

Logic in Computer Science-- Assignment1

作业解答:

1.1 $\neg (p \wedge q) \vdash \neg q \vee \neg p$, 证明如下:

1	$\neg (p \wedge q)$	<i>premise</i>
2	p	<i>assumption</i>
3	q	<i>assumption</i>
4	$p \wedge q$	$\wedge i$ 2, 3
5	\perp	$\neg e$ 1, 4
6	$\neg q$	\perp
7	$p \rightarrow \neg q$	$\rightarrow i$ 2, 6
8	$p \vee \neg p$	<i>LEM</i>
9	p	<i>assumption</i>
10	$\neg q$	$\rightarrow e$ 7, 9
11	$\neg q \vee \neg p$	<i>vi</i> 10
12	$\neg p$	<i>assumption</i>
13	$\neg p \vee \neg q$	<i>vi</i> 12
14	$\neg q \vee \neg p$	$\vee e$ 1, 2 – 6, 7, 8, 9 – 11, 12 – 13

1.2 $\neg(p \wedge q) \vdash \neg q \vee \neg p$, 证明如下:

1	$\neg q \vee \neg p$	<i>premise</i>
2	$\neg q$	<i>assumption</i>
3	$p \wedge q$	<i>assumption</i>
4	q	$\wedge e\ 2, 3$
5	\perp	$\neg e\ 2, 4$
6	$\neg(p \wedge q)$	$\neg i\ 3 - 5$
7	$\neg p$	<i>assumption</i>
8	$p \wedge q$	<i>assumption</i>
9	p	$\wedge e\ 1, 8$
10	\perp	$\neg e\ 7, 9$
11	$\neg(p \wedge q)$	$\neg i\ 8 - 10$
12	$\neg(p \wedge q)$	$\vee e\ 1, 2 - 6, 7 - 11$

2.1 $p \rightarrow q \vdash \neg q \rightarrow \neg p$ 证明如下:

1	$p \rightarrow q$	<i>premise</i>
2	$\neg q$	<i>assumption</i>
3	$\neg p$	<i>MT</i> 1, 2
4	$\neg q \rightarrow \neg p$	$\rightarrow i\ 2 - 3$

2.2 $p \rightarrow q \vdash \neg q \rightarrow \neg p$ 证明如下:

1	$\neg q \vee \neg p$	<i>premise</i>
2	p	<i>assumption</i>
3	$\neg\neg q$	<i>MT 1, 2</i>
4	q	$\neg\neg e$ 3
5	$p \rightarrow q$	$\rightarrow i$ 2 – 4

3.1 $p \wedge q \rightarrow p \vdash r \vee \neg r$, 证明如下:

1	$p \wedge q \rightarrow p$	<i>premise</i>
2	$r \vee \neg r$	<i>LEM</i>

3.2 $p \wedge q \rightarrow p \vdash r \vee \neg r$, 证明如下:

1	$r \vee \neg r$	<i>premise</i>
2	$p \wedge q$	<i>assumption</i>
3	p	$\wedge e$ 1, 2
4	$p \wedge q \rightarrow p$	$\rightarrow i$ 2 – 3

附录: (latex编写源格式)

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1  1.1\quad\neg (p\wedge q) \vdash \neg q\vee \neg p, \text{证明如下: } \\
2  \begin{equation} \\
3  \begin{aligned} \\
4  &1\quad\neg (p\wedge q) \quad\quad\quad\text{premise}\\
5  &2\quad\quad\quad p\quad\quad\quad\text{assumption}\\
6  &3\quad\quad\quad q\quad\quad\quad\text{assumption}\\
7  &4\quad\quad\quad p \wedge q\quad\quad\quad;\wedge e\;2,3\\
8  &5\quad\quad\quad \bot\quad\quad\quad\neg e\;1,4\\
9  &6\quad\quad\quad\neg q\quad\quad\quad\bot\\
10 &7\quad\quad\quad p\rightarrow\neg q\quad\quad\quad\rightarrow i\;2,6\\
11 &8\quad\quad\quad p\vee\neg p\quad\quad\quad\text{LEM}\\
12 &9\quad\quad\quad p\quad\quad\quad\text{assumption}

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13 &10\qquad\qquad\neg q\qquad\qquad\;;\;\rightarrow e\;;7,9\\
14 &11\qquad\qquad\neg q \vee\neg p\qquad\qquad vi1\;;10\\
15 &12\qquad\qquad\neg p\qquad\qquad\;;\;;assumption\\
16 &13\qquad\qquad\neg p \vee\neg q\qquad\qquad vi1\;;12\\
17 &14\qquad\qquad\neg q \vee\neg p\qquad\;;\;;\vee e\;;1,2-6,7,8,9-11,12-13\\
18 \end{aligned}
19 \end{equation}

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1 1.2\qquad\neg (p\wedge q) \dashv\neg q\vee \neg p, \;;证明如下: \\
2
3 \begin{equation}
4 \begin{aligned}
5
6 &1\qquad\qquad\neg q\vee \neg p\qquad\qquad\text{premise}\\
7 &2\qquad\qquad\neg q\qquad\qquad\text{assumption}\\
8 &3\qquad\qquad p\wedge q\;;\;;\;;\qquad\qquad\text{assumption}\\
9 &4\qquad\qquad q\qquad\qquad\qquad\wedge e \;;2,3\\
10 &5\qquad\qquad\bot\qquad\qquad\neg e\;;2,4\\
11 &6\qquad\qquad\;;\;;\neg (p\wedge q) \qquad\qquad\;;\;;\neg i\;;3-5\\
12 &7\qquad\qquad\neg p\qquad\qquad\text{assumption}\\
13 &8\qquad\qquad p\wedge q\;;\;;\;;\qquad\qquad\text{assumption}\\
14 &9\qquad\qquad p\qquad\qquad\qquad\wedge e \;;1,8\\
15 &10\qquad\qquad\bot\qquad\qquad\neg e\;;7,9\\
16 &11\qquad\qquad\neg (p\wedge q) \qquad\qquad\;;\;;\neg i\;;8-10\\
17 &12\qquad\qquad\neg (p\wedge q) \qquad\qquad\;;\;;\vee e\;;1,2-6,7-11\\
18 \end{aligned}
19 \end{equation}

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1 2.1\qquad p\rightarrow q \vdash\neg q\rightarrow \neg p\;;证明如下: \\
2 \begin{equation}
3 \begin{aligned}
4
5 &1\qquad p\rightarrow q\qquad\qquad\;;\;;\text{premise}\\
6 &2\qquad\neg q \qquad\qquad\qquad\text{assumption}\\
7 &3\qquad\neg p \qquad\qquad\qquad\text{MT}\;;1,2\\
8 &4\qquad\neg q\rightarrow \neg p \qquad\qquad\rightarrow i\;;2-3
9 \end{aligned}
10 \end{equation}

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