Logic in Computer Science-- Assignment1

作业解答：

1.1 $\neg(p \wedge q) \vdash \neg q \vee \neg p$, 证明如下：

1	$\neg(p \wedge q)$	$premise$
2	p	$assumption$
3	q	$assumption$
4	$p \wedge q$	$\wedge i\ 2,3$
5	$\bot$	$\neg e\ 1,4$
6	$\neg q$	$\bot$
7	$p \rightarrow \neg q$	$\rightarrow i\ 2,6$
8	$p \vee \neg p$	LEM
9	p	$assumption$
10	$\neg q$	$\rightarrow e\ 7,9$
11	$\neg q \vee \neg p$	$vi1\ 10$
12	$\neg p$	$assumption$
13	$\neg p \vee \neg q$	$vi1\ 12$
14	$\neg q \vee \neg p$	$\vee e\ 1, 2-6, 7, 8, 9-11, 12-13$

1.2 $\neg(p \wedge q) \dashv \neg q \vee \neg p$, 证明如下：

1	$\neg q \vee \neg p$	$premise$
2	$\neg q$	$assumption$
3	$p \wedge q$	$assumption$
4	q	$\wedge e\ 2,3$
5	$\bot$	$\neg e\ 2,4$
6	$\neg(p \wedge q)$	$\neg i\ 3-5$
7	$\neg p$	$assumption$
8	$p \wedge q$	$assumption$
9	p	$\wedge e\ 1,8$
10	$\bot$	$\neg e\ 7,9$
11	$\neg(p \wedge q)$	$\neg i\ 8-10$
12	$\neg(p \wedge q)$	$\vee e\ 1,2-6,7-11$

2.1 $p \to q \vdash \neg q \to \neg p$ 证明如下：

1	$p \to q$	$premise$
2	$\neg q$	$assumption$
3	$\neg p$	$MT\ 1,2$
4	$\neg q \to \neg p$	$\to i\ 2-3$

2.2 $p \to q \dashv \neg q \to \neg p$ 证明如下：

1	$\neg q \vee \neg p$	$premise$
2	p	$assumption$
3	$\neg\neg q$	$MT\ 1,2$
4	q	$\neg\neg e\ 3$
5	$p \to q$	$\to i\ 2-4$

3.1 $p \wedge q \to p \vdash r \vee \neg r$, 证明如下：

1	$p \wedge q \to p$	$premise$
2	$r \vee \neg r$	LEM

3.2 $p \wedge q \to p \dashv r \vee \neg r$, 证明如下：

1	$r \vee \neg r$	$premise$
2	$p \wedge q$	$assumption$
3	p	$\wedge e\ 1,2$
4	$p \wedge q \to p$	$\to i\ 2-3$

附录：（latex编写源格式）

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1.1\qquad\neg（p\wedge q） \vdash\neg q\vee \neg p, \;证明如下：\\
\begin{equation}
\begin{aligned}
&1\qquad \neg（p\wedge q）\qquad\qquad premise\\
&2\qquad\qquad p\qquad\qquad\qquad assumption\\
&3\qquad\qquad q\qquad\qquad\qquad assumption\\
&4\qquad\qquad p \wedge q\qquad\qquad\;\wedge i\;2,3\\
&5\qquad\qquad \bot\qquad\qquad\qquad\neg e\:1,4\\
&6\qquad\qquad\neg q\qquad\qquad\qquad\bot\\
 &7\qquad\qquad p\rightarrow\neg q\qquad\qquad\rightarrow i\;2,6\\
 &8\qquad\qquad p\vee\neg p\qquad\qquad LEM\\
 &9\qquad\qquad p\qquad\qquad\qquad assumption\\
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&10\qquad\qquad\neg q\qquad\qquad\;\;\rightarrow e\;7,9\\
&11\qquad\qquad\neg q \vee\neg p\qquad\qquad vi1\;10\\
&12\qquad\qquad\neg p\qquad\qquad\;\;\;assumption\\
&13\qquad\qquad\neg p \vee\neg q\qquad\qquad vi1\;12\\
&14\qquad\qquad\neg q \vee\neg p\qquad\;\;\;\vee e\;1,2-6,7,8,9-11,12-13\\
\end{aligned}
\end{equation}
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1.2\qquad\neg（p\wedge q） \dashv\neg q\vee \neg p, \;证明如下：\\

\begin{equation}
\begin{aligned}

&1\qquad\qquad \neg q\vee \neg p\qquad premise\\
&2\qquad\qquad \neg q\qquad\qquad assumption\\
&3\qquad\qquad p\wedge q\;\;\;\;\qquad assumption\\
&4\qquad\qquad q\qquad \qquad\;\wedge e \;2,3\\
&5\qquad\qquad \bot\qquad\qquad\neg e\:2,4\\
&6\qquad\;\;\neg（p\wedge q）\qquad\;\;\neg i\;3-5\\
&7\qquad\qquad \neg p\qquad\qquad assumption\\
&8\qquad\qquad p\wedge q\;\;\;\;\qquad assumption\\
&9\qquad\qquad p\qquad \qquad\;\wedge e \;1,8\\
&10\qquad\qquad \bot\qquad\qquad\neg e\:7,9\\
&11\qquad\neg（p\wedge q）\qquad\;\;\neg i\;8-10\\
&12\qquad\neg（p\wedge q）\qquad\;\;\vee e\;1,2-6,7-11\\
\end{aligned}
\end{equation}
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2.1\qquad p\rightarrow q \vdash\neg q\rightarrow \neg p\:证明如下：\\
\begin{equation}
\begin{aligned}

&1\qquad p\rightarrow q\qquad\;\;\; premise\\
&2\qquad \neg q \qquad\qquad\; assumption\\
&3\qquad \neg p \qquad\qquad MT\;1,2\\
&4\qquad \neg q\rightarrow \neg p \qquad \rightarrow i\;2-3
\end{aligned}
\end{equation}
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2.2\qquad p\rightarrow q \dashv\neg q\rightarrow \neg p\:证明如下：\\
\begin{equation}
\begin{aligned}

&1\qquad \neg q\vee \neg p\qquad\;\; premise\\
&2\qquad p\qquad\qquad\;\;\;\;\ assumption\\
&3\qquad \neg \neg q\qquad\qquad MT\;1 ,2\\
&4\qquad q \qquad\qquad\;\;\;\;\; \neg\neg e\;3\\
&5\qquad p\rightarrow q \qquad\;\;\;\; \rightarrow i\;2-4
 \end{aligned}
 \end{equation}
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3.1\qquad p\wedge q\rightarrow p \vdash r\vee \neg r, \;证明如下：\\
\begin{equation}
\begin{aligned}

&1\qquad p\wedge q\rightarrow p\qquad premise\\
&2\qquad r\vee \neg r\qquad\;\;\;\;\; LEM\\
\end{aligned}
\end{equation}
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3.2\qquad p\wedge q\rightarrow p \dashv r\vee \neg r, \;证明如下：\\
\begin{equation}
\begin{aligned}

&1\qquad r\vee \neg r\qquad\qquad premise\\
&2\qquad p\wedge q\qquad\qquad\ assumption\\
&3\qquad p\qquad\qquad\qquad \wedge e\;1,2\\
&4\qquad p\wedge q\rightarrow p\qquad\rightarrow i\;2-3
\end{aligned}
 \end{equation}
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