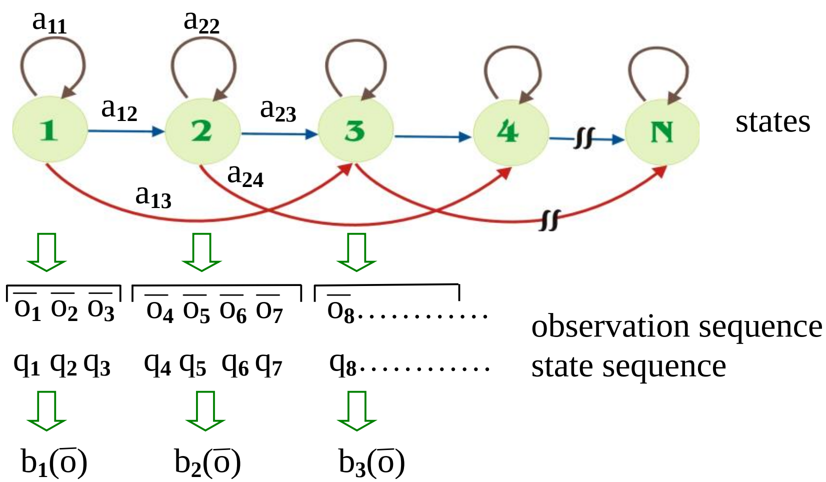
2.0 Fundamentals of Speech Recognition

References for 2.0

1.3, 3.3, 3.4, 4.2, 4.3, 6.4, 7.2, 7.3, of Bechetti

Hidden Markov Models (HMM)



- **Formulation**

$\bar{o}_t = [x_1, x_2, \ldots x_D]^T$ feature vectors for a frame at time t

$q_t \in \{1,2,3\ldots N\}$ state number for feature vector $\bar{o}_t$

$A = [a_{ij}]$, $a_{ij} = \text{Prob}[\, q_t = j \mid q_{t-1} = i \,]$

state transition probability

$B = [b_j(\bar{o}),\ j = 1,2,\ldots N]$ observation (emission) probability

$b_j(\bar{o}) = \sum_{k=1}^{M} c_{jk} b_{jk}(\bar{o})$ Gaussian Mixture Model (GMM)

$b_{jk}(\bar{o})$: multi-variate Gaussian distribution

for the k-th mixture (Gaussian) of the j-th state

M : total number of mixtures

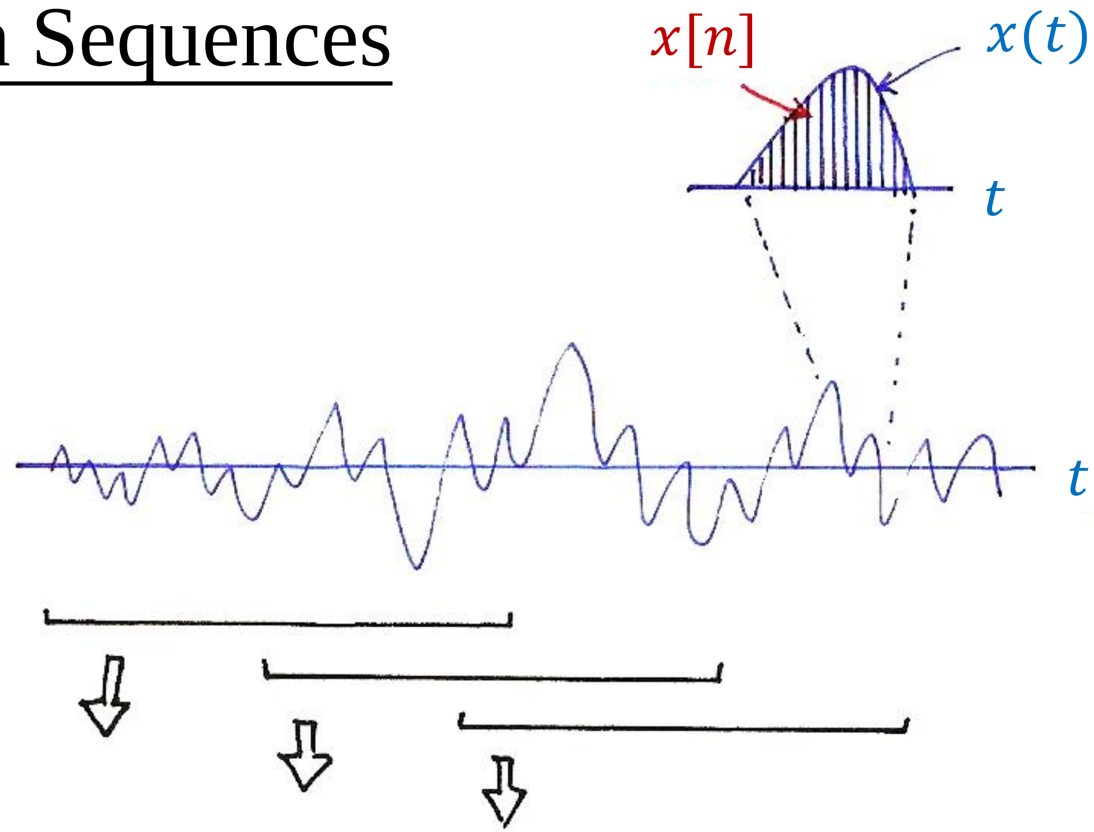
$\sum_{k=1}^{M} c_{jk} = 1$

$\pi = [\, \pi_1, \pi_2, \ldots \pi_N \,]$ initial probabilities

$\pi_i = \text{Prob}[q_1 = i]$

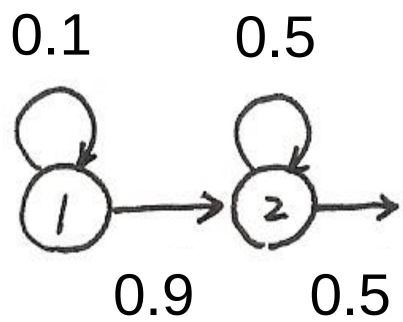
HMM : $(A, B, \pi) = \lambda$

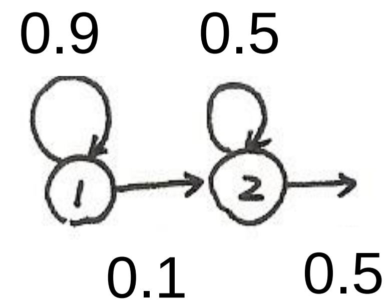
Observation Sequences



$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_D \end{bmatrix} = o_1 \quad \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_D \end{bmatrix} = o_2 \quad \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_D \end{bmatrix} = o_3$$

