

Problem 1

Consider the following program with two unspecified lines.

```
for  $j = 1$  to  $n$  :  
  (*)  
  while  $i > 1$  :  
    print  $i$   
    (**)  
  end while  
end for
```

Give an asymptotic upper bound on the running time, in terms of n for the given program when the missing lines are specified as follows:

- (a) (*) : $i = n$ (**) : $i = i - 1$
 - (b) (*) : $i = n$ (**) : $i = i/2$
 - (c) (*) : $i = j$ (**) : $i = i - 2$
 - (d) (*) : $i = j$ (**) : $i = i/2$
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Problem 2

Let $\Sigma = \{0, 1\}$

- (a) Recursively define a function $\text{str2num} : \Sigma^+ \rightarrow \mathbb{N}$ that converts a non-empty word over Σ to the number that one obtains by viewing the word as a binary number. For example $\text{str2num}(1100) = 12$, $\text{str2num}(0111) = 7$, $\text{str2num}(0000) = 0$.
 - (b) Recursively define a function $\text{num2str} : \mathbb{N} \rightarrow \Sigma^+$ that converts a number to its (shortest) binary representation. *Hint: you may want to use div and %.*
 - (c) Writing your functions as code in the natural way,
 - (i) Give an asymptotic upper bound in terms of $\text{length}((w))$ on the running time to compute $\text{str2num}(w)$.
 - (ii) Give an asymptotic upper bound in terms of n on the running time to compute $\text{num2str}(n)$.
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Problem 3

Consider the procedure given in lectures to simulate a die using a fair coin:

- (A) Flip a coin 3 times.
- (B) If the outcome was:
 - HHH: Output 1

- HHT: Output 2
- HTH: Output 3
- HTT: Output 4
- THH: Output 5
- THT: Output 6
- TTH: Go to (A)
- TTT: Go to (A)

What is the expected number of coin flips to obtain an output?

Problem 4

We want to tile a $2 \times n$ rectangle with 2×1 tiles so that the rectangle is completely covered and no tiles are overlapping. For example, here are two different ways to tile a 2×3 rectangle:



How many different ways (ignoring symmetry) are there of tiling a $2 \times n$ rectangle with 2×1 tiles in this way?

$$T(n): \quad T(1)=1, \quad T(2)=2$$

Problem 5

A tennis doubles match consists of two teams of two players per team. Ordering between teams, and within teams is not considered.

- How many different tennis doubles matches can be made with 4 players?
- How many different tennis doubles matches can be made with 5 players?
- How many different tennis doubles matches can be made from n players?