Prof. Emdad Khan

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Lab#9

Group 1

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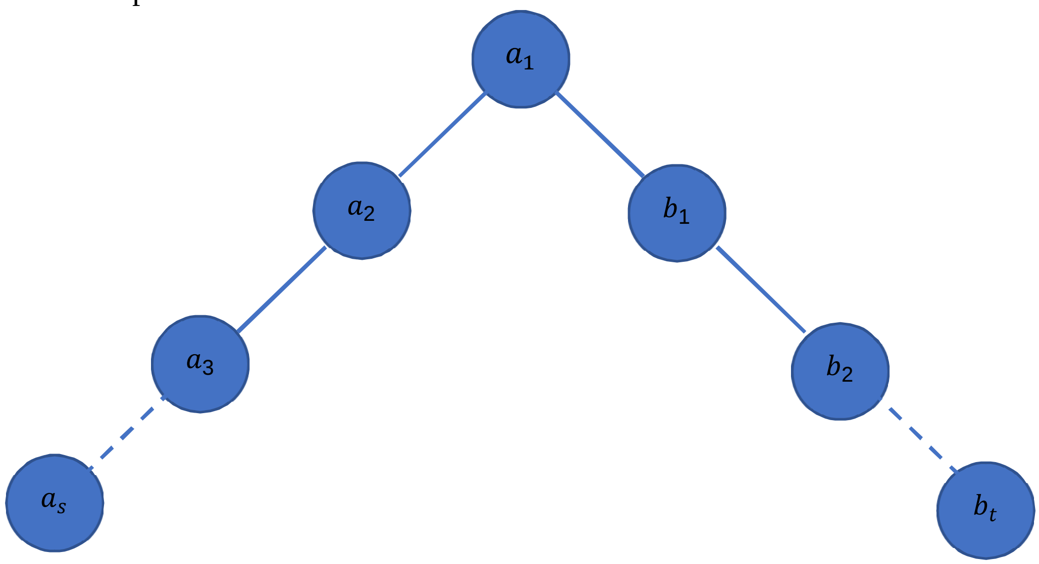
1. Problem 1

A BST with the described sequence has the all a_i elements as left branches of their roots, and all b_i elements as right branches of their roots. Also, element a_1 is its root. The tree should look as the shape below:



Since $s = t \; and \; s + t = n$

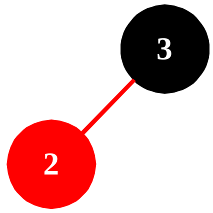
$\therefore 2s = 2t = n \rightarrow s = t = \frac{n}{2}$

And since any search will lead to going through exactly one branch of the two above (if the searched value is < a_1 the search will be limited to the left branch, and otherwise limited to the right branch), then the search and insertions will be similar to applying a search/insertion to a linked list, which has a worst case running time of $O(n)$.

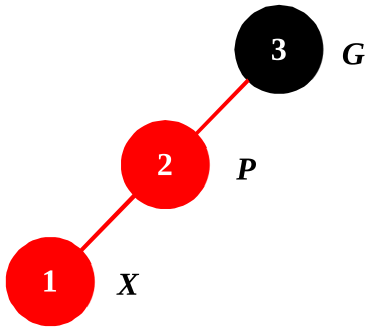
Since the length of any branch above is equal to $\frac{n}{2}$, then worst case running time is $O(\frac{n}{2})$, which is also $O(n)$.

2. Problem 2: Insertion for red-black tree for values: 3, 2, 1, 4, 5, 6.

a. Insert 3, 2:

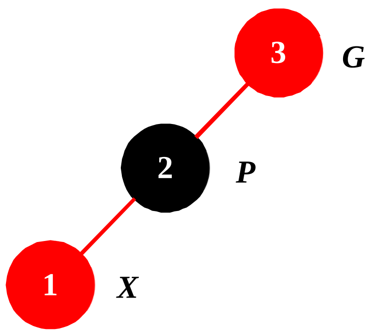


b. Insert 1:

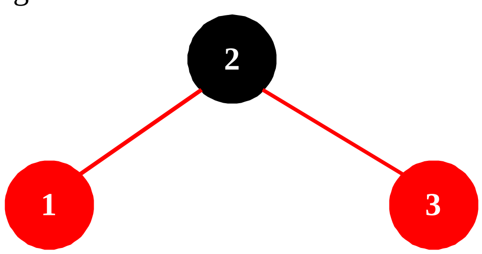


Red-red violation due to an outer grandchild.