State Transition Probabilities

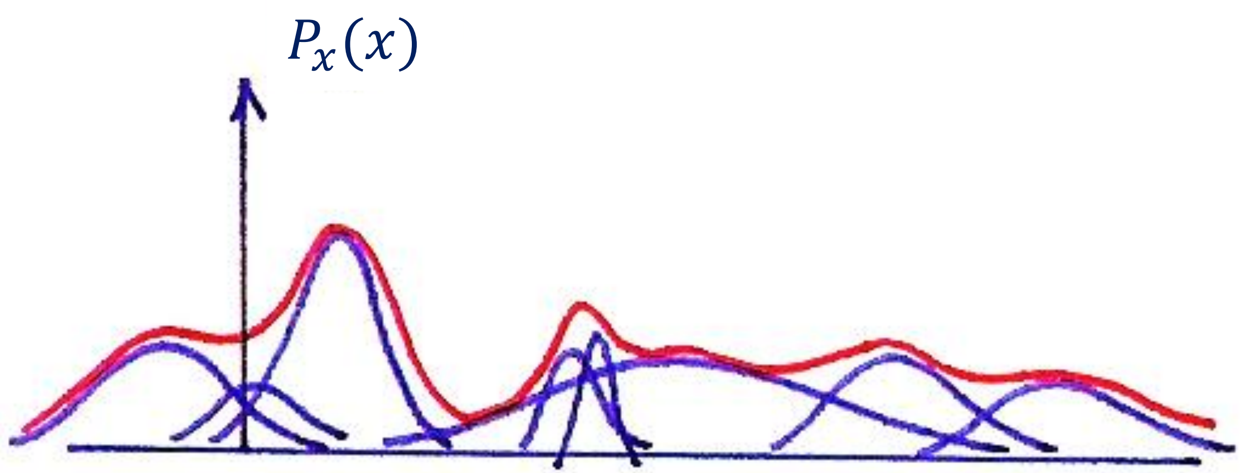




1 2 2 ⋯

1 1 1 1 1 1 1 1 2 2 ⋯

1-dim Gaussian Mixtures



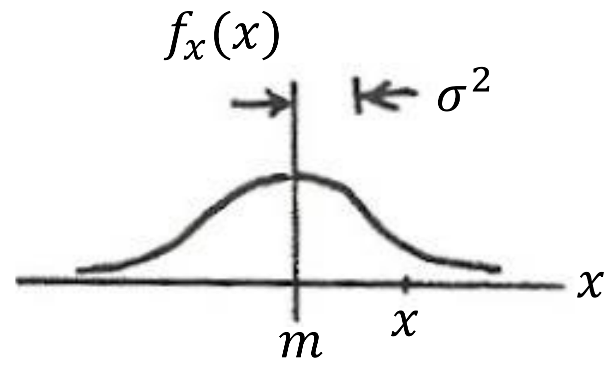
- **Gaussian Random Variable X**

$$f_X(x) = \frac{1}{(2\pi\sigma^2)^{1/2}}\, e^{-(x-m)^2/2\sigma^2}$$

- **Multivariate Gaussian Distribution for n Random Variables**

$$\overline{X} = [\, X_1, X_2, \ldots\ldots, X_n \,]^t$$

$$f_{\overline{X}}(\overline{x}) = \frac{1}{(2\pi)^{n/2}\Delta^{1/2}}\, e^{-\frac{1}{2}[\,(\overline{x}-\overline{\mu})^t \Sigma^{-1} (\overline{x}-\overline{\mu})\,]}$$



$$\overline{\mu} = [\, \mu_{X_1}, \mu_{X_2}, \ldots\ldots, \mu_{X_n} \,]^t$$

$$\Sigma = [\, \sigma_{ij} \,] \text{, covariance matrix}$$

$$\sigma_{ij} = E[\,(X_i - \mu_{X_i})(X_j - \mu_{X_j})\,]$$

$$\Sigma = \begin{array}{cc} j & \\ \begin{bmatrix} & \vdots & \\ \cdots & \sigma_{ij} & \cdots \\ & \vdots & \end{bmatrix} & i \end{array}$$

$$\Delta : \text{determinant of } \Sigma$$

$$\sigma_{ij} = E[(x_i - \mu_{x_i})\,(x_j - \mu_{x_j})]$$

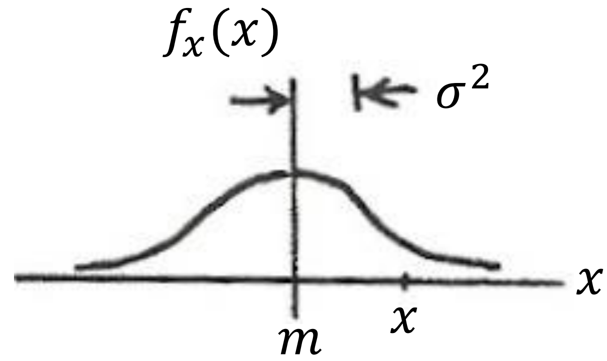
Multivariate Gaussian Distribution

$$(\bar{x}-\bar{\mu})^t \Sigma^{-1} (\bar{x}-\bar{\mu}) = \left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} - \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix} \right)^t \Sigma^{-1} (\bar{x}-\bar{\mu})$$

$$= \begin{bmatrix} x_1-\mu_1 & x_2-\mu_2 & \cdots & x_n-\mu_n \end{bmatrix} \Sigma^{-1} \begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \\ \vdots \\ x_n-\mu_n \end{bmatrix}$$

$$= (x_1-\mu_1)^2 + (x_2-\mu_2)^2 + \cdots\cdots, \quad \text{if } \Sigma = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & \ddots \end{bmatrix}$$

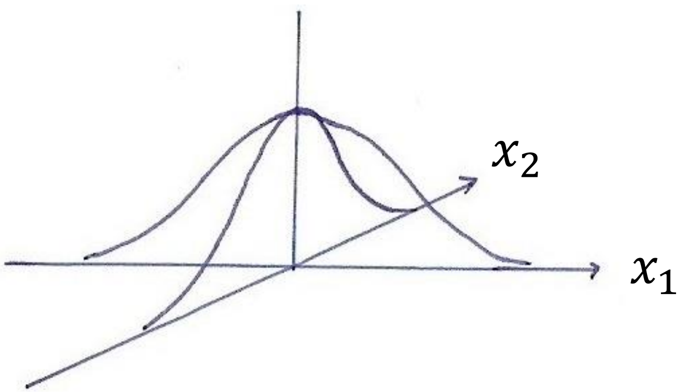
$$= \frac{(x_1-\mu_1)^2}{\sigma_{11}^2} + \frac{(x_2-\mu_2)^2}{\sigma_{22}^2} + \cdots, \quad \text{if } \Sigma = \begin{bmatrix} \sigma_{11}^2 & & 0 \\ & \sigma_{22}^2 & \\ 0 & & \ddots \\ & & & \sigma_{nn}^2 \end{bmatrix}$$

