

Logic in Computer Science--Assignment 2

1. Convert the following formula into CNF

$$\neg(r \rightarrow (\neg((p \vee q) \wedge (\neg p \rightarrow (q \wedge r))))))$$

$$\begin{aligned} & \text{IMPL_FREE}(\neg(r \rightarrow (\neg((p \vee q) \wedge (\neg p \rightarrow (q \wedge r)))))) \\ &= \neg(\text{IMPL_FREE}(r \rightarrow (\neg((p \vee q) \wedge (\neg p \rightarrow (q \wedge r)))))) \\ &= \neg(\text{IMPL_FREE}(\neg r) \vee \text{IMPL_FREE}(\neg((p \vee q) \wedge \text{IMPL_FREE}(\neg p \rightarrow (q \wedge r)))))) \\ &= \neg(\neg r \vee \text{IMPL_FREE}(\neg((p \vee q) \wedge \text{IMPL_FREE}(\neg p \rightarrow (q \wedge r)))))) \\ &= \neg(\neg r \vee (\neg((p \vee q) \wedge \text{IMPL_FREE}(\neg p \rightarrow (q \wedge r)))))) \\ &= \neg(\neg r \vee (\neg((p \vee q) \wedge (\neg \neg p \vee (q \wedge r)))))) \\ & \text{NNF}(\neg(\neg r \vee (\neg((p \vee q) \wedge (\neg \neg p \vee (q \wedge r)))))) \\ &= \text{NNF}(\neg \neg r) \wedge \text{NNF}(\neg \neg((p \vee q) \wedge (\neg \neg p \vee (q \wedge r)))) \\ &= r \wedge ((p \vee q) \wedge \text{NNF}(\neg \neg p \vee (q \wedge r))) \\ &= r \wedge ((p \vee q) \wedge (p \vee (q \wedge r))) \\ & \text{CNF}(r \wedge ((p \vee q) \wedge (p \vee (q \wedge r)))) \\ &= r \wedge \text{CNF}(p \vee q) \wedge \text{CNF}(p \vee (q \wedge r)) \\ &= r \wedge \text{DISTR}(p, q) \wedge \text{DISTR}(p, (q \wedge r)) \\ &= r \wedge (p \vee q) \wedge \text{DISTR}(p, q) \wedge \text{DISTR}(p, r) \\ &= r \wedge (p \vee q) \wedge (p \vee q) \wedge (p \vee r) \end{aligned}$$

2. $(p \wedge q \wedge s \rightarrow \perp) \wedge (q \wedge s \rightarrow p) \wedge (T \rightarrow s) \wedge (s \rightarrow q)$

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Step	Mark	Condition
1	{s}	$T \rightarrow s$
2	{s, q}	$s \rightarrow q$ 且 s被标记
3	{s, q, p}	$q \wedge s \rightarrow p$ 且 s、q被标记
4	{s, q, p, \perp }	$p \wedge q \wedge s \rightarrow \perp$

由于p,q,s 均被标记第一个子句中，q1 等价于 \perp ，所以霍恩公式是不可满足的