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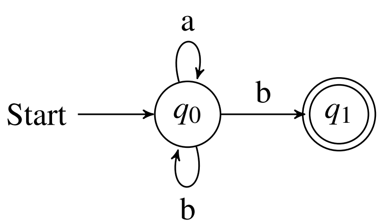
ASSIGNMENT 2

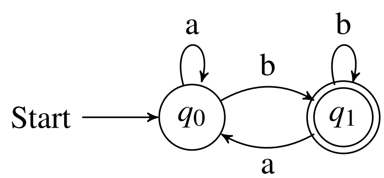
1 Required Exercises

1.1 Exercise 1

1. $L\left((\text{a}|\text{b})^{*}\,\text{b}\right)$

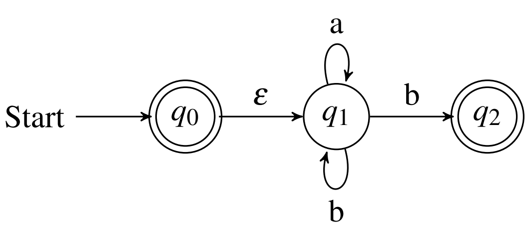


(a) The NFA

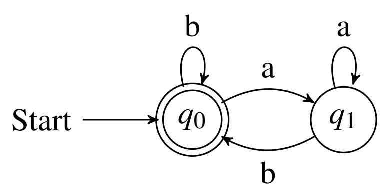


(b) The DFA

2. $L\left(\left((\varepsilon|\text{a})^{*}\,\text{b}\right)^{*}\right)$

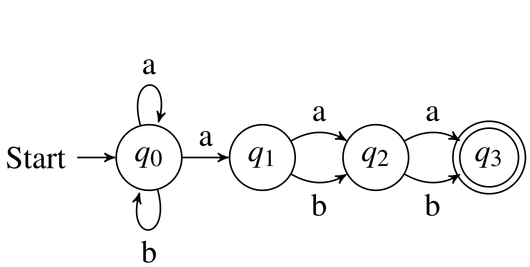


(a) The NFA

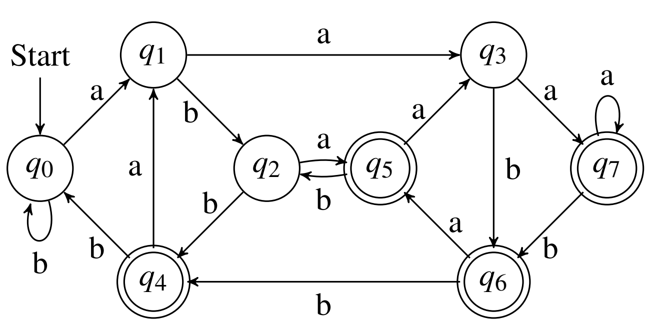


(b) The DFA

3. $L\left((\text{a}|\text{b})^{*}\,\text{a}\,(\text{a}|\text{b})\,(\text{a}|\text{b})\right)$