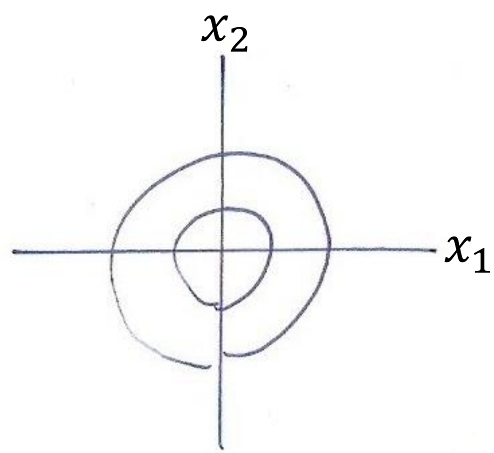


$$\Sigma = \begin{array}{c c} j & \\ \begin{bmatrix} & \vdots & \\ \cdots & \sigma_{ij} & \cdots \\ & \vdots & \end{bmatrix} & i \end{array}$$

$$\sigma_{ij} = E[(x_i - \mu_{x_i})\,(x_j - \mu_{x_j})]$$

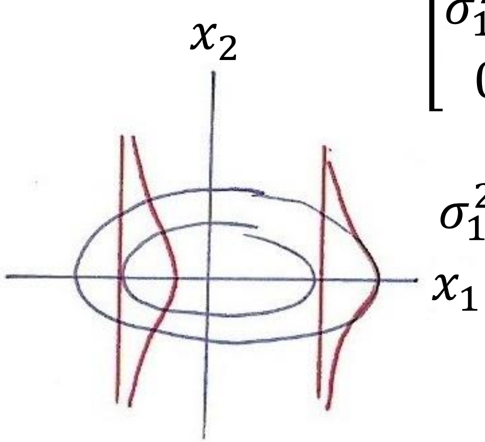
2-dim Gaussian





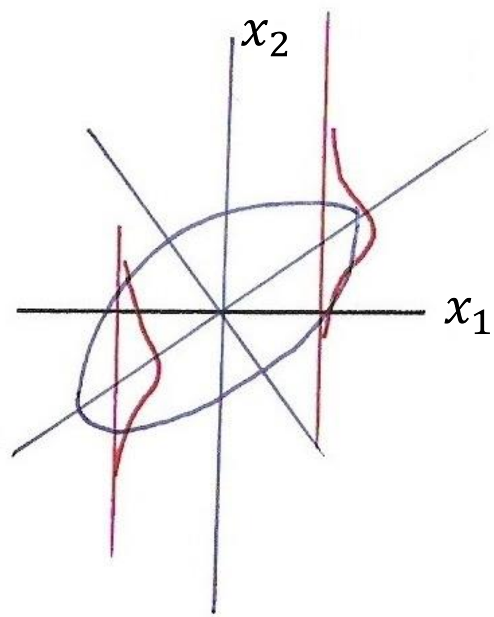
$$\begin{bmatrix} \sigma_{11}^2 & 0 \\ 0 & \sigma_{22}^2 \end{bmatrix}$$

$$\sigma_{11}^2 = \sigma_{22}^2$$



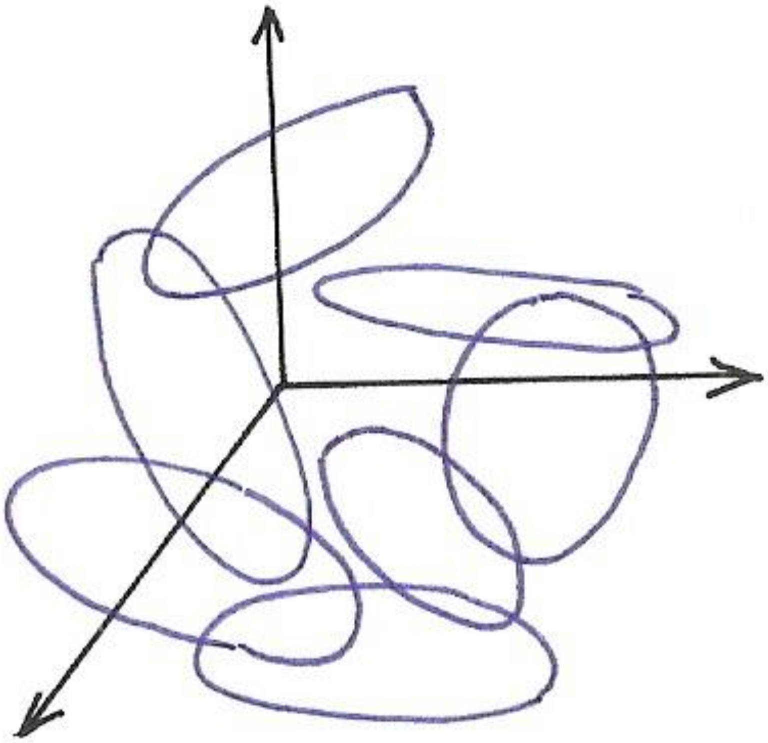
$$\begin{bmatrix} \sigma_{11}^2 & 0 \\ 0 & \sigma_{22}^2 \end{bmatrix}$$

$$\sigma_{11}^2 \neq \sigma_{22}^2$$



$$\begin{bmatrix} \sigma_{11}^2 & \sigma_{12} \\ \sigma_{21} & \sigma_{22}^2 \end{bmatrix}$$

N-dim Gaussian Mixtures



Hidden Markov Models (HMM)

- **Double Layers of Stochastic Processes**
  - hidden states with random transitions for time warping
  - random output given state for random acoustic characteristics
- **Three Basic Problems**

(1) Evaluation Problem:

Given $\overline{O} = (\overline{o}_1, \overline{o}_2, \ldots \overline{o}_t \ldots \overline{o}_T)$ and $\lambda = (A, B, \pi)$

find $\text{Prob}\,[\,\overline{O} \mid \lambda\,]$

(2) Decoding Problem:

Given $\overline{O} = (\overline{o}_1, \overline{o}_2, \ldots \overline{o}_t \ldots \overline{o}_T)$ and $\lambda = (A, B, \pi)$

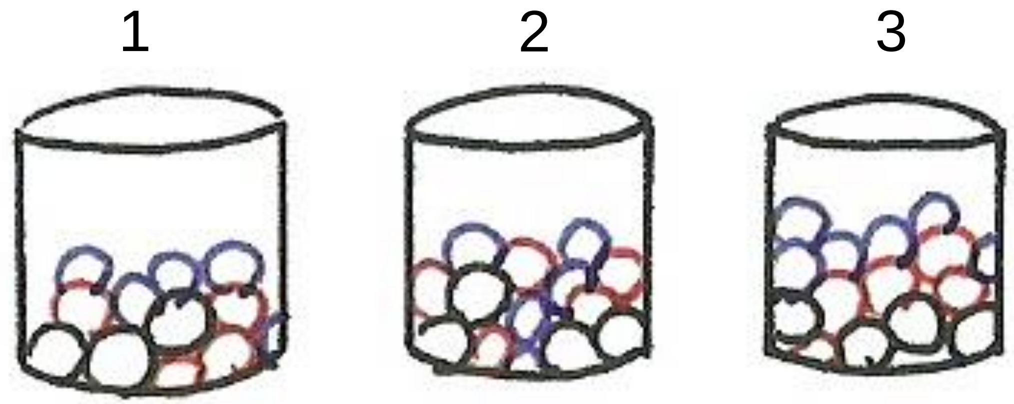
find a best state sequence $\overline{q} = (q_1, q_2, \ldots q_t, \ldots q_T)$

