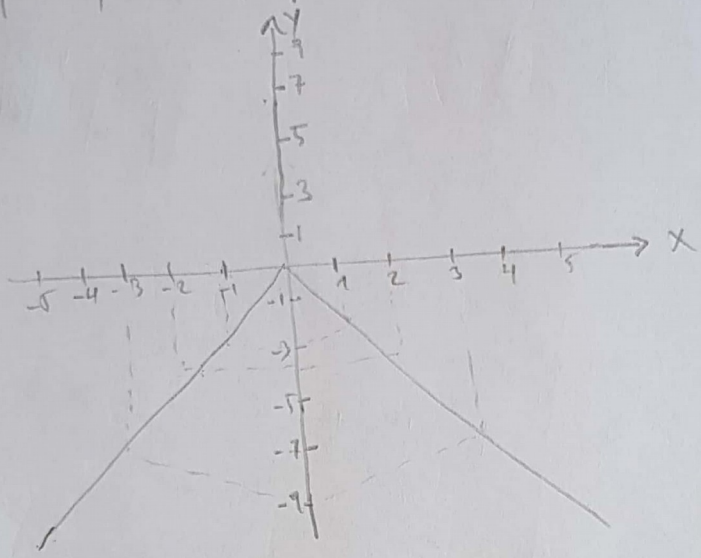
Mark

Problem 1

(1)

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| y | -9 | -4 | -1 | 0 | -1 | -4 | -9 |

