Comparison of the Exponential Distribution with the Central Limit Theorem

Glen Greer, December 2015; Statistical Inference - Simulation Exercise

1 Overview

The Central Limit Theorem (CLT) states that the distribution of averages of *iid* variables (properly normalized) becomes that of a standard normal as the sample size increases. We demonstrate this by taking a large number of samples of the exponential distribution, and compare the sample mean and variance with their theoretical values.

2 Simulations

To investigate the exponential distribution we use $rexp(n, \lambda)$ where λ is the rate parameter. We set $\lambda = 0.2$ and use a sample size of 40 for all of the simulations. We run 1000 simulations and calculate the mean from each.

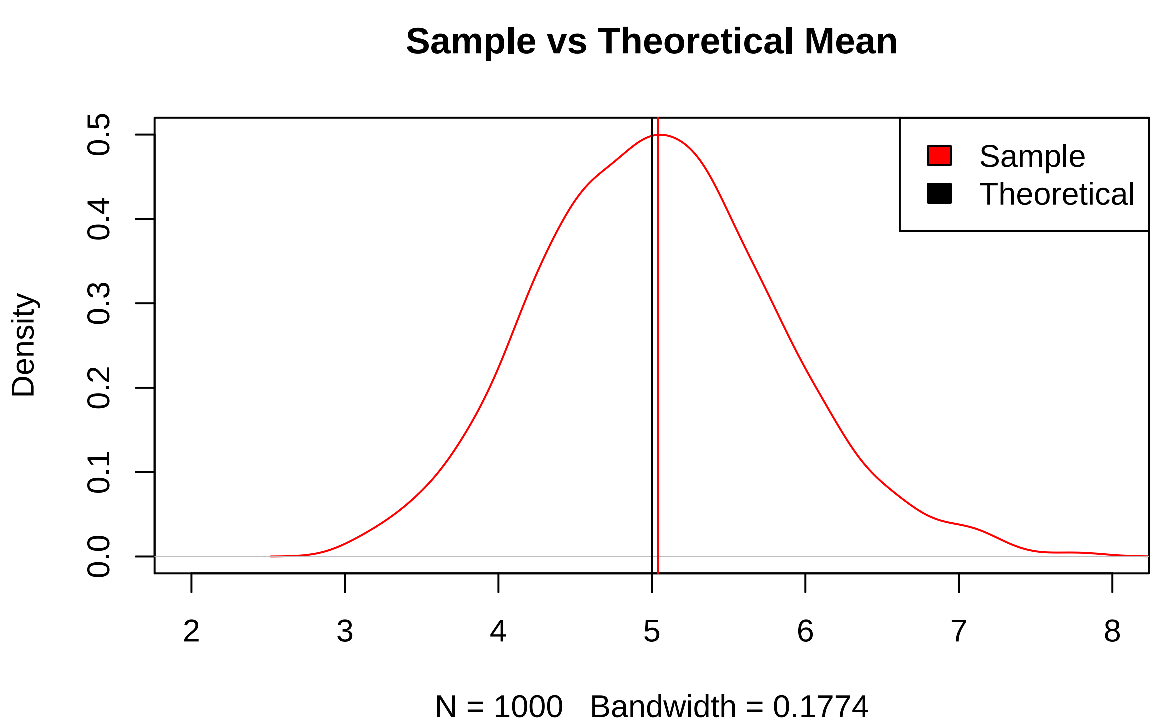
```
lambda <- 0.2; n_sample <- 40; n_simulations <- 1000
exp_means <- NULL
for (i in 1 : n_simulations)
    exp_means <- c(exp_means, mean(rexp(n = n_sample, rate = lambda)))
```

3 Sample Mean vs Theoretical Mean

The theoretical mean of the sample distribution is that of the population mean which is $\frac{1}{\lambda}$ for the exponential distribution.

```
theo_mean <- 1/lambda

plot(density(exp_means), col = "red", xlim = c(2,8), ylim = c(0,0.5),
     main = "Sample vs Theoretical Mean")
abline(v = mean(exp_means), col = "red")
abline(v = theo_mean, col = "black")
legend("topright", legend = c("Sample", "Theoretical"), fill = c("red", "black"))
```



This plot shows the sample mean of 5.0380944 which closely approximates the theoretical mean of 5 for our exponential distribution where $\lambda = 0.2$.

