Review For Final

Structure of the Exam. You will have 2 ½ hours for the exam.

1. True/False and Multiple Choice
2. Very Short Answer
3. Short Answer
4. SCI (3 points)

Review Points

1. *Defintions* Be familiar with definitions of the following concepts

- heap
- red-black tree
- vertex cover
- Hamiltonian graph / Hamiltonian cycle
- connected graph
- simple cycle in a graph
- sparse graph / dense graph
- tree (as a graph)
- P = NP

2. Be familiar with the formulas related to red-black tree (discussed in class): If *T* is a red-black tree with *n* nodes, height *h*, and root *r*, then the following hold true for *T*:

$$\begin{aligned} \mathrm{bh}(r) &\geq \frac{h}{2} \\ n &\geq 2^{\mathrm{bh}(r)} - 1 \\ h &\leq 2\log(n+1) \end{aligned}$$

3. Be able to carry out the steps of the insertion algorithm for red-black trees.

4. Be able to do HeapSort by hand, clearly indicating steps for Phase I and Phase II.

5. Know the brute force algorithm for determining the smallest size of a vertex cover for a graph. Know also the VertexCoverApprox algorithm and be able to explain why its running time is polynomial.

6. Know in detail the BFS algorithm and how it can be used for various tasks (spanning tree/forest, shortest path, num components, connected graph, existence of path between two given vertices)

7. Be able to do simple reasoning about graphs. Know the following results (you do not need to know the proofs).

i. A graph *G* is **connected** if for any two vertices *u*, *v* in *G*, there is a path from *u* to *v*.

ii. **Theorem.** For any graph *G*, if

$$\epsilon > \binom{\nu - 1}{2}$$

then *G* is connected.

iii. **Theorem.** If *G* is a tree, then **$\varepsilon = \nu - 1$.**

Exercise. Suppose G is a graph with $\nu - 1$ edges. Show that the following are equivalent:

a. G is connected
b. G is acyclic
c. G is a tree

iv. **Theorem**. If $\varepsilon < \nu - 1$, then G is not connected.

v. **Theorem**. If $\varepsilon \geq \nu$, G contains a cycle.

vi. **Ore's Theorem**. If in a connected graph G with n vertices (n > 2), the sum of the degrees of nonadjacent vertices is always no smaller than n, then G is Hamiltonian.

8. Know how to prove that the running time of insertion, deletion and search in a heap is O(log n)

9. Be able to carry out the steps of Kruskal's algorithm

10. Know how to do the steps of slow version of Dijkstra.

11. Be able to verify that a decision problem belongs to *NP*.

12. Be able to establish that a decision problem is NP-complete, given that some other (related) problem is already known to be NP-complete (as in the slides).

13. Be familiar with the results about the SubsetSum problem.

14. Be able to illustrate with an example the fact that the HamiltonianCycle problem is polynomial reducible to TSP (as in the slides and Lab 12)

15. Be familiar with the graph package that was developed in the class. You may be asked to implement a method in the Graph class by devising your own subclass of BreadthFirstSearch.

16. Be able to explain why the usual IsPrime algorithm is in reality exponential.

17. NOTE: You will not be required to reproduce any proofs