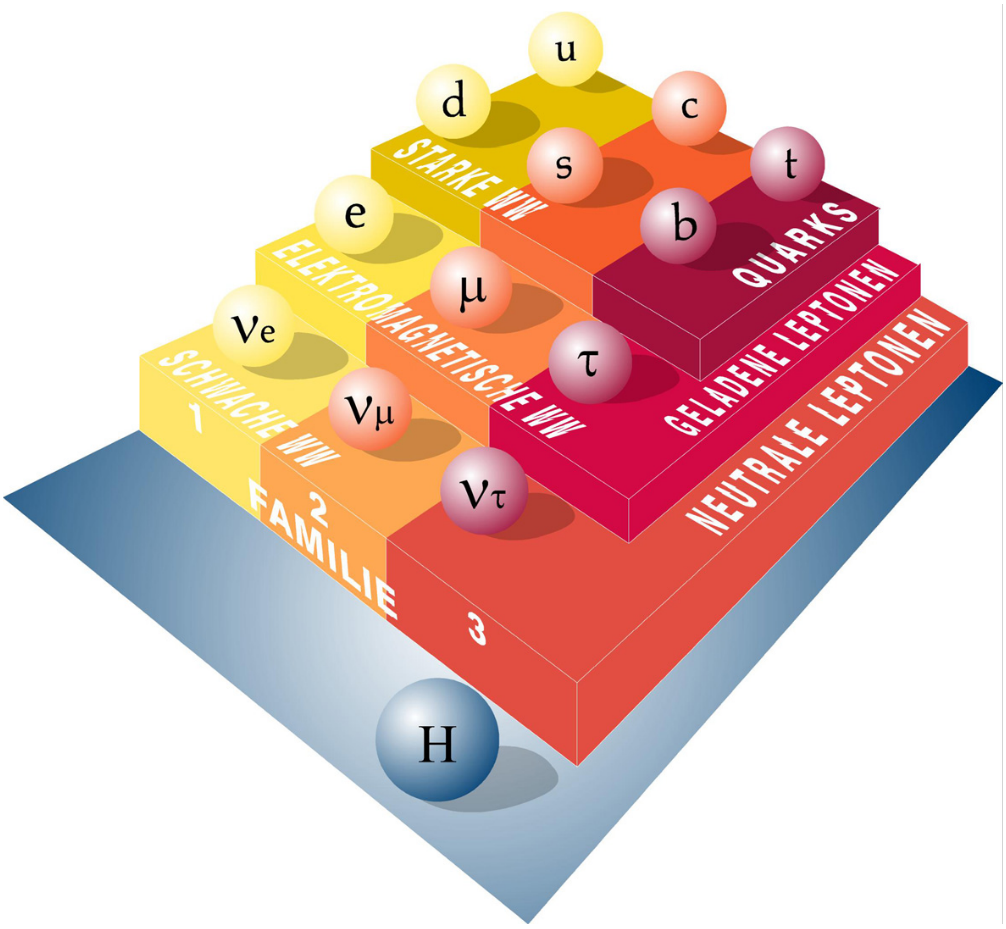
Friedrich-Alexander-Universität Erlangen-Nürnberg

Transcript of the lecture advanced experimental physics B

Particle and astroparticle physics



following the lecture of Prof. Dr. C. van Eldik

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last compiled on 31st March 2020

summer semester 2019 + 2020

Contents

Preface

Before the essential contents of the lectures are presented, some remarks have to be made:
This script – or rather this lecture transcript – was written in the course of the summer semester 2019 accompanying the lecture Advanced experimental physics: Particle and astroparticle physics, read by Prof. Dr. van Eldik. In its peculiarity, this transcript is probably also incorrect and inaccurate.

If you have an overleaf account, you can mark the appropriate places in the code and leave a comment, what needs to be improved. So others can benefit from a correction as well.

In addition, the script is constantly being revised with regard to spelling, structure and content.

