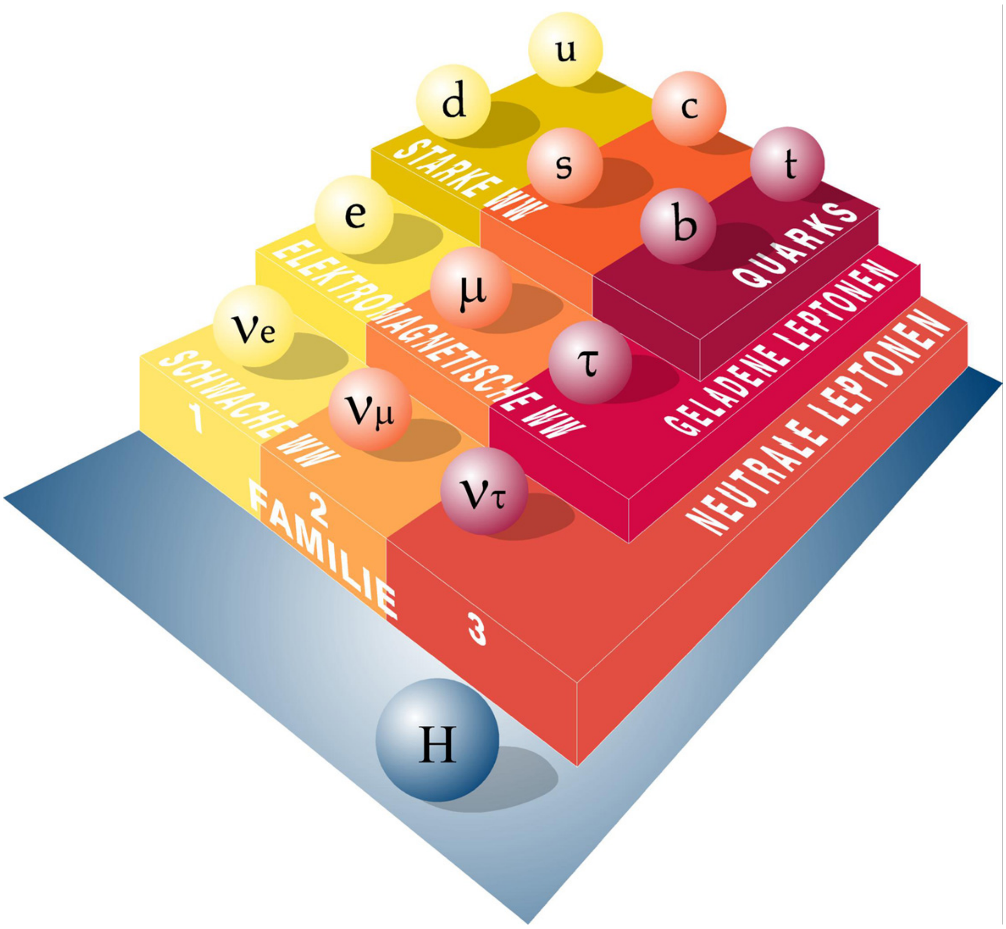
Friedrich-Alexander-Universität
Erlangen-Nürnberg

Transcript of the lecture advanced experimental physics B

Particle and astroparticle physics



following the lecture of Prof. Dr. C. van Eldik

written by Michael Winter

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summer semester 2019 + 2020

Contents

Preface

Before the essential contents of the lectures are presented, some remarks have to be made:
This script – or rather this lecture transcript – was written in the course of the summer semester 2019 accompanying the lecture Advanced experimental physics: Particle and astroparticle physics, read by Prof. Dr. van Eldik. In its peculiarity, this transcript is probably also incorrect and inaccurate.

If you have an overleaf account, you can mark the appropriate places in the code and leave a comment, what needs to be improved. So others can benefit from a correction as well.

In addition, the script is constantly being revised with regard to spelling, structure and content.

