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Time-Bounded Positive Influence in Social Networks

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Abstract The appearance of social networks provides great opportunities for people to communicate, share and disseminate information. Meanwhile, it is quite challenge for utilizing a social networks efficiently in order to increase the commercial profit or alleviate social problems. One feasible solution is to select a subset of individuals that can positively influence the maximum other ones in the given social network, and some algorithms have been proposed to solve the optimal individual subset selection problem. However, most of the existing works ignored the constraint on time. They either assume that the time is infinite or only suitable to solve the snapshot selection problems. Obviously, both of them are impractical in the real system. Due to such reason, we study the problem of selecting the optimal individual subset to diffuse the positive influence when time is bounded. We proved that such a problem is NP-hard, and a heuristic algorithm based on greedy strategy is proposed. The experimental results on both simulation and real-world social networks based on the trace data in Shanghai show that our proposed algorithm outperforms the existing algorithms significantly, especially when the network structure is sparse.

Keywords social network · influence · dominating set

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1 Introduction

Social network is a platform consisting of individuals who share same interests, activities and backgrounds or have real-life connections. With the exponential growth of online social networks, more and more people are involved. According to the recent statistics, there are more than a billion Facebook accounts in the world. Among the billions of online social network individuals, more than 27% and 40% Europeans access social networks at least once within a day and a week, respectively [1]. Such a platform provides great opportunities for people to communicate, share and disseminate information. The popularity of social networks and their influences among individuals have attracted a great deal of attention due to its great potential benefits on both economy and society [2] . For example, in viral marketing, commercial companies can utilize social network as a channel to publicize their products [3]. Many intervention and education programs take advantage of this platform to help with alleviating social problems, such as drinking, smoking and drug problems. The government can also use social networks to stop rumors [4][5] .

In general, a social network can be represented as a graph $G(V, E)$, where V is the set of nodes representing the individuals in a social network, E is the set of edges where each edge represents the social tie between two individuals. Many research works indicate that individuals have tendency to follow the behaviors of their friends and relatives rather than the information obtained from other ways, such as radio, TV and online advertisement [6], and this tendency is growing with the increase of the number of her friends having the same behavior. Thus, one promising way to take advantage of social networks is to select a subset of individuals to positively influence other nodes in a social network, that is, this subset is expected to result in large cascade of further adoptions. The major reason to choose a subset of individuals instead of nodes in a whole network is because of budget limitation which is the primary concern for many applications. For example, a commercial company can promote some individuals in a social network with a fixed budget and ask them to publicize their products. An intervention program can select a subset of binge drinkers to participate such that they will positively influence other binge drinkers subsequently over a social network. The government can also depend on a subset of individuals to spread the truth in order to stop rumors. One efficient way to address the aforementioned problems is to make use of a dominating set [7]. A node set D is called a dominating set of graph G if each node in G not in D has at least one neighbor in D. However, the approaches employing dominating sets only consider individual-to-individual social influence, which is not practical in social networks. It is indicated by [6] that positive (negative) influence from more neighbors/friends will result in more impact to an individual. That is, if one individual will turn from negative to positive with a high probability if she has more positive friends than negative friends. Wang *et al.* [8] take the group-to-individual interaction into account and introduce a more reasonable model in social networks. They studied the Positive Influence Dominating Set (PIDS) problem. A set D is called a PIDS if for each individual in a social network, there are more than half of its neighbors belonging to D. Wang *et al.* proposed a greedy approximation algorithm to derive the least possible set to influence all the nodes. In many previous works, the problems are always formulated as discrete optimization problems (influence maximization problems) with different object functions and constraints. For example, Kempe *et al.* [9] proposed the first influence maximization problem, which is to select a fixed number of candidates such that the expected number of the influenced nodes can be maximized. However, one of the key factors, time constraint, has been overlooked by most existing studies. The goal of the influence maximization problem is to maximize the expected number of influenced nodes after an unbounded time period, while the purpose of the PIDS problem is to find a dominate set D such that all the other nodes can be influenced by D instantly. In many real-world applications, time is an inevitable factor that needs to be considered. For example, in viral marketing, commercial companies are expected to know the effect of popularization within a time limit in order to ensure the normal operation of the chain of funds. Government always wants to stop rumors within a certain time to reduce public panic. Another factor overlooked by most of the existing works is the negative influence. Note that people can have both positive and negative influence on their close friends in social networks and an individual could turn back and forth with the changing of her neighbors. For example, one can trust the truth at the beginning and will have positive impact on her direct friends, on the other hand, she may believe the rumors if most of her friends pass negative information to her.