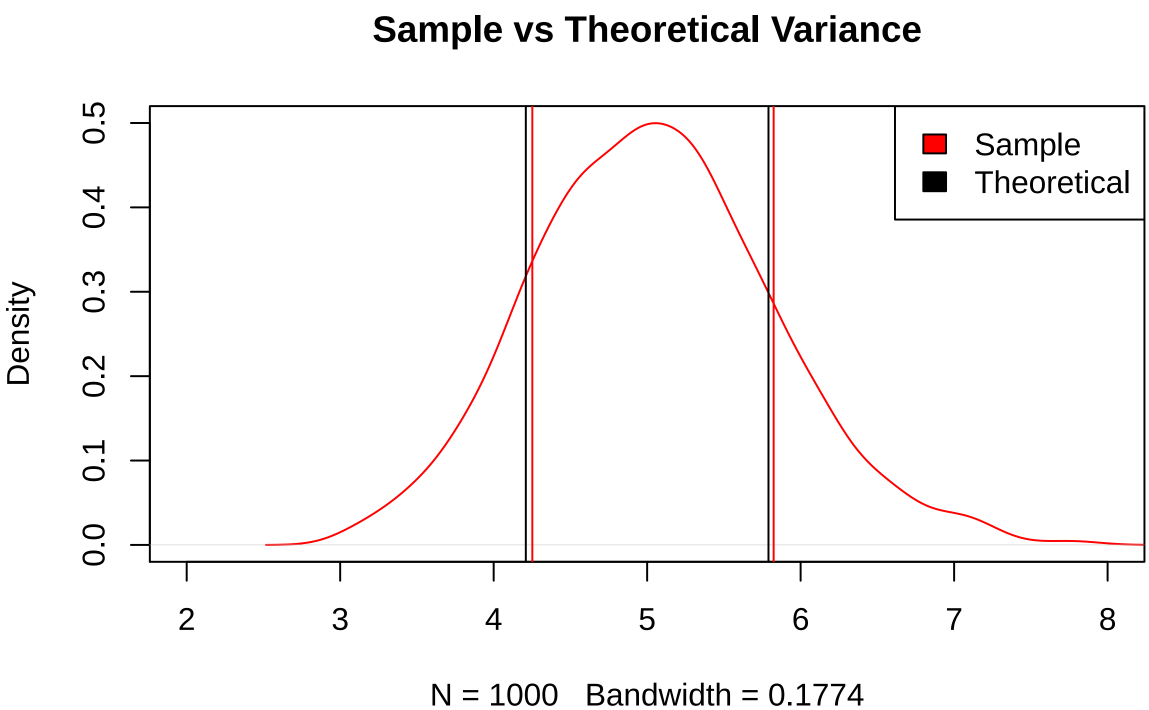
4 Sample Variance vs Theoretical Variance

The theoretical standard deviation of the sample distribution is $\frac{1}{\sqrt{n}}$ times the population's standard deviation of $\frac{1}{\lambda}$ for the exponential distribution.

```
theo_sd <- 1/sqrt(n_sample) * 1/lambda

x <- theo_mean + seq(-4, 4, length = 100) * theo_sd
y <- dnorm(x, theo_mean, theo_sd)

plot(density(exp_means), col = "red", xlim = c(2,8), ylim = c(0,0.5),
     main = "Sample vs Theoretical Variance")
abline(v = mean(exp_means)+c(-1,1)*sd(exp_means), col = "red")
abline(v = theo_mean+c(-1,1)*theo_sd, col = "black")
legend("topright", legend = c("Sample", "Theoretical"), fill = c("red", "black"))
```



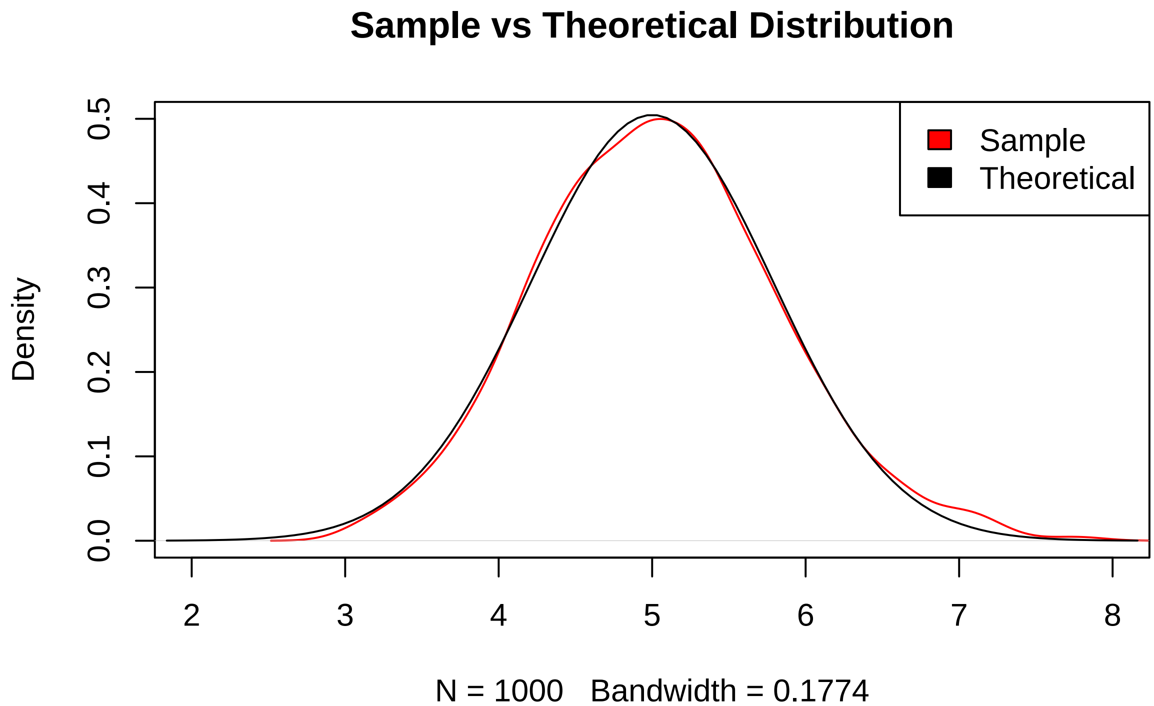
As with the means, the variance is also close with the sample standard distribution of 0.7861737 compared to 0.7905694, our estimated theoretical value.

5 Distribution

The Central Limit Theorm states that the sample distribution should approximate a normal distribution. Using our theoretical values, we overlay a normal distribution over our sample distribution to compare them.

```
x <- theo_mean + seq(-4, 4, length = 100) * theo_sd
y <- dnorm(x, theo_mean, theo_sd)

plot(density(exp_means), col = "red", xlim = c(2,8), ylim = c(0,0.5),
     main = "Sample vs Theoretical Distribution")
lines(x, y, col = "black")
legend("topright", legend = c("Sample", "Theoretical"), fill = c("red", "black"))
```



The normal distribution approximates the sample distribution in this example.