# Graphs

**Problem 1**
True or false?

(a) The complete bipartite graph $K_{5,5}$ has no cycle of length five.

(b) If $T$ is a tree with at least four edges, then $\chi(T) = 3$.

(c) Let $C_n$ denote a cycle on $n$ vertices. For all $n \geq 5$ it holds $\chi(C_n) \neq \chi(C_{n-1})$.

(d) It is possible to remove two edges from $K_6$ so that the resulting graph has a clique number of 4.

**Problem 2**
What is the minimum number of edges that need to be removed from $K_5$ so that the resulting graph has a chromatic number of

(a) 3?

(b) 2?

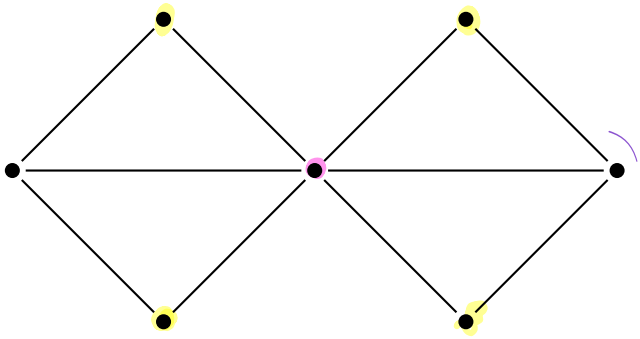
(c) 1?

**Problem 3**
Consider the complete 3-partite graphs $K_{4,1,1}$, $K_{3,2,1}$, $K_{2,2,2}$.

(a) What is the chromatic number of each of these graph?

(b) Which of these graphs are planar?

**Problem 4**
Consider the following graph, $G$:



(a) What is the chromatic number of $G$?

(b) What is the clique number of $G$? 3

(c) Does $G$ have a Hamiltonian path and/or a Hamiltonian cycle? No.

(d) Does $G$ have an Eulerian path and/or an Eulerian cycle? Yes. Ep.

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**Problem 5**
Draw a single graph with 6 vertices and 10 edges that satisfies each of the following:

(a) is planar,

(b) contains a Hamiltonian circuit, and

(c) does not contain an Eulerian path.