Motivated by the aforementioned factors, we consider the problem of how to select as few initial users as possible to influence all of the users in a social network within a certain time limit in this paper. An individual is positively influenced if more than half of its neighbors have positive impact on its and vice versa. In this paper, we first introduce the problem of finding a time-bounded Positive Influence Dominating Set (tPIDS), and propose a greedy algorithm to find the smallest possible size of a subset of the nodes in a social network which can influence the whole network within time t, where t is a parameter specified by users. In summary, the main contributions of this paper are summarized as follows.

1)The time-bounded Positive Influence Dominating Set problem is defined and we show finding a minimum tPIDS is NP-hard.

2)A greedy algorithm is proposed.The algorithm is based on the tPIDS SPREAD graph built on an original graph. The Move-Down operation is defined to adjust a tPIDS in order to find a feasible solution. The complexity of our greedy algorithm is also analyzed.

3)The correctness of our proposed algorithm is proven, that is, the solution returned by our algorithm is valid tPIDS of social networks.

4)We conduct extensive experiments on randomly generated graphs and real-world social networks like trace data sets provided by [10]. On loosely structured networks with roughly uniform degree of nodes, the initial influence set can be reduced by 15%-20% and the running time of our algorithm is acceptable. The experimental results indicate that our greedy algorithm is effective and efficient to find a tPIDS.

The rest of the paper is organized as follows. Section II summarizes the state-of-art works on the influence maximization problem. Section III provides the problem definition. Section IV proposes a greedy algorithm and proves that the result returned by our algorithm is feasible tPIDS of social networks. Section V presents the experimental results. Section VI concludes the paper.

2 Related Works

Domingos and Richardson first proposed the influence maximization problem [11]. The milestone work for the influence maximization problem is [9]. The problem is formulated as a discrete optimization problem which is to find k most influential nodes such that the expected number of the influenced nodes by the k seeds is maximized. Two fundamental information diffusion models, the Independent Cascade (IC) model and the Linear Threshold (LT) model are introduced. They first showed the problem is NP-hard and proposed two greedy algorithms with performance ratio of $1 - \frac{1}{e}$ for both the IC and LT models. The experimental results demonstrate their greedy algorithms significantly outperform two straightforward algorithms, classic degree and centrality-based heuristics. Kimura and Saito [12] proposed an efficient algorithm to calculate influence spread under shortest-path based influence cascade models. Leskovec *et al.* [13] proposed a Cost-Effective Lazy Forward (CELF) scheme to select seeds. CELF significantly reduces the number of evaluations on the influence spread ability of nodes by utilizing the submodularity property of the influence maximization objective. CELF speeds up the previous greedy algorithm more than a hundred times. Chen *et al.* [14] further improved the CELF algorithm in terms of scalability. They proposed an efficient heuristic algorithm which can keep the same approximation ratio but improve the running time significantly. Liu *et al.* [15] utilized the supervised Monte Carlo method to estimate the influence spread for nodes with specified precision by the sampling technology. Sheldon *et al.* [16] proposed a mixed integer programming formulation to maximize spread of cascades in networks. Although the objective function is not submodular anymore, their proposed preprocessing techniques can significantly reduce the computation time. Jung *et al.* [17] proposed a novel algorithm IRIE that incorporates Influence Ranking (IR) with a fast Influence Estimation (IE) method in order to obtain scalability and robustness. The Independent Cascade(IC) model and its extended version, the Independent Cascade Negative (IC-N) model, were considered. Wang *et al.* [18] proposed a novel greedy algorithm for selecting top-K influential nodes based on the community structure of a graph. Due to the fact that individuals within the same community have close relationships thus are more likely to influence each other while individuals among different communities are less likely to influence each other, the community based greedy algorithm chooses influential nodes within communities. All the aforementioned models try to maximize the expected number of the influenced nodes and do not take the time factor into account.