// brute force solution



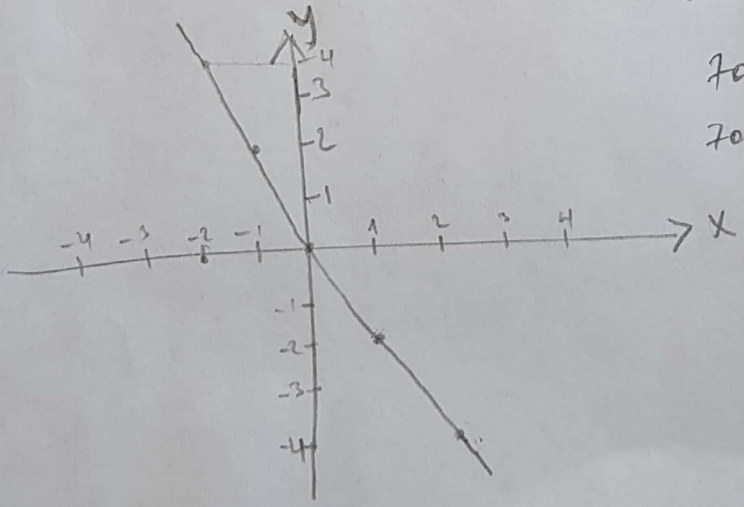

-1 -3
-2 -4
-3 -9

Using proof of Derivatives

$$f'(x) = \frac{dy}{dx} x^2$$

$$= -2x$$

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 4 | 2 | 0 | -2 | -4 |




for $x < 0$, increasing

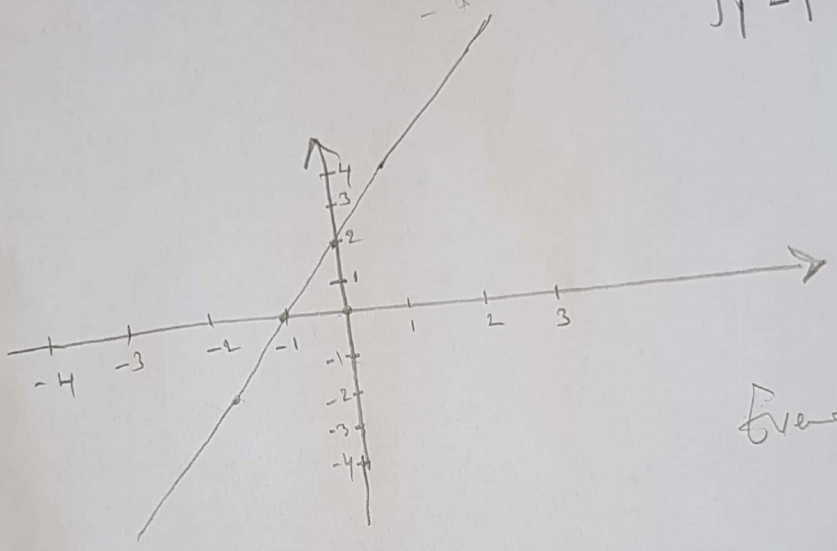
for $x > 0$, decreasing

(2) $f(x) = x^2 + 2x + 1$

$f'(x) = \frac{dy}{dx}x^2 + \frac{dy}{dx}2x + \frac{dy}{dx}$

$f'(x) = 2x + 2$

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | -2 | 0 | 2 | 4 | 6 |

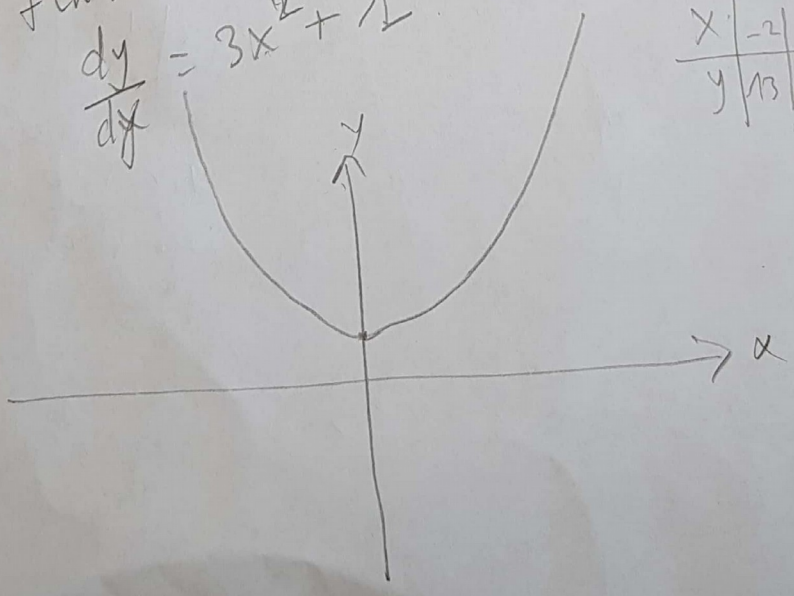


Eventually Non Decreasing.

(3) $f(x) = x^3 + x$

$\frac{dy}{dx} = 3x^2 + 1$

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 13 | 4 | 1 | 4 | 13 |



Increasing.

Mark

problem 2.

① $f(x) = 2x^2$

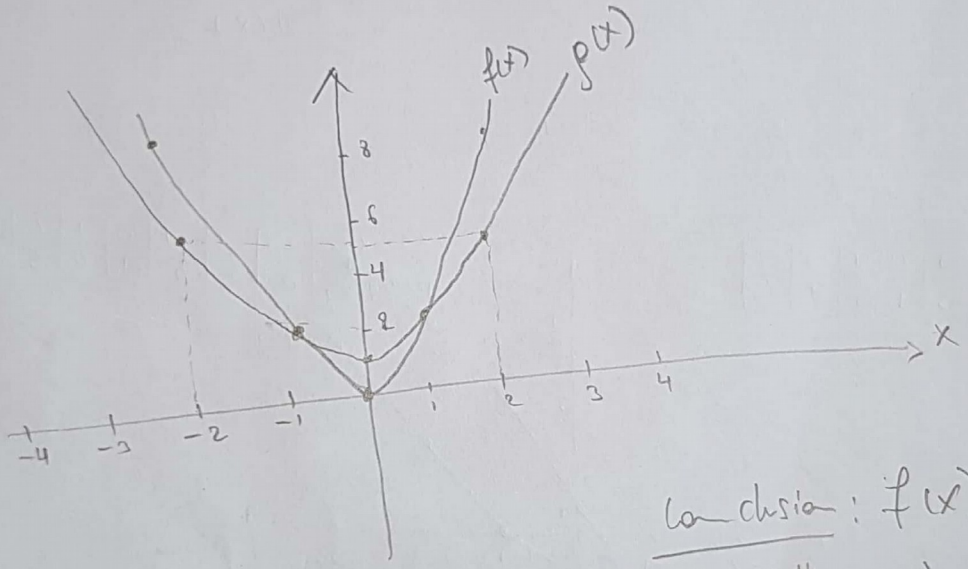
$g(x) = x^2 + 1$

$f(x)$:

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 8 | 2 | 0 | 2 | 8 |

$g(x)$:

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 5 | 2 | 1 | 2 | 5 |

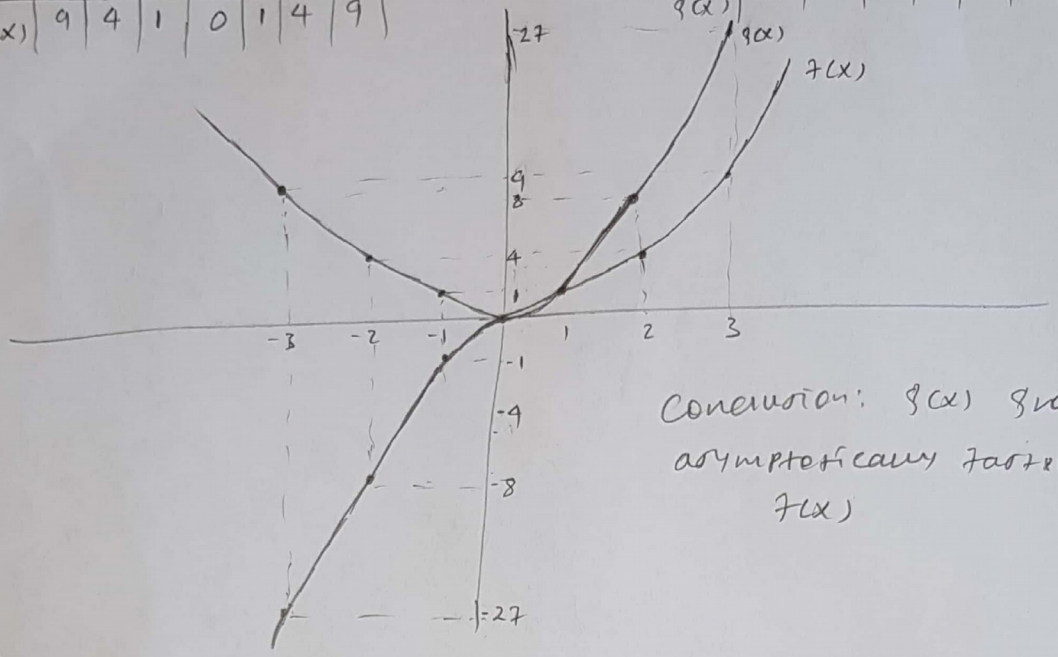



Conclusion: $f(x)$ is asymptotically growing faster than $g(x)$.

(2)

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 9 | 4 | 1 | 0 | 1 | 4 | 9 |

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $g(x)$ | -27 | -8 | -1 | 0 | 1 | 8 | 27 |

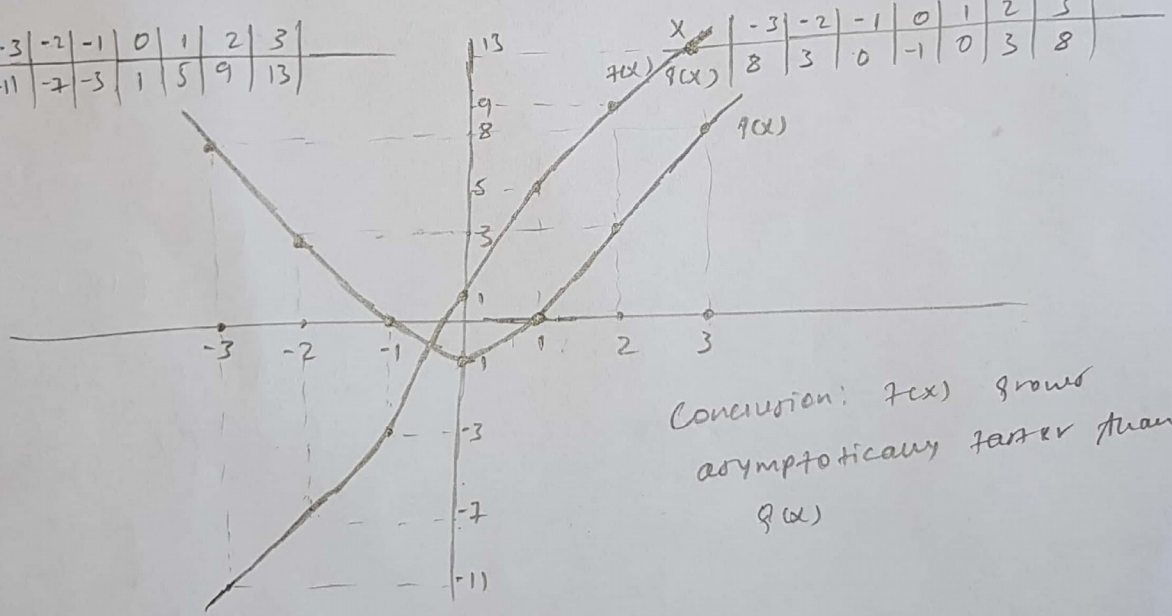



Conclusion: $g(x)$ grows asymptotically faster than $f(x)$

(3)

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | -11 | -7 | -3 | 1 | 5 | 9 | 13 |

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $g(x)$ | 8 | 3 | 0 | -1 | 0 | 3 | 8 |




Conclusion: $f(x)$ grows asymptotically faster than $g(x)$