Prof. Emdad Khan

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Lab#5

Group 1

Group members:

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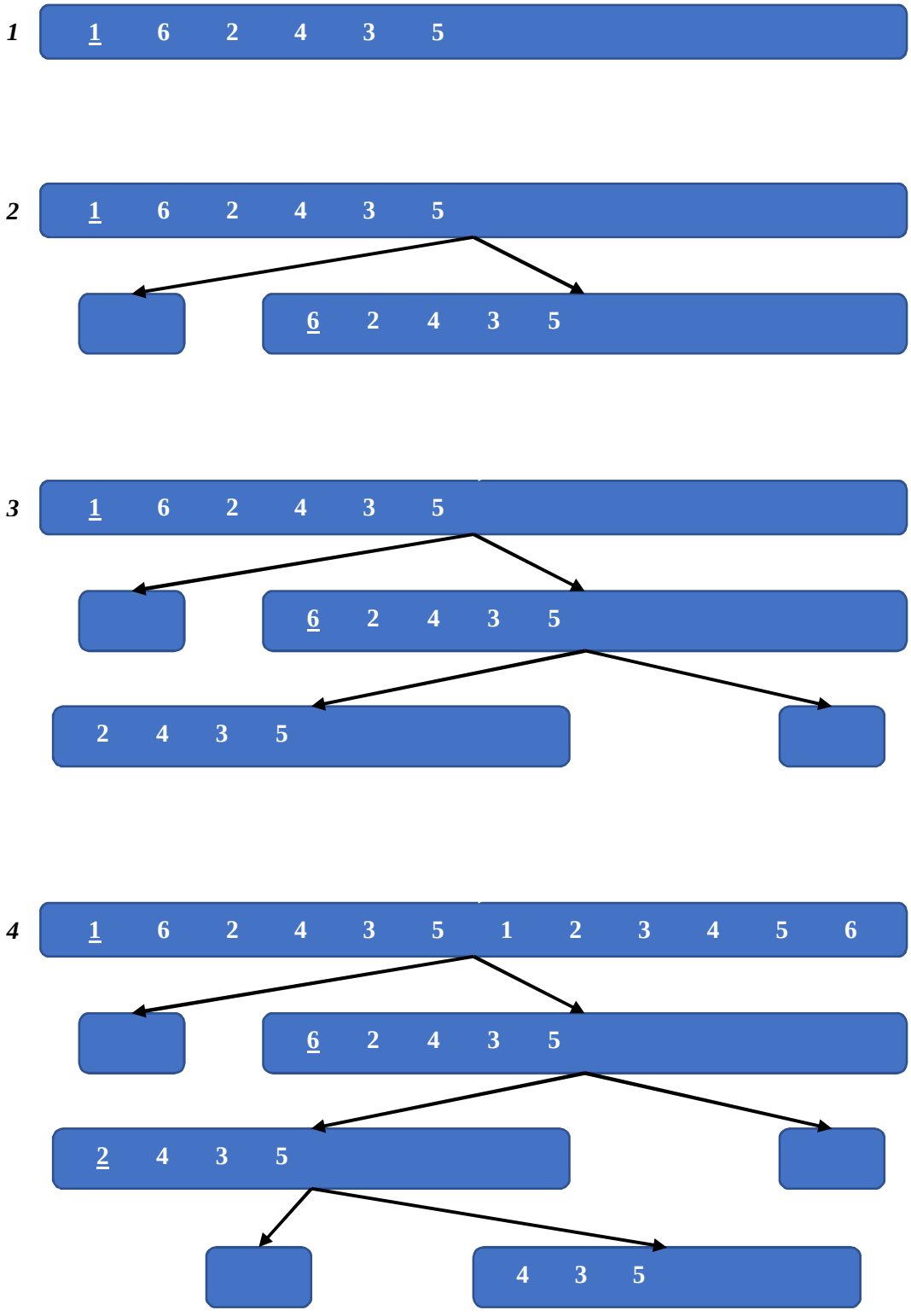
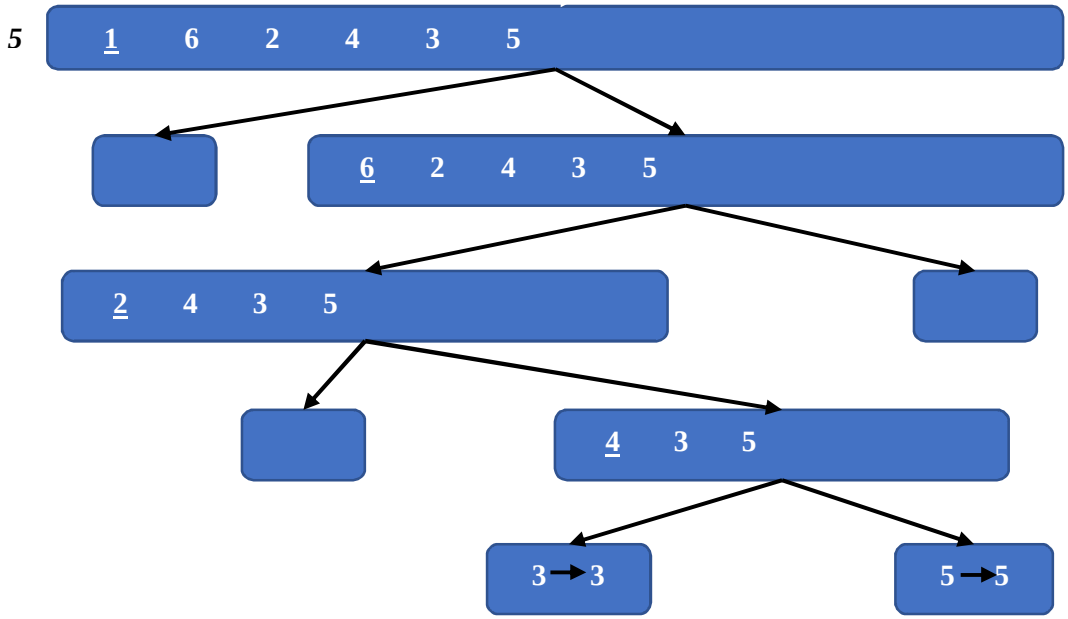
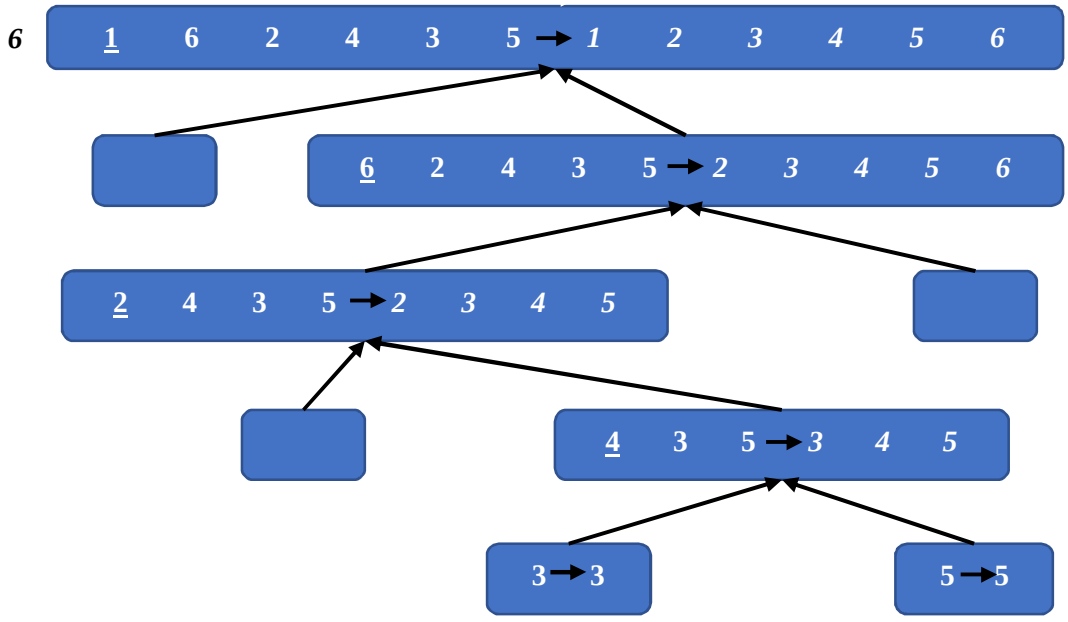
1. Problem 1