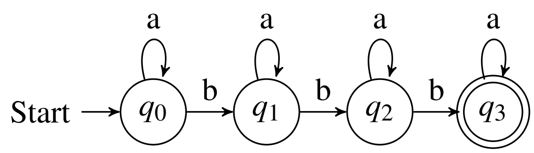


(a) The NFA

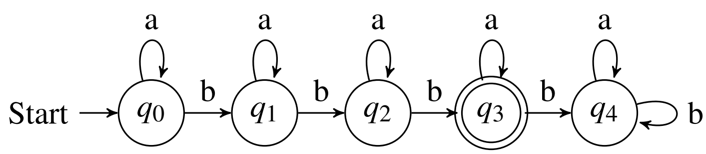


(b) The DFA

4. $L\left(\mathrm{a}^*\mathrm{ba}^*\mathrm{ba}^*\mathrm{ba}^*\right)$



(a) The NFA

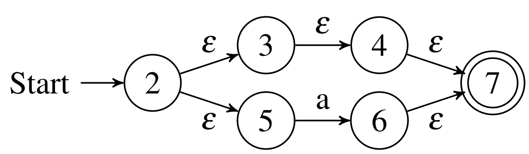


(b) The DFA

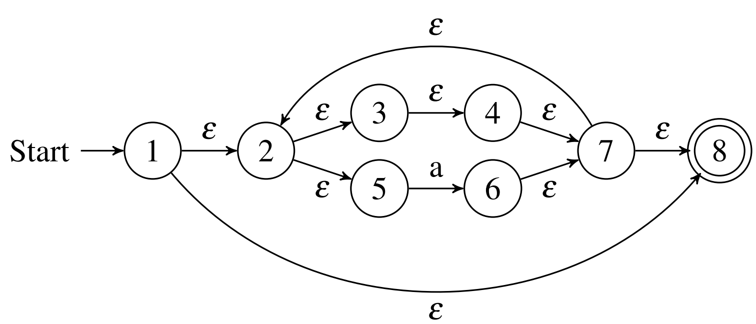
1.2 Exercise 2

1. $\left((\varepsilon|\mathrm{a})^*\,\mathrm{b}\right)^*$

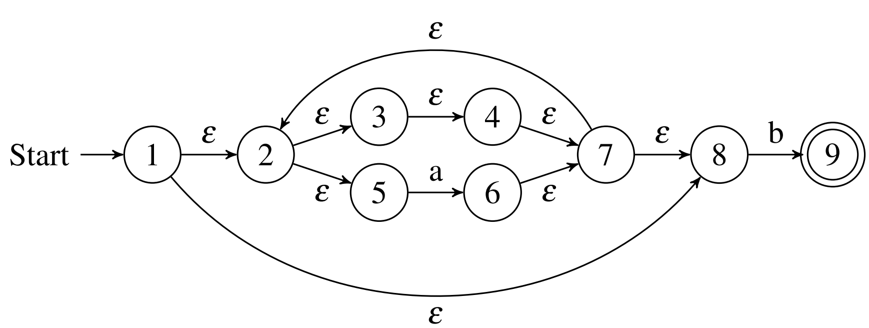
   (a) Construct the NFA for $R_1 = \varepsilon|\mathrm{a}$.



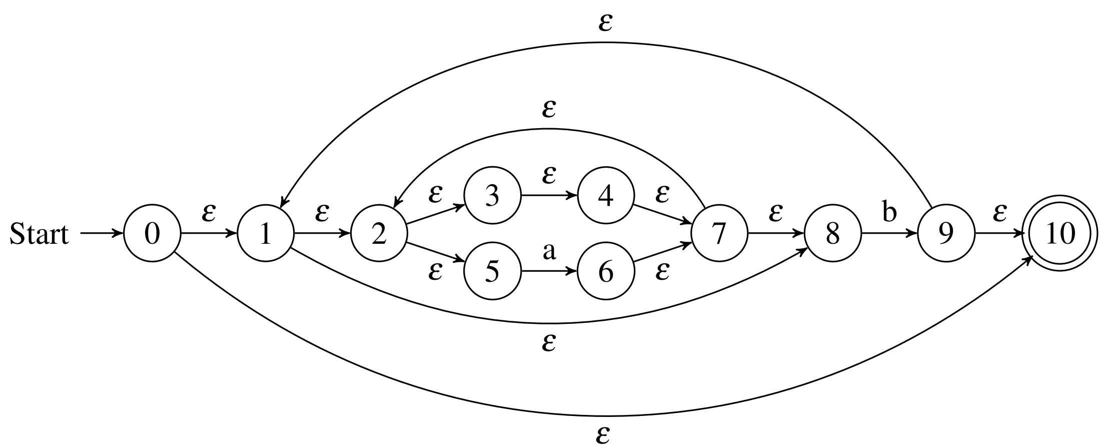
   (b) Construct the NFA for $R_2 = (\varepsilon|\mathrm{a})^* = R_1^*$.



   (c) Construct the NFA for $R_3 = (\varepsilon|\mathrm{a})^*\,\mathrm{b} = R_2\mathrm{b}$.