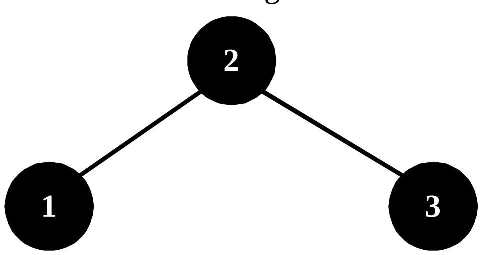
- Change color of P & G:



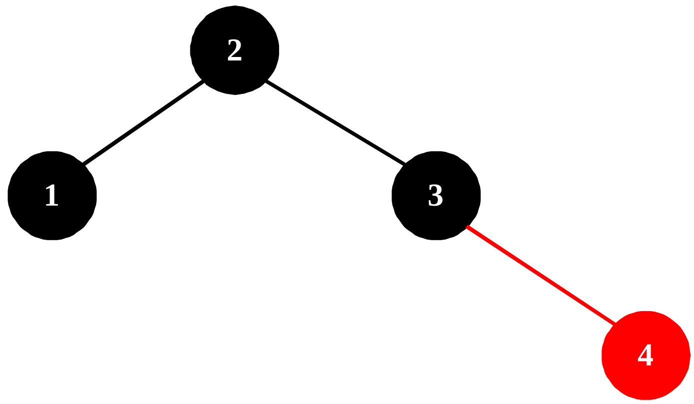
- Rotate P & G, lifting X:



- Parent node 2 has both children red. Change colors of 1 & 3:

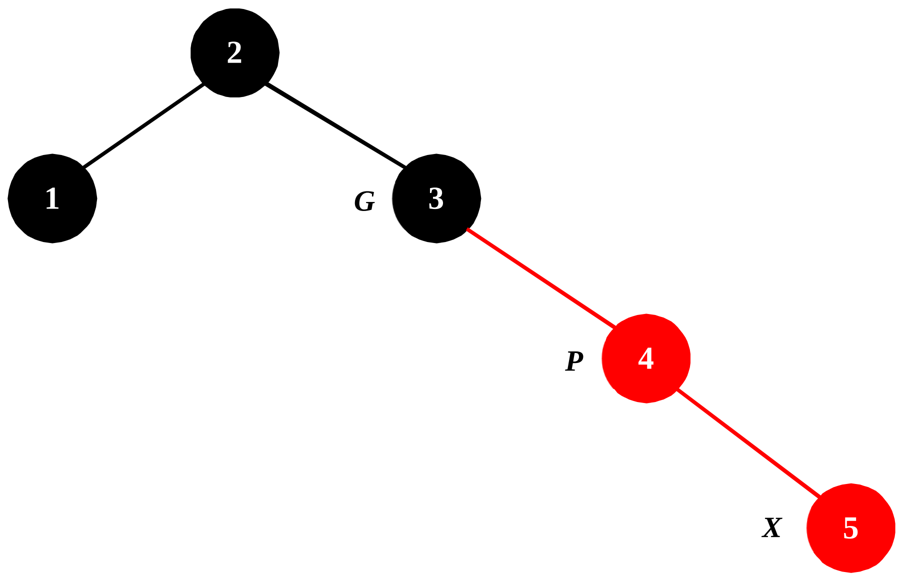


c. Insert 4:



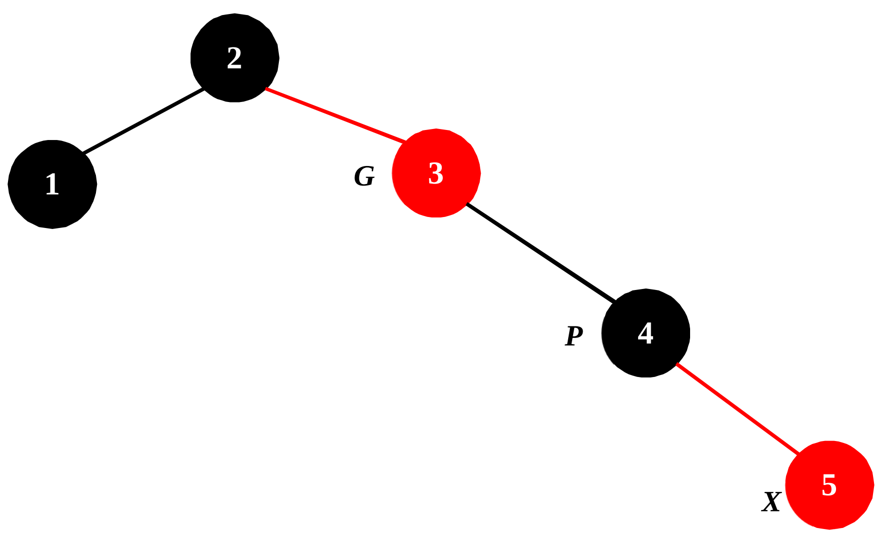
- All black-red rules check out. Continue.

