

Logic in Computer Science--Assignment 3

$$\forall x(P(x) \rightarrow \neg Q(x)) \vdash \neg(\exists x(P(x) \wedge Q(x)))$$

1	$\forall x(P(x) \rightarrow \neg Q(x))$	premise
2	$\exists x(P(x) \wedge Q(x))$	assumption
3	x_0	
4	$P(x_0) \wedge Q(x_0)$	assumption
5	$P(x_0)$	$\wedge e$ 4
6	$Q(x_0)$	$\wedge e$ 4
7	$P(x_0) \rightarrow \neg Q(x_0)$	$\forall x e$ 1
8	$\neg Q(x_0)$	$\rightarrow e$ 5,7
9	\perp	$\neg e$ 6,8
10	\perp	$\exists x e$ 2,3-9
11	$\neg(\exists x(P(x) \wedge Q(x)))$	$\neg i$ 2-10

$$\forall x(P(x) \leftrightarrow x=b) \vdash P(b) \wedge \forall x \forall y(P(x) \wedge P(y) \rightarrow x=y)$$

1	$\forall x(P(x) \rightarrow x=b)$	premise
2	$\forall x(x=b \rightarrow P(x))$	premise
3	$b=b \rightarrow P(b)$	$\forall x e$ 2
4	$b=b$	reflexivity
5	$P(b)$	$\rightarrow e$ 4,3
6	x_0, y_0	
7	$P(x_0) \wedge P(y_0)$	assumption
8	$P(x_0)$	$\wedge e$ 7
9	$P(x_0) \rightarrow x_0 = b$	$\forall x e$ 1
10	$x_0 = b$	$\rightarrow e$ 8,9
11	$y_0 = b$	$\wedge e$ 7; $\forall x e$ 1; $\rightarrow e$ 8,9
12	$x_0 = y_0$	transitivity 10,11
13	$P(x_0) \wedge P(y_0) \rightarrow x_0 = y_0$	$\rightarrow i$ 7-12
14	$\forall x \forall y(P(x) \wedge P(y) \rightarrow x=y)$	$\forall x \forall y i$ 6-13
15	$P(b) \wedge \forall x \forall y(P(x) \wedge P(y) \rightarrow x=y)$	$\wedge i$ 5,14