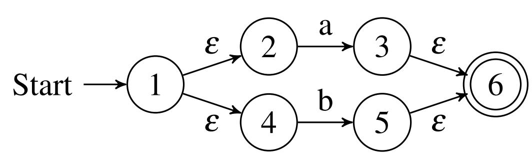


(d) Construct the NFA for $\big((\varepsilon|\mathrm{a})^*\,\mathrm{b}\big)^* = R_3^*$.

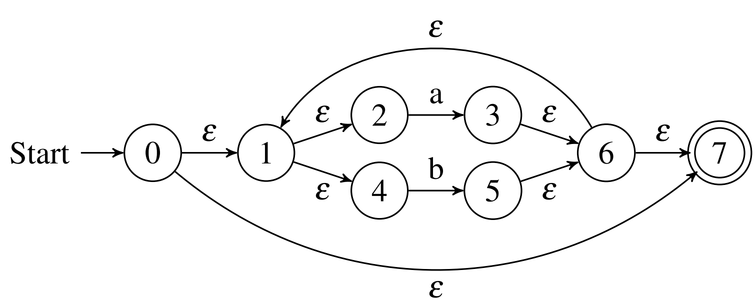


2. $(\mathrm{a}|\mathrm{b})^*\,\mathrm{a}\,(\mathrm{a}|\mathrm{b})\,(\mathrm{a}|\mathrm{b})$

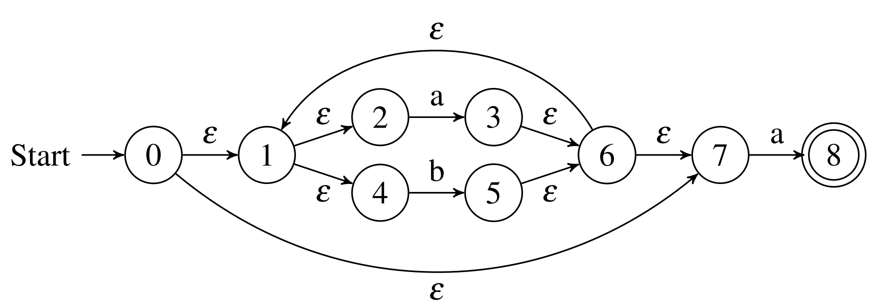
(a) Construct the NFA for $R_1 = \mathrm{a}|\mathrm{b}$.



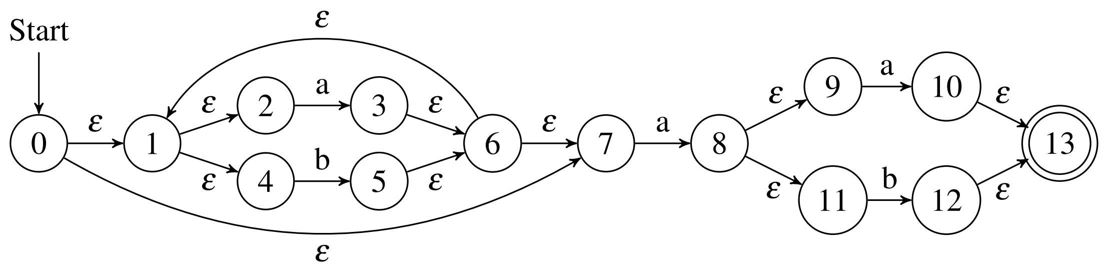
(b) Construct the NFA for $R_2 = (\mathrm{a}|\mathrm{b})^* = R_1^*$.