Another vein of the influence maximization problem is to select a positive influence dominating set. Dominating set, a subset D of node set V in graph G such that every node not in D is adjacent to at least one member of D, has been widely studied and utilized in many real life applications [19]. A positive influence dominating set is a variation of a dominating set, which is more suitable for social networks. A PIDS is a subset of graph G, called D, such that any node v in G is dominated by at least $\lceil \frac{deg(v)}{2} \rceil$ nodes in D. Wang *et al.* [8] first defined the PIDS problem in general random graphs. They proved the NP-hard of the Minimum Positive Influence Dominating Set (MPIDS) problem and proposed a greedy algorithm with approximation ratio $H(\delta)$ to construct an MPIDS in $O(n^3)$ time, where H is the harmonic function and δ is the maximum node degree. Zhang *et al.* [20] investigated the MPIDS problem in power-law graphs and proposed a greedy algorithm with a constant approximation ratio. Raei *et al.* [21] further improved the greedy algorithm with time complexity $O(n^2)$ for a graph with power-law degree distribution. Dihn *et al.* [22] extended the PIDS problem to a more general version, called the Total Positive Influence Dominating Set (TPIDS) problem. This problem is to seek a minimum sized D such that every node in a graph has a fraction ρ of neighbors in D. They proved a minimized TPIDS cannot be approximated with factor $1 - \epsilon \ln \max\{\delta, V^{\frac{1}{2}}\}$ for general graphs where δ is the maximum degree in a graph. A simple proof was provided to show the problem can be approximated within factor $\ln \delta + O(1)$. For a power law graph, a constant factor approximation algorithm was proposed [22]. Note that, all the PIDS related problems are to seek a dominating set which can instantly affect all the nodes in a network, thus the size of a PIDS is large, which is not practical. In this paper, we will introduce a novel positive influence dominating set formulation by taking the time limit into account.

2.1 Problem Definition

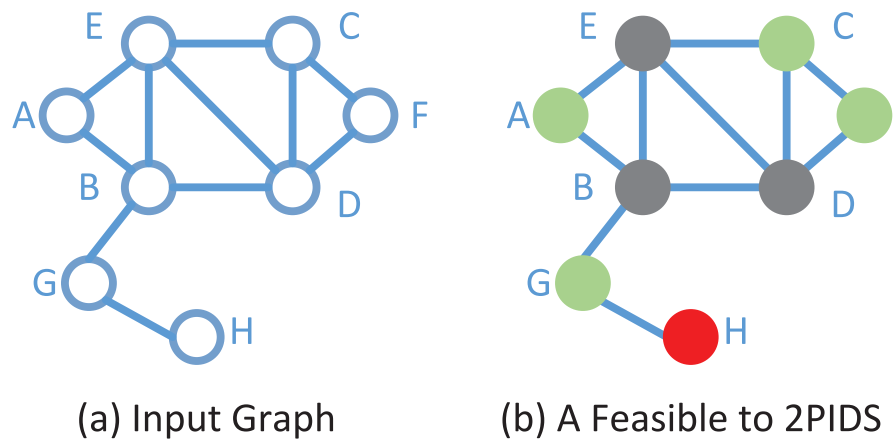
Assume a social network can be represented by an undirected graph $G = (V, E)$, where $v \in V$ is an individual in the social network and $e \in E$ represents the existence of social relationship between two individuals. Let $deg(v)$ be the number of the neighbors of v. That is, there are in total $deg(v)$ individuals who have direct social relationship with v. A set D is called a dominating set of G if each node not in D has at least one neighbor in D.

Definition 1 (Positive Influence Dominating Set (PIDS)) Given an undirected graph $G = (V, E)$, a node v is influenced by set D if at least $\lceil \frac{deg(v)}{2} \rceil$ of its neighbors is in the influence set D. A set P is called an influenced set of D if each $v \in P$ is influenced by D, defined as $P \models D$. A set D is a Positive Influence Dominating Set (PIDS) of G if $V \models D$.

