldots V_k$ with $n_1 \ldots n_k$ vertices and $m_1 \ldots m_k$ edges, where:

$\sum_{i=1}^{k} m_i = \sum_{i=1}^{k} (n_i - 1) = n - k$

But since G is connected and simple (given), then we have a missing number of edges, let it be $m' = m - \sum_{i=1}^{k} m_i = n - 1 - n + k = k - 1$.

Also, since G is connected, then $k - 1$ edges must connect k vertices. Each of which must belong to a different subgraph. That is, edge (i) must connect vertex from V_i to a vertex from V_{i+1}.

B. Consider the graph below:

It is connected, and it contains $\varepsilon = 3$ edges and $\nu = 4$ vertices.

Therefore, $\binom{\nu-1}{2} = \frac{(\upsilon-1)(\upsilon-2)}{2} = \frac{(3)(2)}{2} = 3 = \varepsilon$

So, the mentioned inequality does not hold for all connected graphs.

Alternatively, it should be stated that if the mentioned inequality holds, then the checked graph is CERTAINLY connected, but not the other way around.

C. A graph with two vertices needs minimum of 1 edge to be connected.

A graph with three vertices needs minimum of 2 edge to be connected.

A graph with n vertices needs minimum of $n - 1$ edge to be connected.

4. It is proven that if a graph G is a connected tree, then $m = n - 1$. If $m > n - 1$ then it has at least one cycle.
   If G is not connected, then it contains $G_1, G_2, \ldots G_k$ components with $m_1, m_2, \ldots m_k$ edges and $n_1, n_2, \ldots n_k$ vertices. If any component G_i has $m > n - 1$ edges, then it contains a cycle. Consequently, graph G contains a cycle.

5. Problem 5

Let tree S have number of vertices n_S and edges $m_S = n_S - 1$.
Let tree T have number of vertices n_T and edges $m_T = n_T - 1$.
Let their union with edge (x, y) be U with number of vertices n_U and edges m_U.
To calculate m_U:

$m_U = m_S + m_T + |\{(x, y)\}|$

$m_U = m_S + m_T + 1$

$m_U = n_S - 1 + n_T - 1 + 1$

$\therefore m_U = n_S + n_T - 1 \rightarrow (1)$

$Since\ n_U = n_S + n_T \rightarrow (2)$

$From\ (1)\ and\ (2){:}\ m_U = n_U - 1$

$\therefore$ U is a tree.

6. Problem 6

   The implementation is in the submitted folder. Please check `ShortPathLength.java`.

7. False.
   A disconnected graph *can* be dense. Proof:
   A disconnected graph G could have components g_1, g_2,… , g_k.
   Since it is possible that all components are complete graphs, then their total number of edges is:
   $$m_G = \sum_{i=1}^{k} m_i = \sum_{i=1}^{k} \frac{n_i(n_i - 1)}{2}$$
   $\therefore m_G \text{ is } O(n^2). \text{ That is, } G \text{ can be dense.}$