I. Recap particle physics

I.1. Units

In general natural units are used: $\hbar = \mathrm{c} = 1$

$\rightarrow$ energy, mass and momentum: $[E]$
$\rightarrow$ length and time: $[E]^{-1}$
$\rightarrow$ cross section: $[E]^{-2}$

> **Example:**
>
> $$\hbar \mathrm{c} \approx 200\,\mathrm{MeV\,fm} \mathrel{\widehat{=}} 1 \longrightarrow 1\,\mathrm{fm} \mathrel{\widehat{\approx}} 5\,\mathrm{GeV}^{-1}$$
> $$\hbar = 6.6 \times 10^{-25}\,\mathrm{GeV\,s} \mathrel{\widehat{=}} 1 \longrightarrow 1\,\mathrm{GeV}^{-1} \mathrel{\widehat{\approx}} 6.6 \times 10^{-25}\,\mathrm{s}$$

I.2. Relativistic kinematics

We will use four vector notation, i.e.

$$\boldsymbol{A} = \left(A^0, \vec{A}\right) \longrightarrow A^\mu \qquad \text{contravariant} \tag{I.1}$$
$$\left(A^0, -\vec{A}\right) \longrightarrow A_\mu \qquad \text{covariant.} \tag{I.2}$$

Lorentz trafos Remember that the Lorentz transformation $\underline{\underline{\Lambda}}$:

$$A^\mu \mapsto A'^\mu = \Lambda^\mu{}_\nu A^\nu \tag{I.3}$$

The transformation along the z-axis is a Lorentz-boost

$$\Lambda^\mu{}_\nu \rightarrow \underline{\underline{\Lambda}} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma/\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma/\beta & 0 & 0 & \gamma \end{pmatrix} \qquad \text{with } \beta = \frac{v}{c},\ \gamma = \frac{1}{\sqrt{1-\beta^2}}\,. \tag{I.4}$$

The Lorentz transformation is a **unitary** transformation: ”rotation in 4-space“

Momentum four vector

$$x^\mu \to \boldsymbol{x} = (t, \vec{x}), \qquad E = \gamma m c^2 = \gamma m, \qquad \vec{p} = \gamma m \vec{v} = \gamma m \vec{\beta} \tag{I.5}$$

So it is easy to see that the momentum vector $\vec{p}$ is measured in units of energy. Thus we may choose the following four vector as momentum four vector (and indeed it is)

$$p^\mu \to \boldsymbol{p} = (E, \vec{p}), \qquad \Rightarrow E^2 = \vec{p}^2 + m^2 \,. \tag{I.6}$$

Basic invariant under Lorentz transformation is the scalar product

$$\boldsymbol{A} \cdot \boldsymbol{B} = A^\mu B_\mu = g_{\mu\nu} A^\mu B^\nu = A_\mu B^\mu = A^0 B^0 - \vec{A}\vec{B} \tag{I.7}$$

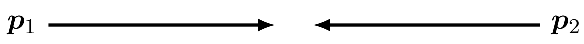
$$\text{with } g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \text{ as metric tensor.}$$

Example: scalar product of four momentum $\boldsymbol{p}$

$$p^2 = p^\mu p_\mu = E^2 - \vec{p}^2 = m^2 \,, \tag{I.8}$$

since $E^2 = \vec{p}^2 + m^2$. Therefore the (rest) mass of the particle is the same in all frames of reference.

Example: total energy $\sqrt{s}$ of particle collision



$$s \coloneqq (\boldsymbol{p}_1 + \boldsymbol{p}_2)^2 = (\boldsymbol{p}_1 + \boldsymbol{p}_2)^\mu (\boldsymbol{p}_1 + \boldsymbol{p}_2)_\mu \tag{I.9}$$

is Lorentz invariant since it is a scalar product. Hence it is the same in all reference frames.

Four vector derivatives

$$\partial_\mu = \frac{\partial}{\partial x^\mu} \to \left(\partial_t, \vec{\boldsymbol{\nabla}}\right) \tag{I.10}$$

transforms like a covariant vector. Whereas

$$\partial^\mu = \frac{\partial}{\partial x_\mu} \to \left(t, -\vec{\boldsymbol{\nabla}}\right) \tag{I.11}$$

transforms like a controvariant vector. The "scalar product" of these two is the d'Alembert operator

$$\Box \coloneqq \partial_\mu \partial^\mu = \partial_t^2 - \boldsymbol{\Delta} \,, \tag{I.12}$$

with the Laplace operator $\boldsymbol{\Delta} = \vec{\boldsymbol{\nabla}}^2$.