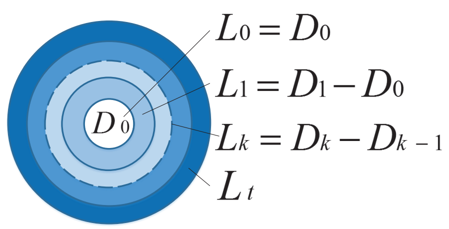
Definition 2 (Time-bounded Positive Influence Dominating Set (tPIDS)) Given an undirected graph $G = (V, E)$, a set D_0 is a t Positive Influence Dominating Set (tPIDS) of G if $V \models D_{t-1} \models \cdots D_i \cdots \models D_1 \models D_0$ where $D_i \subset V$ $(1 \leq i \leq t-1)$ and $D_i = \arg\max |P|_{P \models D_{i-1}}$. That is, D_i is the largest set among all the set influenced by D_{i-1}.

Note that in a tPIDS, all the nodes influenced by D_i will be included in D_{i+1}. That is, $D_i \subseteq D_{i+1}$.

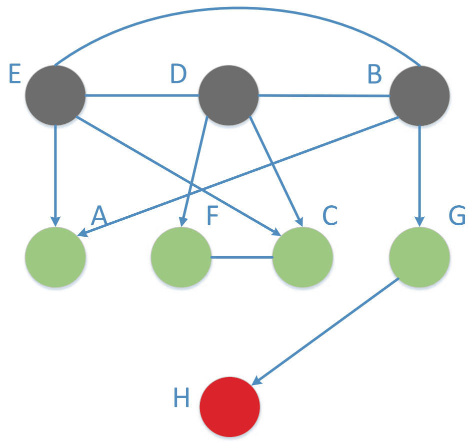
The goal of constructing a $tPIDS$ is to find a set D_0 which can positively influence the whole network within time t. For example, a commercial company wants to find a group of customers to propagate the produces such that all the customers in a social network will be positively influenced within a certain time. An intervention program wants to educate a set of binge drinkers such that positive effect of the intervention will conquer the negative effect within a certain time. Government wants to select a set of people in a social network to stop a rumor in a short time. Apparently, V is the largest tPIDS of $G = (V, E)$. However, it is costly to include all or too many individuals in a tPIDS. Our considered problem is to find a Minimum tPIDS (MtPIDS). That is, we want to use the least possible cost to positively influence the whole network in a certain time. The left graph



(a) An example of 2PIDS



(b) An example of layers



(c) An example of t-SPREAD graph

Fig. 1 Examples

in Fig.1(a) is an input graph $G = (V, E)$ where $V = \{A, B, C, D, E, F, G, H\}$. Let $D_0 = \{B, D, E\}$ which is the black node set in the right graph Fig.1(a). The green nodes A,C,F and G in the graph are influenced by D_0 and will be included in D_1. So $D_1 = \{A, B, C, D, E, F, G\}$. The red node H is influenced by D_1 and will be included in D_2. So $D_2 = \{A, B, C, D, E, F, G, H\} = V$. Thus D_0 is a feasible 2PIDS with size 3.

Theorem 1 *MtPIDS is NP-hard.*

Proof: It has been shown in [8] that finding a minimum PIDS is NP-hard. Since finding MtPIDS is a general version of finding a minimum PIDS, MtPIDS is NP-hard.

Definition 3 **(t-SPREAD)** Given an influence set $D_k \subseteq V$, D_{k+1} is the set with the maximum size such that $D_{k+1} \models D_k$. Let $L_{k+1} = D_{k+1} - D_k$,then L_{k+1} is the set of influence nodes of D_k. Note that $L_0 = D_0$. Then the transition from D_k to D_{k+1} is called a SPREAD. Given a graph $G = (V, E)$, G is said to be t-SPREAD to D_0 if V can be obtained from an initial set D_0 ($D_0 \subseteq V$) in t times of SPREAD.

If graph G is t-SPREAD to D_0, then V can be divided into $t+1$ layers (Fig.1(b)). Let $L_0, L_1, \ldots, L_t$ be the set of nodes in each layer. For an arbitrary node u in set V, there exists an integer k ($0 \leq k \leq t$), such that u belongs to set L_k. Let $deg(u)$ be the degree of node u, and $N(u)$ be the set of neighbors of u. According to the definition of tPIDS, there exists an integer p ($p \geq \lceil \frac{deg(u)}{2} \rceil$) and a set $H(u)$ such that $H(u) \subseteq N(u)$, $H(u) \subseteq D_{k-1}$ and $|H(u)| = p$. That is to say, if u belongs to L_k, then at least half of its neighbors are in the inner layers of L_k.