(3) Learning Problem:

Given $\overline{O}$, find best values for parameters in λ

such that $\text{Prob}\,[\,\overline{O} \mid \lambda\,] = \max$

Simplified HMM



RGBGGBBGRRR……

Feature Extraction (Front-end Signal Processing)

- **Pre-emphasis**

  $H(z) = 1 - az^{-1}, \quad 0 \ll a < 1$

  $x[n] = x'[n] - ax'[n-1]$

  - pre-emphasis of spectrum at higher frequencies

- **Endpoint Detection (Speech/Silence Discrimination)**
  - short-time energy

    $$E_n = \sum_{m=-\infty}^{\infty} (x[m])^2 w[m-n]$$

  - adaptive thresholds

- **Windowing**

  $$Q_n = \sum_{m=-\infty}^{\infty} T\{x[m]\} w[m-n]$$

  $T\{\bullet\}$: some operator

  $w[m]$: window shape

  - Rectangular window

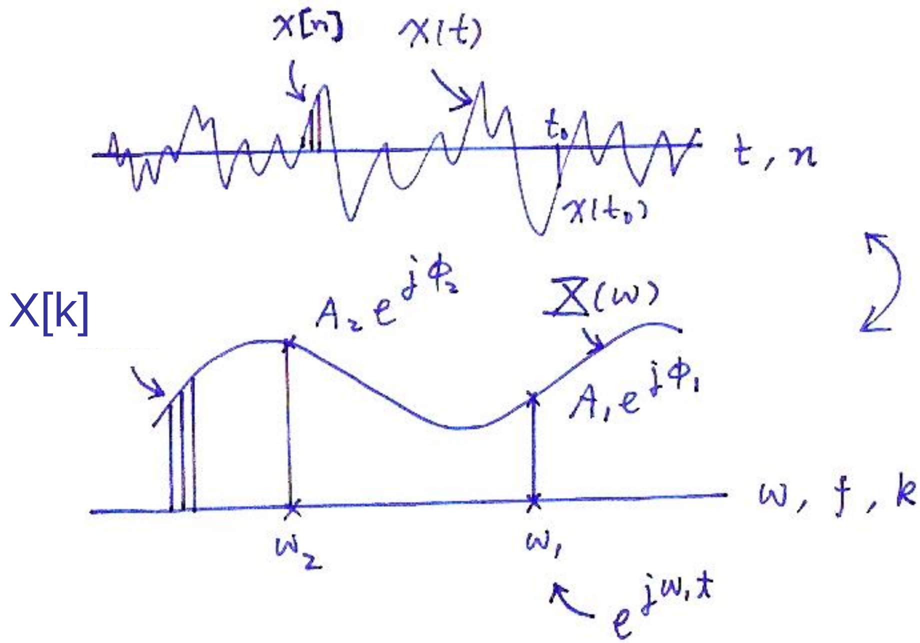
  $$w[m] = \begin{cases} 1, & 0 < m \leq L-1 \\ 0, & \text{else} \end{cases}$$

  Hamming window

  $$w[m] = \begin{cases} 0.54 - 0.46 \cos[\frac{2\pi m}{L}], & 0 \leq m \leq L-1 \\ 0, & \text{else} \end{cases}$$

  window length/shift/shape

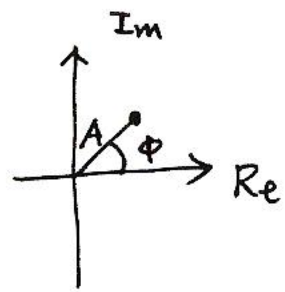
Time and Frequency Domains



time domain

1-1 mapping
Fourier Transform
Fast Fourier Transform (FFT)

frequency domain



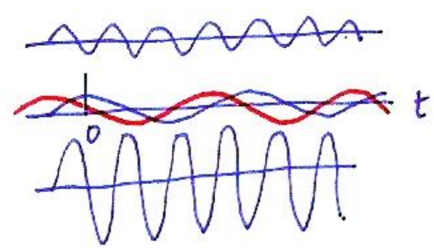
$$Re\{e^{j\omega_1 t}\} = \cos(\omega_1 t)$$

$$Re\{(A_1\, e^{j\phi_1})\,(e^{j\omega_1 t})\} = A_1 \cos(\omega_1 t + \phi_1)$$

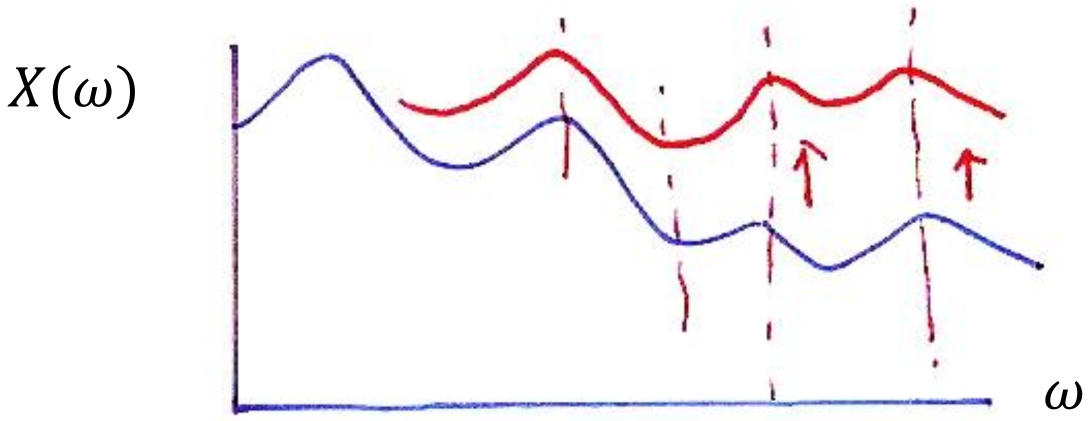
$$\vec{X} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$$