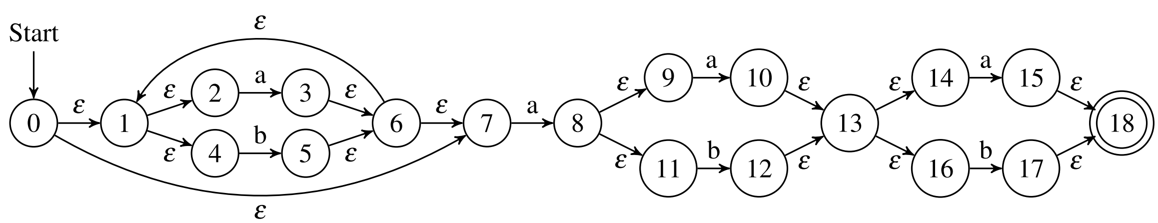
(c) Construct the NFA for $R_3 = (\mathrm{a}|\mathrm{b})^*\,\mathrm{a} = R_2\mathrm{a}$.



(d) Construct the NFA for $R_4 = (\mathrm{a}|\mathrm{b})^*\,\mathrm{a}\,(\mathrm{a}|\mathrm{b}) = R_3R_1$.

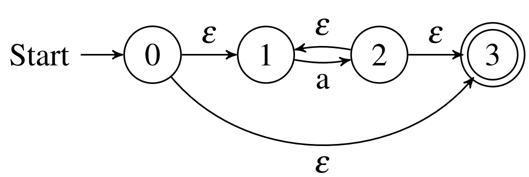


(e) Construct the NFA for $(\mathrm{a}|\mathrm{b})^*\,\mathrm{a}\,(\mathrm{a}|\mathrm{b})\,(\mathrm{a}|\mathrm{b}) = R_4R_1$.

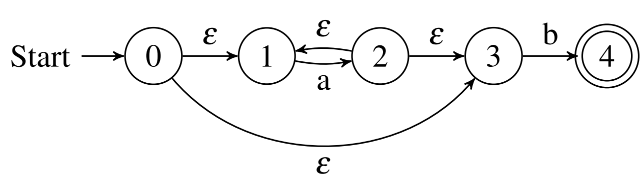


3. $\mathrm{a}^*\mathrm{b}\mathrm{a}^*\mathrm{b}\mathrm{a}^*\mathrm{b}\mathrm{a}^*$

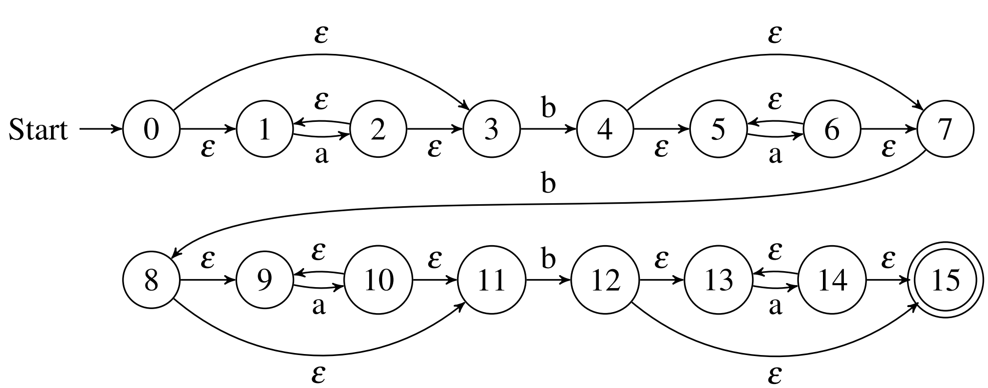
(a) Constrcut the NFA for $R_1 = \mathrm{a}^*$.



(b) Constrcut the NFA for $R_2 = \mathrm{a}^*\mathrm{b} = R_1\mathrm{b}$.



(c) Constrcut the NFA for $\mathrm{a}^*\mathrm{b}\mathrm{a}^*\mathrm{b}\mathrm{a}^*\mathrm{b}\mathrm{a}^* = R_2R_2R_2R_1$.



