Prof. Emdad Khan

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Lab#12

Group 1

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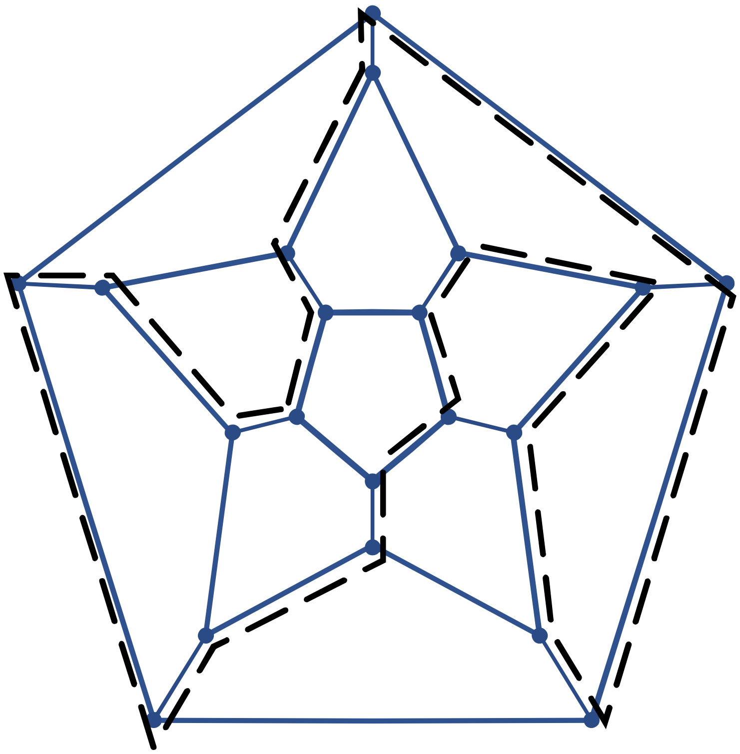
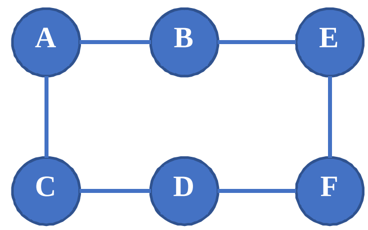
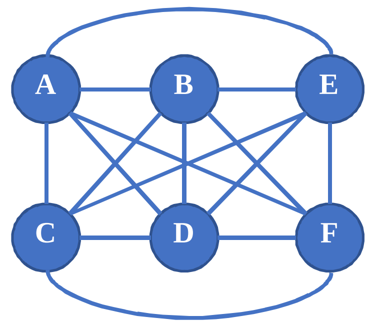
1. Problem 1
   Yes, the graph has a Hamiltonian cycle. A spanning simple cycle is shown below:



2. Problem 2





- Given the graph shown (top), $G = (V, E)$, which has 6 vertices and 6 edges.
    - G is also a subgraph of K_6, the complete graph having 15 edges (bottom graph).
- We obtain an instance (H, c, k) of the TSP, such that:
    - $H = K_6$ (bottom graph).
    - $c(e) = \begin{cases} 0 & if\ e\ \in E \\ 1 & otherwise \end{cases}$
    - $k = 0$
- We note that defining (H, c, k) from G can be done in polynomial time.
- We need to show that :
    - If and only if (H, c, k) has a Hamiltonian cycle with $\sum c(e) = 0 \leq k$, then G has a Hamiltonian cycle.
    - If G has a Hamiltonian cycle C, then C is Hamiltonian in H, too.
- Note that G has a Hamiltonian cycle $C = A - B - E - F - D - C - A$. Also, C is a Hamiltonian cycle in H because every vertex in H is in C and C has no cycles.
- Note that each edge e of C is in G (i.e., $e \in E$ for all edges of C).

  $\therefore c(e) = 0$ for all edges in C, i.e. $\sum c(e) = 0 \leq k \rightarrow (1)$

From (1) we conclude that a solution to the HC problem having input $G = (V, E)$ gives rise to a solution to the TSP problem we defined from G with input data (H, c, k).

- Notice that the cycle $C' = A - B - E - F - D - C - A$ is a Hamiltonian cycle in H and the sum of its edge weights is 0 (which is the value of k).
- By definition of $c(e)$, we know that edge weights are zero only if they belong to E, and since the weights of edges in C' are all equal to 0 (because their sum is zero), then C' is a Hamiltonian cycle in G. $\rightarrow$ (2)
- From (1) and (2) we conclude that the Hamiltonian cycle problem is polynomial reducible to the TSP problem.

3. Problem 3

To show that TSP is NP-complete we need to prove the following:

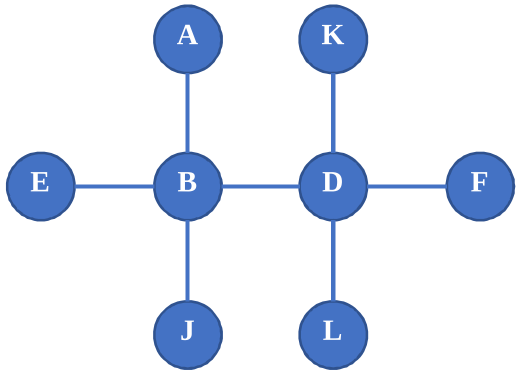
1. TSP is NP-problem:
   A solution to the TSP can be verified using the following algorithm:

Algorithm Verify_TSP_Solution	*Running time*
Input: Graph G = V, E, a path C, a non-negative constant k	
Output: *true* if C is a cycle and sum of its edges cost is ≤ k , *false* otherwise	
for every vertex v_C in V_C do	
calculate deg(v_C)	*O(m.n)*
if (deg(v_C) != 2) return *false*	
perform BFS on C, calculate numberOfComponents and sumOfEdgeCost	*O(m + n)*
if (numberOfComponents != 1 || sumOfEdgeCost > k) then return *false*	*O(1)*
return *true*	*O(1)*

   ∴ Running time is $O(m.n) = O(n^3)$ → Polynomial → TSP is NP

2. Since the Hamiltonian cycle problem (which is an NP-problem) is polynomial reducible to TSP, and from No. 1 above: we conclude that the TSP is NP-complete.

4. Problem 4
   Consider the graph, G below:



Graph G = {AB, KD, EB, BD, DF, BJ, DL}.

a. The smallest vertex cover for G is U = {B, D}, ∴ s = |U| = 2.

b. We apply the VertexCoverApprox algorithm to it as follows:

C = new empty set
G still has edges
   ∴ select edge AB
   add vertices A, B to C → C = {A, B}
   remove edges incident to A or B from G → G = {KD, DF, DL}
G still has edges
   ∴ select edge KD
   add vertices K, D to C → C = {A, B, K, D}
   remove edges incident to K or D → G = {}
G has no edges
return C = {A, B, K, D}

Notice that |C| = 4 = 2 * |U| = 2*s. ∴ Applying the VertexCoverApprox algorithm results in size = 2*s.