$$\vec{X} = \sum_i a_i\, x_i$$

$$x(t) = \sum_i a_i \;\; x_i\,(t)$$



Pre-emphasis



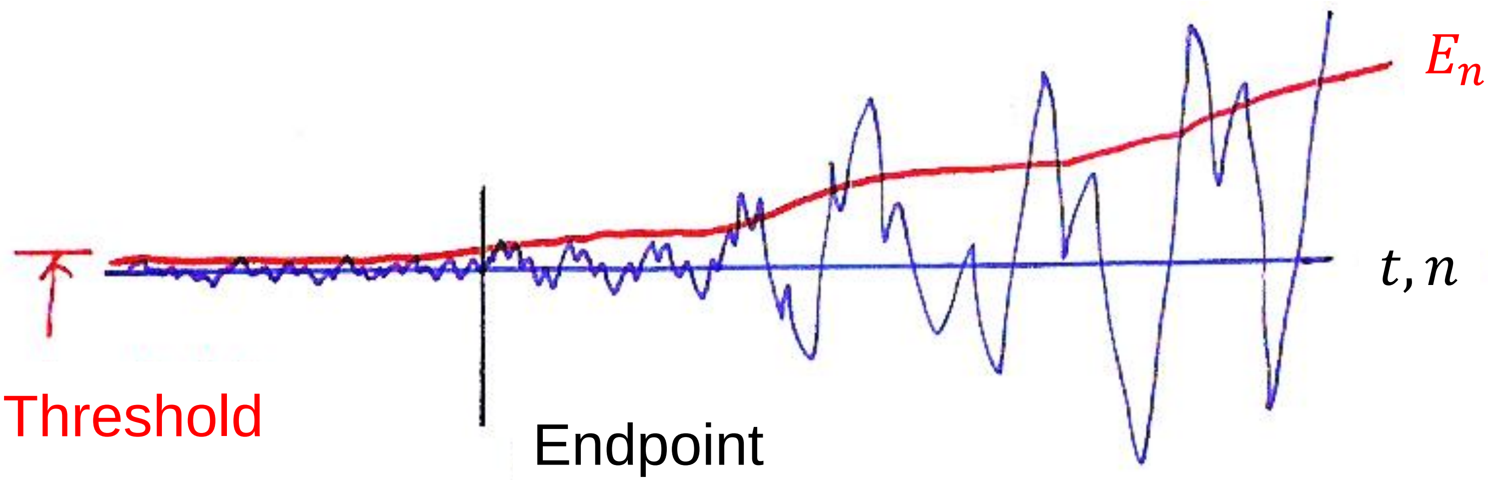
- **Pre-emphasis**

$H(z) = 1 - az^{-1}$, $0 << a < 1$

$x[n] = x'[n] - ax'[n-1]$

pre-emphasis of spectrum at higher frequencies

Endpoint Detection



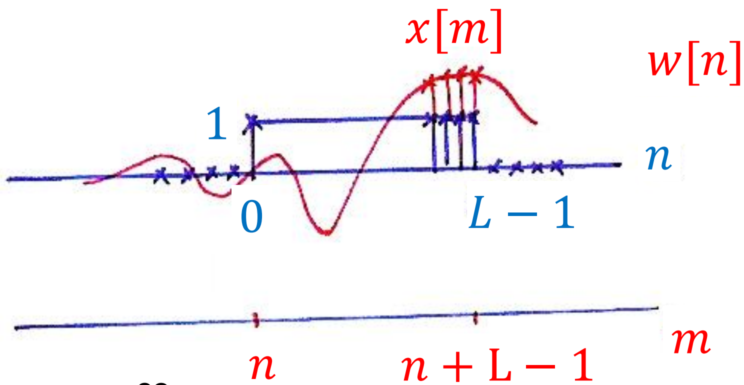
- **Endpoint Detection (Speech/Silence Discrimination)**
  - short-time energy

$$E_n = \sum_{m=-\infty}^{\infty} (x[m])^2 \, w[m-n]$$

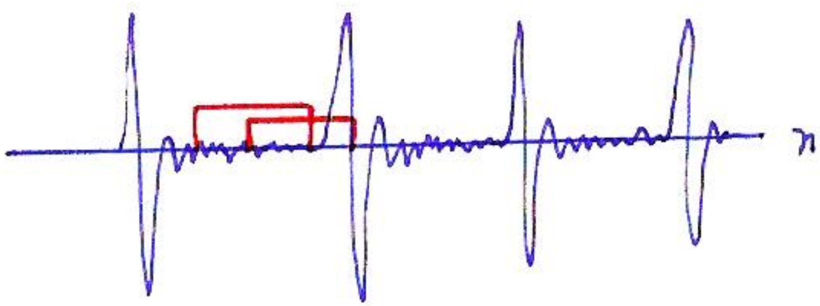
  - adaptive thresholds

Endpoint Detection

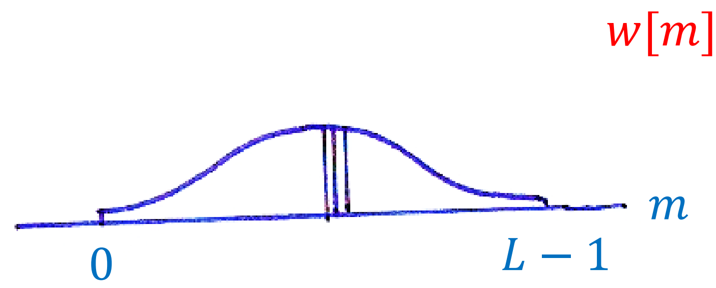
⊙ Rectangular Window



$$E_n = \sum_{-\infty}^{\infty} (x[m])^2\, w[m-n]$$



⊙ Hamming Window



Hamming window

$$w[m] = \begin{cases} 0.54 - 0.46\cos[\frac{2\pi m}{L}], 0 \le m \le L-1 \\ 0, \text{ else} \end{cases}$$

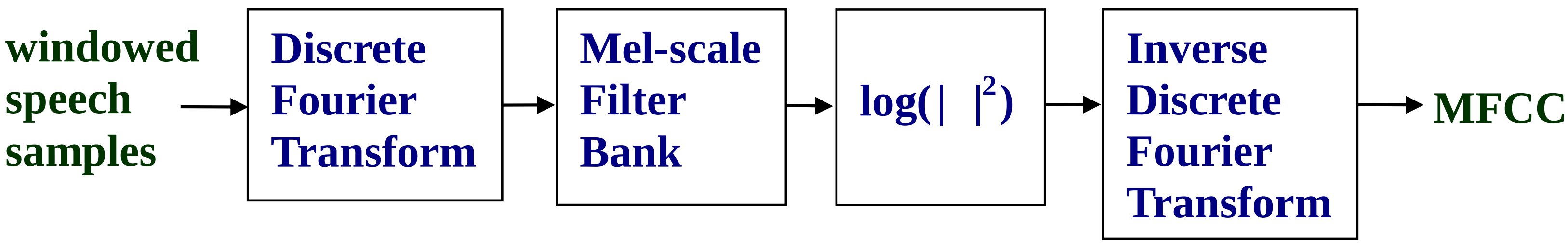
$$Q_n = \sum_{m=-\infty}^{\infty} T\{x[m]\}\, w[m-n]$$

$T\{\bullet\}$: some operator

w$\{m\}$: window shape

Feature Extraction (Front-end Signal Processing)