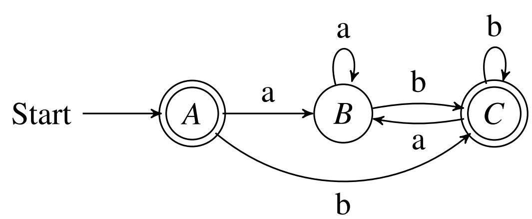
1.3 Exercise 3

1. $A = \varepsilon\text{-closure}(\{0\}) = \{0,1,2,3,4,5,7,8,10\}$.
   $B = \delta_D(A,\mathrm{a}) = \varepsilon\text{-closure}(\{6\}) = \{2,3,4,5,6,7,8\}$.
   $C = \delta_D(A,\mathrm{b}) = \varepsilon\text{-closure}(\{9\}) = \{1,2,3,4,5,7,8,9,10\}$.
   $D = \delta_D(B,\mathrm{a}) = \varepsilon\text{-closure}(\{6\}) = B$.
   $E = \delta_D(B,\mathrm{b}) = \varepsilon\text{-closure}(\{9\}) = C$.
   $F = \delta_D(C,\mathrm{a}) = \varepsilon\text{-closure}(\{6\}) = B$.
   $G = \delta_D(C,\mathrm{b}) = \varepsilon\text{-closure}(\{9\}) = C$.

   Therefore, the transition table of the DFA, whose starting state is A and finite states are $\{A,C\}$, is as follows.

NFA STATE	DFA STATE	a	b
$\{0,1,2,3,4,5,7,8,10\}$	A	B	C
$\{2,3,4,5,6,7,8\}$	B	B	C
$\{1,2,3,4,5,7,8,9,10\}$	C	B	C

   Its transition diagram is depicted as follows.

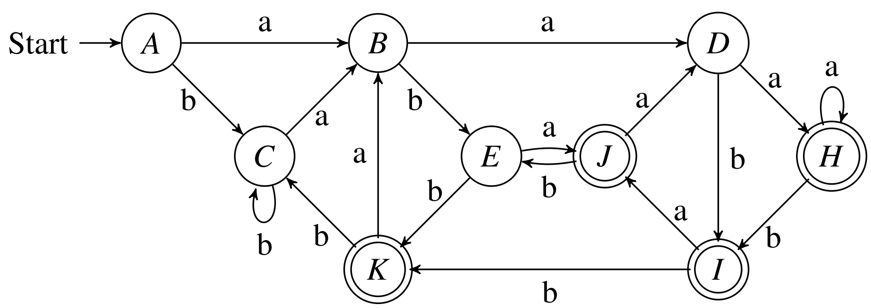


2. $A = \varepsilon\text{-closure}(\{0\}) = \{0,1,2,4,7\}$.
   $B = \delta_D(A,\mathrm{a}) = \varepsilon\text{-closure}(\{3,8\}) = \{1,2,3,4,6,7,8,9,11\}$.
   $C = \delta_D(A,\mathrm{b}) = \varepsilon\text{-closure}(\{5\}) = \{1,2,4,5,6,7\}$.
   $D = \delta_D(B,\mathrm{a}) = \varepsilon\text{-closure}(\{3,8,10\}) = \{1,2,3,4,6,7,8,9,10,11,13,14,16\}$.
   $E = \delta_D(B,\mathrm{b}) = \varepsilon\text{-closure}(\{5,12\}) = \{1,2,4,5,6,7,12,13,14,16\}$.
   $F = \delta_D(C,\mathrm{a}) = \varepsilon\text{-closure}(\{3,8\}) = B$.
   $G = \delta_D(C,\mathrm{b}) = \varepsilon\text{-closure}(\{5\}) = C$.
   $H = \delta_D(D,\mathrm{a}) = \varepsilon\text{-closure}(\{3,8,10,15\}) = \{1,2,3,4,6,7,8,9,10,11,13,14,15,16,18\}$.
   $I = \delta_D(D,\mathrm{b}) = \varepsilon\text{-closure}(\{5,12,17\}) = \{1,2,4,5,6,7,12,13,14,16,17,18\}$.
   $J = \delta_D(E,\mathrm{a}) = \varepsilon\text{-closure}(\{3,8,15\}) = \{1,2,3,4,6,7,8,9,11,15,18\}$.
   $K = \delta_D(E,\mathrm{b}) = \varepsilon\text{-closure}(\{5,17\}) = \{1,2,4,5,6,7,17,18\}$.
   $L = \delta_D(H,\mathrm{a}) = \varepsilon\text{-closure}(\{3,8,10,15\}) = H$.
   $M = \delta_D(H,\mathrm{b}) = \varepsilon\text{-closure}(\{5,12,17\}) = I$.
   $N = \delta_D(I,\mathrm{a}) = \varepsilon\text{-closure}(\{3,8,15\}) = J$.
   $O = \delta_D(I,\mathrm{b}) = \varepsilon\text{-closure}(\{5,17\}) = K$.
   $P = \delta_D(J,\mathrm{a}) = \varepsilon\text{-closure}(\{3,8,10\}) = D$.
   $Q = \delta_D(J,\mathrm{b}) = \varepsilon\text{-closure}(\{5,12\}) = E$.
   $R = \delta_D(K,\mathrm{a}) = \varepsilon\text{-closure}(\{3,8\}) = B$.
   $S = \delta_D(K,\mathrm{b}) = \varepsilon\text{-closure}(\{5\}) = C$.

