Fundamentals (Numbers, Sets, Words, Functions, and Relations)

Problem 1
How many numbers are there between 100 and 1000 that are

(a) divisible by 3? $\lfloor \frac{1000}{3} \rfloor - \lfloor \frac{99}{3} \rfloor = 333 - 33 = 300.$

(b) divisible by 5? $\lfloor \frac{1000}{5} \rfloor - \lfloor \frac{99}{5} \rfloor = 200 - 19 = 181$

(c) divisible by 15? $\lfloor \frac{1000}{15} \rfloor - \lfloor \frac{99}{15} \rfloor = 66 - 6 = 60.$

Problem 2
Let $\Sigma = \{a, b, c\}$ and $\Phi = \{a, c, e\}$.

$(A \cup B) \cap (A \cap B)^c$

$= (A \cup B) \setminus (A \cap B)$

(a) How many words are in the set Σ^2? $3 \times 3 = 9$

(b) What are the elements of $\Sigma^2 \setminus \Phi^*$? bb, ab, ba, bc, cb

(c) Is it true that $\Sigma^* \setminus \Phi^* = (\Sigma \setminus \Phi)^*$? Why? ✗

$\{b\}^*$

Problem 3
Prove that $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$

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$a \geq c \geq a$

$a + 0.5 \geq c \geq a - 0.5$

Problem 4
Consider the relation $R \subseteq \mathbb{R} \times \mathbb{R}$ defined by aRb if, and only if, $b + 0.5 \geq a \geq b - 0.5$. Is R

(a) reflexive? $b + 0.5 \geq b \geq b - 0.5$ ✓

(b) antireflexive? ✗

(c) symmetric? ✓ $b + 0.5 \geq a \geq b - 0.5.$

(d) antisymmetric? ✗ $b + 1 \geq a + 0.5 \geq b$

$b \geq a - 0.5 \geq b - 1$

(e) transitive? ✗

$(a, b), (b, c)$ (a, c) $a \geq c$

$a + 0.5 \geq b \geq a - 0.5$ a

$a = b$? $b + 0.5 \geq c \geq b - 0.5.$

$1.1 + 0.5 \geq 1 \geq 1.1 - 0.5$ $a \neq c$?

$1 \geq 1.1 \geq$

$a + 1 \geq b + 0.5 \geq a$ $a \geq b - 0.5 \geq a - 1$

Problem 5
For each of the following statements, provide a valid proof if it is true for all sets S and all relations $R_1 \subseteq S \times S$ and $R_2 \subseteq S \times S$. If the statement is not always true, provide a counterexample.

(a) If R_1 and R_2 are symmetric, then $R_1 \cap R_2$ is symmetric. ✓

(b) If R_1 and R_2 are antisymmetric, then $R_1 \cup R_2$ is antisymmetric.

Suppose $(a, b) \in R_1 \cap R_2$. Then $(a, b) \in R_1$, $(a, b) \in R_2$

$\because R_1, R_2$ sy. $\therefore (a, b) \in R_1$ R_2 $(b, a) \in R_2$ ..

$(a, b), (b, a) \in R_1 \cup R_2.$ $\because$ ansy (a, b)