Definition 4 (t-SPREAD graph) For each node u in V, draw a directed edge from each node in $H(u)$ to u, an undirected edge from each node in $N(u)$ which is in the same layer with u to u and a directed edge from u to the rest of the nodes in $N(u)$ to form a t-SPREAD graph.

An example of t-SPREAD graph based on the graph in Fig.1(a) is shown on Fig.1(c). For each node u in a t-SPREAD graph, let R_u be its in-degree, C_u be its out-degree, and P_u be the number of its neighbors in the same layer with u, then $R_u + C_u + P_u = deg(u)$. We also have

$$\begin{cases} R_u \geq C_u + P_u & \text{if } u \notin D_0 \\ P_u \geq C_u & \text{Otherwise} \end{cases} \tag{1}$$

If u belongs to D_0, $P_u \geq C_u$ means that at least half of the neighbors of u are in the same layer with u, thus u is a influence node of D_0 and would be influenced by D_0. If u is not in D_0, suppose that u belongs to L_k, $R_u \geq C_u + P_u$ means that over half of neighbors of u are in the inner layer of L_k, that is, u is a influence node of D_{k-1} and would be influenced by D_{k-1}.

Definition 5 (Redundant Node) For any node u in D_0, u is called a *redundant node* if it satisfies any of the following conditions:
1)$C_u = 0$ and for each $v \in N(u) \cap D_0$, $P_v - 1 \geq C_v + 1$.
2)For each $m \in N(u) \cap D_1$, $R_m - 1 \geq C_m + (1 + P_m)$. If $v \in N(u) \cap D_0$, $P_v - 1 \geq C_v + 1$. Suppose that v is one of the neighbors of u, which is a *redundant point* in D_0. If v satisfies the above conditions, then we say that v supports u.

If all the neighbors of u in D_0 satisfy $P - 1 \geq C + 1$ and all the neighbors of u in D_1 satisfy $R - 1 \geq C + P + 1$, after moving u down, all the neighbors of u would still satisfy Formula (1). In other word, moving u down would not break the structure of SPREAD graph. That is to say, redundant nodes are the candidate nodes to be moved down. The goal of defining redundant node is to find some nodes in D_0 to be moved down. Note that moving down these nodes should not result in other nodes dissatisfying Formula (1). For a redundant node v_i in D_0 and a node v_j in L_k, we say v_j supports v_i if v_j still satisfies Formula (1) even when moving v_i down to any layer between L_t and L_k. In the definition of redundant node, only nodes in L_0 and L_1 that support some redundant nodes in D_0 are taken into account. According to Theorem 4., moving down a redundant node will certainly influence the value of P, R and C of neighbors in L_0 and L_1. That is to say, a node must be a redundant node if all the neighbors in L_0 and L_1 support it, thus it can be moved down so as to reduce the size of D_0.

The problem definition of MtPIDS is as follows:
Input: A given graph $G = (V, E)$ and a time limit t.
Output: A tPIDS of G with the smallest size.

3 Algorithm Description

3.1 Move-Down Algorithm

Once the t-SPREAD graph is constructed, the node set in each layer is determined. During the process of construction, if the size of D_0 is minimized, then an optimal result is obtained for the tPIDS problem. We employ a top-down approach to construct the t-SPREAD graph. To be specific, all the nodes in the initial graph G are included in D_0, and then all the redundant nodes in D_0 are moved down to a certain layer between L_1 and L_t. Once all the redundant nodes are moved down, the final result of the tPIDS problem is approximately optimal. The key point of the move-down operation is how to choose a candidate node to be moved down in each step. According to Theorem 5, when we moving down a redundant node, some other redundant nodes in D_0 might become non-redundant nodes, while some non-redundant nodes might become redundant nodes. Suppose that when moving down a redundant node in D_0, say v_i, n_i nodes in D_0 become redundant nodes and m_i nodes become non-redundant nodes. A cost function is defined as $Cost(i) = m_i - n_i$. Intuitively, the smaller the $Cost(i)$ is, the more redundant nodes can be found. In our greedy algorithm, each time we choose a redundant node v_i to minimize the cost function until there are no redundant nodes any more. Then the size of D_0 is as small as possible and D_0 is the approximate optimal solution to the tPIDS problem.