I.3. Elementary particles

Fermions (spin 1/2) Lepton sector:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \qquad \text{with charges } \begin{matrix} Q = 0 \\ Q = -e \end{matrix} \tag{I.13}$$

They come with a certain mass hierarchy $m_e < m_\mu < m_\tau$ and the assumption in the standard model that $m_\nu = 0$ regardless of the neutrino flavour. However we know $m_\nu > 0$ but very small.

Quark sector:

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}, \quad \begin{pmatrix} t \\ b \end{pmatrix} \qquad \text{with charges } \begin{matrix} Q = \frac{2}{3}e \\ Q = -\frac{1}{3}e \end{matrix} \tag{I.14}$$

Free particles are bounded states with $qq'q''$ as baryons and $q\bar{q}$ as mesons. Additionally, only colour-neutral particle states are observed.

Both sectors come with the respective 6 anti-particles.

Bosons (spin 1) exchange particles of interactions

γ: em. interaction, $m = 0$, $Q = 0$ couples to electric charge
Z^0: weak interaction, $m = 91\,\text{GeV}$, $Q = 0$, couples to weak charge
$W^\pm$: weak interaction, $m = 80\,\text{GeV}$, $Q = \pm e$, couples to weak charge
g: strong interaction, $m = 0$, $Q = 0$
8 gluons in total, carry colour charge
couple to quarks and to one another ("self-coupling")

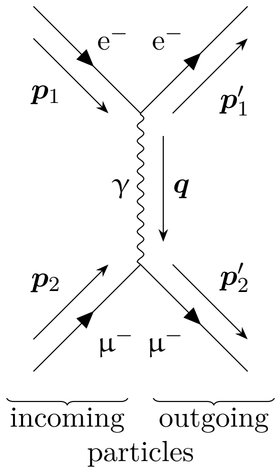
Scalar (spin 0)

H^0: Higgs boson, brings mass to Z^0, $W^\pm$ ("spontaneous symmetry breaking")
$m = 125\,\text{GeV}$

I.4. Feynman diagrams

These give pictorial representations of particle reactions. Perturbative expansion of scattering "in a potential" into leading order and higher order terms, if necessary.

Example: electromagnetic scattering (leading order)



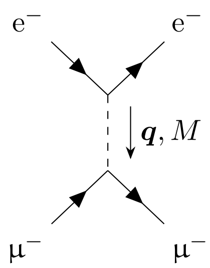
with $\boldsymbol{q}$: 4-momentum transfer

We will describe this in $\boldsymbol{q}^2$ (which is Lorentz invariant)

$$\boldsymbol{q}^2 = (\boldsymbol{p}_1 - \boldsymbol{p}'_1)^2 \neq 0 \tag{I.15}$$

The inequity to zero means that it is off mass-shell
$\Rightarrow$ virtual particle because it carries to much/less energy

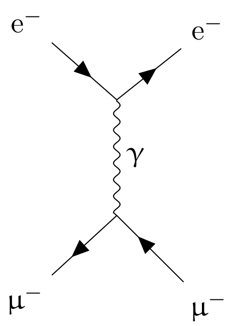
Next question to ask is: What is the **transition amplitude**?



$$amplitude \propto g_1 \frac{1}{\boldsymbol{q}^2 - M^2} g_2 \tag{I.16}$$

with $g_{1,2}$: coupling strength between fermion and exchange particle
$1/\boldsymbol{q}^2 - M^2$: boson propagator for exchange particle of mass M

Here the electromagnetic and the weak interaction contribute, i.e.