I. Recap particle physics

I.1. Units

In general natural units are used: $\hbar = \mathrm{c} = 1$

→ energy, mass and momentum: $[E]$
→ length and time: $[E]^{-1}$
→ cross section: $[E]^{-2}$

> **Example:**
>
> $$\hbar c \approx 200\,\mathrm{MeV\,fm} \mathrel{\hat{=}} 1 \longrightarrow 1\,\mathrm{fm} \mathrel{\hat{\approx}} 5\,\mathrm{GeV}^{-1}$$
> $$\hbar = 6.6 \times 10^{-25}\,\mathrm{GeV\,s} \mathrel{\hat{=}} 1 \longrightarrow 1\,\mathrm{GeV}^{-1} \mathrel{\hat{\approx}} 6.6 \times 10^{-25}\,\mathrm{s}$$

I.2. Relativistic kinematics

We will use four vector notation, i.e.

$$\boldsymbol{A} = \left(A^0, \vec{A}\right) \longrightarrow A^\mu \qquad \text{contravariant} \tag{I.1}$$

$$\left(A^0, -\vec{A}\right) \longrightarrow A_\mu \qquad \text{covariant.} \tag{I.2}$$

Lorentz trafos Remember that the Lorentz transformation $\underline{\underline{\Lambda}}$:

$$A^\mu \mapsto A'^\mu = \Lambda^\mu{}_\nu A^\nu \tag{I.3}$$

The transformation along the z-axis is a Lorentz-boost

$$\Lambda^\mu{}_\nu \to \underline{\underline{\Lambda}} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma/\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma/\beta & 0 & 0 & \gamma \end{pmatrix} \qquad \text{with } \beta = \frac{v}{c},\ \gamma = \frac{1}{\sqrt{1-\beta^2}}\,. \tag{I.4}$$

The Lorentz transformation is a **unitary** transformation: ”rotation in 4-space“

Momentum four vector

$$x^\mu \to \boldsymbol{x} = (t, \vec{x}) \,, \qquad E = \gamma m c^2 = \gamma m, \qquad \vec{p} = \gamma m \vec{v} = \gamma m \vec{\beta} \tag{I.5}$$

So it is easy to see that the momentim vector $\vec{p}$ is measured in units of energy. Thus we may choose the following four vector as momentum four vector (and indeed it is)

$$p^\mu \to \boldsymbol{p} = (E, \vec{p}) \,, \qquad \Rightarrow E^2 = \vec{p}^2 + m^2 \,. \tag{I.6}$$

Basic invariant under Lorentz transformation is the scalar product

$$\boldsymbol{A} \cdot \boldsymbol{B} = A^\mu B_\mu = g_{\mu\nu} A^\mu B^\nu = A_\mu B^\mu = A^0 B^0 - \vec{A}\vec{B} \tag{I.7}$$

$$\text{with } g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \text{ as metric tensor.}$$

> **Example:** scalar product of four momentum $\boldsymbol{p}$
>
> $$p^2 = p^\mu p_\mu = E^2 - \vec{p}^2 = m^2 \,, \tag{I.8}$$
>
> since $E^2 = \vec{p}^2 + m^2$. Therefore the (rest) mass of the particle is the same in all frames of reference.

> **Example:** total energy $\sqrt{s}$ of particle collision
>
> $$\boldsymbol{p}_1 \longrightarrow \qquad \longleftarrow \boldsymbol{p}_2$$
>
> $$s \coloneqq (\boldsymbol{p}_1 + \boldsymbol{p}_2)^2 = (\boldsymbol{p}_1 + \boldsymbol{p}_2)^\mu (\boldsymbol{p}_1 + \boldsymbol{p}_2)_\mu \tag{I.9}$$
>
> is Lorentz invariant since it is a scalar product. Hence it is the same in all reference frames.

Four vector derivatives

$$\partial_\mu = \frac{\partial}{\partial x^\mu} \to \left(\partial_t, \vec{\boldsymbol{\nabla}}\right) \tag{I.10}$$

transforms like a covariant vector. Whereas

$$\partial^\mu = \frac{\partial}{\partial x_\mu} \to \left(t, -\vec{\boldsymbol{\nabla}}\right) \tag{I.11}$$

transforms like a contravariant vector. The ”scalar product“ of these two is the d'Alembert operator

$$\Box \coloneqq \partial_\mu \partial^\mu = \partial_t^2 - \boldsymbol{\Delta} \,, \tag{I.12}$$

with the Laplace operator $\boldsymbol{\Delta} = \vec{\boldsymbol{\nabla}}^2$.

I.3. Elementary particles

Fermions (spin 1/2) Lepton sector:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \qquad \text{with charges} \begin{matrix} Q = 0 \\ Q = -e \end{matrix} \tag{I.13}$$

They come with a certain mass hierarchy $m_e < m_\mu < m_\tau$ and the assumption in the standard model that $m_\nu = 0$ regardless of the neutrino flavour. However we know $m_\nu > 0$ but very small.

