Squaring the Circle and Cubing a Sphere via EMx: A Ternary Operator Framework for Classical Geometric Limits

Shawn Hohol

November 14, 2025

Abstract

Classical Euclidean constructions forbid two specific transformations: (1) squaring a circle, and (2) cubing a sphere's volume, using only compass and straightedge. This paper reformulates both problems within the EMx framework—a signed-zero, ternary operator calculus incorporating lift, flux, curvature, normalization, symmetry, index, and no-clone constraints. EMx does not violate classical impossibility theorems. Instead, it provides an alternative computational substrate in which *physical processes* and *operator sequences* approximate transcendental quantities through gated cycles, NULL reservoirs, and harmonic closure conditions. We derive EMx operator expressions for π, $4/3\pi r^3$, and volume equivalence mappings, identify the remainder channel as $\varnothing$, and specify the exact operator barriers that explain why the problems are not solvable inside strictly Euclidean closure.

1 Introduction

Two classical problems are central in the history of geometry:

1. **Squaring the circle:** Construct a square of area πr^2 from a circle of radius r using only compass and straightedge.
2. **Cubing the sphere:** Construct a cube of volume equal to the sphere volume $V_s = \frac{4}{3}\pi r^3$.

Compass-and-straightedge constructions are restricted to field extensions generated by nested square roots. Neither π nor $\sqrt[3]{2}$ is algebraically accessible in that model.

EMx introduces a different computational architecture based on:

- ternary signed states (T_0–T_4),
- core operators O_1–O_{10},
- the NULL reservoir $\varnothing$,
- and the No-Clone operator Ω.

EMx allows **structured physical processes** (flux, curvature, normalization, iteration) to approximate transcendental quantities outside the compass/straightedge closure.

This report provides the formal EMx framing.

2 Classical Formulation of the Problems

2.1 Squaring the circle

Given a circle of radius r:

$$A_{\text{circle}} = \pi r^2.$$

A square with side s has

$$A_{\text{square}} = s^2.$$

Squaring the circle requires:

$$s = r\sqrt{\pi}.$$

Thus the geometric problem reduces to constructing $\sqrt{\pi}$.

2.2 Cubing the sphere

Sphere volume:

$$V_s = \frac{4}{3}\pi r^3.$$

Cube volume:

$$V_c = a^3.$$

Cubing the sphere requires:

$$a = r\left(\frac{4\pi}{3}\right)^{1/3}.$$

Both require transcendental or non-radical expressions outside the Euclidean field.

3 EMx Structural Framework

EMx works over:

$$T_0 \to T_1 \to T_2 \to T_3 \to T_4$$

with operators:

$$O_1\ (\Delta),\ O_2\ (\nabla),\ O_3\ (\curlywedge),\ O_4\ (\oint),\ O_5\ (\Pi),\ O_6\ (\mathcal{N}),\ O_7\ (\mathcal{S}),\ O_8\ (\mathcal{W}),\ O_9\ (\Omega),\ O_{10}\ (\Sigma).$$

Key to this paper:

- O_3: curvature (circle geometry),
- O_6: normalization (equal-area mapping),
- O_7: symmetry exchange (shape transitions),
- O_4: closure (loop equivalence),
- O_{10}: iteration (harmonic approximation of π),
- $\varnothing$: remainder channel capturing irrational gaps.

4 Squaring the Circle in EMx

4.1 Circle as a curvature operator

A circle of radius r corresponds to curvature:

$$O_3(r) : T_1 \to T_3.$$

Area emerges by applying closure:

$$O_4(O_3(r)) = \pi r^2.$$

4.2 Square as a projection operator

A square of side s is produced via T_2 projection:

$$O_5(s) : T_3 \to T_2.$$

4.3 Normalization requirement

Squaring the circle requires:

$$O_6(O_3(r)) = O_5(s).$$

Thus the condition for equal area:

$$\mathcal{N}(\pi r^2) = s^2.$$

4.4 The EMx obstruction

Applying EMx's operator algebra:

$$O_6(O_4(O_3(r))) = O_5(s) \iff s = r\sqrt{\pi}.$$

$\sqrt{\pi}$ is not generated by EMx's rational + root operator combinations. In EMx:

$$\sqrt{\pi} \in \varnothing$$

because π is produced through harmonic iteration:

$$\pi = \lim_{k\to\infty} \frac{C_k}{D_k},$$

with

$$C_{k+1} = C_k + \Delta_k, \quad D_{k+1} = D_k + \delta_k$$

driven by O_{10}.

Thus:

$$\text{circle} \longrightarrow \text{square} = O_6 \circ O_5 \circ O_3 \circ O_4$$

produces **bounded approximation**, with all irrational residuals stored in $\varnothing$.

5 Cubing the Sphere in EMx

5.1 Sphere as O_3 curvature in 3D

Sphere volume is generated by:

$$V_s = O_4(O_3(r^3)).$$

5.2 Cube as O_5 projection in 3D

$$V_c = O_5(a^3).$$

To equate the two:

$$a = r\left(\frac{4\pi}{3}\right)^{1/3}.$$

5.3 EMx obstruction

Cube root of π:

$$\left(\frac{4\pi}{3}\right)^{1/3} \in \varnothing.$$

Thus:

$$O_6(O_4(O_3(r))) = O_5(a)$$

is impossible in strict EMx algebra without iteration and null correction.

6 EMx Resolution: Approximation via Harmonic Iteration

EMx does not "solve" the classical problems. It transforms them.

6.1 EMx representation of π

Define an iterative operator:

$$\pi_k = O_{10}(k),$$

where each iteration updates:

$$\pi_{k+1} = \pi_k + \epsilon_k, \quad \epsilon_k \in \varnothing.$$

π is stored as:

$$\pi = \pi_k \oplus \varnothing.$$

6.2 Constructive EMx approximants

Area-squaring approximate square:

$$s_k = r\sqrt{\pi_k}$$

Sphere-cubing approximate cube edge:

$$a_k = r\left(\frac{4\pi_k}{3}\right)^{1/3}.$$

These are produced through:

$$O_6(\pi_k r^2) \to s_k^2, \qquad O_6\left(\frac{4}{3}\pi_k r^3\right) \to a_k^3.$$

7 Interpretation in EMx T-Sets

- Circle $\in T_3$ (curvature class).
- Square $\in T_2$ (projection class).
- Sphere $\in T_4$ (3D curvature).
- Cube $\in T_3$ (3D projection).
- Transformations between them require:

$$O_3 \to O_5 \text{ with } O_6,\ O_{10},\ \varnothing.$$

Thus EMx represents them as **different orbit classes** under O_8:

$$\mathcal{W}(\text{circle}) \neq \mathcal{W}(\text{square}), \qquad \mathcal{W}(\text{sphere}) \neq \mathcal{W}(\text{cube}).$$

8 No-Clone Theorem and Uniqueness Constraints

Exact circle→square or sphere→cube would require:

$$O_9 : \text{duplicate state fidelity across topology.}$$

But:

$$\Omega(\text{circle}) \neq \Omega(\text{square}),$$

so EMx forbids perfect mapping between incompatible states.

9 Conclusion

EMx reframes the classical construction problems as:

$$\text{incompatible orbit classes}$$

under the operator algebra. Rather than attempting exact transformations, EMx provides:

- finite approximants,
- harmonic iteration,
- bounded normalization,
- explicit NULL remainder,
- class labels for incompatible topology.

Squaring the circle and cubing the sphere are therefore:

$$\text{impossible in exact Euclidean form but feasible as EMx-computable approximations.}$$