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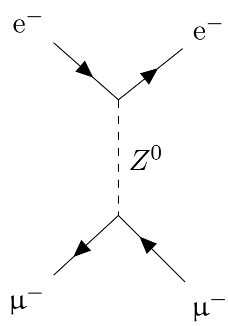
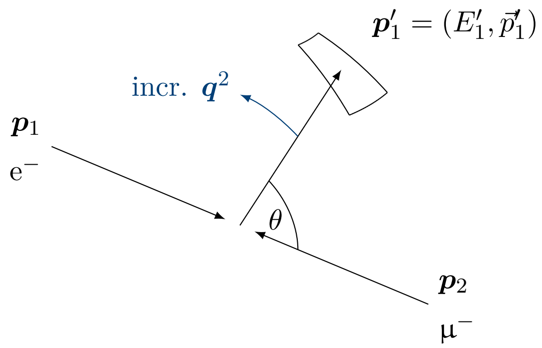


$$= e \frac{1}{\boldsymbol{q}^2} e + g_1 \frac{1}{\boldsymbol{q}^2 - M_Z^2} g_2$$

Last question: What is the **cross section**?
It is proportional to the transition probability

$$\sigma \propto W, \qquad W \propto |amplitude_1 + amplitude_2|^2 \propto \frac{\mathrm{e}^4}{\boldsymbol{q}^4} \tag{I.17}$$

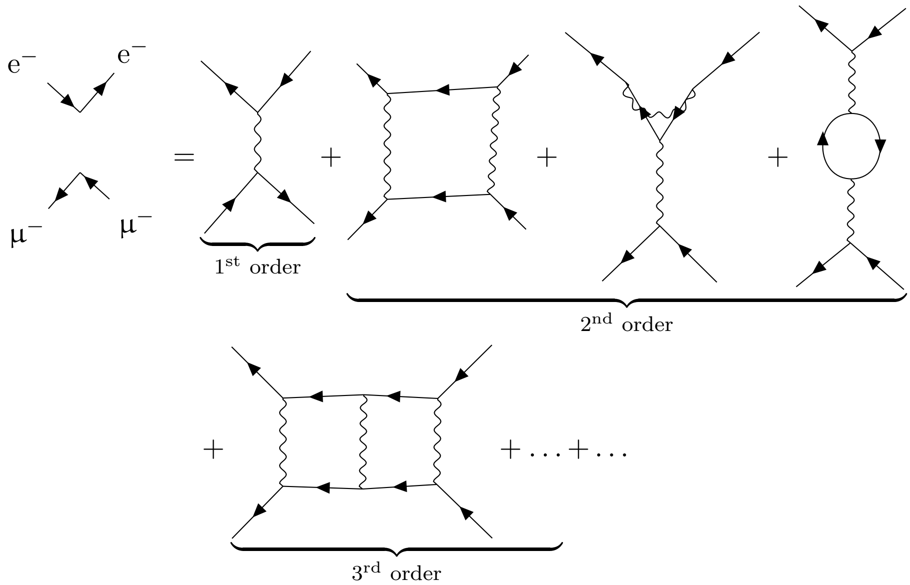
The factor $1/\boldsymbol{q}^4$ leads to a rapid decrease of the cross section as momentum transfer increases. In the centre of mass system this can be depicted as:



Then $W \propto {}^{1}/_{\boldsymbol{q}^4} \longrightarrow$ mostly forward scattering ($\theta \approx 0°$).
cf.: Rutherford scattering

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto \frac{1}{\sin^4(\theta/2)}\,. \tag{I.18}$$

The above example sketches only the leading order terms. Thus the question arises how one would calculate the full scattering amplitude?



$$\textit{full amplitude} = \underset{\propto \mathrm{e}^2 \propto \alpha}{\text{leading order}} + \underset{\propto \mathrm{e}^4 \propto \alpha^2}{2^{\text{nd}}\text{ order}} + \underset{\propto \mathrm{e}^6 \propto \alpha^3}{3^{\text{rd}}\text{ order}} \tag{I.19}$$

with $\alpha = \dfrac{\mathrm{e}^2}{4\pi\hbar c\varepsilon_0} = \dfrac{1}{137}$

Thus in QED it is sufficient to only calculate the first order. It is sufficent to understand the basics of the process (cross section, angular distributions). For comparison to precision measurements sometimes more than 100 diagrams have to calculated.

