(b) $(a,b) \in R \cup R^{\leftarrow}$, $\therefore (a,b) \in R$ or $(a,b) \in R^{\leftarrow}$. $(b,a) \in R$. or $(b,a) \in R^{\leftarrow}$

Logic

Problem 1

Let F be the set of well-formed formulas with propositional variables from PROP. Define a relation, $R \subseteq F \times F$ by $(\varphi, \psi) \in R$ if $\varphi \models \psi$. Prove or give a counter-example to disprove:

if $(\varphi, \psi) \in R$. $(\psi, \varphi) \in R$.

(R, AS, T) $\varphi \models \psi$: $\varphi \to \psi$ is tautology (φ,φ) R ✓ R,) $\varphi = \psi$

(a) R is a partial order.

if $(\varphi \to \psi)$, $(\psi \to \theta)$ $\varphi \models \psi$ $\varphi \to \psi$ $\psi \to \theta$ φ true. ψ true. θ tru $\varphi \to \psi$, $\psi \to \varphi$ $\varphi = \varphi$ AR ✓.

(R.S.T)

(b) $R \cup R^{\leftarrow}$ is an equivalence relation. Suppose $(a,b) \in R$

$(b,c) \in R \cap R^{\leftarrow}$

(c) $R \cap R^{\leftarrow}$ is an equivalence relation.

Suppose $(a,b) \in R \cap R^{\leftarrow}$. $\therefore (a,b) \in R$, $(a,b) \in R^{\leftarrow}$ (

$(a,a) \in R \cap R^{\leftarrow}$ R ✓. $\therefore$ S ✓.

Problem 2

Prove that $\neg N$ follows logically from $H \wedge \neg R$ and $(H \wedge N) \to R$.

Problem 3

Consider the formulae $\phi_1 = (r \to p)$ and $\phi_2 = (p \to (q \vee \neg r))$. Transform the formula $\phi = (\neg q \to (\phi_1 \wedge \phi_2))$ into

r. p. q Φ.

(a) **DNF**, and

(b) **CNF**.

Simplify the result as much as possible.

Problem 4

Let $(T, \wedge, \vee, ', 0, 1)$ be a Boolean Algebra. Define $\oplus : T \times T \to T$ as follows:

$$x \oplus y = (x \wedge y') \vee (x' \wedge y)$$

(a) Prove using the laws of Boolean Algebra that for all $x \in T$, $x \oplus 1 = x'$.

(b) Prove using the laws of Boolean Algebra that $x \wedge (y \oplus z) = (x \wedge y) \oplus (x \wedge z)$.

(c) Find a Boolean Algebra (and x, y, z) which demonstrates that $x \oplus (y \wedge z) \neq (x \oplus y) \wedge (x \oplus z)$

Problem 5

(a) How many well-formed formulas can be constructed from one $\vee$; one $\wedge$; two parenthesis pairs (,); and the three literals p, $\neg p$, and q?

(b) Under the equivalence relation defined by **logical equivalence**, how many equivalence classes do the formulas in part (a) form?