- **Mel Frequency Cepstral Coefficients (MFCC)**



  - Mel-scale Filter Bank

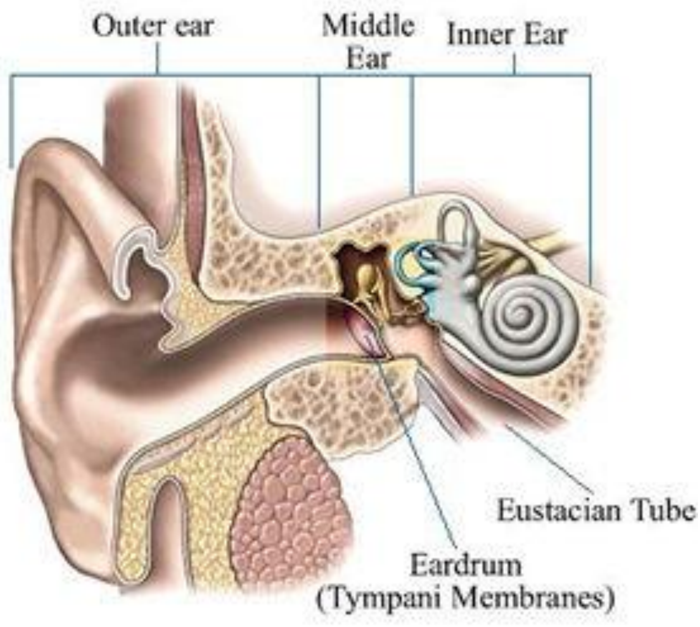
    triangular shape in frequency/overlapped

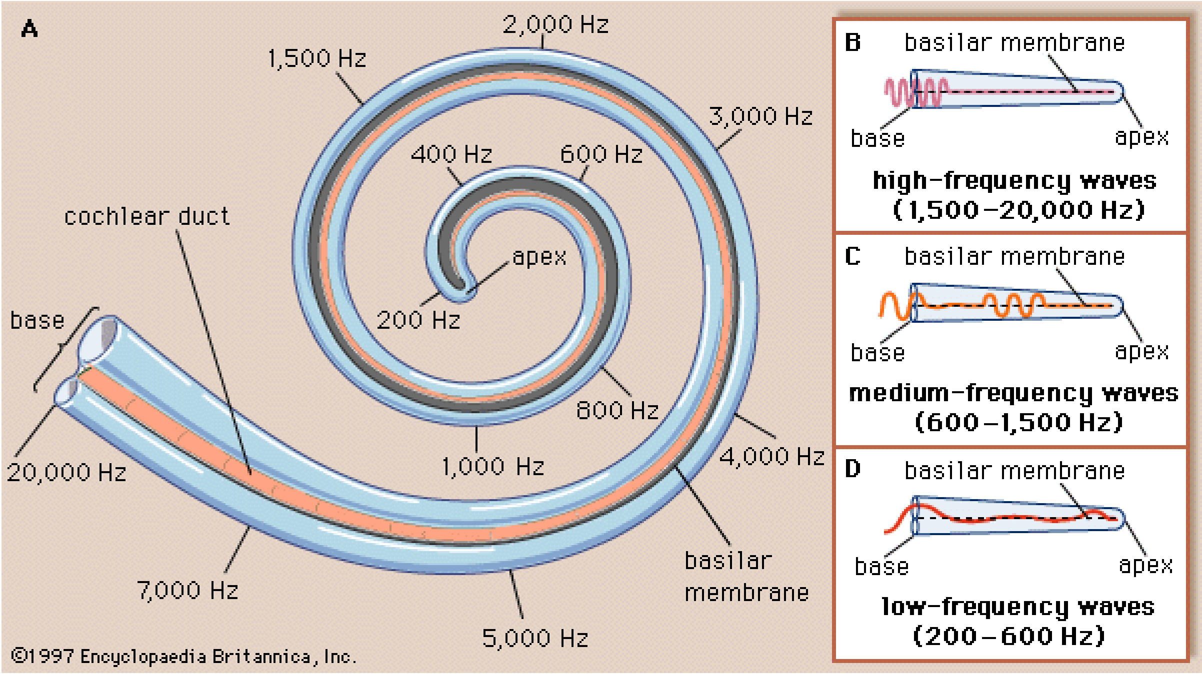
    uniformly spaced below 1 kHz

    logarithmic scale above 1 kHz

- **Delta Coefficients**
  - 1st/2nd order differences

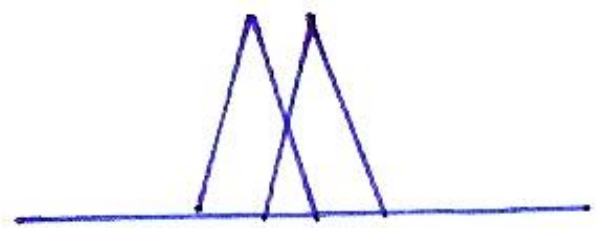
Peripheral Processing for Human Perception (P.34 of 7.0)