Quark sector:

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}, \quad \begin{pmatrix} t \\ b \end{pmatrix} \qquad \text{with charges} \begin{matrix} Q = \frac{2}{3}e \\ Q = -\frac{1}{3}e \end{matrix} \tag{I.14}$$

Free particles are bounded states with $qq'q''$ as baryons and $q\overline{q}$ as mesons. Additionally, only colour-neutral particle states are observed.

Both sectors come with the respective 6 anti-particles.

Bosons (spin 1) exchange particles of interactions

γ: em. interaction, $m = 0$, $Q = 0$ couples to electric charge
Z^0: weak interaction, $m = 91\,\text{GeV}$, $Q = 0$, couples to weak charge
$W^\pm$: weak interaction, $m = 80\,\text{GeV}$, $Q = \pm e$, couples to weak charge
g: strong interaction, $m = 0$, $Q = 0$
8 gluons in total, carry colour charge
couple to quarks and to one another (”self-coupling“)

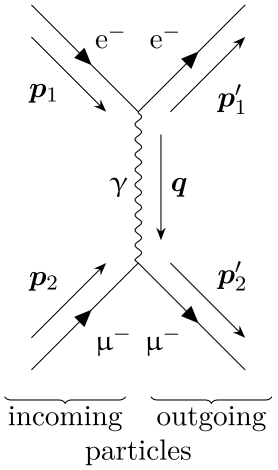
Scalar (spin 0)

H^0: Higgs boson, brings mass to Z^0, $W^\pm$ (”spontaneous symmetry breaking“)
$m = 125\,\text{GeV}$

I.4. Feynman diagrams

These give pictorial representations of particle reactions. Perturbative expansion of scattering ”in a potential“ into leading order and higher order terms, if necessary.

Example: electromagnetic scattering (leading order)



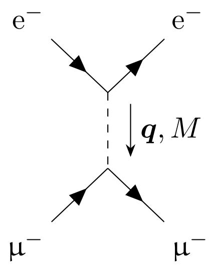
with $\boldsymbol{q}$: 4-momentum transfer

We will describe this in $\boldsymbol{q}^2$ (which is Lorentz invariant)

$$\boldsymbol{q}^2 = (\boldsymbol{p}_1 - \boldsymbol{p}_1')^2 \neq 0 \tag{I.15}$$

The inequity to zero means that it is off mass-shell
⇒ virtual particle because it carries to much/less energy

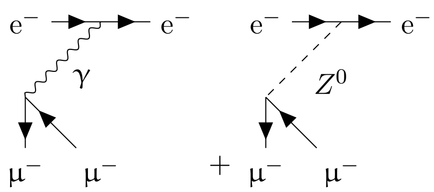
Next question to ask is: What is the **transition amplitude**?



$$\mathit{amplitude} \propto g_1 \frac{1}{\boldsymbol{q}^2 - M^2} g_2 \tag{I.16}$$

with $g_{1,2}$: coupling strength between fermion and exchange particle
$1/\boldsymbol{q}^2 - M^2$: boson propagator for exchange particle of mass M

Here the electromagnetic and the weak interaction contribute, i.e.

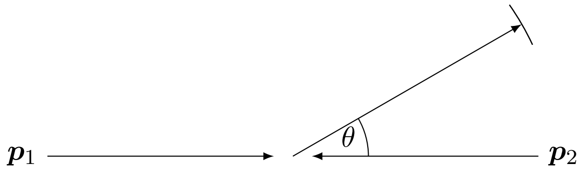


$$= e\frac{1}{\boldsymbol{q}^2}e + g_1 \frac{1}{\boldsymbol{q}^2 - M_Z^2} g_2$$

Last question: What is the **cross section**?
It is proportional to the transition probability

$$\sigma \propto W, \qquad W \propto |\mathit{amplitude}_1 + \mathit{amplitude}_2|^2 \propto \frac{\mathrm{e}^4}{\boldsymbol{q}^4} \tag{I.17}$$

The factor $1/\boldsymbol{q}^4$ leads to a rapid decrease of the cross section as momentum transfer increases.


$\boldsymbol{p}_1$
θ
$\boldsymbol{p}_2$

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