Therefore, the transition table of the DFA, whose starting state is A and finite states are $\{H,I,J,K\}$, is as follows.

NFA STATE	DFA STATE	a	b
$\{0,1,2,4,7\}$	A	B	C
$\{1,2,3,4,6,7,8,9,11\}$	B	D	E
$\{1,2,4,5,6,7\}$	C	B	C
$\{1,2,3,4,6,7,8,9,10,11,13,14,16\}$	D	H	I
$\{1,2,4,5,6,7,12,13,14,16\}$	E	J	K
$\{1,2,3,4,6,7,8,9,10,11,13,14,15,16,18\}$	H	H	I
$\{1,2,4,5,6,7,12,13,14,16,17,18\}$	I	J	K
$\{1,2,3,4,6,7,8,9,11,15,18\}$	J	D	E
$\{1,2,4,5,6,7,17,18\}$	K	B	C

Its transition diagram is depicted as follows.

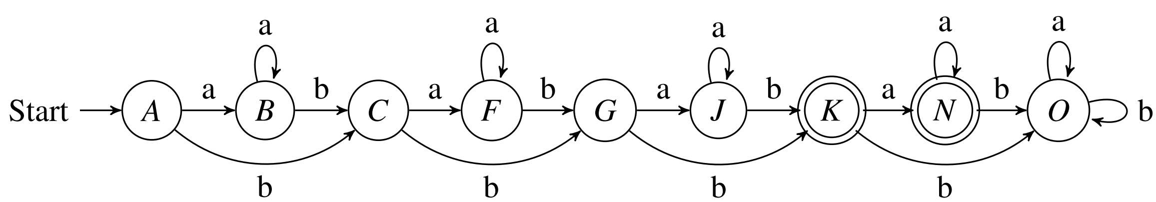


3. $A = \varepsilon\text{-closure}(\{0\}) = \{0,1,3\}$.
$B = \delta_D(A,\text{a}) = \varepsilon\text{-closure}(\{2\}) = \{1,2,3\}$.
$C = \delta_D(A,\text{b}) = \varepsilon\text{-closure}(\{4\}) = \{4,5,7\}$.
$D = \delta_D(B,\text{a}) = \varepsilon\text{-closure}(\{2\}) = B$.
$E = \delta_D(B,\text{b}) = \varepsilon\text{-closure}(\{4\}) = C$.
$F = \delta_D(C,\text{a}) = \varepsilon\text{-closure}(\{6\}) = \{5,6,7\}$.
$G = \delta_D(C,\text{b}) = \varepsilon\text{-closure}(\{8\}) = \{8,9,11\}$.
$H = \delta_D(F,\text{a}) = \varepsilon\text{-closure}(\{6\}) = F$.
$I = \delta_D(F,\text{b}) = \varepsilon\text{-closure}(\{8\}) = G$.
$J = \delta_D(G,\text{a}) = \varepsilon\text{-closure}(\{10\}) = \{9,10,11\}$.
$K = \delta_D(G,\text{b}) = \varepsilon\text{-closure}(\{12\}) = \{12,13,15\}$.
$L = \delta_D(J,\text{a}) = \varepsilon\text{-closure}(\{10\}) = J$.
$M = \delta_D(J,\text{b}) = \varepsilon\text{-closure}(\{12\}) = K$.
$N = \delta_D(K,\text{a}) = \varepsilon\text{-closure}(\{14\}) = \{13,14,15\}$.
$O = \delta_D(K,\text{b}) = \emptyset$.
$P = \delta_D(N,\text{a}) = \varepsilon\text{-closure}(\{14\}) = N$.
$Q = \delta_D(N,\text{b}) = \emptyset = O$.
$R = \delta_D(O,\text{a}) = \emptyset = O$.
$S = \delta_D(O,\text{b}) = \emptyset = O$.