II. Covariant description of relativistic particles

II.1. Non-relativistic quantum mechanics

Recall the energy-momentum relation

$$E = \frac{1}{2m}|\vec{p}|^2 \,. \tag{II.1}$$

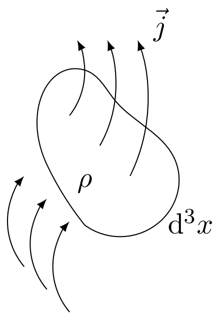
By identifying $E \to \mathrm{i}\hbar\partial_t$, $\vec{p} \to -\mathrm{i}\hbar\vec{\boldsymbol{\nabla}}$, we get an operator equation, i.e. the Schrödinger equation

$$\boxed{\left(\mathrm{i}\partial_t + \vec{\boldsymbol{\nabla}}^2\frac{1}{2m}\right)\phi(\vec{x},t) = 0} \tag{II.2}$$

with $\phi(\vec{x},t) \in \mathbb{C}$ as the one-particle wave function.

We can get the statistical interpretation via $|\phi|\,\mathrm{d}^3x \;\hat{=}$ probability to find particle in volume d^3x at time t and we can obtain the localisation probability density

$$\to \qquad \rho = \rho(\vec{x},t) \coloneqq |\phi(\vec{x},t)|^2 \,. \tag{II.3}$$



From electrodynamics we get the continuity equation:

$$\boxed{\partial_t\rho + \vec{\boldsymbol{\nabla}}\vec{j} = 0} \tag{II.4}$$

with $\vec{j}(\vec{x},t)$ as the **probability density current**.

How does $\vec{j}$ depend on the wave function ϕ?

$$(-\mathrm{i}\phi^*)\cdot(eq.\ (\text{II.2})) = \phi^*\partial_t\phi - \frac{\mathrm{i}}{2m}\phi^*\vec{\boldsymbol{\nabla}}^2\phi = 0$$
$$(-\mathrm{i}\phi)\cdot(eq.\ (\text{II.2}))^* = -\phi\partial_t\phi^* - \frac{\mathrm{i}}{2m}\phi\vec{\boldsymbol{\nabla}}^2\phi^* = 0$$

By subtracting these two equations we get

$$\Rightarrow \quad \underbrace{(\phi^*\partial_t\phi + \phi\partial_t\phi^*)}_{\partial_t(\phi^*\phi)=\partial_t|\phi|^2} - \frac{\mathrm{i}}{2m}\underbrace{\left(\phi^*\vec{\boldsymbol{\nabla}}^2\phi - \phi\vec{\boldsymbol{\nabla}}^2\phi^*\right)}_{\vec{\boldsymbol{\nabla}}\left(\phi^*\vec{\boldsymbol{\nabla}}\phi-\phi\vec{\boldsymbol{\nabla}}\phi^*\right)} = 0 \tag{II.5}$$

and by using the continuity eq. (II.4) we get an explicit expression for the probability density current

$$\boxed{\vec{j} = -\frac{\mathrm{i}}{2m}\left(\phi^*\vec{\boldsymbol{\nabla}}\phi - \phi\vec{\boldsymbol{\nabla}}\phi^*\right)}. \tag{II.6}$$

The probability density current, based on the Schrödinger equation, is not covariant. This means that the Schrödinger equation treats time and space different.

II.2. Relativistic particles: Klein-Gordon equation

Now we use the **relativistic** energy-momentum relation

$$E^2 = \vec{p}^2 + m^2 \qquad E \to \mathrm{i}\hbar\partial_t, \quad \vec{p} \to \mathrm{i}\hbar\vec{\boldsymbol{\nabla}}$$

$$\Rightarrow \quad \boxed{\left(\partial_t^2 - \boldsymbol{\Delta} + m^2\right)\phi(\vec{x},t) = 0}. \tag{II.7}$$

This is the so-called **Klein-Gordon equation**. By using four vector notation

$$p^\mu \to \begin{pmatrix} E \\ \vec{p} \end{pmatrix} \to \begin{pmatrix} \mathrm{i}\partial_t \\ -\mathrm{i}\vec{\boldsymbol{\nabla}} \end{pmatrix} \to \mathrm{i}\partial^\mu \quad \Rightarrow \quad \boxed{\left(\partial_\mu\partial^\mu + m^2\right)\phi = 0}, \tag{II.8}$$

which denotes the Klein-Gordon equation in covariant form. Since the scalar product and the mass are Lorentz invariant, this KG equation is too.
$\to$ Klein-Gordon equation describes spin-0 particles.