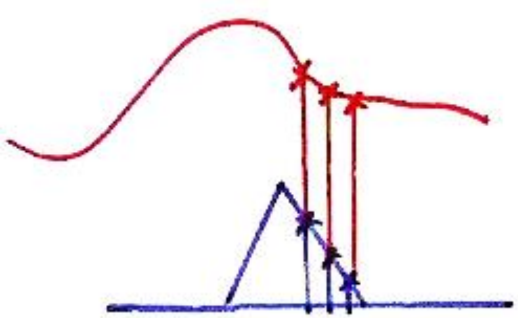


Mel-scale Filter Bank

ω

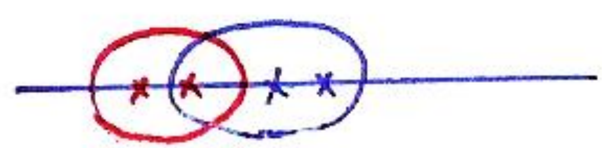


$\log \omega$



$X(\omega)$

ω



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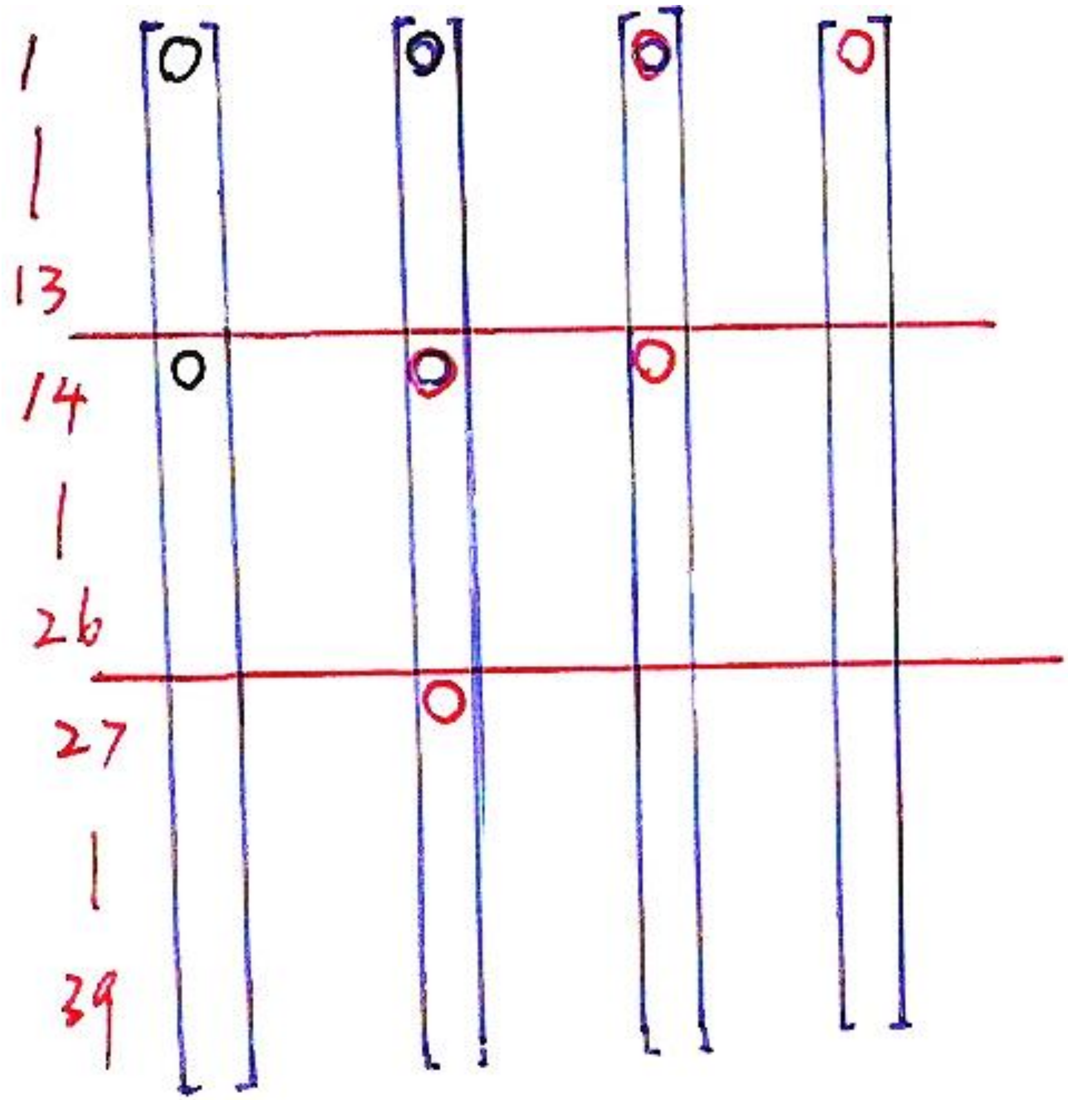
Delta Coefficients



Language Modeling: N-gram

$W = (w_1, w_2, w_3, \ldots, w_i, \ldots w_R)$ a word sequence

- Evaluation of P(W)

$$P(W) = P(w_1) \prod_{i=2}^{R} P(w_i|w_1, w_2, \ldots w_{i-1})$$

- Assumption:

$$P(w_i|w_1, w_2, \ldots w_{i-1}) = P(w_i|w_{i-N+1}, w_{i-N+2}, \ldots w_{i-1})$$

Occurrence of a word depends on previous N–1 words only

N-gram language models

$N = 2$	:	bigram	$P(w_i \mid w_{i-1})$
$N = 3$	:	tri-gram	$P(w_i \mid w_{i-2}, w_{i-1})$
$N = 4$	:	four-gram	$P(w_i \mid w_{i-3}, w_{i-2}, w_{i-1})$
$\vdots$			
$N = 1$	:	unigram	$P(w_i)$

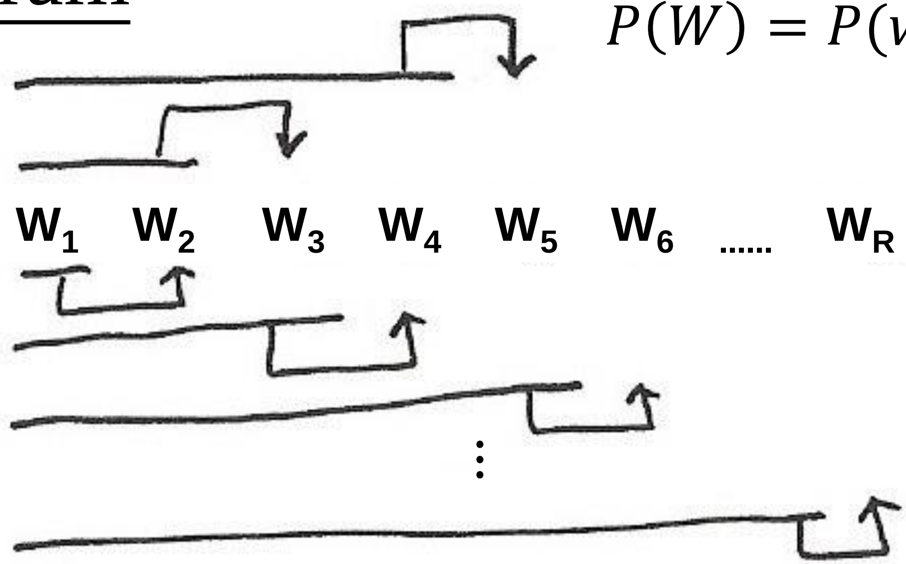
probabilities estimated from a training text database

example : tri-gram model

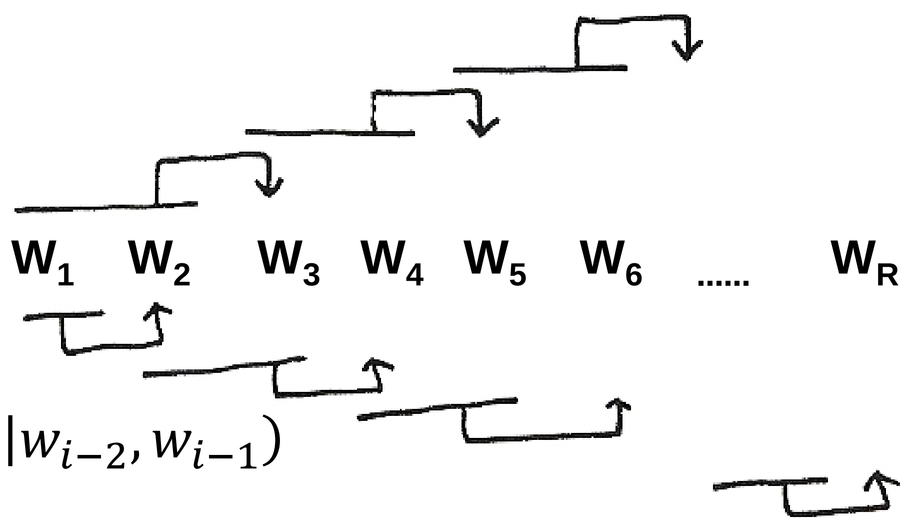
$$P(W) = P(w_1)\, P(w_2|w_1) \prod_{i=3}^{N} P(w_i|w_{i-2}, w_{i-1})$$