d. Insert 5:

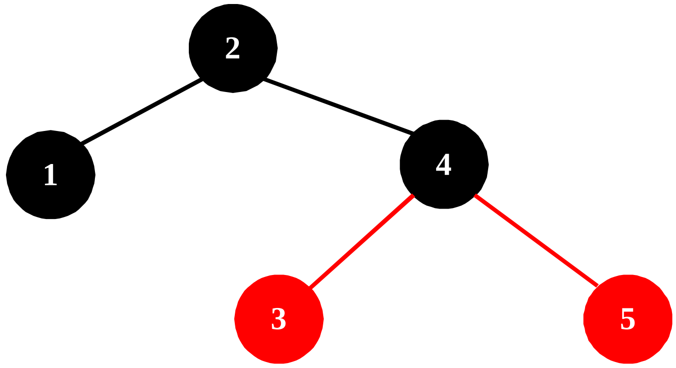


Red-red violation due to an outer grandchild:

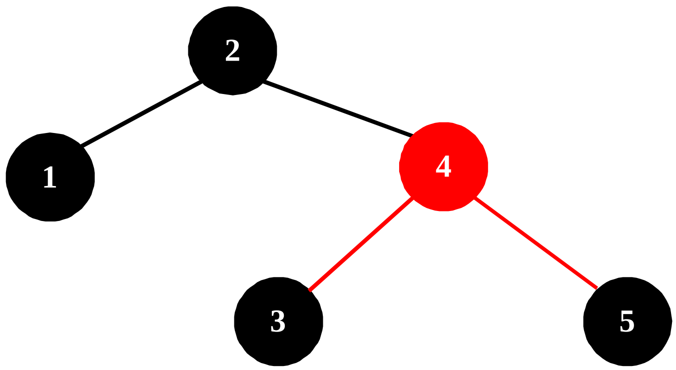
- Change color of P & G:



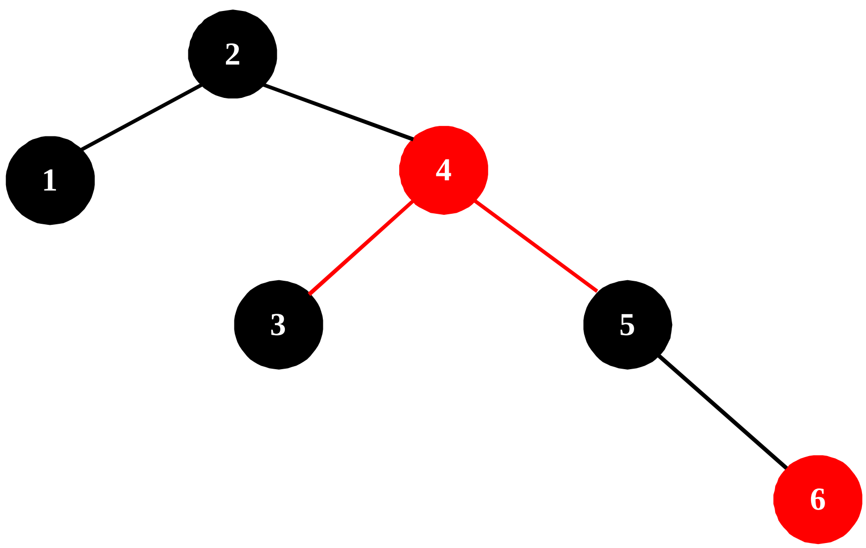
- Rotate G & P, lifting X:



- All children of 4 are red. Flip colors of 4, 3, 5:



e. Insert 6:



- All black-red rules check out. Insertion ended.

3. Problem 3: array-based Heap Sort of the array: A = [1, 4, 3, 9, 12, 2, 4]:

Phase I: Heapification:

1	4	3	9	12	2	4
0	*1*	*2*	*3*	*4*	*5*	*6*

No upheaping needed.