The continuity equation thus is

$$\partial_t\rho + \vec{\boldsymbol{\nabla}}\vec{j} = 0 \quad \to \quad \boxed{\partial_\mu j^\mu = 0}. \tag{II.9}$$

How do ρ and $\vec{j}$ depend on ϕ?

$$\rho = \mathrm{i}(\phi^*\partial_t\phi - \phi\partial_t\phi^*), \quad \vec{j} = -\mathrm{i}\left(\phi^*\vec{\boldsymbol{\nabla}}\phi - \phi\vec{\boldsymbol{\nabla}}\phi^*\right) \tag{II.10}$$

and the for vector current is

$$\boxed{j^\mu = \mathrm{i}(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*)}. \tag{II.11}$$

The fundamental free-particle solution for the KGe is given by

$$\begin{aligned} \phi(\vec{x},t) \equiv \phi(\boldsymbol{x}) &= N\cdot\exp(\mathrm{i}(\vec{p}\vec{x} - Et)) \\ &= N\exp(-\mathrm{i}p_\mu x^\mu) = N\exp(-\mathrm{i}\boldsymbol{p}\boldsymbol{x}) \end{aligned} \tag{II.12}$$

with N as the normalisation.

By inserting this into the probability density current (eq. (II.10)), we get

$$\boxed{j^\mu = 2p^\mu |N|^2} \tag{II.13}$$

for free particles. Note that espacially $\rho = 2E|N|^2$.

→ Localisation probability $\rho\, \mathrm{d}^3x$, but under Lorentz boost $\mathrm{d}^3x \to \frac{1}{\gamma}\, \mathrm{d}^3x$, since $\rho \propto E \propto \gamma$. This effect is compensated for the probability density current $\vec{j} \propto \vec{p}$.

→ Particle current $\vec{j}$ in the direction of the particle momentum $\vec{p}$.

What are the eigenvalues of the free-particle solution?

$$\begin{aligned}
&\left(\partial_\mu \partial^\mu + m^2\right) \exp(-\mathrm{i}\boldsymbol{px}) = 0 \\
\Rightarrow \quad &\left((-\mathrm{i})^2 p_\mu p^\mu + m^2\right) \exp(-\mathrm{i}\boldsymbol{px}) = 0 \\
\Rightarrow \quad &-p_\mu p^\mu + m^2 = 0, \quad p_\mu p^\mu = m^2 = E^2 - \vec{p}^2 \\
\Rightarrow \quad &\boxed{E = \pm\sqrt{\vec{p}^2 + m^2}}
\end{aligned} \tag{II.14}$$

So we get two solutions for the particle energy. However, the negative solution $E < 0 \Rightarrow \rho < 0$ is unphysical! There is one way out, the Feynman-Stückelberg approach. Here we interpret ρ as a charge density.

→ Can be negative, since particles can have negative charge.

→ Interpret j^μ as charge current.

⇒ $j^\mu \to j'^\mu = Qj^\mu$ with Q as particle charge.

Example: electron, $Q = -\mathrm{e}$

By assuming an electron with spin 0, we get

$$j^\mu(\mathrm{e}^-) = -\mathrm{ie}(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*)\,. \tag{II.15}$$

Free particle solution ($E > 0$):