N-gram

$$P(W) = P(w_1)\prod_{i=2}^{R} P\left(w_i | w_1, w_2, \cdots, w_{i-1}\right)$$



⊙ tri-gram



$$P(W) = P(w_1)P(w_2|w_1)\prod_{i=3}^{N} P\left(w_i | w_{i-2}, w_{i-1}\right)$$

Language Modeling

- Evaluation of N-gram model parameters

unigram

$$P(w^{i}) = \frac{N(w^{i})}{\sum_{j=1}^{V} N(w^{j})}$$

w^{i} : a word in the vocabulary

V : total number of different words in the vocabulary

$N(\bullet)$ number of counts in the training text database

bigram

$$P(w^{j}|w^{k}) = \frac{N(<w^{k},w^{j}>)}{N(w^{k})}$$

$< w^{k}, w^{j} >$: a word pair

trigram

$$P(w^{j}|w^{k},w^{m}) = \frac{N(<w^{k},w^{m},w^{j}>)}{N(<w^{k},w^{m}>)}$$

smoothing – estimation of probabilities of rare events by statistical approaches

… this ………	50000
… this is ……	500
… this is a …	5

$$\text{Prob [is| this]} = \frac{500}{50000}$$

$$\text{Prob [a| this is]} = \frac{5}{500}$$

bigram

$$P\left(w^j \middle| w^k\right) = \frac{N(\langle w^k, w^j \rangle)}{N(w^k)}$$

$\langle w^k, w^j \rangle$: a word pair

trigram

$$P\left(w^j \middle| w^k, w^m\right) = \frac{N(\langle w^k, w^m, w^j \rangle)}{N(\langle w^k, w^m \rangle)}$$

Large Vocabulary Continuous Speech Recognition

$W = (w_1, w_2, \ldots w_R)$ a word sequence

$\overline{O} = (\overline{o}_1, \overline{o}_2, \ldots \overline{o}_T)$ feature vectors for a speech utterance

$$W^* = \underset{W}{\text{Arg Max}}\, \text{Prob}(W|\overline{O}) \quad \text{MAP principle}$$

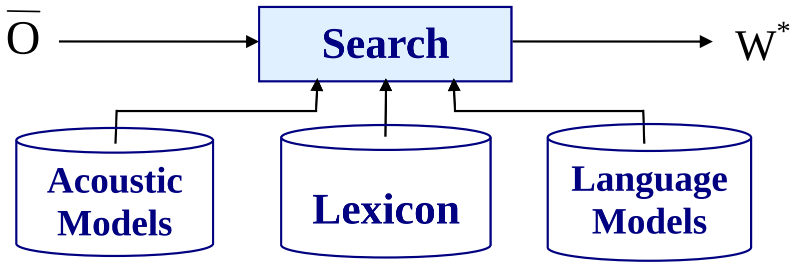
$$\text{Prob}(W|\overline{O}) = \frac{\text{Prob}(\overline{O}|W) \bullet P(W)}{P(\overline{O})} = \max$$

A Posteriori Probability

Maximum A Posteriori (MAP) Principle

$$\underbrace{\text{Prob}(\overline{O}|W)}_{\text{by HMM}} \bullet \underbrace{P(W)}_{\text{by language model}} = \max$$

- **A Search Process Based on Three Knowledge Sources**



- Acoustic Models : HMMs for basic voice units (e.g. phonemes)
- Lexicon : a database of all possible words in the vocabulary, each word including its pronunciation in terms of component basic voice units
- Language Models : based on words in the lexicon

Maximum A Posteriori Principle (MAP)

$$W : \{\ w_1\ ,\ \ w_2\ \ ,\ \ w_3\ \}$$

$\uparrow$ sunny $\quad$ $\uparrow$ rainy $\quad$ $\uparrow$ cloudy

$$\begin{array}{r} P(w_1) \\ P(w_2) \\ +\ P(w_3) \\ \hline 1.0 \end{array}$$

$$\vec{O} = (\vec{o}_1, \vec{o}_2, \vec{o}_3, \cdots)$$

weather parameters

- Problem

given $\vec{O}$ today, to predict W for tomorrow

Maximum A Posteriori Principle (MAP)

⊙ Approach 1

Comparing $P(w_1)$, $P(w_2)$, $P(w_3)$

$\vec{O}$ not used?

⊙ Approach 2

A Posteriori Probability
事後機率

Likelihood function

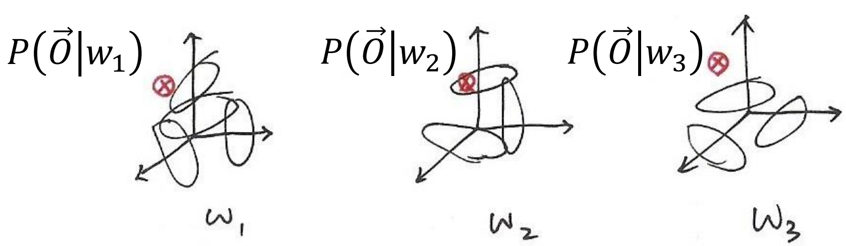
Prior Probability
事前機率

compute $P\left(\begin{matrix} w_1 \\ w_2 \\ w_3 \end{matrix}\middle|\vec{O}\right) = \dfrac{P\left(\vec{O}\middle|\begin{matrix} w_1 \\ w_2 \\ w_3 \end{matrix}\right) \cdot P\left(\begin{matrix} w_1 \\ w_2 \\ w_3 \end{matrix}\right)}{P(\vec{O})}$, $P\left(w_i\middle|\vec{O}\right) = \dfrac{P(\vec{O}|w_i)\,P(w_i)}{P(\vec{O})}$, $i = 1, 2, 3$

unknown

observation

compare $P\left(\vec{O}\middle|\begin{matrix} w_1 \\ w_2 \\ w_3 \end{matrix}\right) \cdot P\left(\begin{matrix} w_1 \\ w_2 \\ w_3 \end{matrix}\right)$, $P\left(\vec{O}\middle|w_i\right) \cdot P(w_i)$, $i = 1, 2, 3$



Syllable-based One-pass Search

- **Finding the Optimal Sentence from an Unknown Utterance Using 3 Knowledge Sources:Acoustic Models, Lexicon and Language Model**
- **Based on a Lattice of Syllable Candidates**