Steps of Quick sort for the array $[1, 6, 2, 4, 3, 5]$ with leftmost value as pivot.




5
1 6 2 4 3 5
6 2 4 3 5
2 4 3 5
4 3 5
3 → 3
5 → 5


6
1 6 2 4 3 5 → 1 2 3 4 5 6
6 2 4 3 5 → 2 3 4 5 6
2 4 3 5 → 2 3 4 5
4 3 5 → 3 4 5
3 → 3
5 → 5

2. Problem 2

a. $n = 9 \rightarrow {}^{3n}/_{4} = {}^{27}/_{4} = 6.75$

This table contains the investigation of good and bad elements:

Pivot, x	No. of elements $\geq x$	No. of elements $\leq x$	Comment
5	2	6	Good pivot
1	8	0	Bad pivot
4	3	5	Good pivot
3	4	4	Good pivot
6	1	7	Bad pivot
2	6	2	Good pivot
7	0	8	Bad pivot
1	8	0	Bad pivot
3	4	4	Good pivot

Elements that are good pivots are: $[5, 4, 3, 2, 3]$.

b. $No. of\ good\ pivots = 5 > n/2$

$\therefore$ According to the example above, it is true that at least half of the elements are good pivots.

3. Problem 3

Algorithm findElementEqualToItsIndex (A, start, end)	*Count of operations*
Input: sorted array A, starting position start, ending position end	
Output: true if element A[m] = m is found, false otherwise	
mid = (start + end) / 2	*3*
if (A[mid] = mid) **then**	*3*
return true	*1*
if (A[mid] < mid and start != end) **then**	*3*
return *findElementEqualToItsIndex* (A, mid + 1, end)	*3 + T(n/2)*
if (A[mid] > mid and start != end) **then**	*3*
return *findElementEqualToItsIndex* (A, start, mid)	*2 + T(n/2)*
return false	*1*

$$T(n) = \begin{cases} 7 & if\ n = 1 \\ T\left(\frac{n}{2}\right) + 10 & otherwise \end{cases}$$

$a = 1,\ b = 2,\ c = 10,\ d = 7,\ k = 0$

$a = 1 = b^k = 2^0 = 1 \rightarrow from\ the\ master\ formula: T(n)\ is\ \Theta(n^k \log n)$

$\therefore T(n)\ is\ o(\log n),\ since\ all\ \log n\ functions\ are\ o(n).$

4. Problem 4
   One pivot selection strategy that can be used to guarantee a Quick Sort with running time of $O(n \log n)$ is to use the Quick Select algorithm to select the $(n/2)^{th}$ smallest element, which is the median. The quick select algorithm has an expected running time of $O(n)$, and it guarantees a good pivot is found for the Quick Sort.

5. Problem 5

$Array$: $[1, 12, 8, 7, -2, -3, 6]$

$n = 7, Median\ index = \frac{n}{2} = \left\lceil \frac{7}{2} \right\rceil = 4 \rightarrow k = 4$

1. $Pivot = 1, L = \{-2, -3\}, E = \{1\}, G = \{12, 8, 7, 6\}.$

   Check: $k \leq |L|? \rightarrow k = 4 \not\leq |L| = 2 \rightarrow Not\ found \dots continue$

   Check: $|L| < k \leq |L| + |E|? \rightarrow 2 < 4 \not\leq 2 + 1 \rightarrow Not\ found \dots continue$

   Check: $k > |L| + |E|? \rightarrow 4 > 2 + 1 \rightarrow Complete\ with\ the\ G - array$

2. $k' = k - |L| - |E| = 4 - 2 - 1 = 1$

   $Pivot = 12, L = \{8, 7, 6\}, E = \{12\}, G = \{\}$

   Check: $k \leq |L|? \rightarrow k = 1 \leq |L| = 3 \rightarrow Completewith\ the\ L - array$

   Check: $|L| < k \leq |L| + |E|? \rightarrow 3 \not< 1 \leq 3 + 1 \rightarrow Not\ found \dots continue$

3. $k = 1$

   $Pivot = 8, L = \{7, 6\}, E = \{8\}, G = \{\}$

   Check: $k \leq |L|? \rightarrow k = 1 \leq |L| = 2 \rightarrow Completewith\ the\ L - array$

   Check: $|L| < k \leq |L| + |E|? \rightarrow 2 \not< 1 \leq 2 + 1 \rightarrow Not\ found \dots continue$

4. $k = 1$

   $Pivot = 7, L = \{6\}, E = \{7\}, G = \{\}$

   Check: $k \leq |L|? \rightarrow k = 1 \leq |L| = 1 \rightarrow Completewith\ the\ L - array$

   Check: $|L| < k \leq |L| + |E|? \rightarrow 1 \not< 1 \leq 1 + 1 \rightarrow Not\ found \dots continue$

5. $k = 1$

   $Pivot = 6, L = \{\}, E = \{6\}, G = \{\}$

   Check: $k \leq |L|? \rightarrow k = 1 \not\leq |L| = 0 \rightarrow Not\ found \dots continue$

   Check: $|L| < k \leq |L| + |E|? \rightarrow 0 < 1 \leq 0 + 1 \rightarrow Found!$

   $\therefore Required\ element\ is\ E = \{6\}$