1	4	3	9	12	2	4
0	*1*	*2*	*3*	*4*	*5*	*6*

Swap A[1] = 4 with A[(1-1)/2] = A[0] = 4 and take next element:

4	1	3	9	12	2	4
0	*1*	*2*	*3*	*4*	*5*	*6*

No upheaping needed. Take next element:

4	1	3	9	12	2	4
0	*1*	*2*	*3*	*4*	*5*	*6*

Swap A[3] = 9 with A[(3-1)/2] = A[1] = 1:

4	9	3	1	12	2	4
0	*1*	*2*	*3*	*4*	*5*	*6*

Swap A[1] = 9 with A[(1-1)/2] = A[0] = 4 and take next element:

9	4	3	1	12	2	4
0	*1*	*2*	*3*	*4*	*5*	*6*

Swap A[4] with A[(4-1)/2] = A[1] = 4:

9	12	3	1	4	2	4
0	*1*	*2*	*3*	*4*	*5*	*6*

Swap A[1] = 12 with A[(1-1)/2] = A[0] = 9 and take next element:

12	9	3	1	4	2	4
0	*1*	*2*	*3*	*4*	*5*	*6*

No upheaping needed. Take next element:

12	9	3	1	4	2	4
0	*1*	*2*	*3*	*4*	*5*	*6*

Swap A[6] = 4 with A[(6-1)/2)] = A[2] = 3:

12	9	4	1	4	2	3
0	*1*	*2*	*3*	*4*	*5*	*6*

No more upheaping is needed.

Phase II: In-place sorting:

Take out A[0] = 12 and replace it with A[n-1] = A[6] = 3:

$temp$ = 12

3	9	4	1	4	2	__
0	*1*	*2*	*3*	*4*	*5*	*6*

Downheaping A[0] = 3:

Swap A[0] = 3 with the largest child, A[(2*0)+1] = A[1] = 9:

$temp$ = 12

9	3	4	1	4	2	__
0	*1*	*2*	*3*	*4*	*5*	*6*

Swap A[1] = 3 with the largest child, A[(2*1)+2] = A[4] = 4:

$temp$ = 12

9	4	4	1	3	2	__
0	*1*	*2*	*3*	*4*	*5*	*6*

No more downheaping is possible. Let temp = A[0] = 9 and let A[0] = 2:

$temp$ = 9

2	4	4	1	3	__	12
0	*1*	*2*	*3*	*4*	*5*	*6*

Swap A[0] = 2 with the largest child, A[(2*0)+1] = A[1] = 4:

$temp$ = 9

4	2	4	1	3	__	12
0	*1*	*2*	*3*	*4*	*5*	*6*

Swap A[1] = 2 with the largest child, A[(2*1)+2] = A[4] = 3:

$temp = 9$

4	3	4	1	2	__	12
0	*1*	*2*	*3*	*4*	*5*	*6*

No more downheaping possible. Let A[5] = temp = 9, let temp = A[0] = 4, let A[0] = A[4] = 2:

$temp = 4$

2	3	4	1	__	9	12
0	*1*	*2*	*3*	*4*	*5*	*6*

Swap A[0] = 2 with its largest child, A[(0*2)+2] = A[2] = 4

$temp = 4$

4	3	2	1	__	9	12
0	*1*	*2*	*3*	*4*	*5*	*6*

No more downheaping possible. Let A[4] = temp = 4, temp = A[0] = 4, A[0] = A[3] = 1:

$temp = 4$

1	3	2	__	4	9	12
0	*1*	*2*	*3*	*4*	*5*	*6*

Swap A[0] with its largest child, A[0*2+1] = A[1] = 3:

$temp = 4$

3	1	2	__	4	9	12
0	*1*	*2*	*3*	*4*	*5*	*6*

No more downheaping is possible. Let A[3] = temp = 4, temp = A[0] = 3, A[0] = A[2] = 2:

$temp = 3$

2	1	__	4	4	9	12
0	*1*	*2*	*3*	*4*	*5*	*6*

No more downheaping is possible. Let A[2] = temp = 3, temp = A[0] = 2, A[0] = A[1] = 1:

$temp = 2$

1	__	3	4	4	9	12
0	*1*	*2*	*3*	*4*	*5*	*6*

No more downheaping is possible. Let A[1] = temp = 2. Since only one element is left, add it to the sorted array

1	2	3	4	4	9	12
0	*1*	*2*	*3*	*4*	*5*	*6*

Sorted array is: [1, 2, 3, 4, 4, 9, 12].

4. Problem 4:

Algorithm subsetSum (S, k)

Input: Set S of positive integers, positive integer k

Output: true if there is a subset of S whose sum is k; false, otherwise.

sum = 0

for (i = 0 to S.size – 1) ***do***

    for (j = 0 to i) ***do***

        if (sum + S[j] ≤ k) ***then*** sum += S[j]

        if (sum = k) ***then*** return true

return false