Therefore, the transition table of the DFA, whose starting state is A and finite states are $\{K,N\}$, is as follows.

NFA STATE	DFA STATE	a	b
$\{0,1,3\}$	A	B	C
$\{1,2,3\}$	B	B	C
$\{4,5,7\}$	C	F	G
$\{5,6,7\}$	F	F	G
$\{8,9,11\}$	G	J	K
$\{9,10,11\}$	J	J	K
$\{12,13,15\}$	K	N	O
$\{13,14,15\}$	N	N	O
$\emptyset$	O	O	O

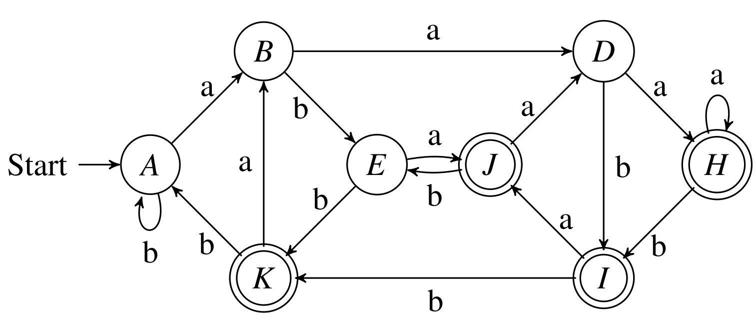
Its transition diagram is depicted as follows.



2 Optional Exercises

2.1 Exercise 1

- Regular expression 2:
  1. Initial partition: $G_1 = \{A,B,C,D,E\}$, $G_2 = \{H,I,J,K\}$.
  2. Handling group G_1: b splits it into two groups $G_3 = \{A,B,C,D\}$, $G_4 = \{E\}$.
  3. Handling group G_3: a splits it into two groups $G_5 = \{A,B,C\}$, $G_6 = \{D\}$.
  4. Handling group G_5: b splits it into two groups $G_7 = \{A,C\}$, $G_8 = \{B\}$.
  5. Handling group G_2: a splits it into two groups $G_9 = \{H,I,J\}$, $G_{10} = \{K\}$.
  6. Handling group G_9: b splits it into two groups $G_{11} = \{H,J\}$, $G_{12} = \{I\}$.
  7. Handling group G_{11}: a splits it into two groups $G_{13} = \{H\}$, $G_{14} = \{J\}$.
  8. Picking A, B, D, E, H, I, J, K as the representatives to construct the minimum-state DFA.



- Regular expression 3:
  1. Initial partition: $G_1 = \{A, B, C, F, G, J, O\}$, $G_2 = \{K, N\}$.
  2. Handling group G_1: a splits it into two groups $G_3 = \{A, B, C, F, O\}$, $G_4 = \{G, J\}$.
  3. Handling group G_3: b splits it into two groups $G_5 = \{A, B, O\}$, $G_6 = \{C, F\}$.
  4. Handling group G_5: b splits it into two groups $G_7 = \{A, B\}$, $G_8 = \{O\}$.
  5. Picking A, C, G, K, O as the representatives to construct the minimum-state DFA.