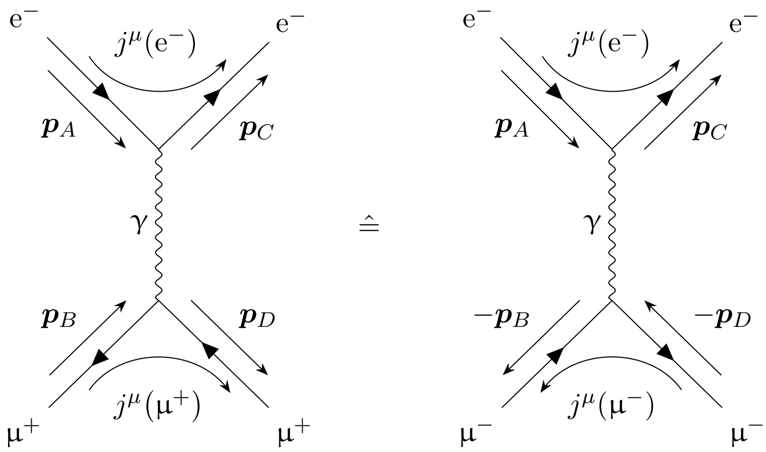
$$j^\mu(\mathrm{e}^-) = -2\mathrm{e}p^\mu |N|^2 \to -2\mathrm{e}|N|^2 \begin{pmatrix} E \\ \vec{p} \end{pmatrix} \tag{II.16}$$

For $E < 0$: Consider anti muon with $E > 0$

$$\boldsymbol{j}\left(\mu^+\right) = 2\mathrm{e}\boldsymbol{p}|N|^2 = 2\mathrm{e}|N|^2 \begin{pmatrix} E \\ \vec{p} \end{pmatrix} = -2\mathrm{e}|N|^2 \begin{pmatrix} -E \\ -\vec{p} \end{pmatrix} = -\boldsymbol{j}(\mu^-)\,. \tag{II.17}$$

So we get a muon with $E < 0$, that moves backwards.

→ solution with $E < 0$ can be used to describe antiparticles with $E' = -E$



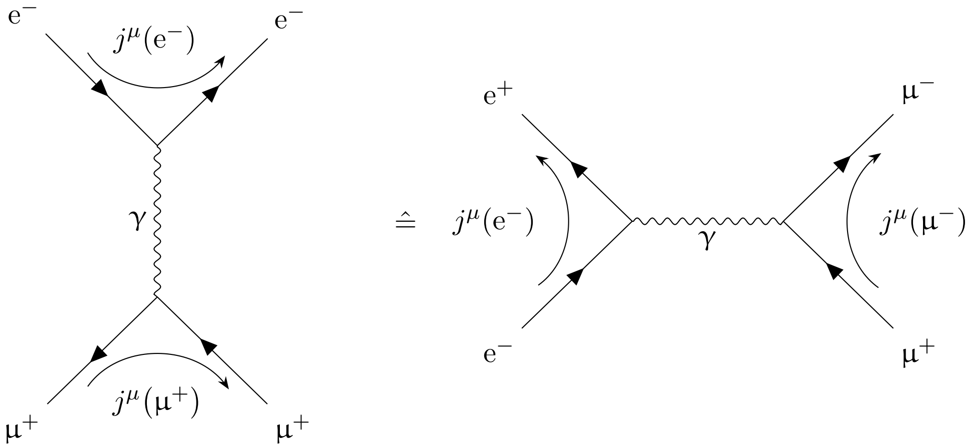
Consequence: Can use particle states with $p^\mu \to -p^\mu$ for description of antiparticles.

II.3. Crossing symmetry

The description of scattering processes is highly symmetric under the exchange of space and time: This originates in the fact that wave equations treat time and space the same way.

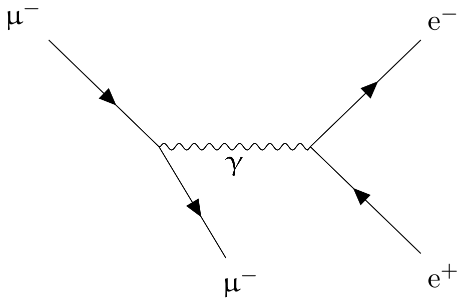
Example: $\mathrm{e}^-\mu^-$ scattering in QED

By exchange of time and space, the Feynman diagram changes as:



This is equivalent to a counter clockwise rotation by 90°. The resulting diagram represents $\mathrm{e}^+\mathrm{e}^-$ annihilation followed by $\mu^+\mu^-$ creation.

By exchanging the incoming anti muon by an outgoing muon in the $\mu^+\mu^-$ annihilation, we get the diagram:



This is μ^--Bremsstrahlung with subsequent pair creation.

All these processes share the same common transition amplitude, as they contain the same basic interaction.

List of Figures