We design a practical and efficient greedy algorithm to solve the tPIDS problem

Algorithm 1 Move-Down Algorithm

Require: Graph $G = (V, E)$, t: the spreading times
Ensure: The set of initial influence nodes D_0
1: $Rdd = \varnothing$, D_0=V, $L_i = \varnothing$ $(1 \leqslant i \leqslant t)$.
2: For each node v_i in node set V, if v_i satisfies the definition of redundant node, then $Rdd = Rdd \cup v_i$.
3: For each node $v_i \in Rdd$, calculate $Cost(i)$.
4: If v_j owns the minimum cost function, move v_j to L_t, and examine whether all of the neighbors of v_j satisfy Formula (1). If not, consider $L_{t-1}, ..., L_k, ..., L_1$ in order until when moving v_j down to L_k, all the neighbors of v_j satisfy Formula (1). Then $L_k = L_k \cup v_j$, $D_0 = D_0 - v_j$.
5: Scan the node set V, add new redundant nodes into Rdd and delete non-redundant nodes from Rdd.
6: Repeat step 3-5 until $Rdd = \varnothing$.
7: Output D_0.

as shown in Algorithm 1. It takes as input the network graph $G = (V, E)$ and the spreading times t. Initially, all the nodes in V are assigned to D_0 $(i.e., D_0 = V)$. Rdd is the set of the redundant nodes in V and L_i is the node set of each layer i $(1 \leqslant i \leqslant t)$. The final result is given by the subset D_0 which is the initial influence set. Firstly, the algorithm scans all the nodes in V. For each node v_i, if it satisfies the definition of redundant node, add v_i to *Rdd* (Step 4). It initializes *Rdd*, thus we know which nodes might be moved down from D_0.

Secondly, the algorithm needs to choose the node to be moved down based on a cost function. It calculates the cost function for each node in *Rdd* and chooses the node, say v_j, to minimize the cost function (Step 5). Particularly, if more than one node share the minimum cost function, then the one with the least degree is chosen intuitively for reason that moving down a node with a smaller degree means less impact on other nodes. Then the algorithm considers the appropriate layer in descending order (from L_t to L_1) to which v_j should be moved down. In more details, first we consider the situation that we move v_j down to the outer most layer (L_t). It should be examined whether all the neighbors of v_j satisfy Formula (1). If not, v_j cannot be moved to L_t and other layers ($L_{t-1}, L_{t-2}, ..., L_k, ..., L_1$) should be taken into account in descending order. That is, in the next step, we attempt to move v_j to L_{t-1} and examine the constraints and so on. Finally, we find that moving v_j down to L_k satisfies the constraints. Then we update the node sets L_k and D_0, that is, $L_k = L_k \cup v_j$ and $D_0 = D_0 - v_j$ (Step 6). After we moving down a redundant node, some redundant nodes might become non-redundant nodes, and vice versa, according to Theorem 5. In the last step, the algorithm updates *Rdd* and repeats the above steps until there is no redundant nodes (Steps 7-8). Whether a node is a redundant node or not depends on the property of its neighbors in L_0 and L_1. Intuitively, if we can reduce the number of the nodes in L_0 and L_1, the constraint on redundant nodes might become weak, thus the number of the redundant nodes might increase. The only way to reduce the nodes in L_0 is to move down some nodes, while the way to reduce nodes in L_1 is to move fewer nodes from L_0 to L_1. In addition, a redundant node must be moved down to L_1 if it cannot be moved to L_2, that is, the neighbors in L_2 of the redundant node do not support it. Naturally, we tend to reduce the nodes in L_2. By that analogy, each time we move down a redundant node, the target layer should be as outer as possible. That is to say, the priority of an outer layer is higher than that of an inner layer. The example of our algorithm is in technique report [23].

Performance Guarantee

According to Theorem 1, the $tPIDS$ problem is NP-Hard. Our greedy algorithm runs in $O(D^2 * |V|^2)$ where D is the maximum degree and $|V|$ is the number of the nodes in a network.

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